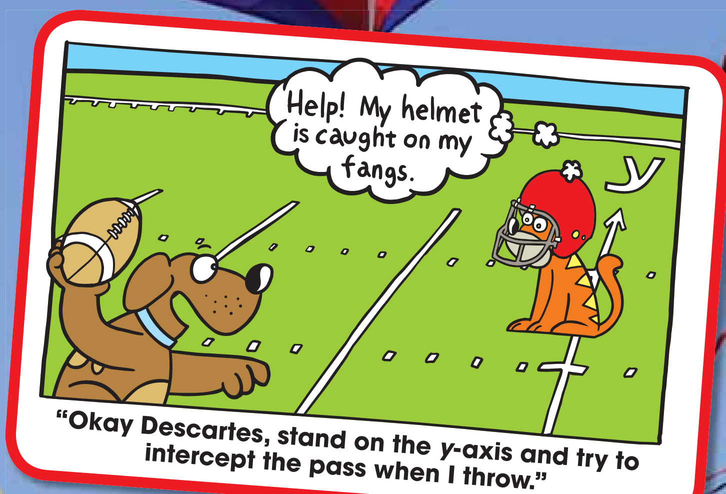
4 Graphing and Writing Linear Equations

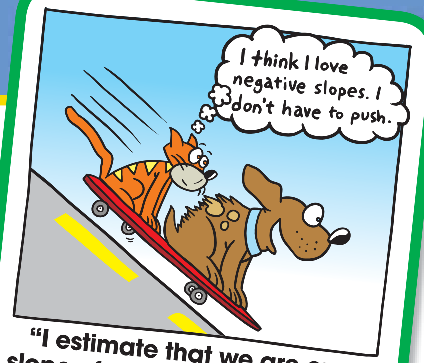


"Okay Descartes, stand on the y-axis and try to intercept the pass when I throw."

"Here's an easy example of a line with a slope of 1."

"You eat one mouse treat the first day. Two treats the second day. And so on. Get it?"

What You Learned Before



"I estimate that we are on a slope of about −0.625. What do you think?"

Evaluating Expressions Using Order of Operations (6.EE.2c)

Example 1 **Evaluate $2xy + 3(x + y)$ when $x = 4$ and $y = 7$.**

$2xy + 3(x + y)$	$= 2(4)(7) + 3(4 + 7)$	Substitute 4 for x and 7 for y.
	$= 8(7) + 3(4 + 7)$	Use order of operations.
	$= 56 + 3(11)$	Simplify.
	$= 56 + 33$	Multiply.
	$= 89$	Add.

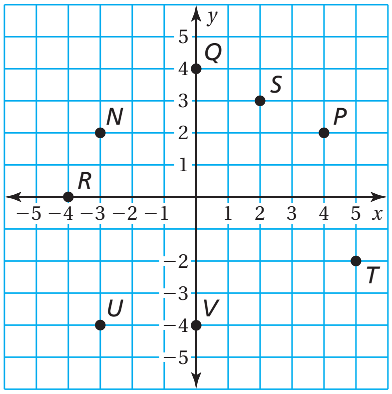
Try It Yourself

Evaluate the expression when $a = \frac{1}{4}$ and $b = 6$.

1. $-8ab$ **2.** $16a^2 - 4b$ **3.** $\frac{5b}{32a^2}$ **4.** $12a + (b - a - 4)$

Plotting Points (6.NS.6c)

Example 2 **Write the ordered pair that corresponds to point U.**



Point U is 3 units to the left of the origin and 4 units down. So, the x-coordinate is -3, and the y-coordinate is -4.

∴ The ordered pair $(-3, -4)$ corresponds to point U.

Example 3 **Which point is located at $(5, -2)$?**

Start at the origin. Move 5 units right and 2 units down.

∴ Point T is located at $(5, -2)$.

Try It Yourself

Use the graph to answer the question.

5. Write the ordered pair that corresponds to point Q.

6. Write the ordered pair that corresponds to point P.

7. Which point is located at $(-4, 0)$?

8. Which point is located in Quadrant II?

4.1 Graphing Linear Equations

Essential Question How can you recognize a linear equation? How can you draw its graph?

1 ACTIVITY: Graphing a Linear Equation

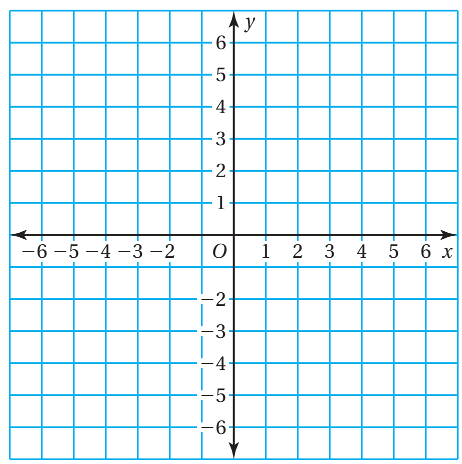
Work with a partner.

a. Use the equation $y = \frac{1}{2}x + 1$ to complete the table. (Choose any two x-values and find the y-values.)

	Solution Points	
x		
$y = \frac{1}{2}x + 1$		

b. Write the two ordered pairs given by the table. These are called *solution points* of the equation.

c. **PRECISION** Plot the two solution points. Draw a line *exactly* through the two points.



d. Find a different point on the line. Check that this point is a solution point of the equation $y = \frac{1}{2}x + 1$.

e. **LOGIC** Do you think it is true that *any* point on the line is a solution point of the equation $y = \frac{1}{2}x + 1$? Explain.

f. Choose five additional x-values for the table. (Choose positive and negative x-values.) Plot the five corresponding solution points. Does each point lie on the line?

	Solution Points				
x					
$y = \frac{1}{2}x + 1$					

g. **LOGIC** Do you think it is true that *any* solution point of the equation $y = \frac{1}{2}x + 1$ is a point on the line? Explain.

h. Why do you think $y = ax + b$ is called a *linear equation*?

Common Core

Graphing Equations

In this lesson, you will

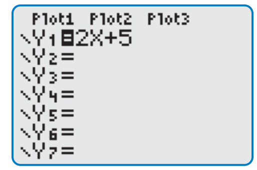
- understand that lines represent solutions of linear equations.
- graph linear equations.

Preparing for Standard 8.EE.5

2 ACTIVITY: Using a Graphing Calculator

Use a graphing calculator to graph $y = 2x + 5$.

a. Enter the equation $y = 2x + 5$ into your calculator.

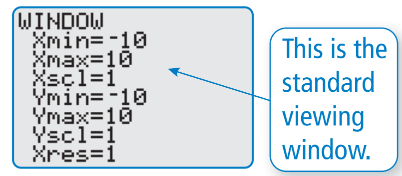


Math Practice 5

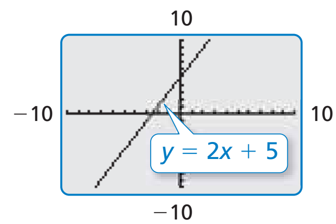
Recognize Usefulness of Tools

What are some advantages and disadvantages of using a graphing calculator to graph a linear equation?

b. Check the settings of the *viewing window*. The boundaries of the graph are set by the minimum and the maximum x- and y-values. The numbers of units between the tick marks are set by the x- and y-scales.

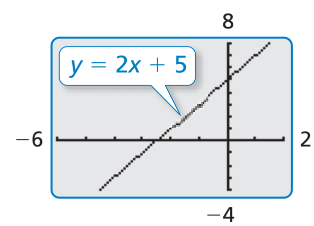


c. Graph $y = 2x + 5$ on your calculator.



d. Change the settings of the viewing window to match those shown.

Compare the two graphs.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you recognize a linear equation? How can you draw its graph? Write an equation that is linear. Write an equation that is *not* linear.

4. Use a graphing calculator to graph $y = 5x - 12$ in the standard viewing window.

   a. Can you tell where the line crosses the x-axis? Can you tell where the line crosses the y-axis?

   b. How can you adjust the viewing window so that you can determine where the line crosses the x- and y-axes?

5. **CHOOSE TOOLS** You want to graph $y = 2.5x - 3.8$. Would you graph it by hand or by using a graphing calculator? Why?

Practice Use what you learned about graphing linear equations to complete Exercises 3 and 4 on page 146.

4.1 Lesson

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Key Vocabulary
linear equation, *p. 144*
solution of a linear equation, *p. 144*

Remember
An ordered pair (x, y) is used to locate a point in a coordinate plane.

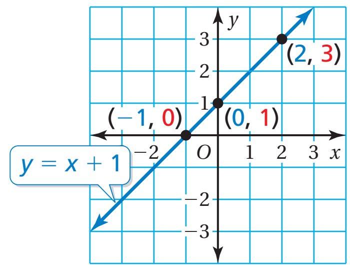
Key Idea

Linear Equations

A **linear equation** is an equation whose graph is a line. The points on the line are **solutions** of the equation.

You can use a graph to show the solutions of a linear equation. The graph below represents the equation $y = x + 1$.

x	y	(x, y)
-1	0	$(-1, 0)$
0	1	$(0, 1)$
2	3	$(2, 3)$



EXAMPLE 1 Graphing a Linear Equation

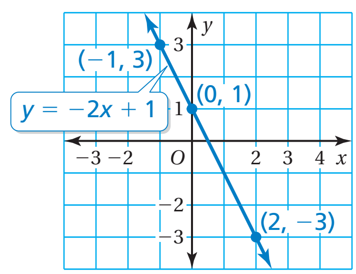
Graph $y = -2x + 1$.

Step 1: Make a table of values.

x	$y = -2x + 1$	y	(x, y)
-1	$y = -2(-1) + 1$	3	$(-1, 3)$
0	$y = -2(0) + 1$	1	$(0, 1)$
2	$y = -2(2) + 1$	-3	$(2, -3)$

Step 2: Plot the ordered pairs.

Step 3: Draw a line through the points.



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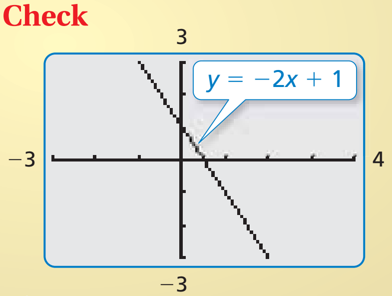
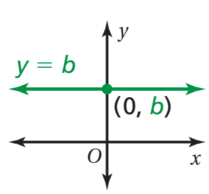


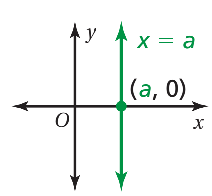
Key Idea

Graphing Horizontal and Vertical Lines

The graph of $y = b$ is a horizontal line passing through $(0, b)$.



The graph of $x = a$ is a vertical line passing through $(a, 0)$.

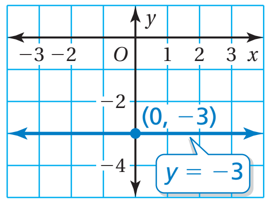


Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Graphing a Horizontal Line and a Vertical Line

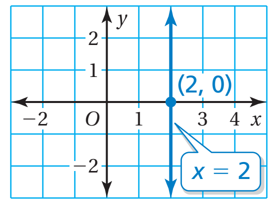
a. Graph $y = -3$.

The graph of $y = -3$ is a horizontal line passing through $(0, -3)$. Draw a horizontal line through this point.



b. Graph $x = 2$.

The graph of $x = 2$ is a vertical line passing through $(2, 0)$. Draw a vertical line through this point.



On Your Own

Now You're Ready
Exercises 5–16

Graph the linear equation. Use a graphing calculator to check your graph, if possible.

1. $y = 3x$
2. $y = -\frac{1}{2}x + 2$
3. $x = -4$
4. $y = -1.5$

EXAMPLE 3 Real-Life Application

The wind speed y (in miles per hour) of a tropical storm is $y = 2x + 66$, where x is the number of hours after the storm enters the Gulf of Mexico.

a. Graph the equation.

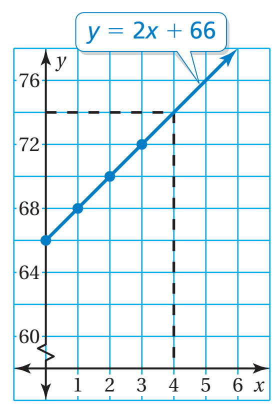
b. When does the storm become a hurricane?

A tropical storm becomes a hurricane when wind speeds are at least 74 miles per hour.

a. Make a table of values.

x	$y = 2x + 66$	y	(x, y)
0	$y = 2(0) + 66$	66	(0, 66)
1	$y = 2(1) + 66$	68	(1, 68)
2	$y = 2(2) + 66$	70	(2, 70)
3	$y = 2(3) + 66$	72	(3, 72)

Plot the ordered pairs and draw a line through the points.



b. From the graph, you can see that $y = 74$ when $x = 4$. So, the storm becomes a hurricane 4 hours after it enters the Gulf of Mexico.

On Your Own

5. **WHAT IF?** The wind speed of the storm is $y = 1.5x + 62$. When does the storm become a hurricane?

4.1 Exercises

Check It Out
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Vocabulary and Concept Check

1. **VOCABULARY** What type of graph represents the solutions of the equation $y = 2x + 4$?

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$y = 0.5x - 0.2$	$4x + 3 = y$	$y = x^2 + 6$	$\frac{3}{4}x + \frac{1}{3} = y$

Practice and Problem Solving

PRECISION Copy and complete the table. Plot the two solution points and draw a line *exactly* through the two points. Find a different solution point on the line.

3.

x		
$y = 3x - 1$		

4.

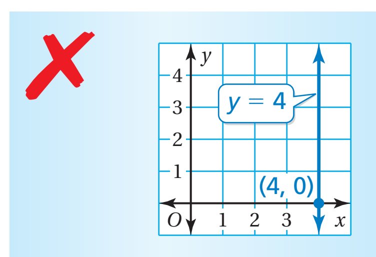
x		
$y = \frac{1}{3}x + 2$		

Graph the linear equation. Use a graphing calculator to check your graph, if possible.

1 2

5. $y = -5x$
6. $y = \frac{1}{4}x$
7. $y = 5$
8. $x = -6$
9. $y = x - 3$
10. $y = -7x - 1$
11. $y = -\frac{x}{3} + 4$
12. $y = \frac{3}{4}x - \frac{1}{2}$
13. $y = -\frac{2}{3}$
14. $y = 6.75$
15. $x = -0.5$
16. $x = \frac{1}{4}$

17. **ERROR ANALYSIS** Describe and correct the error in graphing the equation.



18. **MESSAGING** You sign up for an unlimited text-messaging plan for your cell phone. The equation $y = 20$ represents the cost y (in dollars) for sending x text messages. Graph the equation. What does the graph tell you?

19. **MAIL** The equation $y = 2x + 3$ represents the cost y (in dollars) of mailing a package that weighs x pounds.
    a. Graph the equation.
    b. Use the graph to estimate how much it costs to mail the package.
    c. Use the equation to find exactly how much it costs to mail the package.

Solve for y. Then graph the equation. Use a graphing calculator to check your graph.

20. $y - 3x = 1$

21. $5x + 2y = 4$

22. $-\frac{1}{3}y + 4x = 3$

23. $x + 0.5y = 1.5$

24. **SAVINGS** You have \$100 in your savings account and plan to deposit \$12.50 each month.

a. Graph a linear equation that represents the balance in your account.

b. How many months will it take you to save enough money to buy 10 acres of land on Mars?

25. **GEOMETRY** The sum S of the interior angle measures of a polygon with n sides is $S = (n - 2) \cdot 180°$.

a. Plot four points (n, S) that satisfy the equation. Is the equation a linear equation? Explain your reasoning.

b. Does the value $n = 3.5$ make sense in the context of the problem? Explain your reasoning.

26. **SEA LEVEL** Along the U.S. Atlantic coast, the sea level is rising about 2 millimeters per year. How many millimeters has sea level risen since you were born? How do you know? Use a linear equation and a graph to justify your answer.

27. Problem Solving One second of video on your digital camera uses the same amount of memory as two pictures. Your camera can store 250 pictures.

a. Write and graph a linear equation that represents the number y of pictures your camera can store when you take x seconds of video.

b. How many pictures can your camera store in addition to the video shown?

Fair Game Review What you learned in previous grades & lessons

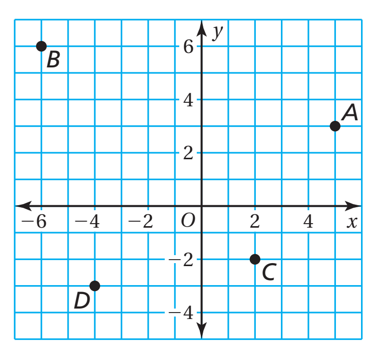
Write the ordered pair corresponding to the point. *(Skills Review Handbook)*

28. point A

29. point B

30. point C

31. point D



32. **MULTIPLE CHOICE** A debate team has 15 female members. The ratio of females to males is 3 : 2. How many males are on the debate team? *(Skills Review Handbook)*

Ⓐ 6 Ⓑ 10 Ⓒ 22 Ⓓ 25

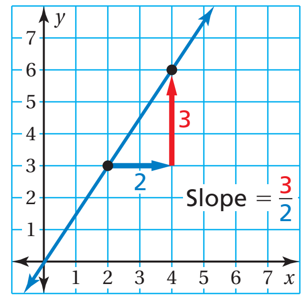
4.2 Slope of a Line

Essential Question How can you use the slope of a line to describe the line?

Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the ratio of the change in y (vertical change) to the change in x (horizontal change).

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



1 ACTIVITY: Finding the Slope of a Line

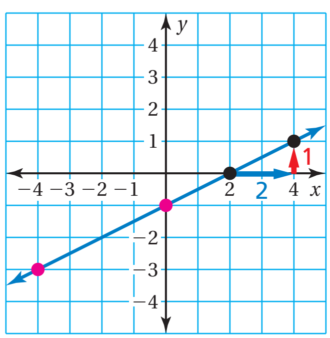
Work with a partner. Find the slope of each line using two methods.

Method 1: Use the two black points. •

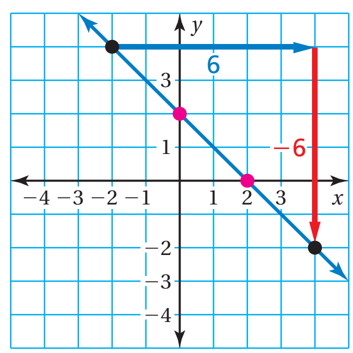
Method 2: Use the two pink points. •

Do you get the same slope using each method? Why do you think this happens?

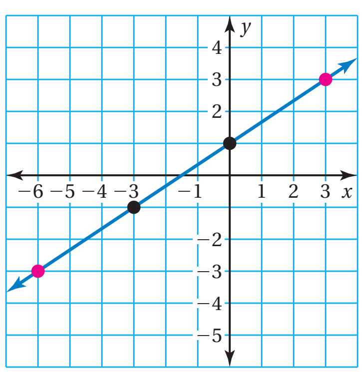
a.



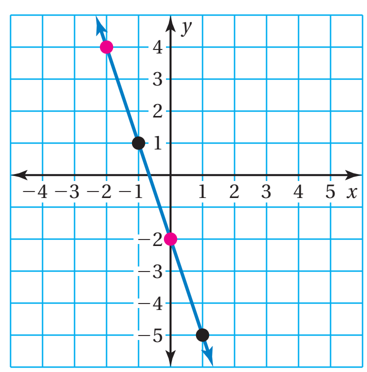
b.



c.



d.



Common Core

Graphing Equations

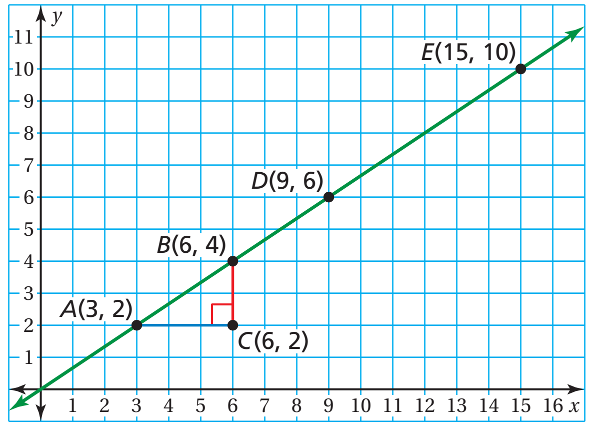
In this lesson, you will

- find slopes of lines by using two points.
- find slopes of lines from tables.

Learning Standard 8.EE.6

2 ACTIVITY: Using Similar Triangles

Work with a partner. Use the figure shown.



a. $\triangle ABC$ is a right triangle formed by drawing a horizontal line segment from point A and a vertical line segment from point B. Use this method to draw another right triangle, $\triangle DEF$.

b. What can you conclude about $\triangle ABC$ and $\triangle DEF$? Justify your conclusion.

c. For each triangle, find the ratio of the length of the vertical side to the length of the horizontal side. What do these ratios represent?

d. What can you conclude about the slope between any two points on the line?

3 ACTIVITY: Drawing Lines with Given Slopes

Work with a partner.

a. Draw two lines with slope $\frac{3}{4}$. One line passes through $(-4, 1)$, and the other line passes through $(4, 0)$. What do you notice about the two lines?

b. Draw two lines with slope $-\frac{4}{3}$. One line passes through $(2, 1)$, and the other line passes through $(-1, -1)$. What do you notice about the two lines?

c. **CONJECTURE** Make a conjecture about two different nonvertical lines in the same plane that have the same slope.

d. Graph one line from part (a) and one line from part (b) in the same coordinate plane. Describe the angle formed by the two lines. What do you notice about the product of the slopes of the two lines?

e. **REPEATED REASONING** Repeat part (d) for the two lines you did *not* choose. Based on your results, make a conjecture about two lines in the same plane whose slopes have a product of -1.

Math Practice 1

Interpret a Solution

What does the slope tell you about the graph of the line? Explain.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use the slope of a line to describe the line?

Practice Use what you learned about the slope of a line to complete Exercises 4–6 on page 153.

4.2 Lesson

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Lesson Tutorials
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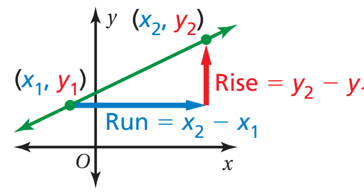
Key Vocabulary
slope, *p. 150*
rise, *p. 150*
run, *p. 150*

Key Idea

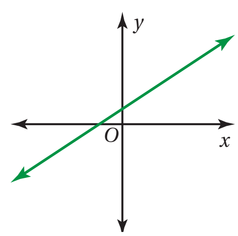
Slope

The **slope** m of a line is a ratio of the change in y (the **rise**) to the change in x (the **run**) between any two points, (x_1, y_1) and (x_2, y_2), on the line.



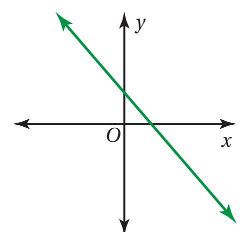
$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Positive Slope



The line rises from left to right.

Negative Slope



The line falls from left to right.

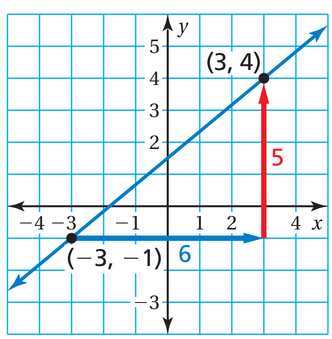
Reading

In the slope formula, x_1 is read as "x sub one," and y_2 is read as "y sub two." The numbers 1 and 2 in x_1 and y_2 are called *subscripts*.

EXAMPLE 1 Finding the Slope of a Line

Describe the slope of the line. Then find the slope.

a.



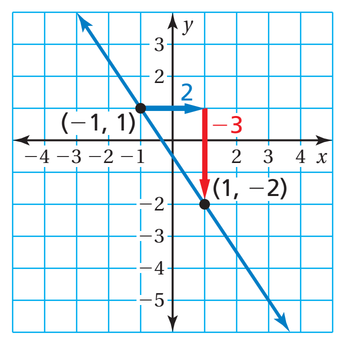
The line rises from left to right. So, the slope is positive. Let $(x_1, y_1) = (-3, -1)$ and $(x_2, y_2) = (3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - (-1)}{3 - (-3)}$$

$$= \frac{5}{6}$$

b.



The line falls from left to right. So, the slope is negative. Let $(x_1, y_1) = (-1, 1)$ and $(x_2, y_2) = (1, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 1}{1 - (-1)}$$

$$= \frac{-3}{2}, \text{ or } -\frac{3}{2}$$

Study Tip

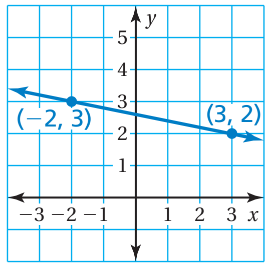
When finding slope, you can label either point as (x_1, y_1) and the other point as (x_2, y_2).

On Your Own

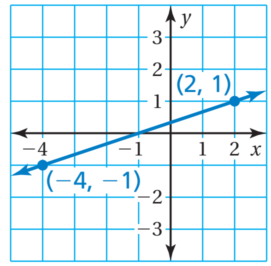
Now You're Ready
Exercises 7–9

Find the slope of the line.

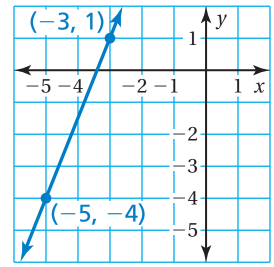
1.



2.

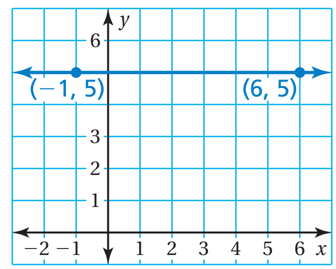


3.



EXAMPLE 2 Finding the Slope of a Horizontal Line

Find the slope of the line.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

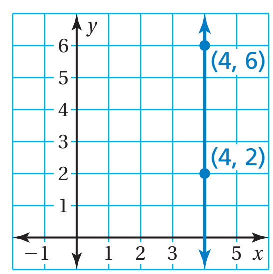
$$= \frac{5 - 5}{6 - (-1)}$$

$$= \frac{0}{7}, \text{ or } 0$$

∴ The slope is 0.

EXAMPLE 3 Finding the Slope of a Vertical Line

Find the slope of the line.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 2}{4 - 4}$$

$$= \frac{4}{0}$$

∴ Because division by zero is undefined, the slope of the line is undefined.

Study Tip

The slope of every horizontal line is 0. The slope of every vertical line is undefined.

On Your Own

Now You're Ready
Exercises 13–15

Find the slope of the line through the given points.

4. (1, −2), (7, −2)

5. (−2, 4), (3, 4)

6. (−3, −3), (−3, −5)

7. (0, 8), (0, 0)

8. How do you know that the slope of every horizontal line is 0? How do you know that the slope of every vertical line is undefined?

EXAMPLE 4 Finding Slope from a Table

The points in the table lie on a line. How can you find the slope of the line from the table? What is the slope?

x	1	4	7	10
y	8	6	4	2

Choose any two points from the table and use the slope formula.

Use the points $(x_1, y_1) = (1, 8)$ and $(x_2, y_2) = (4, 6)$.

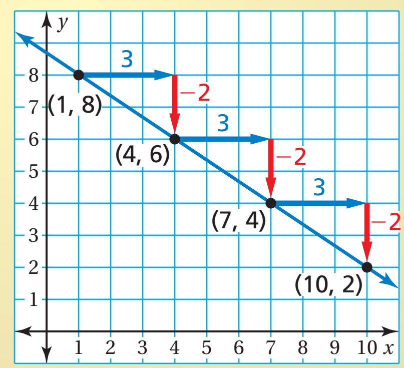
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 8}{4 - 1}$$

$$= \frac{-2}{3}, \text{ or } -\frac{2}{3}$$

The slope is $-\frac{2}{3}$.

Check



On Your Own

Now You're Ready
Exercises 21–24

The points in the table lie on a line. Find the slope of the line.

9.

x	1	3	5	7
y	2	5	8	11

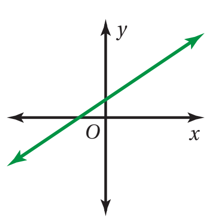
10.

x	−3	−2	−1	0
y	6	4	2	0

Summary

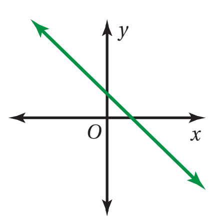
Slope

Positive Slope



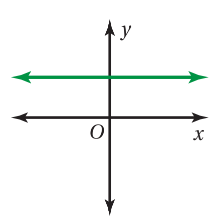
The line rises from left to right.

Negative Slope



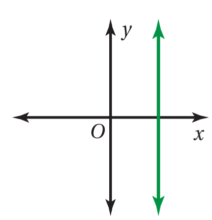
The line falls from left to right.

Slope of 0



The line is horizontal.

Undefined Slope



The line is vertical.

4.2 Exercises

Check It Out
Help with Homework
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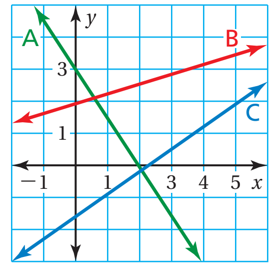
Vocabulary and Concept Check

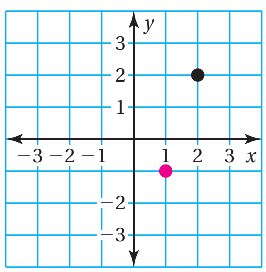


1. **CRITICAL THINKING** Refer to the graph.
   - **a.** Which lines have positive slopes?
   - **b.** Which line has the steepest slope?
   - **c.** Do any lines have an undefined slope? Explain.
2. **OPEN-ENDED** Describe a real-life situation in which you need to know the slope.
3. **REASONING** The slope of a line is 0. What do you know about the line?

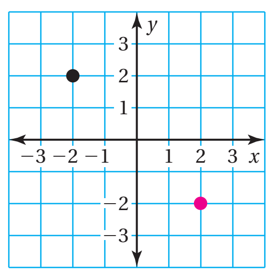
Practice and Problem Solving

Draw a line through each point using the given slope. What do you notice about the two lines?

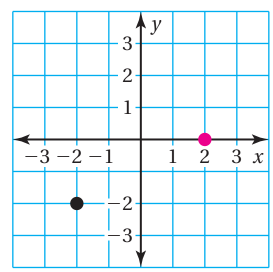
4. slope = 1



5. slope = -3

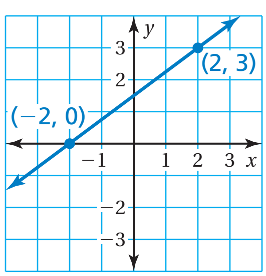


6. slope = $\frac{1}{4}$

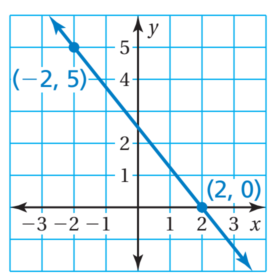


Find the slope of the line.

1 7.



8.



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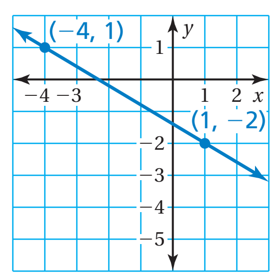

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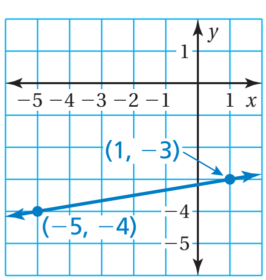
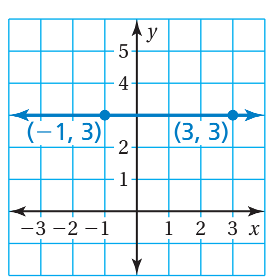
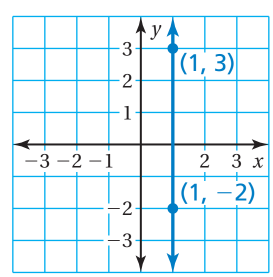


11.



12.



Find the slope of the line through the given points.

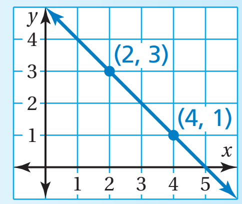
2 3 **13.** $(4, -1), (-2, -1)$ **14.** $(5, -3), (5, 8)$ **15.** $(-7, 0), (-7, -6)$

16. $(-3, 1), (-1, 5)$ **17.** $(10, 4), (4, 15)$ **18.** $(-3, 6), (2, 6)$

19. ERROR ANALYSIS Describe and correct the error in finding the slope of the line.

✗

$$m = \frac{3 - 1}{4 - 2} = \frac{2}{2} = 1$$



20. CRITICAL THINKING Is it more difficult to walk up the ramp or the hill? Explain.

The points in the table lie on a line. Find the slope of the line.

4 **21.**

x	1	3	5	7
y	2	10	18	26

22.

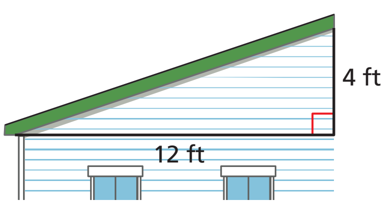
x	−3	2	7	12
y	0	2	4	6

23.

x	−6	−2	2	6
y	8	5	2	−1

24.

x	−8	−2	4	10
y	8	1	−6	−13



25. PITCH Carpenters refer to the slope of a roof as the *pitch* of the roof. Find the pitch of the roof.

26. PROJECT The guidelines for a wheelchair ramp suggest that the ratio of the rise to the run be no greater than 1 : 12.

a. CHOOSE TOOLS Find a wheelchair ramp in your school or neighborhood. Measure its slope. Does the ramp follow the guidelines?

b. Design a wheelchair ramp that provides access to a building with a front door that is 2.5 feet above the sidewalk. Illustrate your design.

Use an equation to find the value of *k* so that the line that passes through the given points has the given slope.

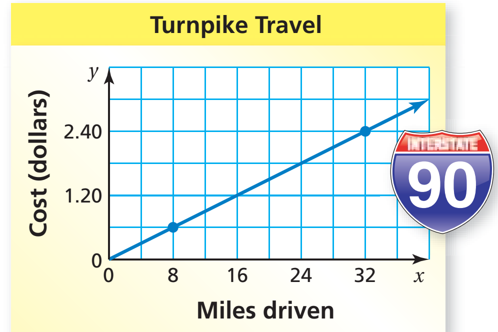
27. $(1, 3), (5, k);\ m = 2$ **28.** $(-2, k), (2, 0);\ m = -1$

29. $(-4, k), (6, -7);\ m = -\frac{1}{5}$ **30.** $(4, -4), (k, -1);\ m = \frac{3}{4}$

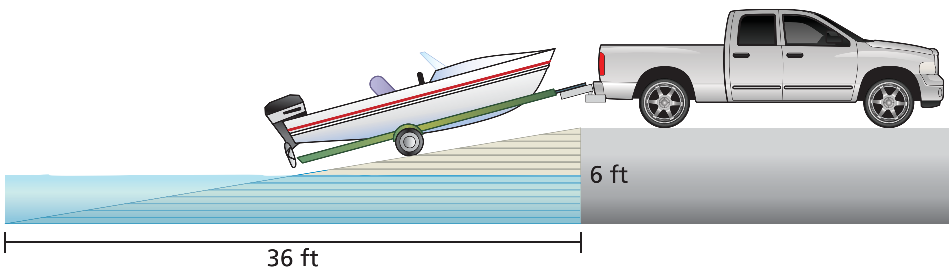
31. **TURNPIKE TRAVEL** The graph shows the cost of traveling by car on a turnpike.

a. Find the slope of the line.

b. Explain the meaning of the slope as a rate of change.



32. **BOAT RAMP** Which is steeper: the boat ramp or a road with a 12% grade? Explain. (*Note:* Road grade is the vertical increase divided by the horizontal distance.)



33. **REASONING** Do the points $A(-2, -1)$, $B(1, 5)$, and $C(4, 11)$ lie on the same line? Without using a graph, how do you know?

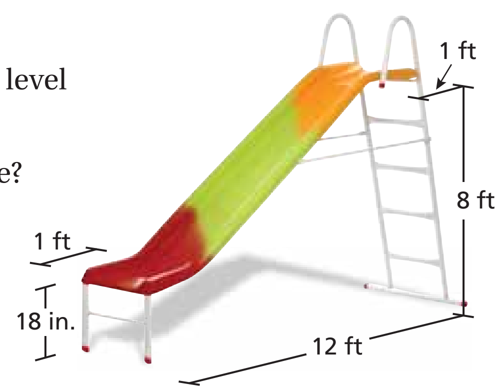
34. **BUSINESS** A small business earns a profit of \$6500 in January and \$17,500 in May. What is the rate of change in profit for this time period?

35. **STRUCTURE** Choose two points in the coordinate plane. Use the slope formula to find the slope of the line that passes through the two points. Then find the slope using the formula $\dfrac{y_1 - y_2}{x_1 - x_2}$. Explain why your results are the same.

36. **Critical Thinking** The top and the bottom of the slide are level with the ground, which has a slope of 0.

a. What is the slope of the main portion of the slide?

b. How does the slope change when the bottom of the slide is only 12 inches above the ground? Is the slide steeper? Explain.



Fair Game Review What you learned in previous grades & lessons

Solve the proportion. *(Skills Review Handbook)*

37. $\dfrac{b}{30} = \dfrac{5}{6}$

38. $\dfrac{7}{4} = \dfrac{n}{32}$

39. $\dfrac{3}{8} = \dfrac{x}{20}$

40. **MULTIPLE CHOICE** What is the prime factorization of 84? *(Skills Review Handbook)*

Ⓐ $2 \times 3 \times 7$ Ⓑ $2^2 \times 3 \times 7$ Ⓒ $2 \times 3^2 \times 7$ Ⓓ $2^2 \times 21$

Extension 4.2 Slopes of Parallel and Perpendicular Lines

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COMMON CORE

Graphing Equations

In this extension, you will

- identify parallel and perpendicular lines.

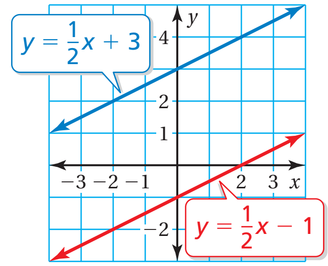
Applying Standard 8.EE.6

Key Idea

Parallel Lines and Slopes

Lines in the same plane that do not intersect are parallel lines. Nonvertical parallel lines have the same slope.

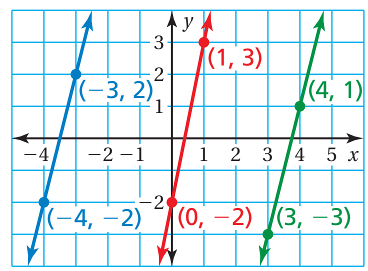
All vertical lines are parallel.



EXAMPLE 1 Identifying Parallel Lines

Which two lines are parallel? How do you know?

Find the slope of each line.



Blue Line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{-4 - (-3)} = \frac{-4}{-1}, \text{ or } 4$$

Red Line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{0 - 1} = \frac{-5}{-1}, \text{ or } 5$$

Green Line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{3 - 4} = \frac{-4}{-1}, \text{ or } 4$$

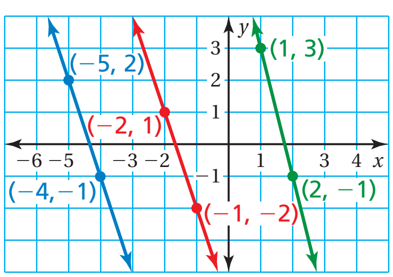
The slopes of the blue and green lines are 4. The slope of the red line is 5.

∴ The blue and green lines have the same slope, so they are parallel.

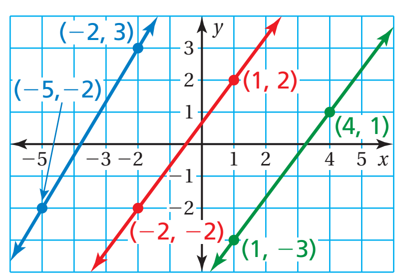
Practice

Which lines are parallel? How do you know?

1.



2.



Are the given lines parallel? Explain your reasoning.

3. $y = -5, y = 3$

4. $y = 0, x = 0$

5. $x = -4, x = 1$

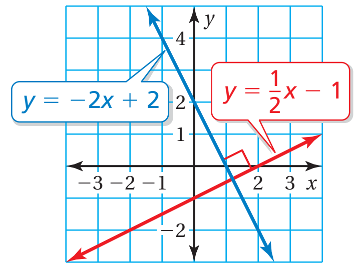
6. **GEOMETRY** The vertices of a quadrilateral are $A(-5, 3)$, $B(2, 2)$, $C(4, -3)$, and $D(-2, -2)$. How can you use slope to determine whether the quadrilateral is a parallelogram? Is it a parallelogram? Justify your answer.

Key Idea

Perpendicular Lines and Slope

Lines in the same plane that intersect at right angles are perpendicular lines. Two nonvertical lines are perpendicular when the product of their slopes is -1.

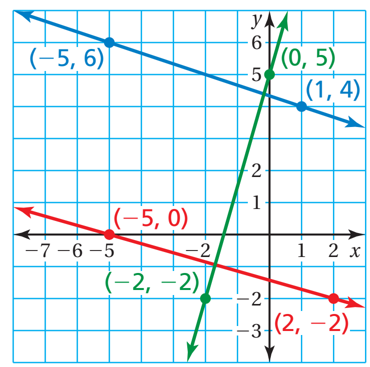
Vertical lines are perpendicular to horizontal lines.



EXAMPLE 2 Identifying Perpendicular Lines

Which two lines are perpendicular? How do you know?

Find the slope of each line.



Blue Line	*Red Line*	*Green Line*
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \dfrac{4 - 6}{1 - (-5)}$	$= \dfrac{-2 - 0}{2 - (-5)}$	$= \dfrac{5 - (-2)}{0 - (-2)}$
$= \dfrac{-2}{6}$, or $-\dfrac{1}{3}$	$= -\dfrac{2}{7}$	$= \dfrac{7}{2}$

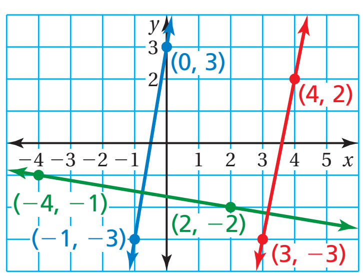
The slope of the red line is $-\frac{2}{7}$. The slope of the green line is $\frac{7}{2}$.

Because $-\frac{2}{7} \cdot \frac{7}{2} = -1$, the red and green lines are perpendicular.

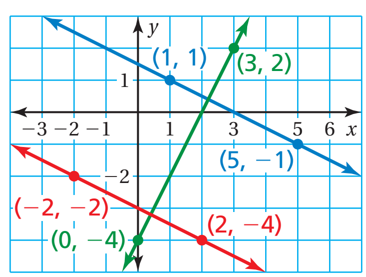
Practice

Which lines are perpendicular? How do you know?

7.



8.



Are the given lines perpendicular? Explain your reasoning.

9. $x = -2, y = 8$

10. $x = -8, x = 7$

11. $y = 0, x = 0$

12. **GEOMETRY** The vertices of a parallelogram are $J(-5, 0)$, $K(1, 4)$, $L(3, 1)$, and $M(-3, -3)$. How can you use slope to determine whether the parallelogram is a rectangle? Is it a rectangle? Justify your answer.

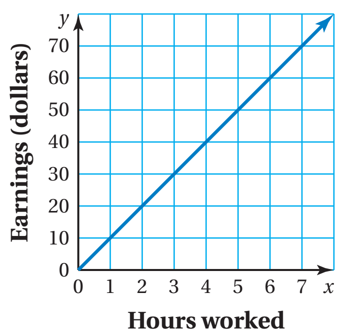
4.3 Graphing Proportional Relationships

Essential Question How can you describe the graph of the equation $y = mx$?

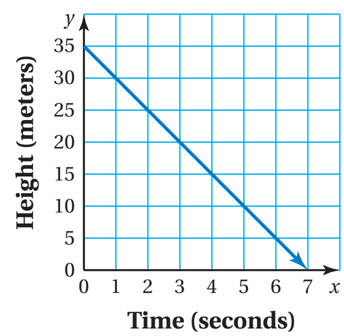
1 ACTIVITY: Identifying Proportional Relationships

Work with a partner. Tell whether *x* and *y* are in a proportional relationship. Explain your reasoning.

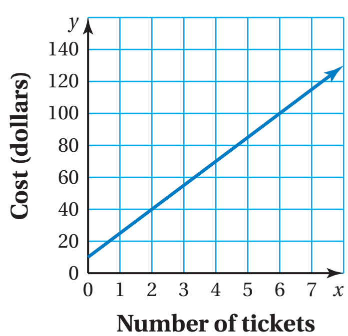
a. **Money**



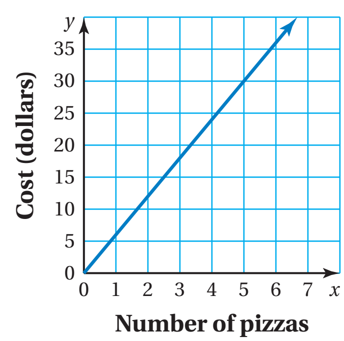
b. **Helicopter**



c. **Tickets**



d. **Pizzas**



e.

Laps, x	1	2	3	4
Time (seconds), y	90	200	325	480

f.

Cups of Sugar, x	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
Cups of Flour, y	1	2	3	4

COMMON CORE

Graphing Equations

In this lesson, you will

- write and graph proportional relationships.

Learning Standards
8.EE.5
8.EE.6

2 ACTIVITY: Analyzing Proportional Relationships

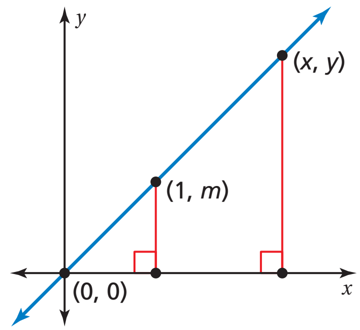
Work with a partner. Use only the proportional relationships in Activity 1 to do the following.

- **Find the slope of the line.**
- **Find the value of *y* for the ordered pair (1, *y*).**

What do you notice? What does the value of *y* represent?

3 ACTIVITY: Deriving an Equation

Work with a partner. Let (x, y) represent any point on the graph of a proportional relationship.



a. Explain why the two triangles are similar.

b. Because the triangles are similar, the corresponding side lengths are proportional. Use the vertical and horizontal side lengths to complete the steps below.

Math Practice 7

View as Components

What part of the graph can you use to find the side lengths?

$$\frac{\square}{\square} = \frac{m}{1}$$ Ratios of side lengths

$$\frac{\square}{\square} = m$$ Simplify.

$$\square = m \cdot \square$$ Multiplication Property of Equality

What does the final equation represent?

c. Use your result in part (b) to write an equation that represents each proportional relationship in Activity 1.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you describe the graph of the equation $y = mx$? How does the value of m affect the graph of the equation?

5. Give a real-life example of two quantities that are in a proportional relationship. Write an equation that represents the relationship and sketch its graph.

Practice Use what you learned about proportional relationships to complete Exercises 3–6 on page 162.

4.3 Lesson

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Lesson Tutorials
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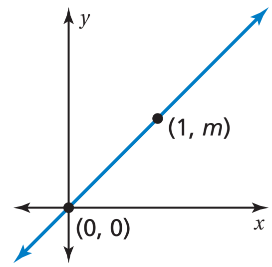
Key Idea

Direct Variation

Words When two quantities x and y are proportional, the relationship can be represented by the direct variation equation $y = mx$, where m is the constant of proportionality.

Graph The graph of $y = mx$ is a line with a slope of m that passes through the origin.



Study Tip

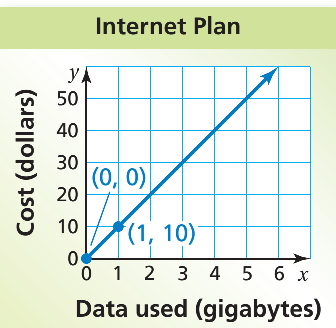
In the direct variation equation $y = mx$, m represents the constant of proportionality, the slope, and the unit rate.

EXAMPLE 1 Graphing a Proportional Relationship

The cost y (in dollars) for x gigabytes of data on an Internet plan is represented by $y = 10x$. Graph the equation and interpret the slope.

The equation shows that the slope m is 10. So, the graph passes through (0, 0) and (1, 10).

Plot the points and draw a line through the points. Because negative values of x do not make sense in this context, graph in the first quadrant only.



∴ The slope indicates that the unit cost is \$10 per gigabyte.

EXAMPLE 2 Writing and Using a Direct Variation Equation

The weight y of an object on Titan, one of Saturn's moons, is proportional to the weight x of the object on Earth. An object that weighs 105 pounds on Earth would weigh 15 pounds on Titan.

a. Write an equation that represents the situation.

Use the point (105, 15) to find the slope of the line.

$y = mx$	Direct variation equation
$15 = m(105)$	Substitute 15 for y and 105 for x.
$\frac{1}{7} = m$	Simplify.

∴ So, an equation that represents the situation is $y = \frac{1}{7}x$.

b. How much would a chunk of ice that weighs 3.5 pounds on Titan weigh on Earth?

$3.5 = \frac{1}{7}x$	Substitute 3.5 for y.
$24.5 = x$	Multiply each side by 7.

∴ So, the chunk of ice would weigh 24.5 pounds on Earth.

Study Tip

In Example 2, the slope indicates that the weight of an object on Titan is one-seventh its weight on Earth.

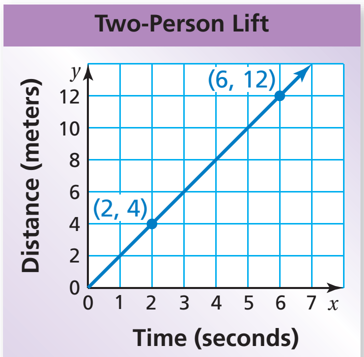
On Your Own

Now You're Ready
Exercises 7–8

1. **WHAT IF?** In Example 1, the cost is represented by $y = 12x$. Graph the equation and interpret the slope.

2. In Example 2, how much would a spacecraft that weighs 3500 kilograms on Earth weigh on Titan?

EXAMPLE 3 Comparing Proportional Relationships



The distance y (in meters) that a four-person ski lift travels in x seconds is represented by the equation $y = 2.5x$. The graph shows the distance that a two-person ski lift travels.

a. Which ski lift is faster?

Interpret each slope as a unit rate.

Four-Person Lift

$y = 2.5x$

The slope is 2.5.

The four-person lift travels 2.5 meters per second.

Two-Person Lift

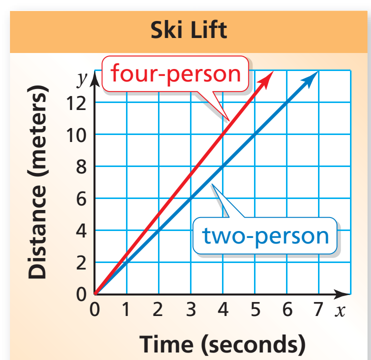
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{8}{4} = 2$$

The two-person lift travels 2 meters per second.

So, the four-person lift is faster than the two-person lift.

b. Graph the equation that represents the four-person lift in the same coordinate plane as the two-person lift. Compare the steepness of the graphs. What does this mean in the context of the problem?



The graph that represents the four-person lift is steeper than the graph that represents the two-person lift. So, the four-person lift is faster.

On Your Own

Now You're Ready
Exercise 9

3. The table shows the distance y (in meters) that a T-bar ski lift travels in x seconds. Compare its speed to the ski lifts in Example 3.

x (seconds)	1	2	3	4
y (meters)	$2\frac{1}{4}$	$4\frac{1}{2}$	$6\frac{3}{4}$	9

4.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

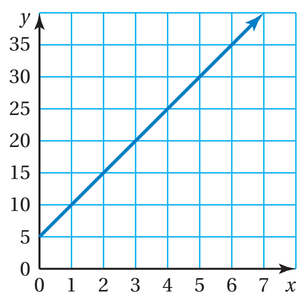
Vocabulary and Concept Check

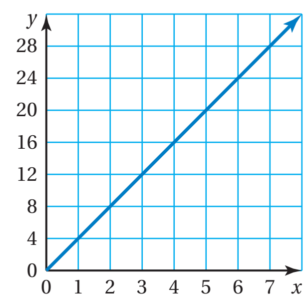
1. **VOCABULARY** What point is on the graph of every direct variation equation?

2. **REASONING** Does the equation $y = 2x + 3$ represent a proportional relationship? Explain.

Practice and Problem Solving

Tell whether x and y are in a proportional relationship. Explain your reasoning. If so, write an equation that represents the relationship.

3.


4.


5.

x	3	6	9	12
y	1	2	3	4

6.

x	2	5	8	10
y	4	8	13	23

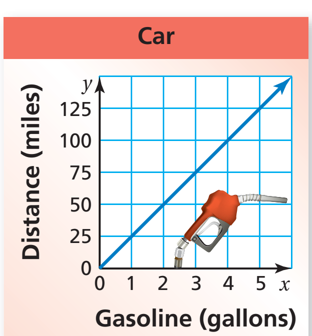
1 7. **TICKETS** The amount y (in dollars) that you raise by selling x fundraiser tickets is represented by the equation $y = 5x$. Graph the equation and interpret the slope.

2 8. **KAYAK** The cost y (in dollars) to rent a kayak is proportional to the number x of hours that you rent the kayak. It costs $27 to rent the kayak for 3 hours.

   a. Write an equation that represents the situation.

   b. Interpret the slope.

   c. How much does it cost to rent the kayak for 5 hours?



3 9. **MILEAGE** The distance y (in miles) that a truck travels on x gallons of gasoline is represented by the equation $y = 18x$. The graph shows the distance that a car travels.

   a. Which vehicle gets better gas mileage? Explain how you found your answer.

   b. How much farther can the vehicle you chose in part (a) travel than the other vehicle on 8 gallons of gasoline?

10. **BIOLOGY** Toenails grow about 13 millimeters per year. The table shows fingernail growth.

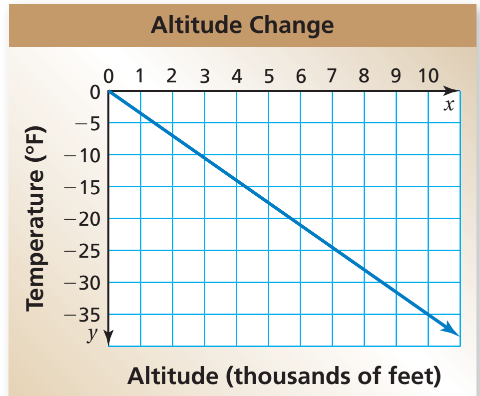
Weeks	1	2	3	4
Fingernail Growth (millimeters)	0.7	1.4	2.1	2.8

a. Do fingernails or toenails grow faster? Explain.

b. In the same coordinate plane, graph equations that represent the growth rates of toenails and fingernails. Compare the steepness of the graphs. What does this mean in the context of the problem?

11. **REASONING** The quantities x and y are in a proportional relationship. What do you know about the ratio of y to x for any point (x, y) on the line?

12. **PROBLEM SOLVING** The graph relates the temperature change y (in degrees Fahrenheit) to the altitude change x (in thousands of feet).



a. Is the relationship proportional? Explain.

b. Write an equation of the line. Interpret the slope.

c. You are at the bottom of a mountain where the temperature is 74°F. The top of the mountain is 5500 feet above you. What is the temperature at the top of the mountain?

13. **Critical Thinking** Consider the distance equation $d = rt$, where d is the distance (in feet), r is the rate (in feet per second), and t is the time (in seconds).

a. You run 6 feet per second. Are distance and time proportional? Explain. Graph the equation.

b. You run for 50 seconds. Are distance and rate proportional? Explain. Graph the equation.

c. You run 300 feet. Are rate and time proportional? Explain. Graph the equation.

d. One of these situations represents *inverse variation*. Which one is it? Why do you think it is called inverse variation?

Fair Game Review What you learned in previous grades & lessons

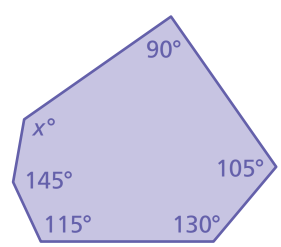
Graph the linear equation. *(Section 4.1)*

14. $y = -\frac{1}{2}x$

15. $y = 3x - \frac{3}{4}$

16. $y = -\frac{x}{3} - \frac{3}{2}$

17. **MULTIPLE CHOICE** What is the value of x? *(Section 3.3)*



(A) 110 (B) 135

(C) 315 (D) 522

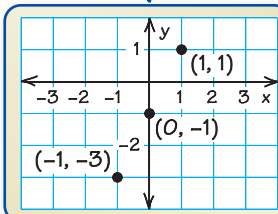
4 Study Help

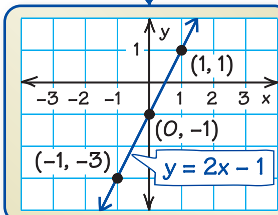
Check It Out
Graphic Organizer
BigIdeasMath.com

You can use a **process diagram** to show the steps involved in a procedure. Here is an example of a process diagram for graphing a linear equation.

Graphing a linear equation	Example: Graph $y = 2x - 1$.
Make a table of values.	see table below
Plot the ordered pairs.	see first graph below
Draw a line through the points.	see second graph below

x	-1	0	1
y	-3	-1	1





On Your Own

Make process diagrams with examples to help you study these topics.

1. finding the slope of a line
2. graphing a proportional relationship

After you complete this chapter, make process diagrams for the following topics.

3. graphing a linear equation using
   a. slope and y-intercept
   b. x- and y-intercepts
4. writing equations in slope-intercept form
5. writing equations in point-slope form

"Here is a process diagram with suggestions for what to do if a hyena knocks on your door."

4.1–4.3 Quiz

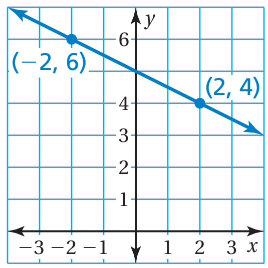
Check It Out
Progress Check
BigIdeasMath.com

Graph the linear equation. *(Section 4.1)*

1. $y = -x + 8$ **2.** $y = \frac{x}{3} - 4$ **3.** $x = -1$ **4.** $y = 3.5$

Find the slope of the line. *(Section 4.2)*

5.



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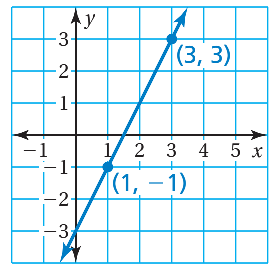

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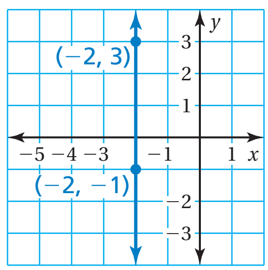

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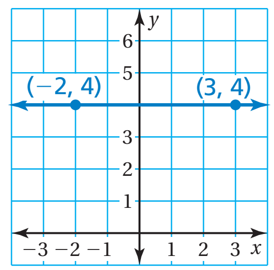


9. What is the slope of a line that is parallel to the line in Exercise 5? What is the slope of a line that is perpendicular to the line in Exercise 5? *(Section 4.2)*

10. Are the lines $y = -1$ and $x = 1$ parallel? Are they perpendicular? Justify your answer. *(Section 4.2)*

11. BANKING A bank charges \$3 each time you use an out-of-network ATM. At the beginning of the month, you have \$1500 in your bank account. You withdraw \$60 from your bank account each time you use an out-of-network ATM. Graph a linear equation that represents the balance in your account after you use an out-of-network ATM x times. *(Section 4.1)*

12. MUSIC The number y of hours of cello lessons that you take after x weeks is represented by the equation $y = 3x$. Graph the equation and interpret the slope. *(Section 4.3)*

13. DINNER PARTY The cost y (in dollars) to provide food for guests at a dinner party is proportional to the number x of guests attending the party. It costs \$30 to provide food for 4 guests. *(Section 4.3)*

a. Write an equation that represents the situation.

b. Interpret the slope.

c. How much does it cost to provide food for 10 guests?

4.4 Graphing Linear Equations in Slope-Intercept Form

Essential Question How can you describe the graph of the equation $y = mx + b$?

1 ACTIVITY: Analyzing Graphs of Lines

Work with a partner.

- **Graph each equation.**
- **Find the slope of each line.**
- **Find the point where each line crosses the *y*-axis.**
- **Complete the table.**

Equation	Slope of Graph	Point of Intersection with *y*-axis
a. $y = -\frac{1}{2}x + 1$		
b. $y = -x + 2$		
c. $y = -x - 2$		
d. $y = \frac{1}{2}x + 1$		
e. $y = x + 2$		
f. $y = x - 2$		
g. $y = \frac{1}{2}x - 1$		
h. $y = -\frac{1}{2}x - 1$		
i. $y = 3x + 2$		
j. $y = 3x - 2$		

k. Do you notice any relationship between the slope of the graph and its equation? between the point of intersection with the *y*-axis and its equation? Compare the results with those of other students in your class.

COMMON CORE

Graphing Equations

In this lesson, you will

- find slopes and *y*-intercepts of graphs of linear equations.
- graph linear equations written in slope-intercept form.

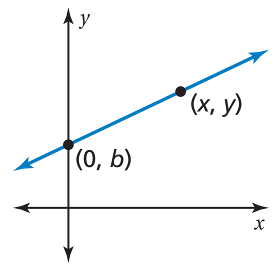
Learning Standard
8.EE.6

2 ACTIVITY: Deriving an Equation

Work with a partner.

a. Look at the graph of each equation in Activity 1. Do any of the graphs represent a proportional relationship? Explain.

b. For a nonproportional linear relationship, the graph crosses the y-axis at some point $(0, b)$, where b does not equal 0. Let (x, y) represent any other point on the graph. You can use the formula for slope to write the equation for a nonproportional linear relationship.



Use the graph to complete the steps.

$\dfrac{y_2 - y_1}{x_2 - x_1} = m$	Slope formula
$\dfrac{y - \square}{x - \square} = m$	Substitute values.
$\dfrac{\square}{\square} = m$	Simplify.
$\dfrac{\square}{\square} \cdot \square = m \cdot \square$	Multiplication Property of Equality
$y - \square = m \cdot \square$	Simplify.
$y = m\square + \square$	Addition Property of Equality

Math Practice 3

Use Prior Results

How can you use the results of Activity 1 to help support your answer?

c. What do m and b represent in the equation?

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you describe the graph of the equation $y = mx + b$?

   a. How does the value of m affect the graph of the equation?

   b. How does the value of b affect the graph of the equation?

   c. Check your answers to parts (a) and (b) with three equations that are not in Activity 1.

4. **LOGIC** Why do you think $y = mx + b$ is called the *slope-intercept form* of the equation of a line? Use drawings or diagrams to support your answer.

Practice Use what you learned about graphing linear equations in slope-intercept form to complete Exercises 4–6 on page 170.

4.4 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary

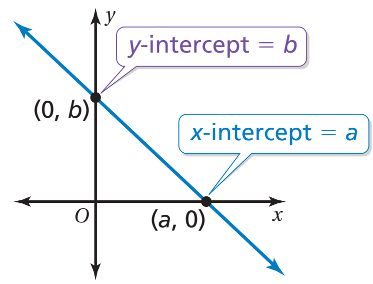
x-intercept, *p. 168*
y-intercept, *p. 168*
slope-intercept form, *p. 168*

Key Ideas

Intercepts

The **x-intercept** of a line is the x-coordinate of the point where the line crosses the x-axis. It occurs when $y = 0$.

The **y-intercept** of a line is the y-coordinate of the point where the line crosses the y-axis. It occurs when $x = 0$.



Slope-Intercept Form

Words A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.

Algebra $y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$

Study Tip

Linear equations can, but do not always, pass through the origin. So, proportional relationships are a special type of linear equation in which $b = 0$.

EXAMPLE 1 Identifying Slopes and y-Intercepts

Find the slope and the y-intercept of the graph of each linear equation.

a. $y = -4x - 2$

$y = -4x + (-2)$ Write in slope-intercept form.

∴ The slope is -4, and the y-intercept is -2.

b. $y - 5 = \frac{3}{2}x$

$y = \frac{3}{2}x + 5$ Add 5 to each side.

∴ The slope is $\frac{3}{2}$, and the y-intercept is 5.

On Your Own

Now You're Ready
Exercises 7–15

Find the slope and the y-intercept of the graph of the linear equation.

1. $y = 3x - 7$

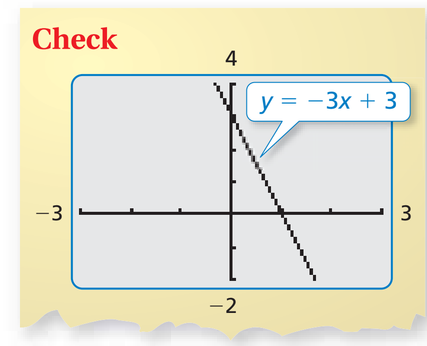
2. $y - 1 = -\frac{2}{3}x$

EXAMPLE 2 Graphing a Linear Equation in Slope-Intercept Form

Graph $y = -3x + 3$. Identify the x-intercept.

Step 1: Find the slope and the y-intercept.

$$y = \underbrace{-3}_{\text{slope}}x + \underbrace{3}_{y\text{-intercept}}$$

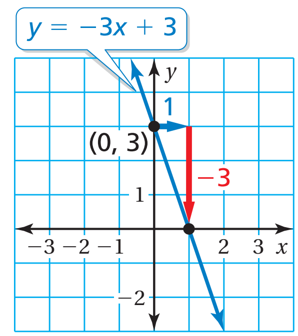


Step 2: The y-intercept is 3. So, plot (0, 3).

Step 3: Use the slope to find another point and draw the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-3}{1}$$

Plot the point that is 1 unit right and 3 units down from (0, 3). Draw a line through the two points.

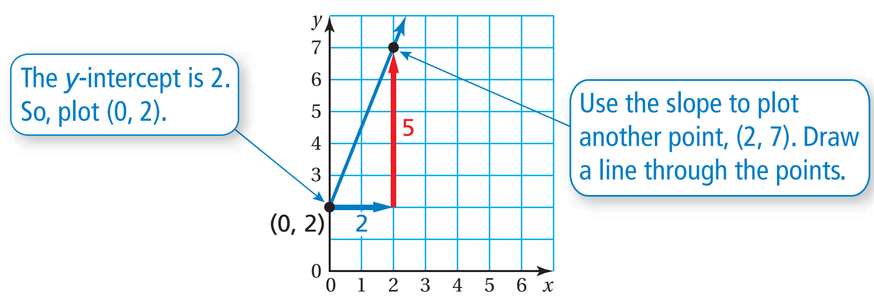


The line crosses the x-axis at (1, 0). So, the x-intercept is 1.

EXAMPLE 3 Real-Life Application

The cost y (in dollars) of taking a taxi x miles is $y = 2.5x + 2$.
(a) Graph the equation. (b) Interpret the y-intercept and the slope.

a. The slope of the line is $2.5 = \frac{5}{2}$. Use the slope and the y-intercept to graph the equation.



b. The slope is 2.5. So, the cost per mile is \$2.50. The y-intercept is 2. So, there is an initial fee of \$2 to take the taxi.

On Your Own

Now You're Ready
Exercises 18–23

Graph the linear equation. Identify the x-intercept. Use a graphing calculator to check your answer.

3. $y = x - 4$

4. $y = -\frac{1}{2}x + 1$

5. In Example 3, the cost y (in dollars) of taking a different taxi x miles is $y = 2x + 1.5$. Interpret the y-intercept and the slope.

4.4 Exercises

Check It Out
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Vocabulary and Concept Check

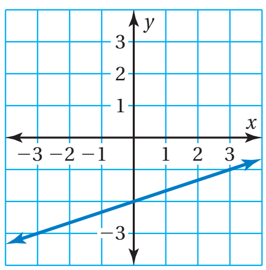
1. **VOCABULARY** How can you find the x-intercept of the graph of $2x + 3y = 6$?
2. **CRITICAL THINKING** Is the equation $y = 3x$ in slope-intercept form? Explain.
3. **OPEN-ENDED** Describe a real-life situation that you can model with a linear equation. Write the equation. Interpret the y-intercept and the slope.

Practice and Problem Solving

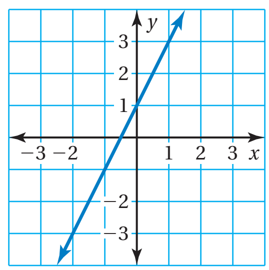
Match the equation with its graph. Identify the slope and the y-intercept.

4. $y = 2x + 1$
5. $y = \frac{1}{3}x - 2$
6. $y = -\frac{2}{3}x + 1$

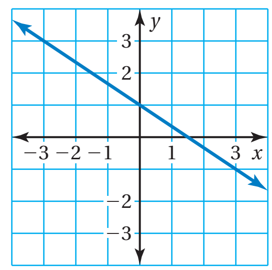
A.



B.



C.



Find the slope and the y-intercept of the graph of the linear equation.

1 7. $y = 4x - 5$
8. $y = -7x + 12$
9. $y = -\frac{4}{5}x - 2$
10. $y = 2.25x + 3$
11. $y + 1 = \frac{4}{3}x$
12. $y - 6 = \frac{3}{8}x$
13. $y - 3.5 = -2x$
14. $y = -5 - \frac{1}{2}x$
15. $y = 11 + 1.5x$

16. **ERROR ANALYSIS** Describe and correct the error in finding the slope and the y-intercept of the graph of the linear equation.

> ✗ $y = 4x - 3$
> The slope is 4, and the y-intercept is 3.

17. **SKYDIVING** A skydiver parachutes to the ground. The height y (in feet) of the skydiver after x seconds is $y = -10x + 3000$.
    a. Graph the equation.
    b. Interpret the x-intercept and the slope.

Graph the linear equation. Identify the x-intercept. Use a graphing calculator to check your answer.

2 **18.** $y = \frac{1}{5}x + 3$ **19.** $y = 6x - 7$ **20.** $y = -\frac{8}{3}x + 9$

21. $y = -1.4x - 1$ **22.** $y + 9 = -3x$ **23.** $y = 4 - \frac{3}{5}x$

24. APPLES You go to a harvest festival and pick apples.

a. Which equation represents the cost (in dollars) of going to the festival and picking x pounds of apples? Explain.

$y = 5x + 0.75$ $y = 0.75x + 5$

b. Graph the equation you chose in part (a).

25. REASONING Without graphing, identify the equations of the lines that are (a) parallel and (b) perpendicular. Explain your reasoning.

$y = 2x + 4$	$y = -\frac{1}{3}x - 1$	$y = -3x - 2$	$y = \frac{1}{2}x + 1$
$y = 3x + 3$	$y = -\frac{1}{2}x + 2$	$y = -3x + 5$	$y = 2x - 3$

26. Critical Thinking Six friends create a website. The website earns money by selling banner ads. The site has 5 banner ads. It costs $120 a month to operate the website.

a. A banner ad earns $0.005 per click. Write a linear equation that represents the monthly income y (in dollars) for x clicks.

b. Graph the equation in part (a). On the graph, label the number of clicks needed for the friends to start making a profit.

Fair Game Review What you learned in previous grades & lessons

Solve the equation for y. *(Section 1.4)*

27. $y - 2x = 3$ **28.** $4x + 5y = 13$ **29.** $2x - 3y = 6$ **30.** $7x + 4y = 8$

31. MULTIPLE CHOICE Which point is a solution of the equation $3x - 8y = 11$? *(Section 4.1)*

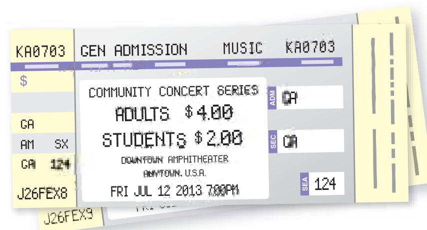
Ⓐ $(1, 1)$ Ⓑ $(1, -1)$ Ⓒ $(-1, 1)$ Ⓓ $(-1, -1)$

4.5 Graphing Linear Equations in Standard Form

Essential Question How can you describe the graph of the equation $ax + by = c$?

1 ACTIVITY: Using a Table to Plot Points

Work with a partner. You sold a total of \$16 worth of tickets to a school concert. You lost track of how many of each type of ticket you sold.



$\dfrac{\square}{\text{adult}}$ • Number of adult tickets + $\dfrac{\square}{\text{student}}$ • Number of student tickets = $\square$

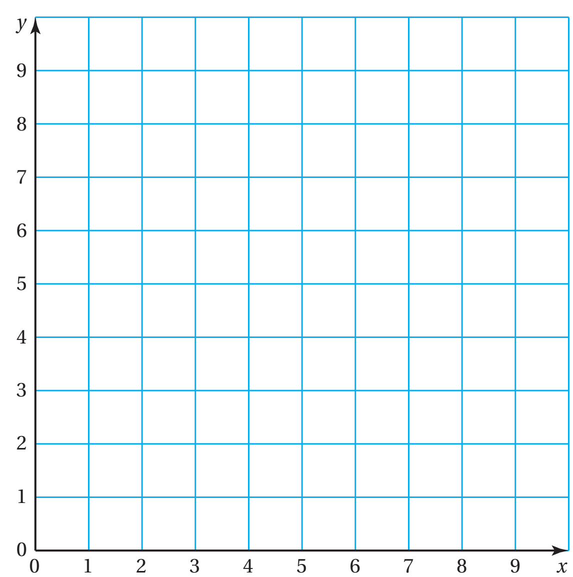
a. Let x represent the number of adult tickets.
Let y represent the number of student tickets.
Write an equation that relates x and y.

b. Copy and complete the table showing the different combinations of tickets you might have sold.

Number of Adult Tickets, x					
Number of Student Tickets, y					

c. Plot the points from the table. Describe the pattern formed by the points.

d. If you remember how many adult tickets you sold, can you determine how many student tickets you sold? Explain your reasoning.



Common Core
Graphing Equations
In this lesson, you will
- graph linear equations written in standard form.

Applying Standard
8.EE.6

2 ACTIVITY: Rewriting an Equation

Work with a partner. You sold a total of \$16 worth of cheese. You forgot how many pounds of each type of cheese you sold.

$\dfrac{\square}{\text{pound}}$ • Pounds of swiss + $\dfrac{\square}{\text{pound}}$ • Pounds of cheddar = $\square$

Math Practice 2

Understand Quantities

What do the equation and the graph represent? How can you use this information to solve the problem?

a. Let x represent the number of pounds of swiss cheese.

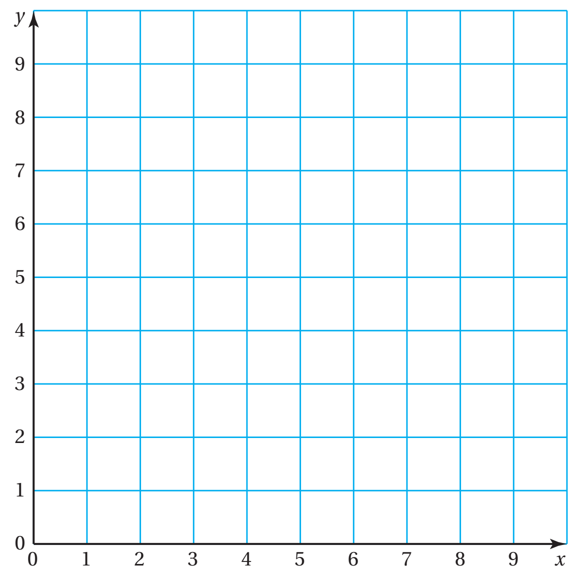
Let y represent the number of pounds of cheddar cheese.

Write an equation that relates x and y.

b. Rewrite the equation in slope-intercept form. Then graph the equation.

c. You sold 2 pounds of cheddar cheese. How many pounds of swiss cheese did you sell?

d. Does the value $x = 2.5$ make sense in the context of the problem? Explain.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you describe the graph of the equation $ax + by = c$?

4. Activities 1 and 2 show two different methods for graphing $ax + by = c$. Describe the two methods. Which method do you prefer? Explain.

5. Write a real-life problem that is similar to those shown in Activities 1 and 2.

6. Why do you think it might be easier to graph $x + y = 10$ without rewriting it in slope-intercept form and then graphing?

Practice Use what you learned about graphing linear equations in standard form to complete Exercises 3 and 4 on page 176.

4.5 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
standard form, *p. 174*

Study Tip
Any linear equation can be written in standard form.

Key Idea

Standard Form of a Linear Equation

The **standard form** of a linear equation is

$$ax + by = c$$

where a and b are not both zero.

EXAMPLE 1 Graphing a Linear Equation in Standard Form

Graph $-2x + 3y = -6$.

Step 1: Write the equation in slope-intercept form.

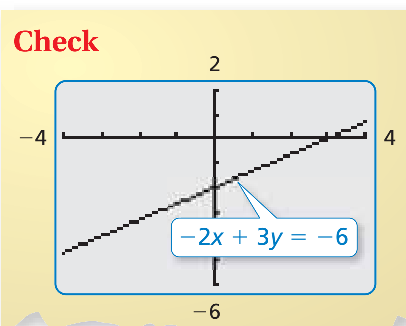
$-2x + 3y = -6$ Write the equation.

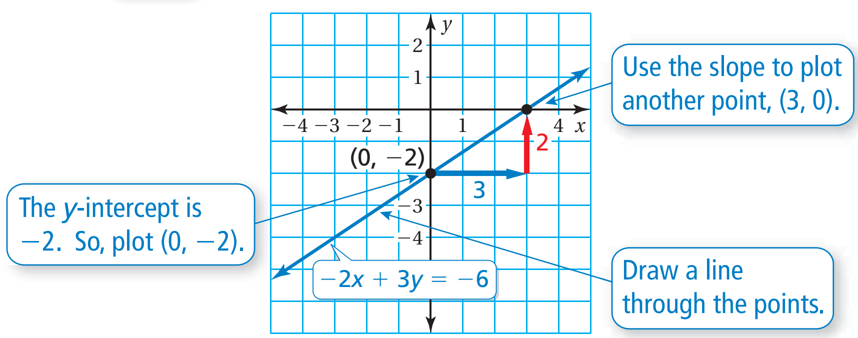
$3y = 2x - 6$ Add $2x$ to each side.

$y = \frac{2}{3}x - 2$ Divide each side by 3.

Step 2: Use the slope and the y-intercept to graph the equation.

$$y = \underbrace{\frac{2}{3}}_{\text{slope}}x + \underbrace{(-2)}_{y\text{-intercept}}$$





On Your Own

Now You're Ready
Exercises 5–10

Graph the linear equation. Use a graphing calculator to check your graph.

1. $x + y = -2$
2. $-\frac{1}{2}x + 2y = 6$
3. $-\frac{2}{3}x + y = 0$
4. $2x + y = 5$

EXAMPLE 2 Graphing a Linear Equation in Standard Form

Graph $x + 3y = -3$ using intercepts.

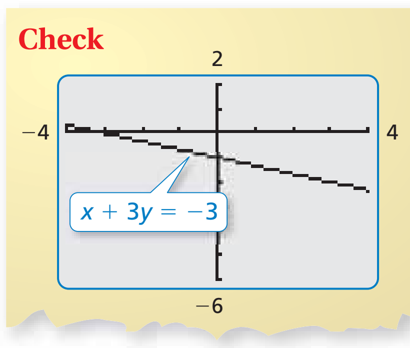
Step 1: To find the x-intercept, substitute 0 for y.

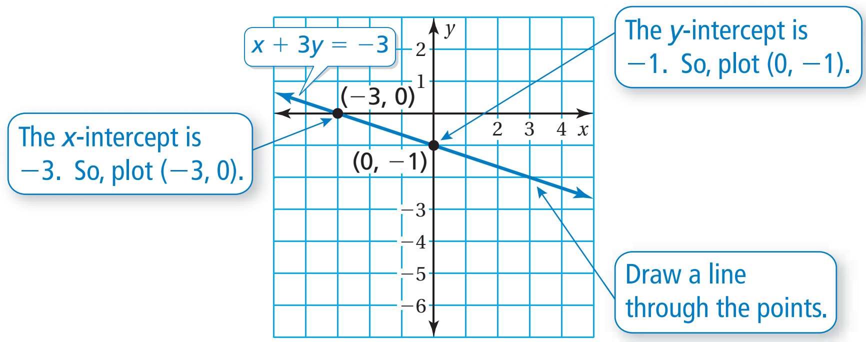
$$x + 3y = -3$$
$$x + 3(0) = -3$$
$$x = -3$$

To find the y-intercept, substitute 0 for x.

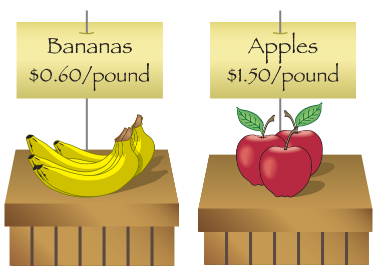
$$x + 3y = -3$$
$$0 + 3y = -3$$
$$y = -1$$

Step 2: Graph the equation.





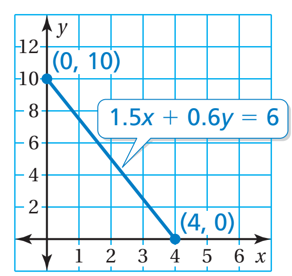
EXAMPLE 3 Real-Life Application



You have \$6 to spend on apples and bananas. (a) Graph the equation $1.5x + 0.6y = 6$, where x is the number of pounds of apples and y is the number of pounds of bananas. (b) Interpret the intercepts.

a. Find the intercepts and graph the equation.

***x*-intercept**	***y*-intercept**
$1.5x + 0.6y = 6$	$1.5x + 0.6y = 6$
$1.5x + 0.6(0) = 6$	$1.5(0) + 0.6y = 6$
$x = 4$	$y = 10$



b. The x-intercept shows that you can buy 4 pounds of apples when you do not buy any bananas. The y-intercept shows that you can buy 10 pounds of bananas when you do not buy any apples.

On Your Own

Now You're Ready
Exercises 16–18

Graph the linear equation using intercepts. Use a graphing calculator to check your graph.

5. $2x - y = 8$

6. $x + 3y = 6$

7. **WHAT IF?** In Example 3, you buy y pounds of oranges instead of bananas. Oranges cost \$1.20 per pound. Graph the equation $1.5x + 1.2y = 6$. Interpret the intercepts.

4.5 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** Is the equation $y = -2x + 5$ in standard form? Explain.
2. **WRITING** Describe two ways to graph the equation $4x + 2y = 6$.

Practice and Problem Solving

Define two variables for the verbal model. Write an equation in slope-intercept form that relates the variables. Graph the equation.

3. $\dfrac{\$2.00}{\text{pound}} \cdot$ Pounds of peaches $+ \dfrac{\$1.50}{\text{pound}} \cdot$ Pounds of apples $=$ \$15

4. $\dfrac{16 \text{ miles}}{\text{hour}} \cdot$ Hours biked $+ \dfrac{2 \text{ miles}}{\text{hour}} \cdot$ Hours walked $=$ 32 miles

Write the linear equation in slope-intercept form.

1 5. $2x + y = 17$
6. $5x - y = \dfrac{1}{4}$
7. $-\dfrac{1}{2}x + y = 10$

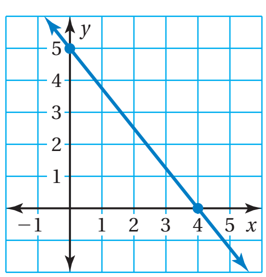
Graph the linear equation. Use a graphing calculator to check your graph.

8. $-18x + 9y = 72$
9. $16x - 4y = 2$
10. $\dfrac{1}{4}x + \dfrac{3}{4}y = 1$

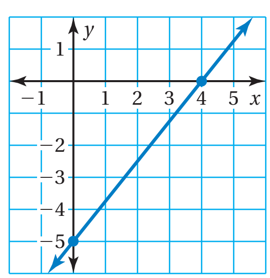
Match the equation with its graph.

11. $15x - 12y = 60$
12. $5x + 4y = 20$
13. $10x + 8y = -40$

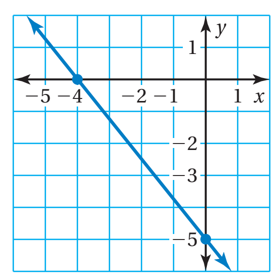
A.



B.



C.

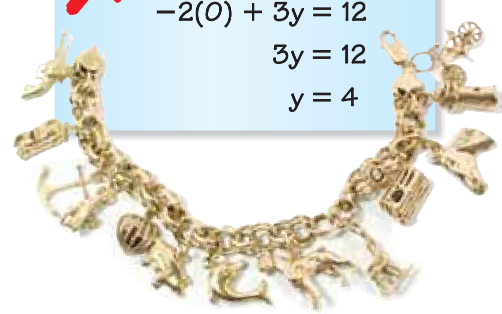


14. **ERROR ANALYSIS** Describe and correct the error in finding the x-intercept.

$$-2x + 3y = 12$$
$$-2(0) + 3y = 12$$
$$3y = 12$$
$$y = 4$$

15. **BRACELET** A charm bracelet costs \$65, plus \$25 for each charm. The equation $-25x + y = 65$ represents the cost y of the bracelet, where x is the number of charms.

   a. Graph the equation.
   b. How much does the bracelet shown cost?



Graph the linear equation using intercepts. Use a graphing calculator to check your graph.

2 **16.** $3x - 4y = -12$ **17.** $2x + y = 8$ **18.** $\frac{1}{3}x - \frac{1}{6}y = -\frac{2}{3}$

19. **SHOPPING** The amount of money you spend on x CDs and y DVDs is given by the equation $14x + 18y = 126$. Find the intercepts and graph the equation.

20. **SCUBA** Five friends go scuba diving. They rent a boat for x days and scuba gear for y days. The total spent is $1000.

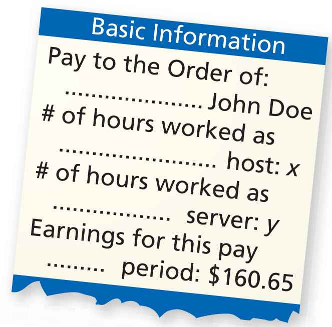
a. Write an equation in standard form that represents the situation.

b. Graph the equation and interpret the intercepts.

21. **MODELING** You work at a restaurant as a host and a server. You earn $9.45 for each hour you work as a host and $7.65 for each hour you work as a server.

a. Write an equation in standard form that models your earnings.

b. Graph the equation.



22. **LOGIC** Does the graph of every linear equation have an x-intercept? Explain your reasoning. Include an example.

23. Critical Thinking For a house call, a veterinarian charges $70, plus $40 an hour.

a. Write an equation that represents the total fee y (in dollars) the veterinarian charges for a visit lasting x hours.

b. Find the x-intercept. Does this value make sense in this context? Explain your reasoning.

c. Graph the equation.

Fair Game Review What you learned in previous grades & lessons

The points in the table lie on a line. Find the slope of the line. *(Section 4.2)*

24.

x	-2	-1	0	1
y	-10	-6	-2	2

25.

x	2	4	6	8
y	2	3	4	5

26. **MULTIPLE CHOICE** Which value of x makes the equation $4x - 12 = 3x - 9$ true? *(Section 1.3)*

Ⓐ -1 Ⓑ 0 Ⓒ 1 Ⓓ 3

4.6 Writing Equations in Slope-Intercept Form

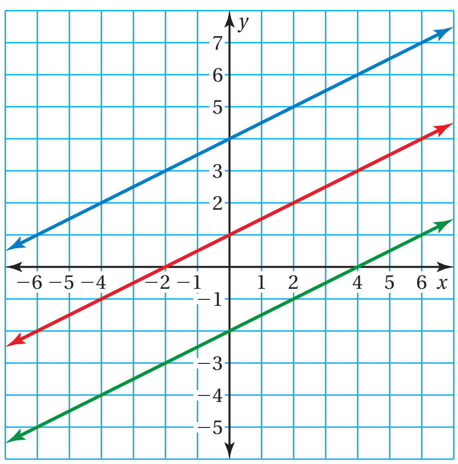
Essential Question How can you write an equation of a line when you are given the slope and the y-intercept of the line?

1 ACTIVITY: Writing Equations of Lines

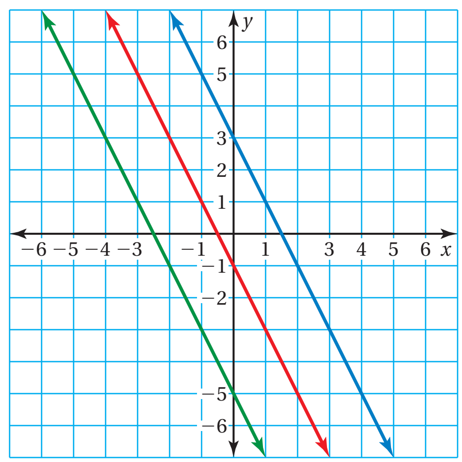
Work with a partner.

- **Find the slope of each line.**
- **Find the y-intercept of each line.**
- **Write an equation for each line.**
- **What do the three lines have in common?**

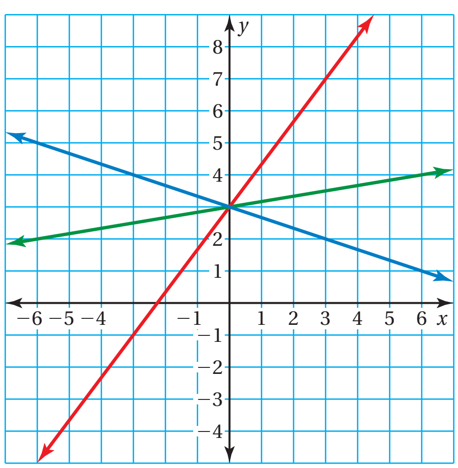
a.



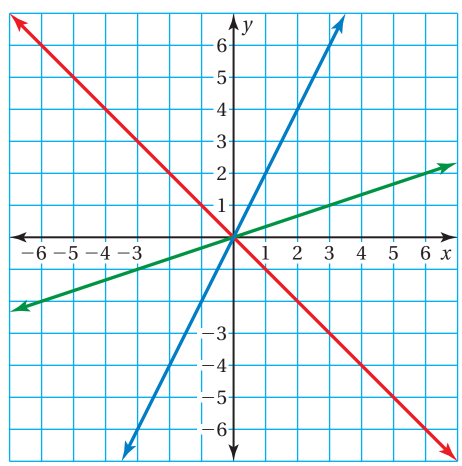
b.



c.



d.



COMMON CORE

Writing Equations

In this lesson, you will

- write equations of lines in slope-intercept form.

Preparing for Standard 8.F.4

2 ACTIVITY: Describing a Parallelogram

Math Practice 1

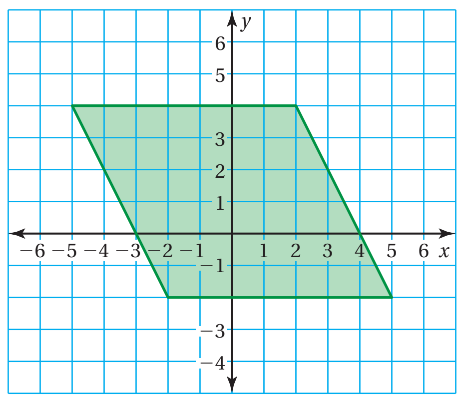
Analyze Givens

What do you need to know to write an equation?

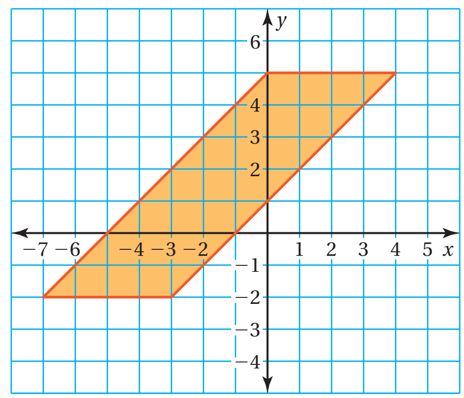
Work with a partner.

- **Find the area of each parallelogram.**
- **Write an equation that represents each side of each parallelogram.**

a.



b.



3 ACTIVITY: Interpreting the Slope and the *y*-Intercept

Work with a partner. The graph shows a trip taken by a car, where *t* is the time (in hours) and *y* is the distance (in miles) from Phoenix.

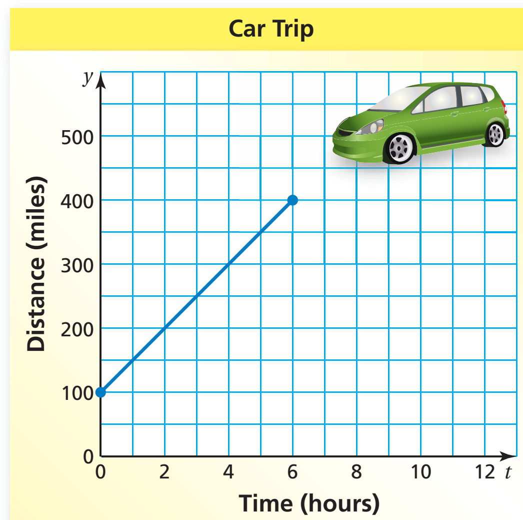
a. Find the y-intercept of the graph. What does it represent?

b. Find the slope of the graph. What does it represent?

c. How long did the trip last?

d. How far from Phoenix was the car at the end of the trip?

e. Write an equation that represents the graph.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you write an equation of a line when you are given the slope and the y-intercept of the line? Give an example that is different from those in Activities 1, 2, and 3.

5. Two sides of a parallelogram are represented by the equations $y = 2x + 1$ and $y = -x + 3$. Give two equations that can represent the other two sides.

Practice Use what you learned about writing equations in slope-intercept form to complete Exercises 3 and 4 on page 182.

4.6 Lesson

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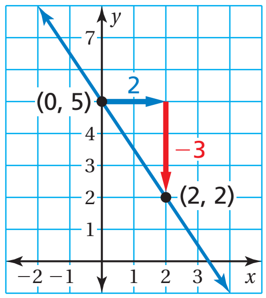
EXAMPLE 1 Writing Equations in Slope-Intercept Form

Write an equation of the line in slope-intercept form.

a.



Find the slope and the y-intercept.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{2 - 0} = \frac{-3}{2}, \text{ or } -\frac{3}{2}$$

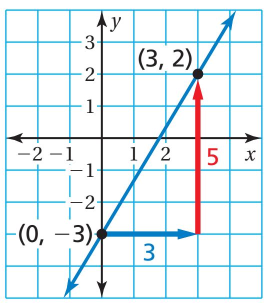
Because the line crosses the y-axis at (0, 5), the y-intercept is 5.

So, the equation is $y = \underbrace{-\frac{3}{2}}_{\text{slope}}x + \underbrace{5}_{y\text{-intercept}}$.

Study Tip

After writing an equation, check that the given points are solutions of the equation.

b.



Find the slope and the y-intercept.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{0 - 3} = \frac{-5}{-3}, \text{ or } \frac{5}{3}$$

Because the line crosses the y-axis at (0, −3), the y-intercept is −3.

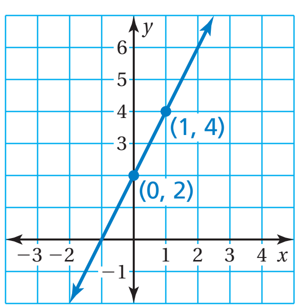
So, the equation is $y = \underbrace{\frac{5}{3}}_{\text{slope}}x + \underbrace{(-3)}_{y\text{-intercept}}$, or $y = \frac{5}{3}x - 3$.

On Your Own

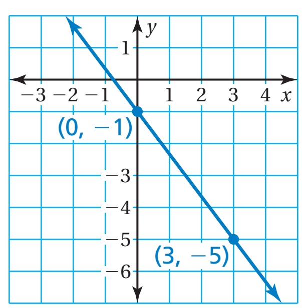
Now You're Ready
Exercises 5–10

Write an equation of the line in slope-intercept form.

1.



2.

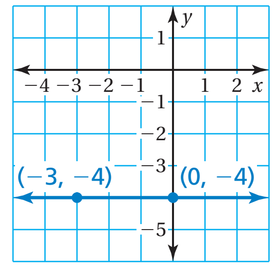


EXAMPLE 2 Writing an Equation

Which equation is shown in the graph?

Ⓐ $y = -4$ Ⓑ $y = -3$

Ⓒ $y = 0$ Ⓓ $y = -3x$



Remember

The graph of $y = a$ is a horizontal line that passes through $(0, a)$.

Find the slope and the y-intercept.

The line is horizontal, so the change in y is 0.

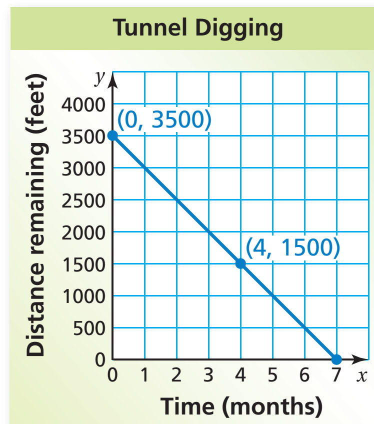
$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{0}{3} = 0$$

Because the line crosses the y-axis at $(0, -4)$, the y-intercept is -4.

∴ So, the equation is $y = 0x + (-4)$, or $y = -4$. The correct answer is Ⓐ.

EXAMPLE 3 Real-Life Application

The graph shows the distance remaining to complete a tunnel. (a) Write an equation that represents the distance y (in feet) remaining after x months. (b) How much time does it take to complete the tunnel?



Engineers used tunnel boring machines like the ones shown above to dig an extension of the Metro Gold Line in Los Angeles. The new tunnels are 1.7 miles long and 21 feet wide.

a. Find the slope and the y-intercept.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{-2000}{4} = -500$$

Because the line crosses the y-axis at $(0, 3500)$, the y-intercept is 3500.

∴ So, the equation is $y = -500x + 3500$.

b. The tunnel is complete when the distance remaining is 0 feet. So, find the value of x when $y = 0$.

$y = -500x + 3500$	Write the equation.
$0 = -500x + 3500$	Substitute 0 for y.
$-3500 = -500x$	Subtract 3500 from each side.
$7 = x$	Divide each side by -500.

∴ It takes 7 months to complete the tunnel.

On Your Own

Now You're Ready
Exercises 13–15

3. Write an equation of the line that passes through $(0, 5)$ and $(4, 5)$.

4. **WHAT IF?** In Example 3, the points are $(0, 3500)$ and $(5, 1500)$. How long does it take to complete the tunnel?

4.6 Exercises

Check It Out
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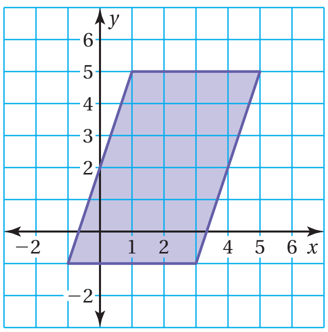
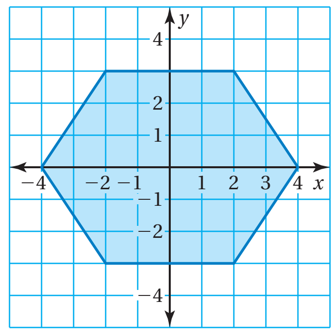
Vocabulary and Concept Check

1. **PRECISION** Explain how to find the slope of a line given the intercepts of the line.
2. **WRITING** Explain how to write an equation of a line using its graph.

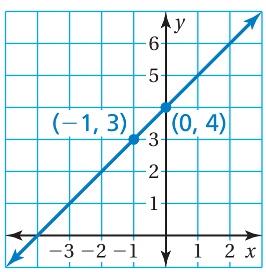
Practice and Problem Solving

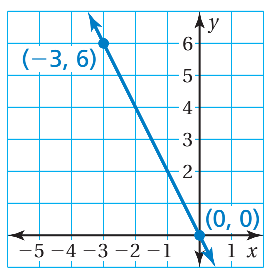
Write an equation that represents each side of the figure.

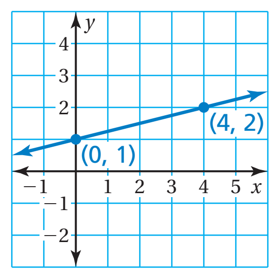
3. 

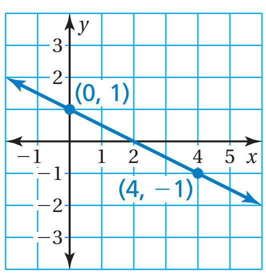
4. 

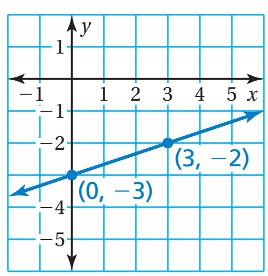
Write an equation of the line in slope-intercept form.

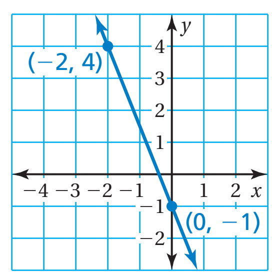
1 5. 

6. 

7. 

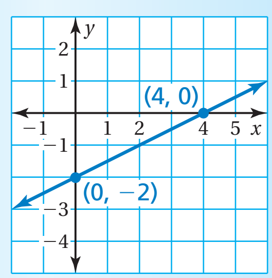
8. 

9. 

10. 

11. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line.

$$y = \frac{1}{2}x + 4$$



12. **BOA** A boa constrictor is 18 inches long at birth and grows 8 inches per year. Write an equation that represents the length y (in feet) of a boa constrictor that is x years old.

Write an equation of the line that passes through the points.

13. (2, 5), (0, 5) **14.** (−3, 0), (0, 0) **15.** (0, −2), (4, −2)

16. **WALKATHON** One of your friends gives you \$10 for a charity walkathon. Another friend gives you an amount per mile. After 5 miles, you have raised \$13.50 total. Write an equation that represents the amount y of money you have raised after x miles.

17. **BRAKING TIME** During each second of braking, an automobile slows by about 10 miles per hour.

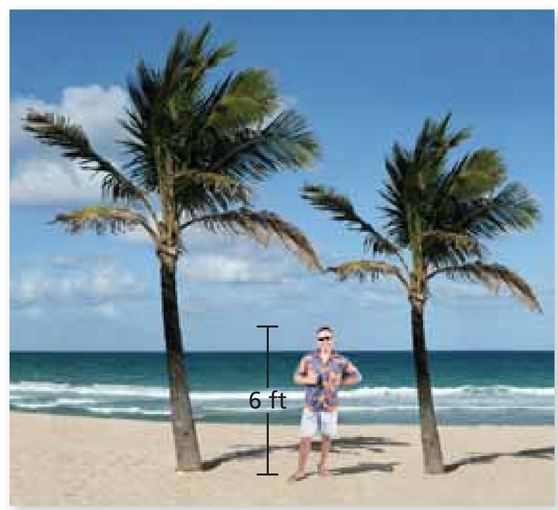
a. Plot the points (0, 60) and (6, 0). What do the points represent?

b. Draw a line through the points. What does the line represent?

c. Write an equation of the line.

18. **PAPER** You have 500 sheets of notebook paper. After 1 week, you have 72% of the sheets left. You use the same number of sheets each week. Write an equation that represents the number y of pages remaining after x weeks.

19. Critical Thinking The palm tree on the left is 10 years old. The palm tree on the right is 8 years old. The trees grow at the same rate.



a. Estimate the height y (in feet) of each tree.

b. Plot the two points (x, y), where x is the age of each tree and y is the height of each tree.

c. What is the rate of growth of the trees?

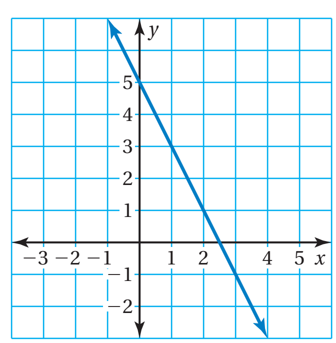
d. Write an equation that represents the height of a palm tree in terms of its age.

Fair Game Review What you learned in previous grades & lessons

Plot the ordered pair in a coordinate plane. *(Skills Review Handbook)*

20. (1, 4) **21.** (−1, −2) **22.** (0, 1) **23.** (2, 7)

24. **MULTIPLE CHOICE** Which of the following statements is true? *(Section 4.4)*



(A) The x-intercept is 5.

(B) The x-intercept is −2.

(C) The y-intercept is 5.

(D) The y-intercept is −2.

4.7 Writing Equations in Point-Slope Form

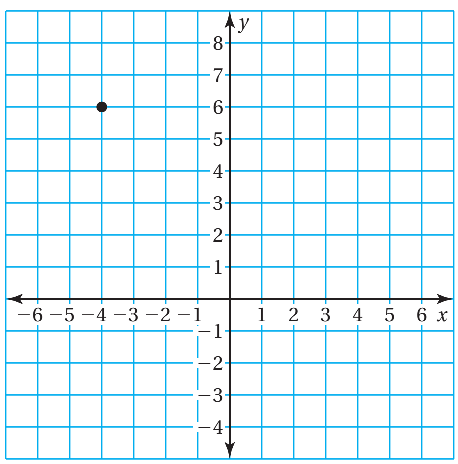
Essential Question How can you write an equation of a line when you are given the slope and a point on the line?

1 ACTIVITY: Writing Equations of Lines

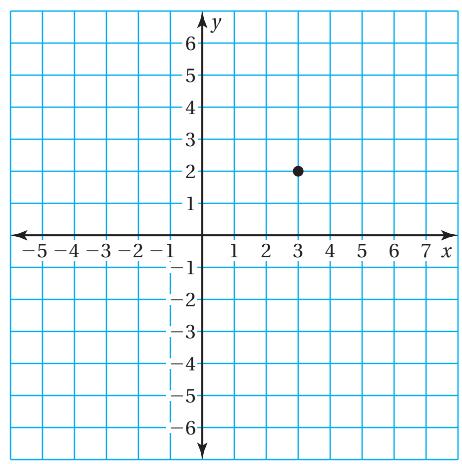
Work with a partner.

- **Sketch the line that has the given slope and passes through the given point.**
- **Find the *y*-intercept of the line.**
- **Write an equation of the line.**

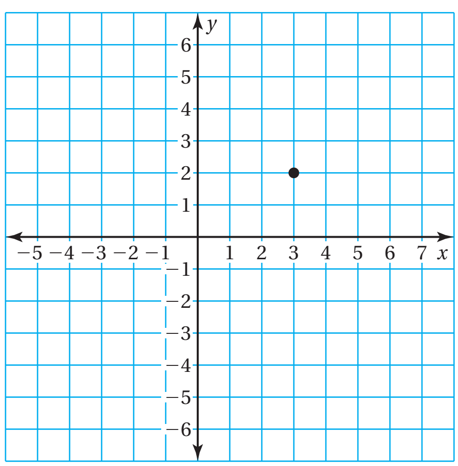
a. $m = -2$



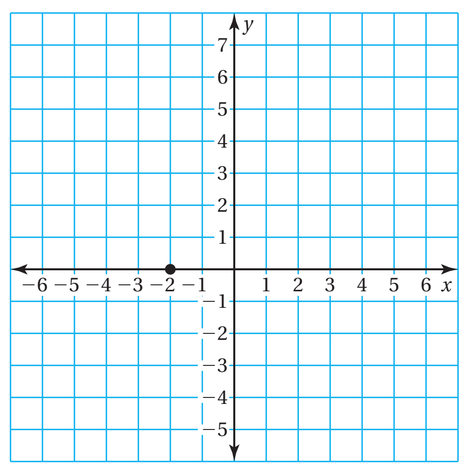
b. $m = \frac{1}{3}$



c. $m = -\frac{2}{3}$



d. $m = \frac{5}{2}$



COMMON CORE

Writing Equations

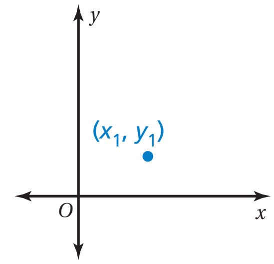
In this lesson, you will

- write equations of lines using a slope and a point.
- write equations of lines using two points.

Preparing for Standard 8.F.4

2 ACTIVITY: Deriving an Equation

Work with a partner.



a. Draw a nonvertical line that passes through the point (x_1, y_1).

b. Plot another point on your line. Label this point as (x, y). This point represents any other point on the line.

c. Label the rise and the run of the line through the points (x_1, y_1) and (x, y).

d. The rise can be written as $y - y_1$. The run can be written as $x - x_1$. Explain why this is true.

e. Write an equation for the slope m of the line using the expressions from part (d).

f. Multiply each side of the equation by the expression in the denominator. Write your result. What does this result represent?

Math Practice 3

Construct Arguments

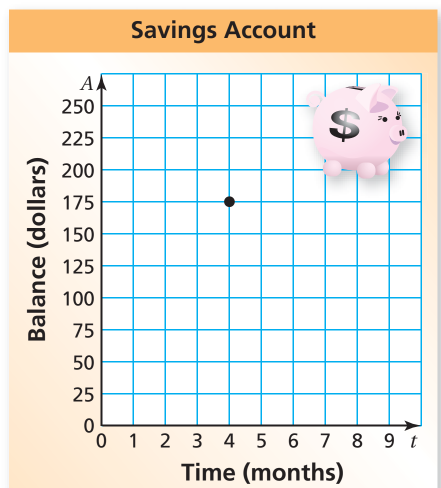
How does a graph help you derive an equation?

3 ACTIVITY: Writing an Equation

Work with a partner.

For 4 months, you saved \$25 a month. You now have \$175 in your savings account.

- Draw a graph that shows the balance in your account after t months.
- Use your result from Activity 2 to write an equation that represents the balance A after t months.



What Is Your Answer?

4. Redo Activity 1 using the equation you found in Activity 2. Compare the results. What do you notice?

5. Why do you think $y - y_1 = m(x - x_1)$ is called the *point-slope form* of the equation of a line? Why do you think it is important?

6. **IN YOUR OWN WORDS** How can you write an equation of a line when you are given the slope and a point on the line? Give an example that is different from those in Activity 1.

Practice Use what you learned about writing equations using a slope and a point to complete Exercises 3–5 on page 188.

Key Vocabulary
point-slope form, *p. 186*

Key Idea

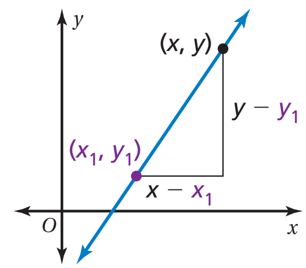
Point-Slope Form

Words A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**. The line passes through the point (x_1, y_1), and the slope of the line is m.

Algebra

$$y - y_1 = \overbrace{m}^{\text{slope}}(x - x_1)$$

passes through (x_1, y_1)



EXAMPLE 1 Writing an Equation Using a Slope and a Point

Write in point-slope form an equation of the line that passes through the point (−6, 1) with slope $\frac{2}{3}$.

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 1 = \frac{2}{3}[x - (-6)]$ Substitute $\frac{2}{3}$ for m, -6 for x_1, and 1 for y_1.

$y - 1 = \frac{2}{3}(x + 6)$ Simplify.

So, the equation is $y - 1 = \frac{2}{3}(x + 6)$.

Check Check that (−6, 1) is a solution of the equation.

$y - 1 = \frac{2}{3}(x + 6)$ Write the equation.

$1 - 1 \stackrel{?}{=} \frac{2}{3}(-6 + 6)$ Substitute.

$0 = 0$ ✓ Simplify.

On Your Own

Now You're Ready
Exercises 6–11

Write in point-slope form an equation of the line that passes through the given point and has the given slope.

1. $(1, 2);\ m = -4$ **2.** $(7, 0);\ m = 1$ **3.** $(-8, -5);\ m = -\frac{3}{4}$

EXAMPLE 2 Writing an Equation Using Two Points

Study Tip

You can use either of the given points to write the equation of the line.

Use $m = -2$ and $(5, -2)$.

$y - (-2) = -2(x - 5)$

$y + 2 = -2x + 10$

$y = -2x + 8$ ✓

Write in slope-intercept form an equation of the line that passes through the points (2, 4) and (5, −2).

Find the slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - 4}{5 - 2} = \dfrac{-6}{3} = -2$

Then use the slope $m = -2$ and the point (2, 4) to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 4 = -2(x - 2)$	Substitute −2 for m, 2 for x_1, and 4 for y_1.
$y - 4 = -2x + 4$	Distributive Property
$y = -2x + 8$	Write in slope-intercept form.

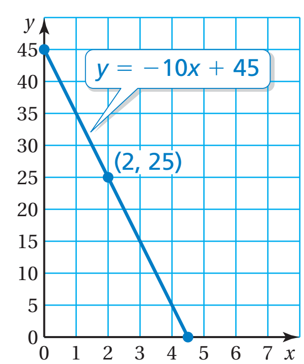
EXAMPLE 3 Real-Life Application

You finish parasailing and are being pulled back to the boat. After 2 seconds, you are 25 feet above the boat. (a) Write and graph an equation that represents your height *y* (in feet) above the boat after *x* seconds. (b) At what height were you parasailing?

a. You are being pulled down at the rate of 10 feet per second. So, the slope is −10. You are 25 feet above the boat after 2 seconds. So, the line passes through (2, 25). Use the point-slope form.

$y - 25 = -10(x - 2)$	Substitute for m, x_1, and y_1.
$y - 25 = -10x + 20$	Distributive Property
$y = -10x + 45$	Write in slope-intercept form.

So, the equation is $y = -10x + 45$.



b. You start descending when $x = 0$. The y-intercept is 45. So, you were parasailing at a height of 45 feet.

On Your Own

Now You're Ready
Exercises 12–17

Write in slope-intercept form an equation of the line that passes through the given points.

4. (−2, 1), (3, −4) **5.** (−5, −5), (−3, 3) **6.** (−8, 6), (−2, 9)

7. WHAT IF? In Example 3, you are 35 feet above the boat after 2 seconds. Write and graph an equation that represents your height *y* (in feet) above the boat after *x* seconds.

4.7 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

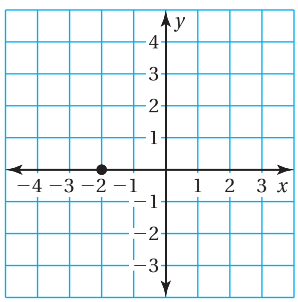
1. **VOCABULARY** From the equation $y - 3 = -2(x + 1)$, identify the slope and a point on the line.

2. **WRITING** Describe how to write an equation of a line using (a) its slope and a point on the line and (b) two points on the line.

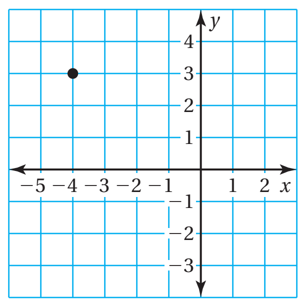
Practice and Problem Solving

Use the point-slope form to write an equation of the line with the given slope that passes through the given point.

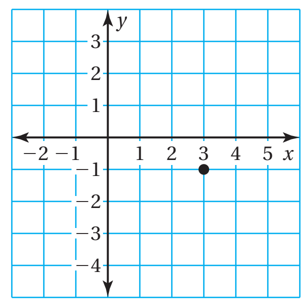
3. $m = \frac{1}{2}$



4. $m = -\frac{3}{4}$



5. $m = -3$



Write in point-slope form an equation of the line that passes through the given point and has the given slope.

1 6. $(3, 0)$; $m = -\frac{2}{3}$

7. $(4, 8)$; $m = \frac{3}{4}$

8. $(1, -3)$; $m = 4$

9. $(7, -5)$; $m = -\frac{1}{7}$

10. $(3, 3)$; $m = \frac{5}{3}$

11. $(-1, -4)$; $m = -2$

Write in slope-intercept form an equation of the line that passes through the given points.

2 12. $(-1, -1)$, $(1, 5)$

13. $(2, 4)$, $(3, 6)$

14. $(-2, 3)$, $(2, 7)$

15. $(4, 1)$, $(8, 2)$

16. $(-9, 5)$, $(-3, 3)$

17. $(1, 2)$, $(-2, -1)$

18. **CHEMISTRY** At 0°C, the volume of a gas is 22 liters. For each degree the temperature T (in degrees Celsius) increases, the volume V (in liters) of the gas increases by $\frac{2}{25}$. Write an equation that represents the volume of the gas in terms of the temperature.

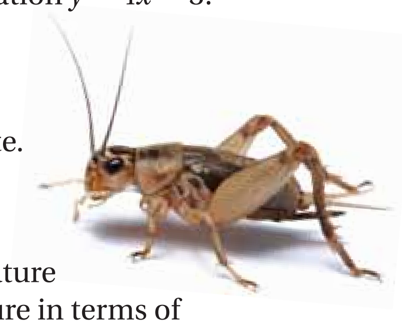
19. **CARS** After it is purchased, the value of a new car decreases \$4000 each year. After 3 years, the car is worth \$18,000.

a. Write an equation that represents the value V (in dollars) of the car x years after it is purchased.

b. What was the original value of the car?

20. **REASONING** Write an equation of a line that passes through the point (8, 2) that is (a) parallel and (b) perpendicular to the graph of the equation $y = 4x - 3$.

21. **CRICKETS** According to Dolbear's law, you can predict the temperature T (in degrees Fahrenheit) by counting the number x of chirps made by a snowy tree cricket in 1 minute. For each rise in temperature of 0.25°F, the cricket makes an additional chirp each minute.

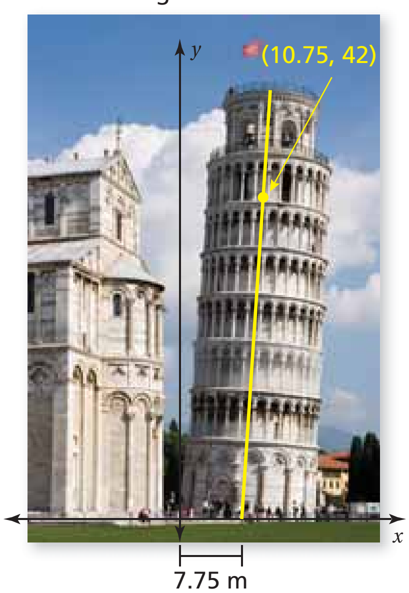


a. A cricket chirps 40 times in 1 minute when the temperature is 50°F. Write an equation that represents the temperature in terms of the number of chirps in 1 minute.

b. You count 100 chirps in 1 minute. What is the temperature?

c. The temperature is 96°F. How many chirps would you expect the cricket to make?

Leaning Tower of Pisa



22. **WATERING CAN** You water the plants in your classroom at a constant rate. After 5 seconds, your watering can contains 58 ounces of water. Fifteen seconds later, the can contains 28 ounces of water.

a. Write an equation that represents the amount y (in ounces) of water in the can after x seconds.

b. How much water was in the can when you started watering the plants?

c. When is the watering can empty?

23. **Problem Solving** The Leaning Tower of Pisa in Italy was built between 1173 and 1350.

a. Write an equation for the yellow line.

b. The tower is 56 meters tall. How far off center is the top of the tower?

Fair Game Review *What you learned in previous grades & lessons*

Graph the linear equation. *(Section 4.4)*

24. $y = 4x$ 25. $y = -2x + 1$ 26. $y = 3x - 5$

27. **MULTIPLE CHOICE** What is the x-intercept of the equation $3x + 5y = 30$? *(Section 4.5)*

(A) -10 (B) -6 (C) 6 (D) 10

4.4–4.7 Quiz

Check It Out
Progress Check
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Find the slope and the *y*-intercept of the graph of the linear equation. *(Section 4.4)*

1. $y = \frac{1}{4}x - 8$

2. $y = -x + 3$

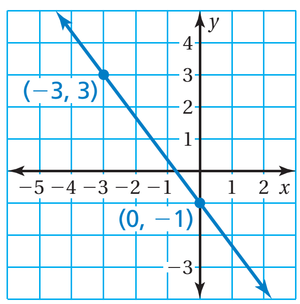
Find the *x*- and *y*-intercepts of the graph of the equation. *(Section 4.5)*

3. $3x - 2y = 12$

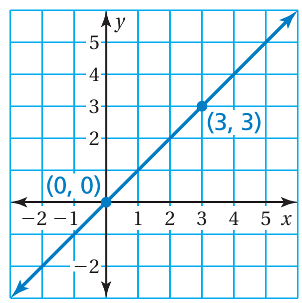
4. $x + 5y = 15$

Write an equation of the line in slope-intercept form. *(Section 4.6)*

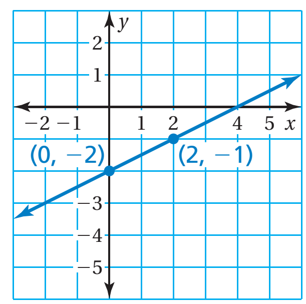
5.



6.



7.



Write in point-slope form an equation of the line that passes through the given point and has the given slope. *(Section 4.7)*

8. $(1, 3)$; $m = 2$

9. $(-3, -2)$; $m = \frac{1}{3}$

10. $(-1, 4)$; $m = -1$

11. $(8, -5)$; $m = -\frac{1}{8}$

Write in slope-intercept form an equation of the line that passes through the given points. *(Section 4.7)*

12. $\left(0, -\frac{2}{3}\right) \left(-3, -\frac{2}{3}\right)$

13. $(4, 0)$, $(0, 4)$

14. STATE FAIR The cost y (in dollars) of one person buying admission to a fair and going on x rides is $y = x + 12$. *(Section 4.4)*

a. Graph the equation.

b. Interpret the y-intercept and the slope.

15. PAINTING You used \$90 worth of paint for a school float. *(Section 4.5)*

a. Graph the equation $18x + 15y = 90$, where x is the number of gallons of blue paint and y is the number of gallons of white paint.

b. Interpret the intercepts.

16. CONSTRUCTION A construction crew is extending a highway sound barrier that is 13 miles long. The crew builds $\frac{1}{2}$ of a mile per week. Write an equation that represents the length y (in miles) of the barrier after x weeks. *(Section 4.6)*

4 Chapter Review

Check It Out
Vocabulary Help
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Review Key Vocabulary

linear equation *p. 144*
solution of a linear equation, *p. 144*
slope, *p. 150*
rise, *p. 150*
run, *p. 150*
x-intercept, *p. 168*
y-intercept, *p. 168*
slope-intercept form, *p. 168*
standard form, *p. 174*
point-slope form, *p. 186*

Review Examples and Exercises

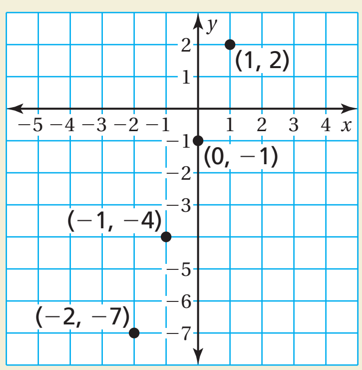
4.1 Graphing Linear Equations *(pp. 142–147)*

Graph $y = 3x - 1$.

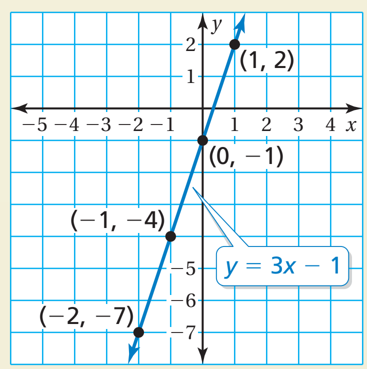
Step 1: Make a table of values.

x	$y = 3x - 1$	y	(x, y)
-2	$y = 3(-2) - 1$	-7	$(-2, -7)$
-1	$y = 3(-1) - 1$	-4	$(-1, -4)$
0	$y = 3(0) - 1$	-1	$(0, -1)$
1	$y = 3(1) - 1$	2	$(1, 2)$

Step 2: Plot the ordered pairs.



Step 3: Draw a line through the points.



Exercises

Graph the linear equation.

1. $y = \frac{3}{5}x$

2. $y = -2$

3. $y = 9 - x$

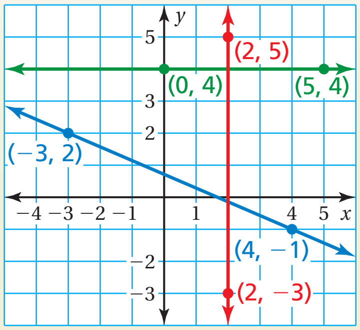
4. $y = 1$

5. $y = \frac{2}{3}x + 2$

6. $x = -5$

4.2 Slope of a Line (pp. 148–157)

Find the slope of each line in the graph.



Red Line: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - (-3)}{2 - 2} = \dfrac{8}{0}$

The slope of the red line is undefined.

Blue Line: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1 - 2}{4 - (-3)} = \dfrac{-3}{7}$, or $-\dfrac{3}{7}$

Green Line: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 4}{5 - 0} = \dfrac{0}{5}$, or 0

Exercises

The points in the table lie on a line. Find the slope of the line.

7.

x	0	1	2	3
y	−1	0	1	2

8.

x	−2	0	2	4
y	3	4	5	6

9. Are the lines $x = 2$ and $y = 4$ parallel? Are they perpendicular? Explain.

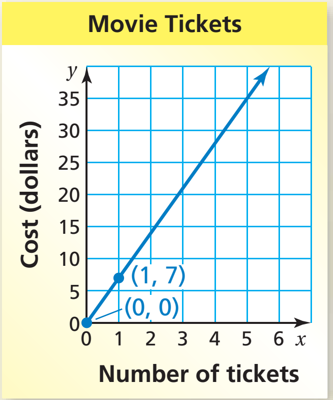
4.3 Graphing Proportional Relationships (pp. 158–163)

The cost *y* (in dollars) for *x* tickets to a movie is represented by the equation $y = 7x$. Graph the equation and interpret the slope.

The equation shows that the slope *m* is 7. So, the graph passes through (0, 0) and (1, 7).

Plot the points and draw a line through the points. Because negative values of *x* do not make sense in this context, graph in the first quadrant only.

The slope indicates that the unit cost is $7 per ticket.



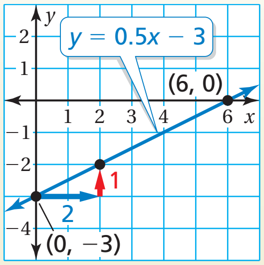
Exercises

10. RUNNING The number *y* of miles you run after *x* weeks is represented by the equation $y = 8x$. Graph the equation and interpret the slope.

11. STUDYING The number *y* of hours that you study after *x* days is represented by the equation $y = 1.5x$. Graph the equation and interpret the slope.

4.4 Graphing Linear Equations in Slope-Intercept Form (pp. 166–171)

Graph $y = 0.5x - 3$. Identify the x-intercept.



Step 1: Find the slope and the y-intercept.

$$y = 0.5x + (-3)$$

slope → 0.5; y-intercept → -3

Step 2: The y-intercept is -3. So, plot $(0, -3)$.

Step 3: Use the slope to find another point and draw the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$

Plot the point that is 2 units right and 1 unit up from (0, –3). Draw a line through the two points.

∴ The line crosses the x-axis at $(6, 0)$. So, the x-intercept is 6.

Exercises

Graph the linear equation. Identify the x-intercept. Use a graphing calculator to check your answer.

12. $y = 2x - 6$ **13.** $y = -4x + 8$ **14.** $y = -x - 8$

4.5 Graphing Linear Equations in Standard Form (pp. 172–177)

Graph $8x + 4y = 16$.

Step 1: Write the equation in slope-intercept form.

$8x + 4y = 16$	Write the equation.
$4y = -8x + 16$	Subtract $8x$ from each side.
$y = -2x + 4$	Divide each side by 4.

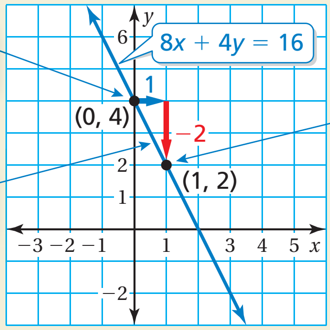
Step 2: Use the slope and the y-intercept to graph the equation.

$$y = -2x + 4$$

slope → -2; y-intercept → 4

The y-intercept is 4. So, plot (0, 4).

Draw a line through the points.



Use the slope to plot another point, (1, 2).

Exercises

Graph the linear equation.

15. $\frac{1}{4}x + y = 3$

16. $-4x + 2y = 8$

17. $x + 5y = 10$

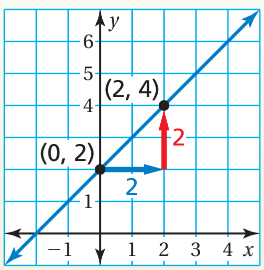
18. $-\frac{1}{2}x + \frac{1}{8}y = \frac{3}{4}$

19. A dog kennel charges \$30 per night to board your dog and \$6 for each hour of playtime. The amount of money you spend is given by $30x + 6y = 180$, where x is the number of nights and y is the number of hours of playtime. Graph the equation and interpret the intercepts.

4.6 Writing Equations in Slope-Intercept Form *(pp. 178–183)*

Write an equation of the line in slope-intercept form.

a.



Find the slope and the y-intercept.

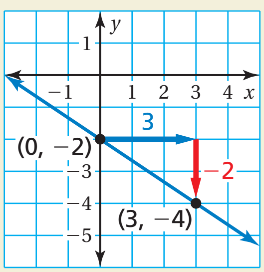
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{2 - 0} = \frac{2}{2}, \text{ or } 1$$

Because the line crosses the y-axis at (0, 2), the y-intercept is 2.

slope y-intercept

So, the equation is $y = 1x + 2$, or $y = x + 2$.

b.



Find the slope and the y-intercept.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-2)}{3 - 0} = \frac{-2}{3}, \text{ or } -\frac{2}{3}$$

Because the line crosses the y-axis at (0, −2), the y-intercept is −2.

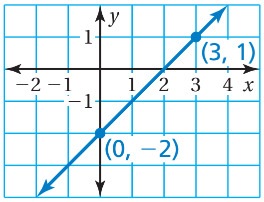
slope y-intercept

So, the equation is $y = -\frac{2}{3}x + (-2)$, or $y = -\frac{2}{3}x - 2$.

Exercises

Write an equation of the line in slope-intercept form.

20.



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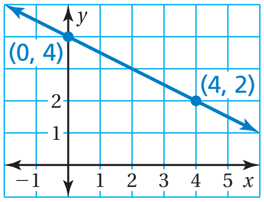
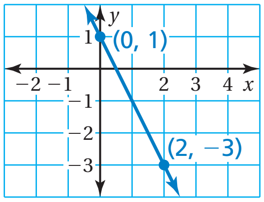
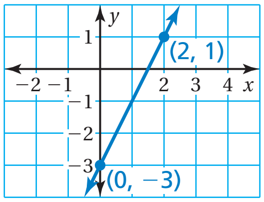


22.



23.

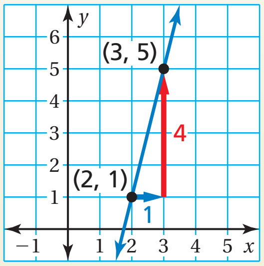


24. Write an equation of the line that passes through (0, 8) and (6, 8).

25. Write an equation of the line that passes through (0, −5) and (−5, −5).

4.7 Writing Equations in Point-Slope Form *(pp. 184–189)*

Write in slope-intercept form an equation of the line that passes through the points (2, 1) and (3, 5).



Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{3 - 2} = \frac{4}{1}, \text{ or } 4$$

Then use the slope and one of the given points to write an equation of the line.

Use $m = 4$ and $(2, 1)$.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 1 = 4(x - 2)$	Substitute 4 for m, 2 for x_1, and 1 for y_1.
$y - 1 = 4x - 8$	Distributive Property
$y = 4x - 7$	Write in slope-intercept form.

So, the equation is $y = 4x - 7$.

Exercises

26. Write in point-slope form an equation of the line that passes through the point (4, 4) with slope 3.

27. Write in slope-intercept form an equation of the line that passes through the points (−4, 2) and (6, −3).

4 Chapter Test

Check It Out
Test Practice
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Find the slope and the y-intercept of the graph of the linear equation.

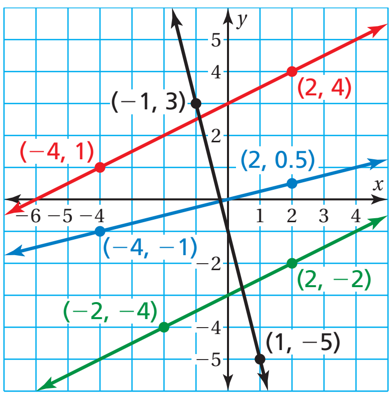
1. $y = 6x - 5$ **2.** $y = 20x + 15$ **3.** $y = -5x - 16$

4. $y - 1 = 3x + 8.4$ **5.** $y + 4.3 = 0.1x$ **6.** $-\frac{1}{2}x + 2y = 7$

Graph the linear equation.

7. $y = 2x + 4$ **8.** $y = -\frac{1}{2}x - 5$ **9.** $-3x + 6y = 12$

10. Which lines are parallel? Which lines are perpendicular? Explain.

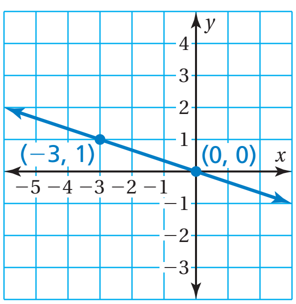


11. The points in the table lie on a line. Find the slope of the line.

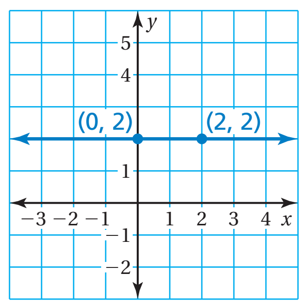
x	y
−1	−4
0	−1
1	2
2	5

Write an equation of the line in slope-intercept form.

12.



13.



Write in slope-intercept form an equation of the line that passes through the given points.

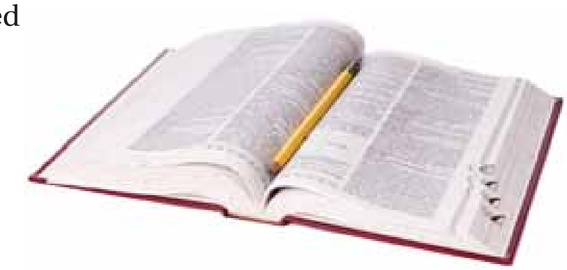
14. $(-1, 5), (3, -3)$ **15.** $(-4, 1), (4, 3)$ **16.** $(-2, 5), (-1, 1)$

17. VOCABULARY The number y of new vocabulary words that you learn after x weeks is represented by the equation $y = 15x$.

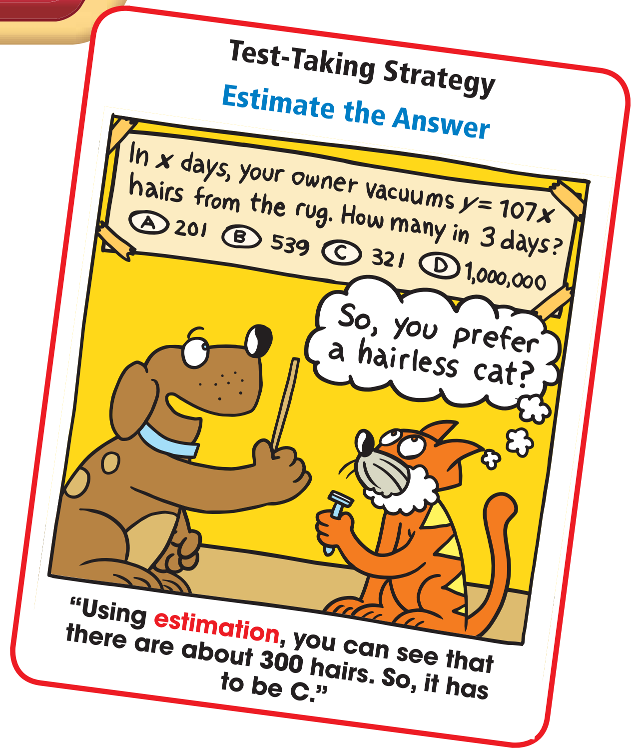
a. Graph the equation and interpret the slope.

b. How many new vocabulary words do you learn after 5 weeks?

c. How many more vocabulary words do you learn after 6 weeks than after 4 weeks?



4 Standards Assessment



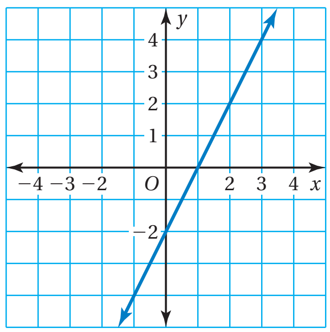
1. Which equation matches the line shown in the graph? *(8.EE.6)*

   A. $y = 2x - 2$

   B. $y = 2x + 1$

   C. $y = x - 2$

   D. $y = x + 1$



2. The equation $6x - 5y = 14$ is written in standard form. Which point lies on the graph of this equation? *(8.EE.6)*

   F. $(-4, -1)$

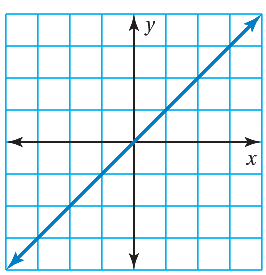
   G. $(-2, 4)$

   H. $(-1, -4)$

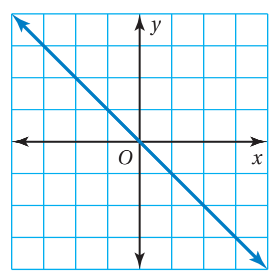
   I. $(4, -2)$

3. Which line has a slope of 0? *(8.EE.6)*

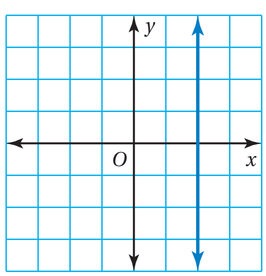
A.



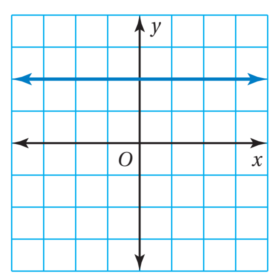
C.



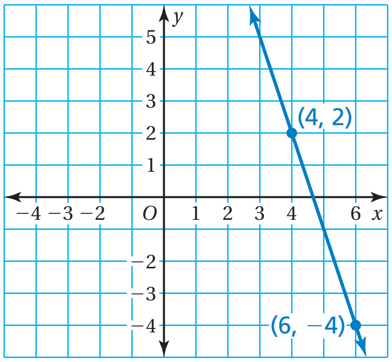
B.



D.



4. Which of the following is the equation of a line perpendicular to the line shown in the graph? *(8.EE.6)*



F. $y = 3x - 10$

G. $y = \frac{1}{3}x + 12$

H. $y = -3x + 5$

I. $y = -\frac{1}{3}x - 18$

5. What is the slope of the line that passes through the points $(2, -2)$ and $(8, 1)$? *(8.EE.6)*

6. A cell phone plan costs \$10 per month plus \$0.10 for each minute used. Last month, you spent \$18.50 using this plan. This can be modeled by the equation below, where m represents the number of minutes used.

$$0.1m + 10 = 18.5$$

How many minutes did you use last month? *(8.EE.7b)*

A. 8.4 min

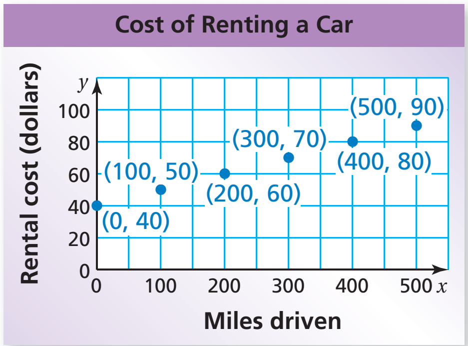
B. 85 min

C. 185 min

D. 285 min

7. It costs \$40 to rent a car for one day. In addition, the rental agency charges you for each mile driven, as shown in the graph. *(8.EE.6)*

Think Solve Explain



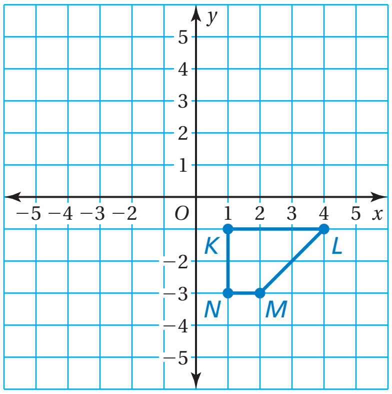
Part A Determine the slope of the line joining the points on the graph.

Part B Explain what the slope represents.

8. What value of x makes the equation below true? *(8.EE.7a)*

$$7 + 2x = 4x - 5$$

9. Trapezoid $KLMN$ is graphed in the coordinate plane shown.



Rotate Trapezoid $KLMN$ 90° clockwise about the origin. What are the coordinates of point M', the image of point M after the rotation? *(8.G.3)*

F. $(-3, -2)$ **H.** $(-2, 3)$

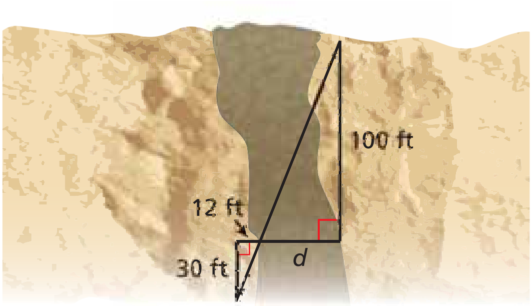
G. $(-2, -3)$ **I.** $(3, 2)$

10. Solve the formula $K = 3M - 7$ for M. *(8.EE.7b)*

A. $M = K + 7$ **C.** $M = \frac{K}{3} + 7$

B. $M = \frac{K + 7}{3}$ **D.** $M = \frac{K - 7}{3}$

11. What is the distance d across the canyon? *(8.G.5)*



F. 3.6 ft **H.** 40 ft

G. 12 ft **I.** 250 ft