3 Angles and Triangles

"Start with any triangle."

"Tear off the angles. You can always rearrange the angles so that they form a straight line."

"Well, there now, you've ruined a perfectly good triangle."

"What does that prove?"

"It proves that you just can't have anything nice."

"Maybe if I sit perfectly still he won't see me."

"Let's use shadows and similar triangles to indirectly measure the height of the giant hyena standing right behind you."

What You Learned Before

"I just remember that **C** comes before **S** and **90** comes before **180**. That makes it easy."

🔴 Adjacent and Vertical Angles (7.G.5)

Example 1 Tell whether the angles are *adjacent* or *vertical*. Then find the value of *x*.
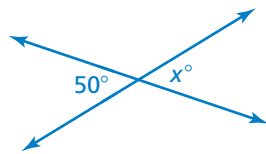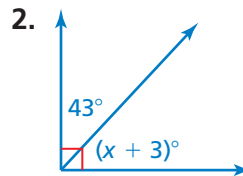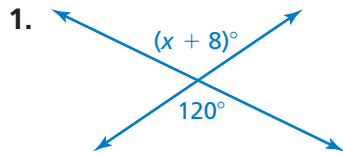

The angles are vertical angles. Because vertical angles are congruent, the angles have the same measure.
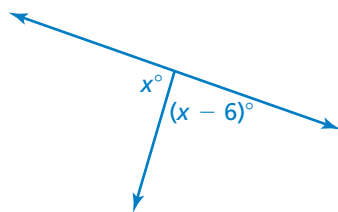
⋮ So, the value of *x* is 50.

Try It Yourself

Tell whether the angles are *adjacent* or *vertical*. Then find the value of *x*.

1.

$(x + 8)°$
$120°$

2.

$43°$
$(x + 3)°$

🔴 Complementary and Supplementary Angles (7.G.5)

Example 2 Tell whether the angles are *complementary* or *supplementary*. Then find the value of *x*.

$x°$ $(x − 6)°$

The two angles make up a straight angle. So, the angles are supplementary angles, and the sum of their measures is 180°.

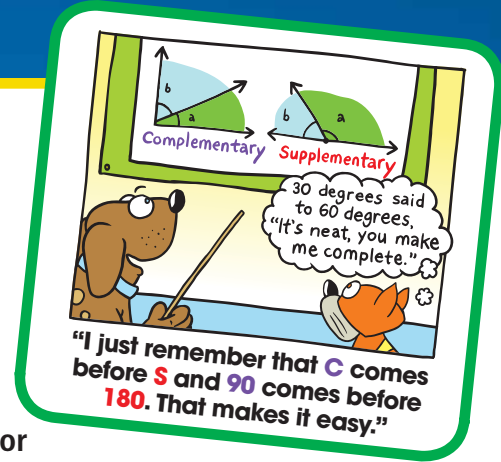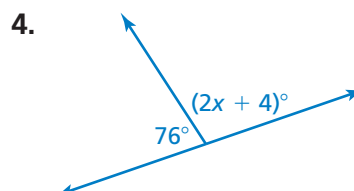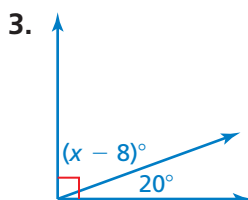|	
$x + (x − 6) = 180$	Write equation.
$2x − 6 = 180$	Combine like terms.
$2x = 186$	Add 6 to each side.
$x = 93$	Divide each side by 2.

Try It Yourself

Tell whether the angles are *complementary* or *supplementary*. Then find the value of *x*.

3.

$(x − 8)°$
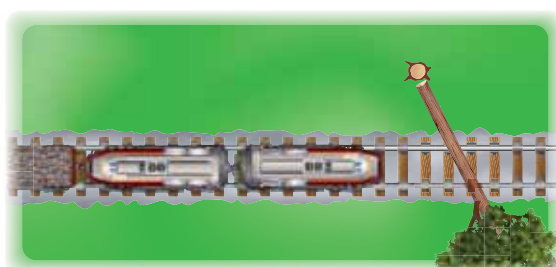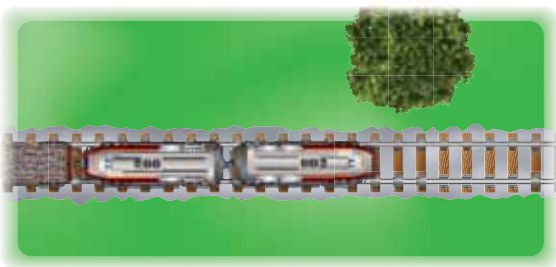$20°$

4.

$(2x + 4)°$
$76°$

3.1 Parallel Lines and Transversals

Essential Question How can you describe angles formed by parallel lines and transversals?

The Meaning of a Word ● Transverse

When an object is **transverse**,   it is lying or extending across something.

1 ACTIVITY: A Property of Parallel Lines

Work with a partner.

- **Discuss what it means for two lines to be parallel. Decide on a strategy for drawing two parallel lines. Then draw the two parallel lines.**

- **Draw a third line that intersects the two parallel lines. This line is called a transversal.**

parallel lines
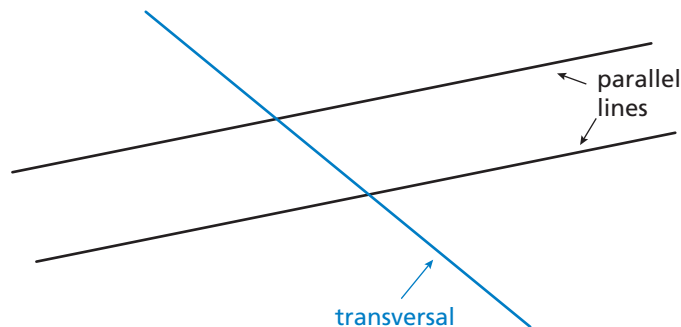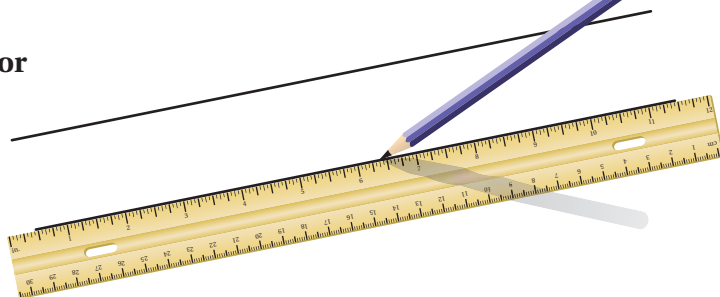
transversal

COMMON CORE

Geometry

In this lesson, you will

- identify the angles formed when parallel lines are cut by a transversal.
- find the measures of angles formed when parallel lines are cut by a transversal.

Learning Standard
8.G.5

a. How many angles are formed by the parallel lines and the transversal? Label the angles.

b. Which of these angles have equal measures? Explain your reasoning.

2 ACTIVITY: Creating Parallel Lines

Work with a partner.

a. If you were building the house in the photograph, how could you make sure that the studs are parallel to each other?

b. Identify sets of parallel lines and transversals in the photograph.

Studs

3 ACTIVITY: Using Technology

Work with a partner. Use geometry software to draw two parallel lines intersected by a transversal.

a. Find all the angle measures.

b. Adjust the figure by moving the parallel lines or the transversal to a different position. Describe how the angle measures and relationships change.
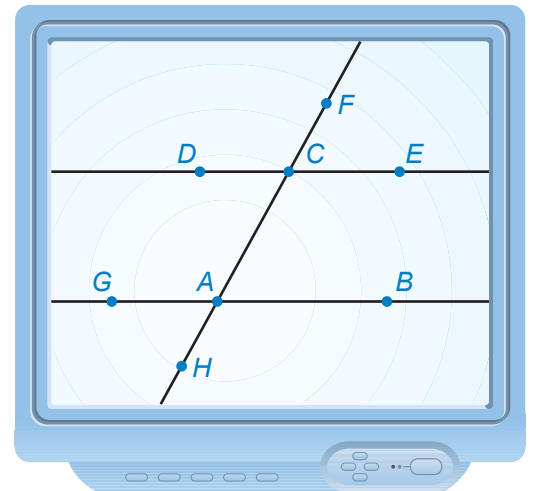
What Is Your Answer?

4. IN YOUR OWN WORDS How can you describe angles formed by parallel lines and transversals? Give an example.

5. Use geometry software to draw a transversal that is perpendicular to two parallel lines. What do you notice about the angles formed by the parallel lines and the transversal?

Practice

Use what you learned about parallel lines and transversals to complete Exercises 3–6 on page 107.

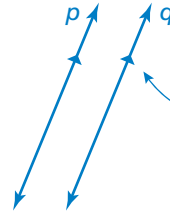# 3.1 Lesson

Key Vocabulary
transversal, *p. 104*
interior angles, *p. 105*
exterior angles, *p. 105*

Lines in the same plane that do not intersect are called *parallel lines*. Lines that intersect at right angles are called *perpendicular lines*.

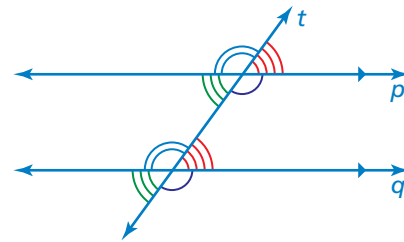Indicates lines ℓ and *m* are perpendicular.
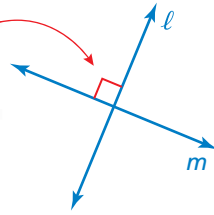
Indicates lines *p* and *q* are parallel.

A line that intersects two or more lines is called a **transversal**. When parallel lines are cut by a transversal, several pairs of congruent angles are formed.

Key Idea

Study Tip
Corresponding angles lie on the same side of the transversal in corresponding positions.

Corresponding Angles

When a transversal intersects parallel lines, corresponding angles are congruent.

Corresponding angles

EXAMPLE 1 Finding Angle Measures
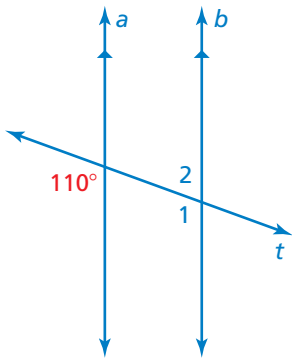
Use the figure to find the measures of (a) ∠1 and (b) ∠2.

a. ∠1 and the 110° angle are corresponding angles. They are congruent.

⋮ So, the measure of ∠1 is 110°.

b. ∠1 and ∠2 are supplementary.

$$\angle 1 + \angle 2 = 180°$$ Definition of supplementary angles

$$110° + \angle 2 = 180°$$ Substitute 110° for ∠1.

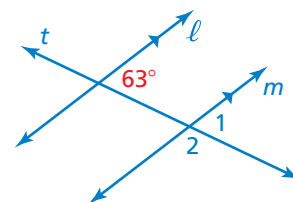$$\angle 2 = 70°$$ Subtract 110° from each side.

⋮ So, the measure of ∠2 is 70°.

On Your Own

Use the figure to find the measure of the angle. Explain your reasoning.

1. ∠1   **2.** ∠2

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 — Using Corresponding Angles
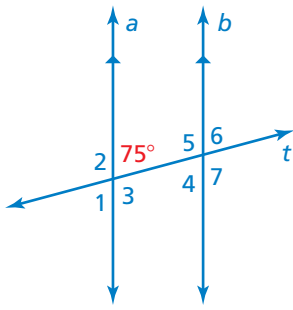
Use the figure to find the measures of the numbered angles.

∠1: ∠1 and the 75° angle are vertical angles. They are congruent.

∴ So, the measure of ∠1 is 75°.

∠2 and ∠3: The 75° angle is supplementary to both ∠2 and ∠3.

$$75° + ∠2 = 180°$$ Definition of supplementary angles

$$∠2 = 105°$$ Subtract 75° from each side.

∴ So, the measures of ∠2 and ∠3 are 105°.

∠4, ∠5, ∠6, and ∠7: Using corresponding angles, the measures of ∠4 and ∠6 are 75°, and the measures of ∠5 and ∠7 are 105°.
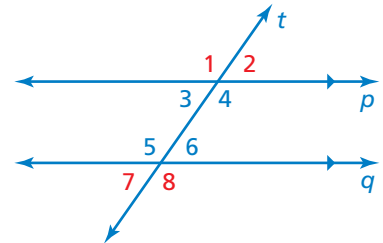
On Your Own

Now You're Ready
Exercises 15–17

3. Use the figure to find the measures of the numbered angles.
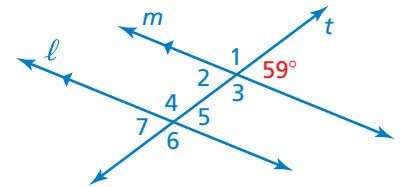
When two parallel lines are cut by a transversal, four **interior angles** are formed on the inside of the parallel lines and four **exterior angles** are formed on the outside of the parallel lines.

∠3, ∠4, ∠5, and ∠6 are interior angles.
∠1, ∠2, ∠7, and ∠8 are exterior angles.

EXAMPLE 3 — Using Corresponding Angles

A store owner uses pieces of tape to paint a window advertisement. The letters are slanted at an 80° angle. What is the measure of ∠1?

(A) 80°   (B) 100°   (C) 110°   (D) 120°

Because all the letters are slanted at an 80° angle, the dashed lines are parallel. The piece of tape is the transversal.

Using corresponding angles, the 80° angle is congruent to the angle that is supplementary to ∠1, as shown.

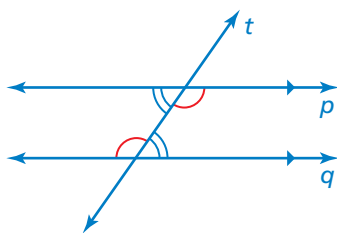∴ The measure of ∠1 is 180° − 80° = 100°. The correct answer is (B).

On Your Own

4. **WHAT IF?** In Example 3, the letters are slanted at a 65° angle. What is the measure of ∠1?

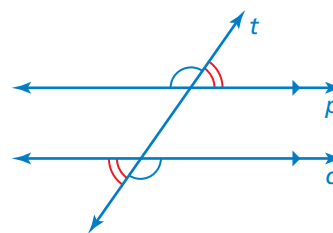## 🔑 Key Idea

Alternate Interior Angles and Alternate Exterior Angles

When a transversal intersects parallel lines, alternate interior angles are congruent and alternate exterior angles are congruent.

Study Tip

Alternate interior angles and alternate exterior angles lie on opposite sides of the transversal.

Alternate interior angles   **Alternate exterior angles**

EXAMPLE 4 Identifying Alternate Interior and Alternate Exterior Angles

The photo shows a portion of an airport. Describe the relationship between each pair of angles.

a. ∠3 and ∠6

∠3 and ∠6 are alternate exterior angles.

So, ∠3 is congruent to ∠6.

b. ∠2 and ∠7

∠2 and ∠7 are alternate interior angles.

So, ∠2 is congruent to ∠7.

On Your Own

In Example 4, the measure of ∠4 is 84°. Find the measure of the angle. Explain your reasoning.

5. ∠3   6. ∠5   7. ∠6

✔ Vocabulary and Concept Check

1. **VOCABULARY** Draw two parallel lines and a transversal. Label a pair of corresponding angles.

2. **WHICH ONE DOESN'T BELONG?** Which statement does *not* belong with the other three? Explain your reasoning. Refer to the figure for Exercises 3–6.

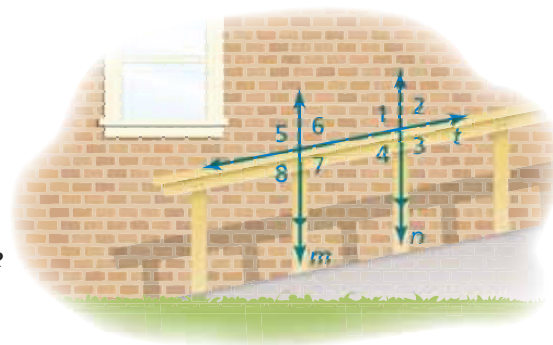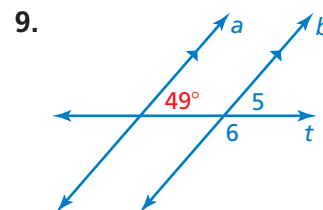 | | |
   |---|---|
   | The measure of ∠2 | The measure of ∠5 |
   | The measure of ∠6 | The measure of ∠8 |

✎ Practice and Problem Solving

In Exercises 3–6, use the figure.

3. Identify the parallel lines.

4. Identify the transversal.

5. How many angles are formed by the transversal?

6. Which of the angles are congruent?

Use the figure to find the measures of the numbered angles.

① 7.

107°

8.

95°

9.

49°
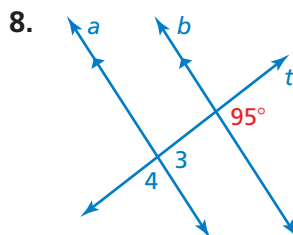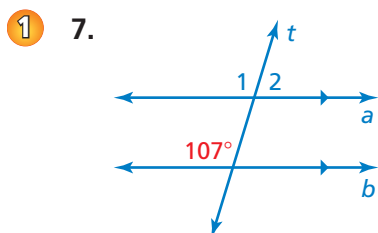
10. **ERROR ANALYSIS** Describe and correct the error in describing the relationship between the angles.

✗ ∠5 is congruent to ∠6.
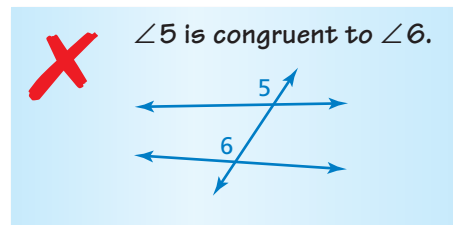
11. **PARKING** The painted lines that separate parking spaces are parallel. The measure of ∠1 is 60°. What is the measure of ∠2? Explain.

12. **OPEN-ENDED** Describe two real-life situations that use parallel lines.

13. **PROJECT** Trace line *p* and line *t* on a piece of paper. Label ∠1. Move the paper so that ∠1 aligns with ∠8. Describe the transformations that you used to show that ∠1 is congruent to ∠8.
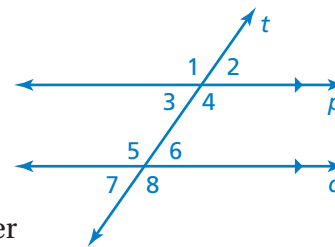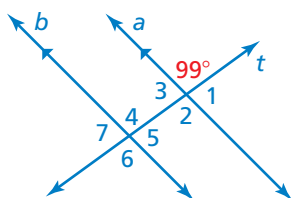
14. **REASONING** Two horizontal lines are cut by a transversal. What is the least number of angle measures you need to know in order to find the measure of every angle? Explain your reasoning.

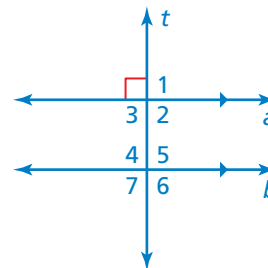**Use the figure to find the measures of the numbered angles. Explain your reasoning.**

② **15.**
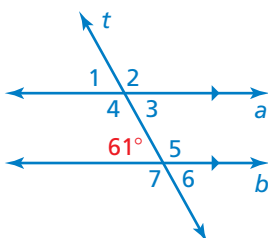
16.

17.

Complete the statement. Explain your reasoning.

③ **18.** If the measure of ∠1 = 124°, then the measure of ∠4 = ▢.

19. If the measure of ∠2 = 48°, then the measure of ∠3 = ▢.

④ **20.** If the measure of ∠4 = 55°, then the measure of ∠2 = ▢.

21. If the measure of ∠6 = 120°, then the measure of ∠8 = ▢.

22. If the measure of ∠7 = 50.5°, then the measure of ∠6 = ▢.

23. If the measure of ∠3 = 118.7°, then the measure of ∠2 = ▢.
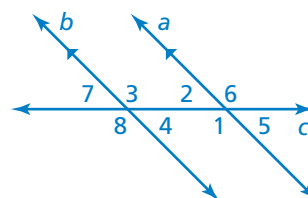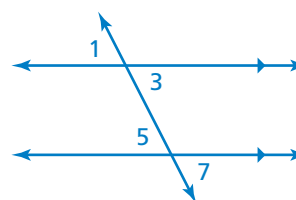
24. **RAINBOW** A rainbow forms when sunlight reflects off raindrops at different angles. For blue light, the measure of ∠2 is 40°. What is the measure of ∠1?
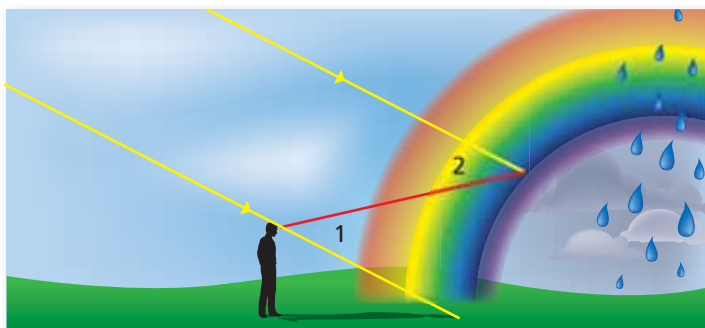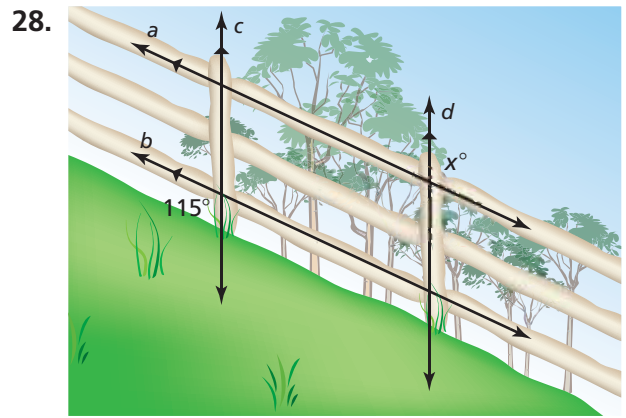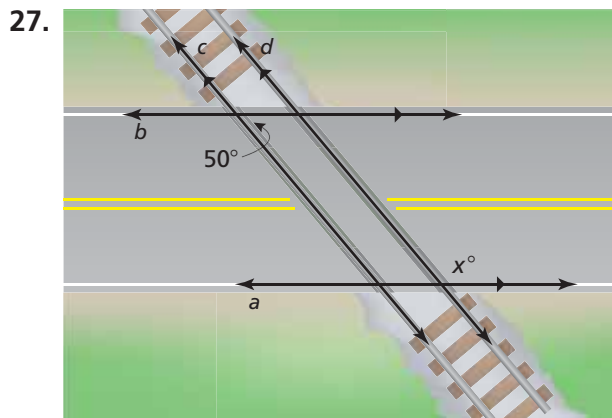
25. **REASONING** When a transversal is perpendicular to two parallel lines, all the angles formed measure 90°. Explain why.

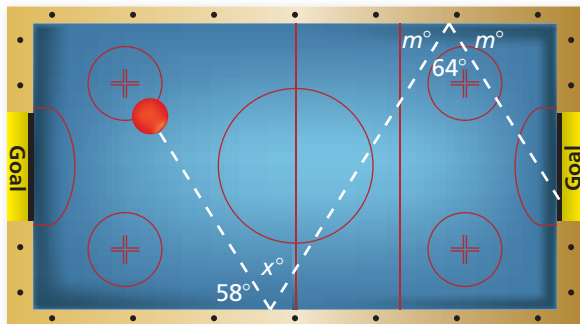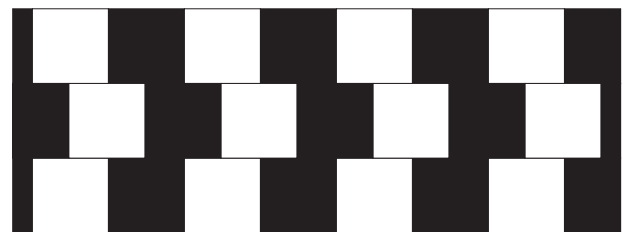26. **LOGIC** Describe two ways you can show that ∠1 is congruent to ∠7.

CRITICAL THINKING Find the value of *x*.

27.

28.

29. OPTICAL ILLUSION Refer to the figure.

a. Do the horizontal lines appear to be parallel? Explain.

b. Draw your own optical illusion using parallel lines.

30. **Geometry** The figure shows the angles used to make a double bank shot in an air hockey game.

a. Find the value of *x*.

b. Can you still get the red puck in the goal when *x* is increased by a little? by a lot? Explain.

Fair Game Review *What you learned in previous grades & lessons*

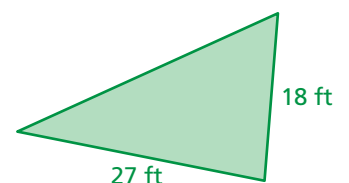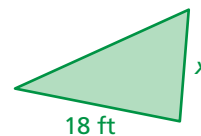**Evaluate the expression.** *(Skills Review Handbook)*

31. $4 + 3^2$

32. $5(2)^2 - 6$

33. $11 + (-7)^2 - 9$

34. $8 \div 2^2 + 1$

35. MULTIPLE CHOICE The triangles are similar. What length does *x* represent? *(Section 2.5)*

Ⓐ 2 ft

Ⓑ 12 ft

Ⓒ 15 ft

Ⓓ 27 ft

Essential Question How can you describe the relationships among the angles of a triangle?

1   ACTIVITY: Exploring the Interior Angles of a Triangle
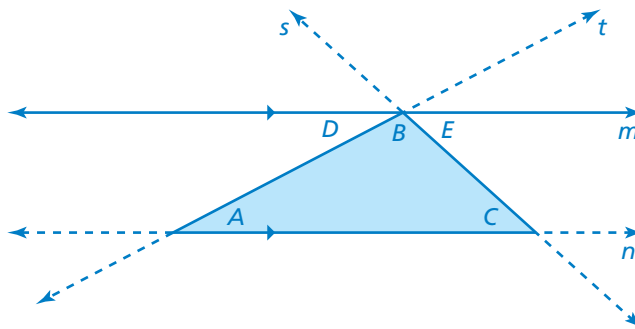
Work with a partner.

a. Draw a triangle. Label the interior angles *A*, *B*, and *C*.

b. Carefully cut out the triangle. Tear off the three corners of the triangle.

c. Arrange angles *A* and *B* so that they share a vertex and are adjacent.

d. How can you place the third angle to determine the sum of the measures of the interior angles? What is the sum?

e. Compare your results with those of others in your class.

f. **STRUCTURE** How does your result in part (d) compare to the rule you wrote in Lesson 1.1, Activity 2?
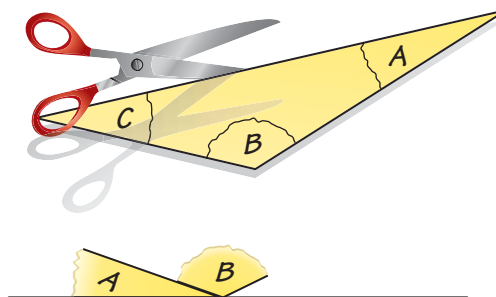
2   ACTIVITY: Exploring the Interior Angles of a Triangle

Work with a partner.

a. Describe the figure.

b. **LOGIC** Use what you know about parallel lines and transversals to justify your result in part (d) of Activity 1.

COMMON CORE

Geometry

In this lesson, you will
- understand that the sum of the interior angle measures of a triangle is 180°.
- find the measures of interior and exterior angles of triangles.

Learning Standard
8.G.5
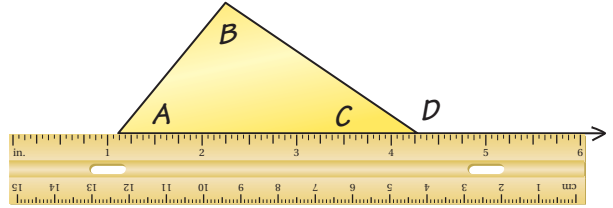
3 ACTIVITY: Exploring an Exterior Angle of a Triangle

Work with a partner.

a. Draw a triangle. Label the interior angles *A*, *B*, and *C*.

b. Carefully cut out the triangle.

c. Place the triangle on a piece of paper and extend one side to form *exterior angle D*, as shown.

d. Tear off the corners that are not adjacent to the exterior angle. Arrange them to fill the exterior angle, as shown. What does this tell you about the measure of exterior angle *D*?
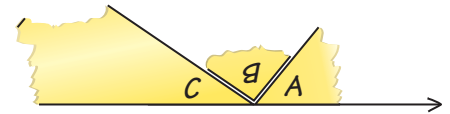
4 ACTIVITY: Measuring the Exterior Angles of a Triangle

Work with a partner.

a. Draw a triangle and label the interior and exterior angles, as shown.

b. Use a protractor to measure all six angles. Copy and complete the table to organize your results. What does the table tell you about the measure of an exterior angle of a triangle?
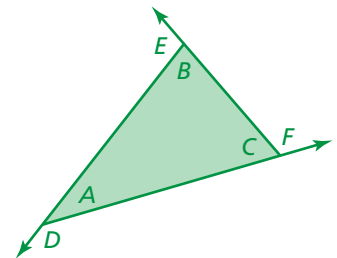
Exterior Angle	$D = $ °	$E = $ °	$F = $ °
Interior Angle	$B = $ °	$A = $ °	$A = $ °
Interior Angle	$C = $ °	$C = $ °	$B = $ °

What Is Your Answer?

5. **REPEATED REASONING** Draw three triangles that have different shapes. Repeat parts (b)–(d) from Activity 1 for each triangle. Do you get the same results? Explain.

6. **IN YOUR OWN WORDS** How can you describe the relationships among angles of a triangle?

Practice Use what you learned about angles of a triangle to complete Exercises 4–6 on page 114.

Key Vocabulary 🔊
interior angles of a
  polygon, *p. 112*
exterior angles of a
  polygon, *p. 112*

The angles inside a polygon are called **interior angles**. When the sides of a polygon are extended, other angles are formed. The angles outside the polygon that are adjacent to the interior angles are called **exterior angles**.
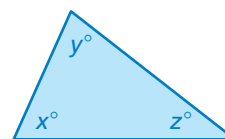
interior angles   exterior angles
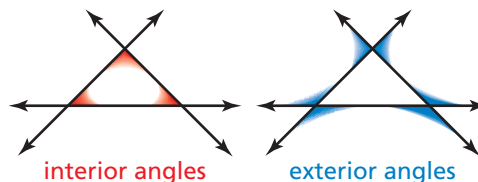
🔑 **Key Idea**

Interior Angle Measures of a Triangle

Words The sum of the interior angle measures of a triangle is 180°.

Algebra $x + y + z = 180$

EXAMPLE 1 **Using Interior Angle Measures**

Find the value of *x*.

a.
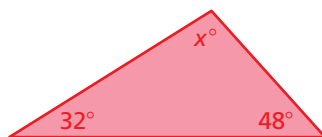
$x + 32 + 48 = 180$
$x + 80 = 180$
$x = 100$

b.
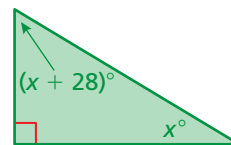
$x + (x + 28) + 90 = 180$
$2x + 118 = 180$
$2x = 62$
$x = 31$

🔴 **On Your Own**

Now You're Ready
Exercises 4–9

Find the value of *x*.

1.

81°
$x°$ 25°

2.

$x°$
$(x - 35)°$ 43°
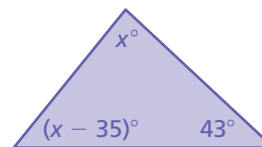
🔑 **Key Idea**

Exterior Angle Measures of a Triangle

Words The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.
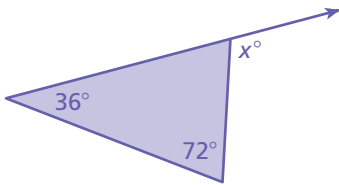
Algebra $z = x + y$

🔊 **Multi-Language Glossary at BigIdeasMath.com**
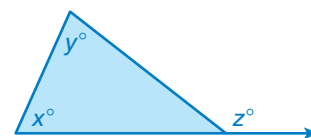
EXAMPLE 2 **Finding Exterior Angle Measures**
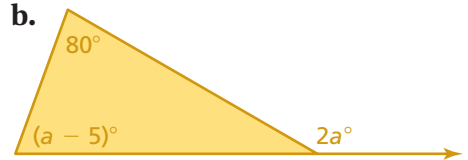
Find the measure of the exterior angle.

a.

$x = 36 + 72$

$x = 108$
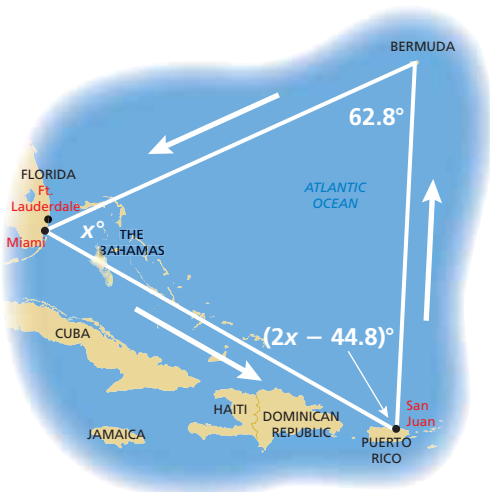
⋮ So, the measure of the exterior angle is 108°.

b.

$2a = (a - 5) + 80$

$2a = a + 75$

$a = 75$

⋮ So, the measure of the exterior angle is $2(75)° = 150°$.

EXAMPLE 3 **Real-Life Application**

An airplane leaves from Miami and travels around the Bermuda Triangle. What is the value of x?

(A) 26.8   (B) 27.2   (C) 54   (D) 64

Use what you know about the interior angle measures of a triangle to write an equation.

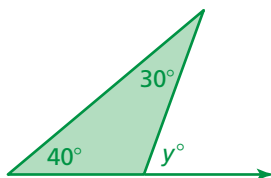|	
$x + (2x - 44.8) + 62.8 = 180$	Write equation.
$3x + 18 = 180$	Combine like terms.
$3x = 162$	Subtract 18 from each side.
$x = 54$	Divide each side by 3.

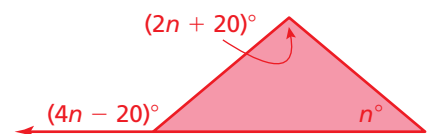⋮ The value of x is 54. The correct answer is (C).

On Your Own
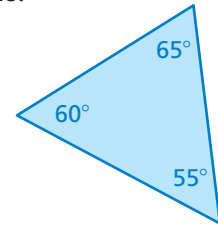
Find the measure of the exterior angle.

3.

4.

5. In Example 3, the airplane leaves from Fort Lauderdale. The interior angle measure at Bermuda is 63.9°. The interior angle measure at San Juan is $(x + 7.5)°$. Find the value of x.

✓ Vocabulary and Concept Check

1. **VOCABULARY** You know the measures of two interior angles of a triangle. How can you find the measure of the third interior angle?

2. **VOCABULARY** How many exterior angles does a triangle have at each vertex? Explain.

3. **NUMBER SENSE** List the measures of the exterior angles for the triangle shown at the right.
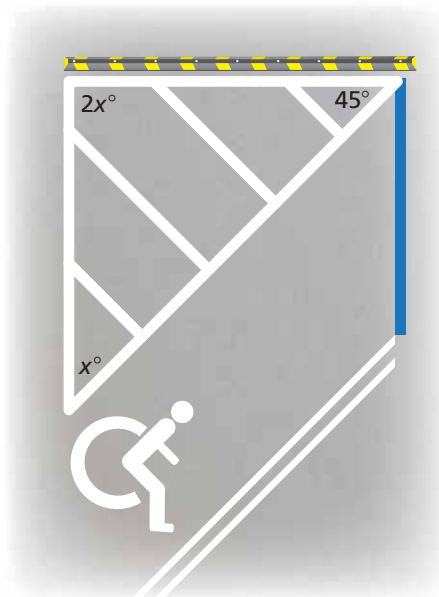
65°
60°
55°

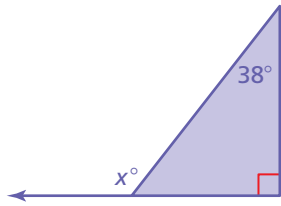## ✎ Practice and Problem Solving

Find the measures of the interior angles.

① 4.
30°
$x°$

5.
65°   40°
$x°$

6.
35° $x°$
45°

7.
$x°$ $(x + 65)°$
25°

8.
$x°$
48° $(x - 44)°$

9.
$x°$
$(x - 11)°$
73°
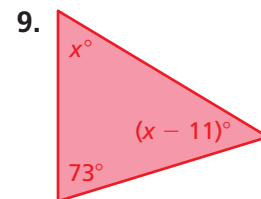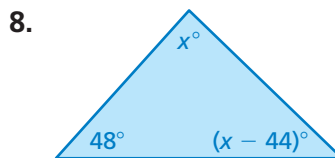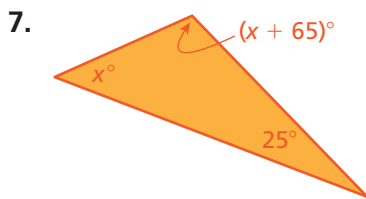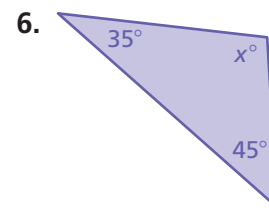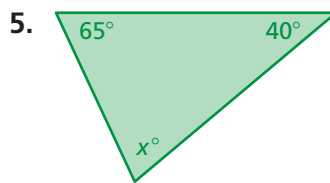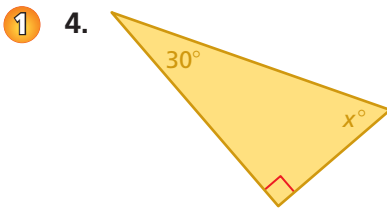
10. **BILLIARD RACK** Find the value of x in the billiard rack.
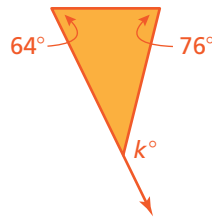
60°
$x°$ $x°$

2$x°$ 45°
$x°$

11. **NO PARKING** The triangle with lines through it designates a no parking zone. What is the value of x?
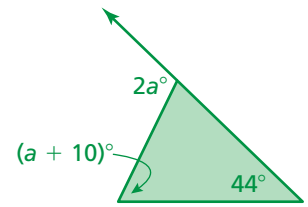
Find the measure of the exterior angle.

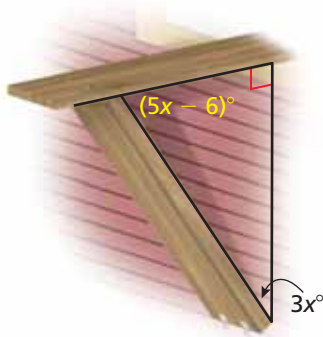**2** **12.**

13.

14.

15. ERROR ANALYSIS Describe and correct the error in finding the measure of the exterior angle.

✗

$$(2x - 12) + x + 30 = 180$$
$$3x + 18 = 180$$
$$x = 54$$

The exterior angle is $(2(54) - 12)° = 96°$.

16. RATIO The ratio of the interior angle measures of a triangle is $2:3:5$. What are the angle measures?

17. CONSTRUCTION The support for a window air-conditioning unit forms a triangle and an exterior angle. What is the measure of the exterior angle?

18. REASONING A triangle has an exterior angle with a measure of $120°$. Can you determine the measures of the interior angles? Explain.

Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

19. Given three angle measures, you can construct a triangle.

20. The acute interior angles of a right triangle are complementary.

21. A triangle has more than one vertex with an acute exterior angle.

22. **Precision** Using the figure at the right, show that $z = x + y$. (*Hint:* Find two equations involving w.)
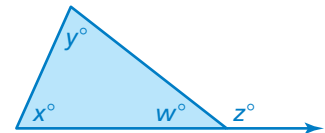
Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 1.2)*

23. $-4x + 3 = 19$

24. $2(y - 1) + 6y = -10$

25. $5 + 0.5(6n + 14) = 3$

26. MULTIPLE CHOICE Which transformation moves every point of a figure the same distance and in the same direction? *(Section 2.2)*

    Ⓐ translation   **Ⓑ** reflection   **Ⓒ** rotation   **Ⓓ** dilation

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **example and non-example chart** to list examples and non-examples of a vocabulary word or item. Here is an example and non-example chart for transversals.

Transversals

Examples	Non-Examples
p q r line p line q line r	a b c line a line b line c
a b c d line a line b line c line d	t p line p line t

On Your Own

Make example and non-example charts to help you study these topics.

1. interior angles formed by parallel lines and a transversal

2. exterior angles formed by parallel lines and a transversal

After you complete this chapter, make example and non-example charts for the following topics.

3. interior angles of a polygon

4. exterior angles of a polygon

5. regular polygons

6. similar triangles

Popular Cat Toys	
Example	Non-Example
Squeaky mouse	Water wings

Right on!
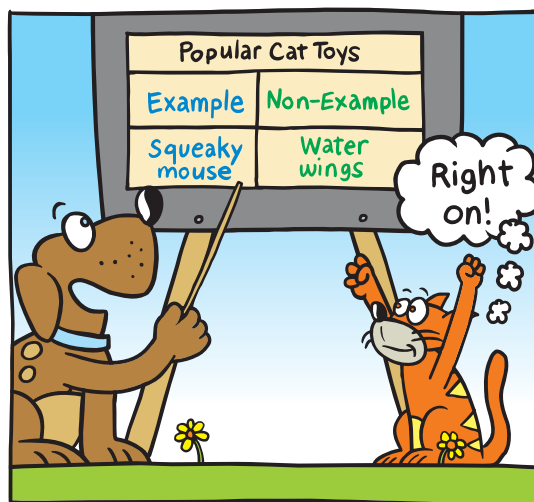
"What do you think of my example & non-example chart for popular cat toys?"
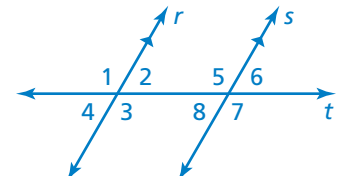
Use the figure to find the measure of the angle. Explain your reasoning. *(Section 3.1)*

1. $\angle 2$
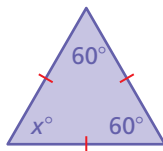
2. $\angle 6$

3. $\angle 4$

4. $\angle 1$
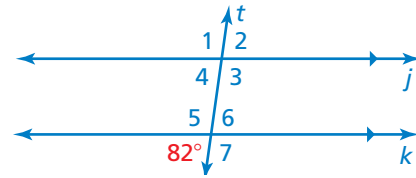
Complete the statement. Explain your reasoning. *(Section 3.1)*

5. If the measure of $\angle 1 = 123°$, then the measure of $\angle 7 = \boxed{}$.

6. If the measure of $\angle 2 = 58°$, then the measure of $\angle 5 = \boxed{}$.

7. If the measure of $\angle 5 = 119°$, then the measure of $\angle 3 = \boxed{}$.

8. If the measure of $\angle 4 = 60°$, then the measure of $\angle 6 = \boxed{}$.

Find the measures of the interior angles. *(Section 3.2)*

9.

10.
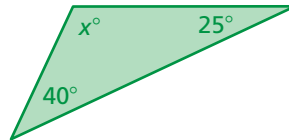
11.
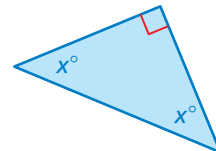
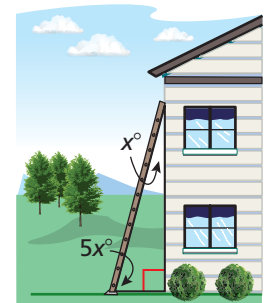**Find the measure of the exterior angle.** *(Section 3.2)*

12.
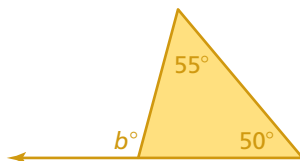
13.
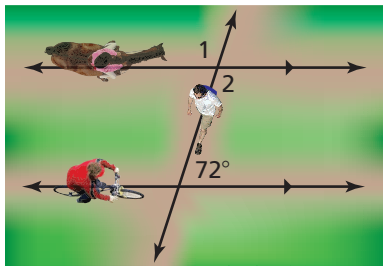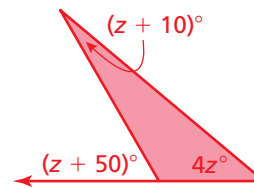
14. **PARK** In a park, a bike path and a horse riding path are parallel. In one part of the park, a hiking trail intersects the two paths. Find the measures of $\angle 1$ and $\angle 2$. Explain your reasoning. *(Section 3.1)*

15. **LADDER** A ladder leaning against a wall forms a triangle and exterior angles with the wall and the ground. What are the measures of the exterior angles? Justify your answer. *(Section 3.2)*

3.3 Angles of Polygons

Essential Question How can you find the sum of the interior angle measures and the sum of the exterior angle measures of a polygon?

### 1 ACTIVITY: Exploring the Interior Angles of a Polygon

Work with a partner. In parts (a)–(e), identify each polygon and the number of sides n. Then find the sum of the interior angle measures of the polygon.
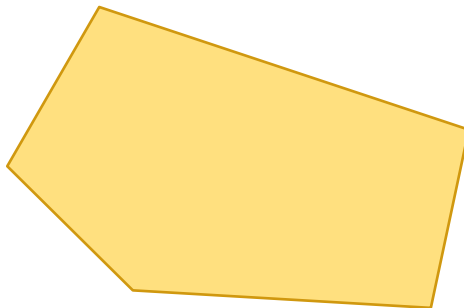
a. Polygon: ▢   Number of sides: $n =$ ▢

Draw a line segment on the figure that divides it into two triangles. Is there more than one way to do this? Explain.

What is the sum of the interior angle measures of each triangle?

What is the sum of the interior angle measures of the figure?
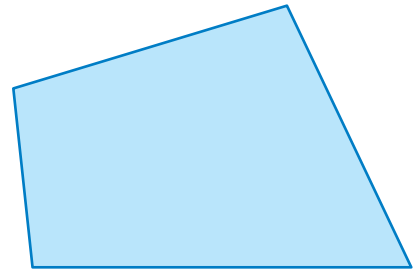
b.

c.
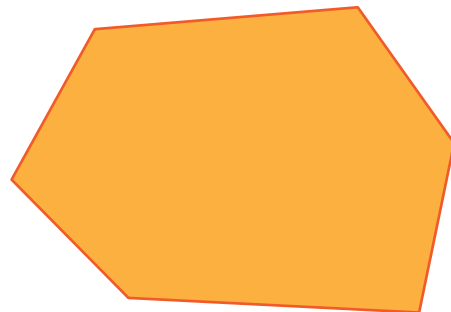
d.
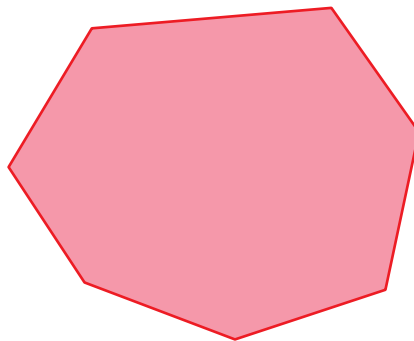
e.
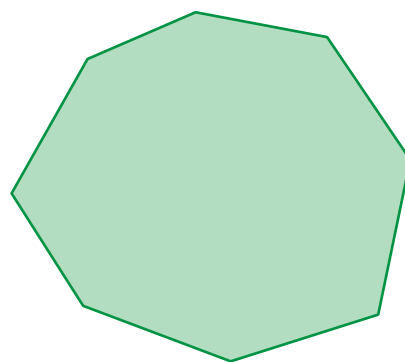
f. **REPEATED REASONING** Use your results to complete the table. Then find the sum of the interior angle measures of a polygon with 12 sides.

Number of Sides, n	3	4	5	6	7	8
Number of Triangles						
Angle Sum, S						

A polygon is **convex** when every line segment connecting any two vertices lies entirely inside the polygon. A polygon is **concave** when at least one line segment connecting any two vertices lies outside the polygon.

Convex

Concave
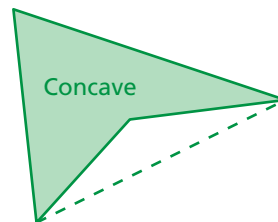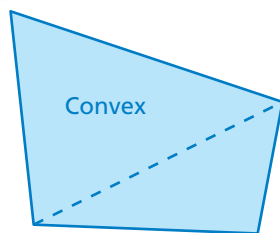
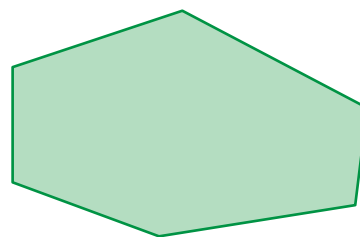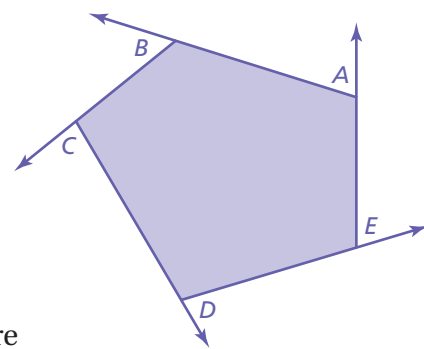## 2 ACTIVITY: Exploring the Exterior Angles of a Polygon

Work with a partner.

a. Draw a convex pentagon. Extend the sides to form the exterior angles. Label one exterior angle at each vertex *A*, *B*, *C*, *D*, and *E*, as shown.

b. Cut out the exterior angles. How can you join the vertices to determine the sum of the angle measures? What do you notice?

c. **REPEATED REASONING** Repeat the procedure in parts (a) and (b) for each figure below.

What can you conclude about the sum of the measures of the exterior angles of a convex polygon? Explain.

What Is Your Answer?

3. **STRUCTURE** Use your results from Activity 1 to write an expression that represents the sum of the interior angle measures of a polygon.

4. **IN YOUR OWN WORDS** How can you find the sum of the interior angle measures and the sum of the exterior angle measures of a polygon?
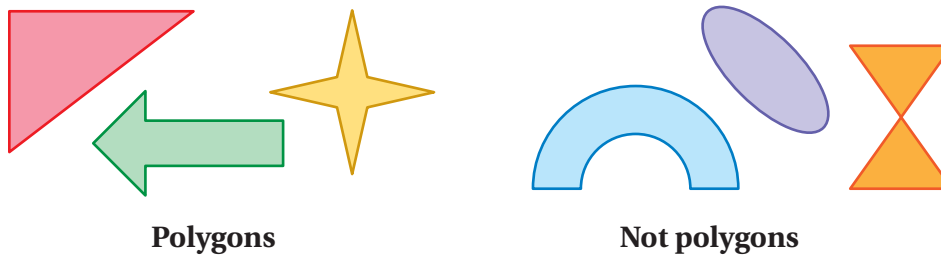
Practice

Use what you learned about angles of polygons to complete Exercises 4–6 on page 123.

3.3   Lesson

A *polygon* is a closed plane figure made up of three or more line segments that intersect only at their endpoints.

Polygons   **Not polygons**

Key Idea

Interior Angle Measures of a Polygon

The sum S of the interior angle measures of a polygon with n sides is

$$S = (n - 2) \cdot 180°.$$

EXAMPLE 1 Finding the Sum of Interior Angle Measures

Find the sum of the interior angle measures of the school crossing sign.

The sign is in the shape of a pentagon. It has 5 sides.

$S = (n - 2) \cdot 180°$ Write the formula.

$ = (5 - 2) \cdot 180°$ Substitute 5 for n.

$ = 3 \cdot 180°$ Subtract.

$ = 540°$ Multiply.

∴ The sum of the interior angle measures is 540°.

Reading

For polygons whose names you have not learned, you can use the phrase "n-gon," where n is the number of sides. For example, a 15-gon is a polygon with 15 sides.

On Your Own

Find the sum of the interior angle measures of the green polygon.

1.   2.
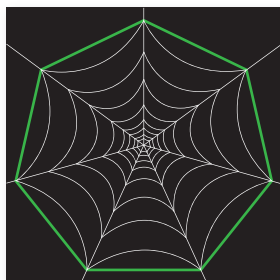
EXAMPLE 2   Finding an Interior Angle Measure of a Polygon
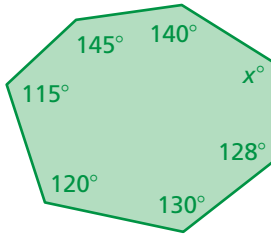
Find the value of x.

Step 1: The polygon has 7 sides. Find the sum of the interior angle measures.

$$S = (n - 2) \cdot 180°$$ Write the formula.

$$= (7 - 2) \cdot 180°$$ Substitute 7 for n.

$$= 900°$$ Simplify. The sum of the interior angle measures is 900°.

Step 2: Write and solve an equation.

$$140 + 145 + 115 + 120 + 130 + 128 + x = 900$$

$$778 + x = 900$$

$$x = 122$$

The value of x is 122.

On Your Own

Now You're Ready
Exercises 12–14

Find the value of x.

3.
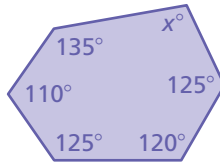
4.
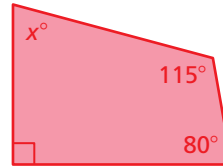
5.
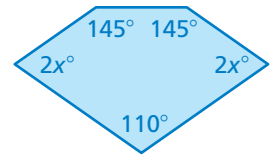
In a **regular polygon**, all the sides are congruent, and all the interior angles are congruent.

EXAMPLE 3   Real-Life Application

A cloud system discovered on Saturn is in the approximate shape of a regular hexagon. Find the measure of each interior angle of the hexagon.

Step 1: A hexagon has 6 sides. Find the sum of the interior angle measures.

$$S = (n - 2) \cdot 180°$$ Write the formula.

$$= (6 - 2) \cdot 180°$$ Substitute 6 for n.

$$= 720°$$ Simplify. The sum of the interior angle measures is 720°.
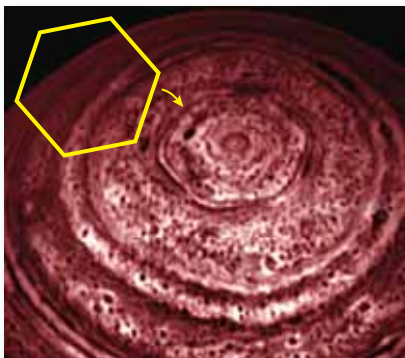
The hexagon is about 15,000 miles across. Approximately four Earths could fit inside it.

Step 2: Divide the sum by the number of interior angles, 6.

$$720° \div 6 = 120°$$

The measure of each interior angle is 120°.

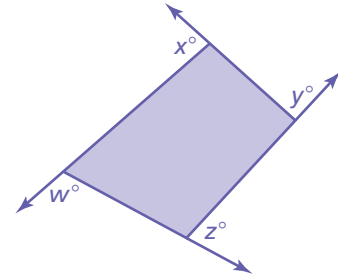**Find the measure of each interior angle of the regular polygon.**

6. octagon   7. decagon   8. 18-gon

Key Idea

Exterior Angle Measures of a Polygon

Words The sum of the measures of the exterior angles of a convex polygon is 360°.
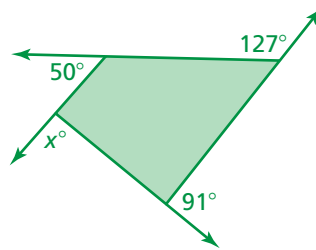
Algebra $w + x + y + z = 360$

EXAMPLE 4   Finding Exterior Angle Measures

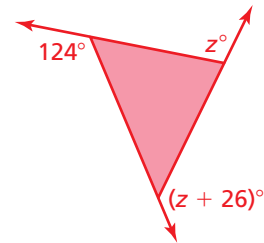**Find the measures of the exterior angles of each polygon.**

a.

Write and solve an equation for x.

$$x + 50 + 127 + 91 = 360$$
$$x + 268 = 360$$
$$x = 92$$

⁝ So, the measures of the exterior angles are 92°, 50°, 127°, and 91°.

b.

Write and solve an equation for z.

$$124 + z + (z + 26) = 360$$
$$2z + 150 = 360$$
$$z = 105$$

⁝ So, the measures of the exterior angles are 124°, 105°, and $(105 + 26)° = 131°$.
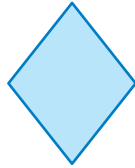
On Your Own

9. Find the measures of the exterior angles of the polygon.
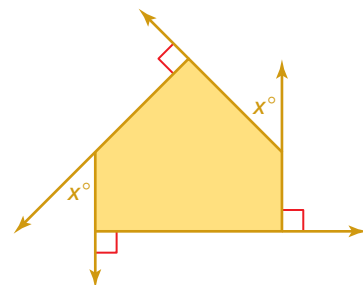
Vocabulary and Concept Check

1. **VOCABULARY** Draw a regular polygon that has three sides.

2. **WHICH ONE DOESN'T BELONG?** Which figure does *not* belong with the other three? Explain your reasoning.

3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the measure of an interior angle of a regular pentagon?	What is the sum of the interior angle measures of a convex pentagon?
What is the sum of the interior angle measures of a regular pentagon?	What is the sum of the interior angle measures of a concave pentagon?

Practice and Problem Solving

Use triangles to find the sum of the interior angle measures of the polygon.

4.
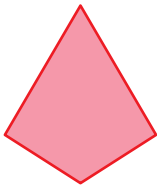
5.

6.

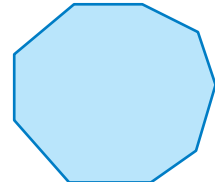**Find the sum of the interior angle measures of the polygon.**

① 7.

8.

9.
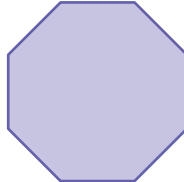
10. **ERROR ANALYSIS** Describe and correct the error in finding the sum of the interior angle measures of a 13-gon.
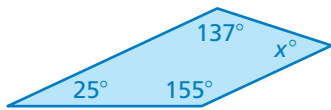
$$\begin{aligned}
S &= n \cdot 180° \\
&= 13 \cdot 180° \\
&= 2340°
\end{aligned}$$

11. **NUMBER SENSE** Can a pentagon have interior angles that measure 120°, 105°, 65°, 150°, and 95°? Explain.
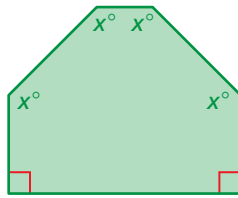
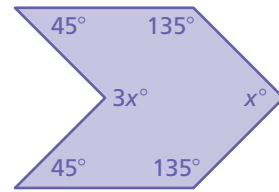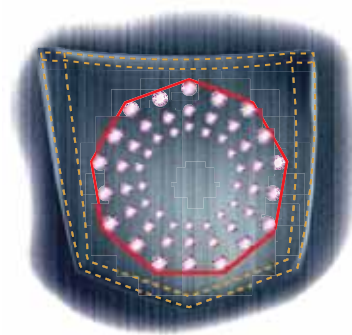**Find the measures of the interior angles.**

② 12.

13.

14.

15. REASONING The sum of the interior angle measures in a regular polygon is 1260°. What is the measure of one of the interior angles of the polygon?

Find the measure of each interior angle of the regular polygon.

③ 16.

17.

18.

19. ERROR ANALYSIS Describe and correct the error in finding the measure of each interior angle of a regular 20-gon.

$$S = (n - 2) \cdot 180°$$
$$= (20 - 2) \cdot 180°$$
$$= 18 \cdot 180°$$
$$= 3240°$$
$$3240° \div 18 = 180$$

The measure of each interior angle is 180°.

20. FIRE HYDRANT A fire hydrant bolt is in the shape of a regular pentagon.

   a. What is the measure of each interior angle?
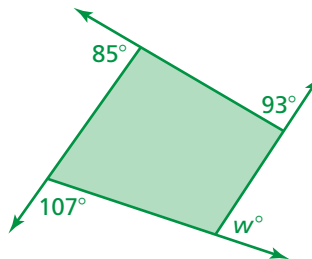
   b. Why are fire hydrants made this way?

21. PROBLEM SOLVING The interior angles of a regular polygon each measure 165°. How many sides does the polygon have?
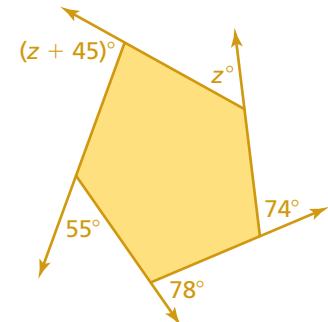
Find the measures of the exterior angles of the polygon.
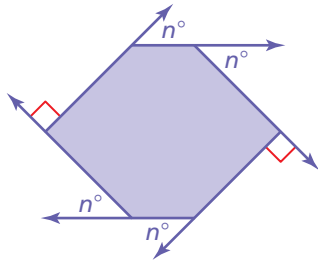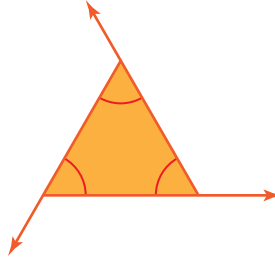
④ 22.
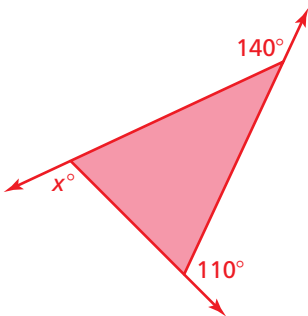
23.

24.

25. REASONING What is the measure of an exterior angle of a regular hexagon? Explain.

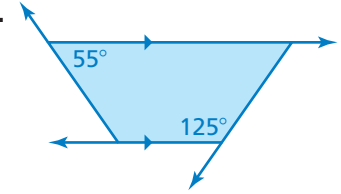**Find the measures of the exterior angles of the polygon.**

26.

27.

28.

29. **STAINED GLASS** The center of the stained glass window is in the shape of a regular polygon. What is the measure of each interior angle of the polygon? What is the measure of each exterior angle?

30. **PENTAGON** Draw a pentagon that has two right interior angles, two 45° interior angles, and one 270° interior angle.

31. **GAZEBO** The floor of a gazebo is in the shape of a heptagon. Four of the interior angles measure 135°. The other interior angles have equal measures. Find their measures.

32. **MONEY** The border of a Susan B. Anthony dollar is in the shape of a regular polygon.

   a. How many sides does the polygon have?

   b. What is the measure of each interior angle of the border? Round your answer to the nearest degree.

33. **Geometry** When tiles can be used to cover a floor with no empty spaces, the collection of tiles is called a *tessellation*.

   a. Create a tessellation using equilateral triangles.

   b. Find two more regular polygons that form tessellations.

   c. Create a tessellation that uses two different regular polygons.

   d. Use what you know about interior and exterior angles to explain why the polygons in part (c) form a tessellation.
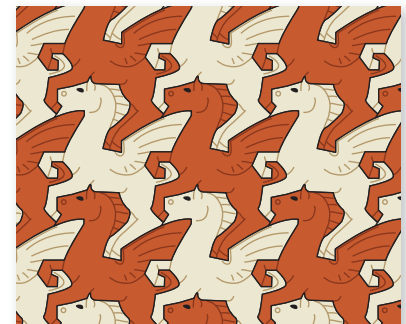
Fair Game Review *What you learned in previous grades & lessons*

Solve the proportion. *(Skills Review Handbook)*

34. $\dfrac{x}{12} = \dfrac{3}{4}$

35. $\dfrac{14}{21} = \dfrac{x}{3}$

36. $\dfrac{9}{x} = \dfrac{6}{2}$

37. $\dfrac{10}{4} = \dfrac{15}{x}$

38. **MULTIPLE CHOICE** The ratio of tulips to daisies is 3 : 5. Which of the following could be the total number of tulips and daisies? *(Skills Review Handbook)*

   Ⓐ 6   Ⓑ 10   Ⓒ 15   Ⓓ 16

3.4 Using Similar Triangles

Essential Question How can you use angles to tell whether triangles are similar?

## 1 ACTIVITY: Constructing Similar Triangles

Work with a partner.

- **Use a straightedge to draw a line segment that is 4 centimeters long.**
- **Then use the line segment and a protractor to draw a triangle that has a 60° and a 40° angle, as shown. Label the triangle *ABC*.**

60°   40°

4 cm
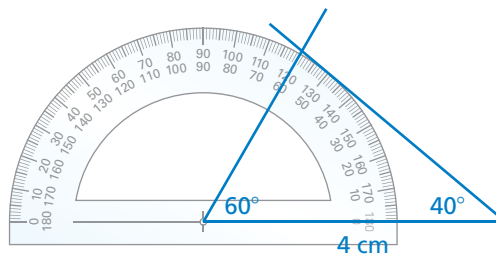
a. Explain how to draw a larger triangle that has the same two angle measures. Label the triangle *JKL*.

b. Explain how to draw a smaller triangle that has the same two angle measures. Label the triangle *PQR*.

c. Are all of the triangles similar? Explain.

2 ACTIVITY: Using Technology to Explore Triangles

Work with a partner. Use geometry software to draw the triangle below.

A

50°   30°

B   *C*
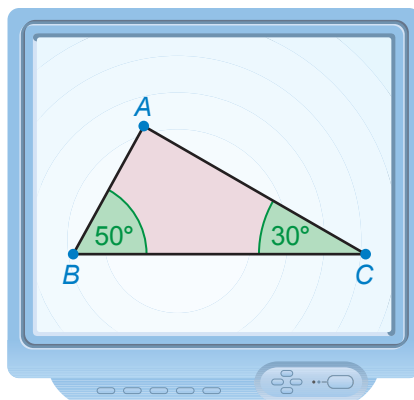
a. Dilate the triangle by the following scale factors.

$$2 \qquad \frac{1}{2} \qquad \frac{1}{4} \qquad 2.5$$

b. Measure the third angle in each triangle. What do you notice?

c. **REASONING** You have two triangles. Two angles in the first triangle are congruent to two angles in the second triangle. Can you conclude that the triangles are similar? Explain.

ACTIVITY: Indirect Measurement

Work with a partner.

a. Use the fact that two rays from the Sun are parallel to explain why $\triangle ABC$ and $\triangle DEF$ are similar.

b. Explain how to use similar triangles to find the height of the flagpole.

F

x ft

Sun's ray

C

Sun's ray

5 ft

A 3 ft B

D 36 ft E
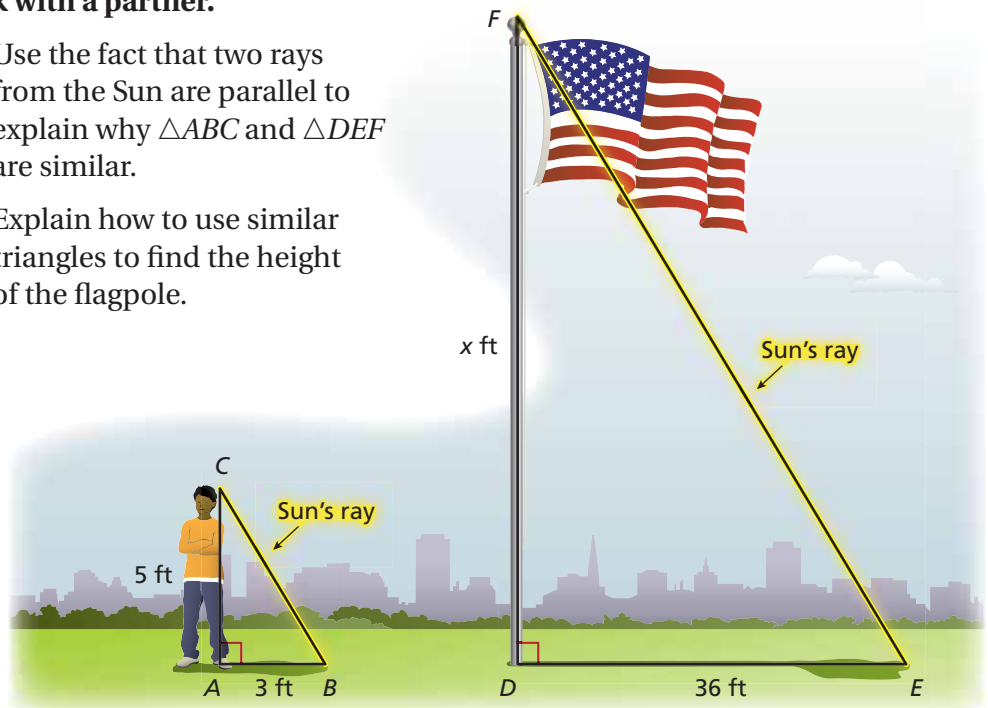
What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use angles to tell whether triangles are similar?

5. **PROJECT** Work with a partner or in a small group.

   a. Explain why the process in Activity 3 is called "indirect" measurement.

   b. **CHOOSE TOOLS** Use indirect measurement to measure the height of something outside your school (a tree, a building, a flagpole). Before going outside, decide what materials you need to take with you.

   c. **MODELING** Draw a diagram of the indirect measurement process you used. In the diagram, label the lengths that you actually measured and also the lengths that you calculated.

6. **PRECISION** Look back at Exercise 17 in Section 2.5. Explain how you can show that the two triangles are similar.

Practice

Use what you learned about similar triangles to complete Exercises 4 and 5 on page 130.

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
indirect measurement,
p. 129

Key Idea

Angles of Similar Triangles

Words When two angles in one triangle are congruent to two angles in another triangle, the third angles are also congruent and the triangles are similar.

Example
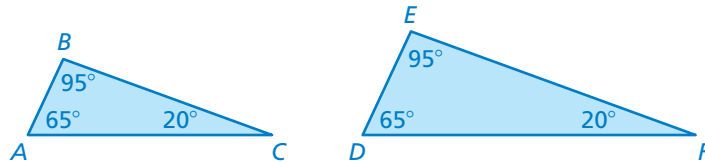
Triangle *ABC* is similar to Triangle *DEF*: $\triangle ABC \sim \triangle DEF$.

EXAMPLE **1** **Identifying Similar Triangles**

Tell whether the triangles are similar. Explain.
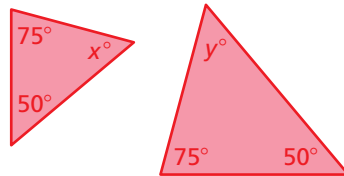
a.

The triangles have two pairs of congruent angles.

∴ So, the third angles are congruent, and the triangles are similar.

b.
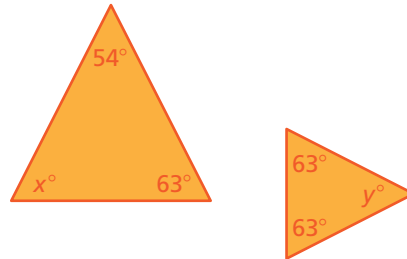
Write and solve an equation to find *x*.

$$x + 54 + 63 = 180$$
$$x + 117 = 180$$
$$x = 63$$

The triangles have two pairs of congruent angles.

∴ So, the third angles are congruent, and the triangles are similar.

c.
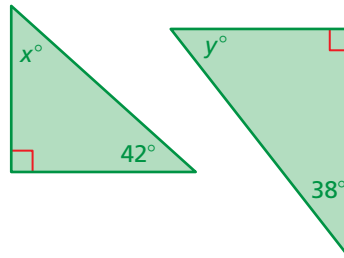
Write and solve an equation to find *x*.

$$x + 90 + 42 = 180$$
$$x + 132 = 180$$
$$x = 48$$

The triangles do not have two pairs of congruent angles.

∴ So, the triangles are not similar.

Multi-Language Glossary at BigIdeasMath.com

Now You're Ready
Exercises 6–9

Tell whether the triangles are similar. Explain.

1.

28°   80°
$x°$
28° $y°$
71°

2.

66°
$x°$
24°
$y°$
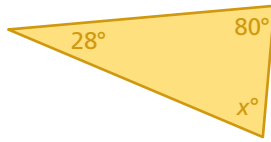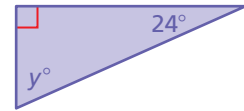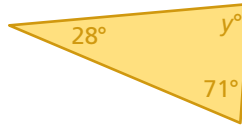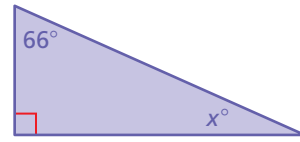
Indirect measurement uses similar figures to find a missing measure when it is difficult to find directly.

EXAMPLE **2** **Using Indirect Measurement**

A
60 ft
B
50 ft
C
40 ft
E
x
D
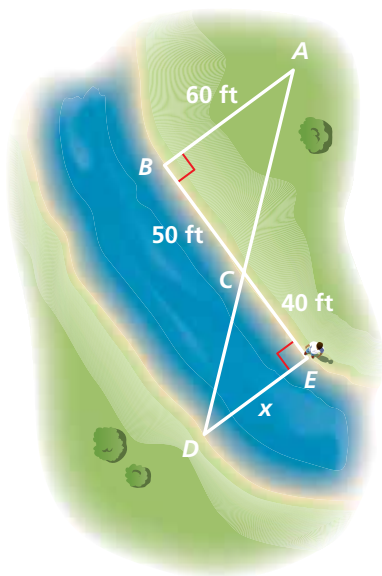
You plan to cross a river and want to know how far it is to the other side. You take measurements on your side of the river and make the drawing shown. **(a)** Explain why $\triangle ABC$ and $\triangle DEC$ are similar. **(b)** What is the distance x across the river?

a. $\angle B$ and $\angle E$ are right angles, so they are congruent. $\angle ACB$ and $\angle DCE$ are vertical angles, so they are congruent.

Because two angles in $\triangle ABC$ are congruent to two angles in $\triangle DEC$, the third angles are also congruent and the triangles are similar.

b. The ratios of the corresponding side lengths in similar triangles are equal. Write and solve a proportion to find x.

$$\frac{x}{60} = \frac{40}{50} \qquad \text{Write a proportion.}$$

$$60 \cdot \frac{x}{60} = 60 \cdot \frac{40}{50} \qquad \text{Multiplication Property of Equality}$$

$$x = 48 \qquad \text{Simplify.}$$

∴ So, the distance across the river is 48 feet.

On Your Own

Now You're Ready
Exercise 13

3. **WHAT IF?** The distance from vertex A to vertex B is 55 feet. What is the distance across the river?

✓ Vocabulary and Concept Check

1. **REASONING** How can you use similar triangles to find a missing measurement?

2. **WHICH ONE DOESN'T BELONG?** Which triangle does *not* belong with the other three? Explain your reasoning.
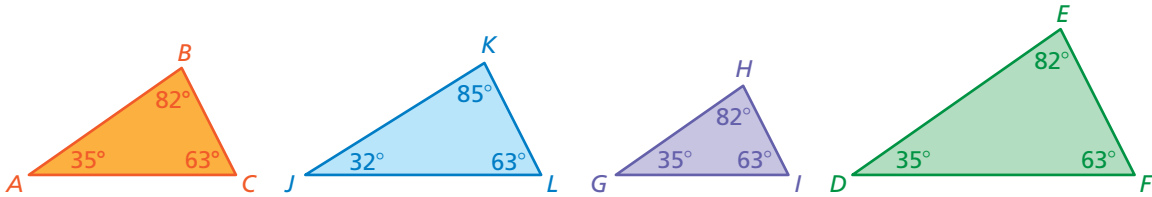

3. **WRITING** Two triangles have two pairs of congruent angles. In your own words, explain why you do not need to find the measures of the third pair of angles to determine that they are congruent.
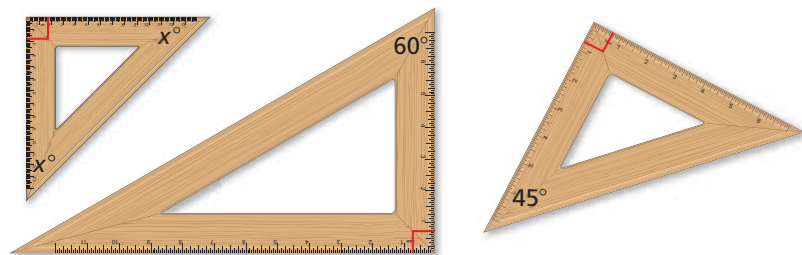
Practice and Problem Solving

Make a triangle that is larger or smaller than the one given and has the same angle measures. Find the ratios of the corresponding side lengths.

4.

5.

Tell whether the triangles are similar. Explain.

① 6.

7.

8.

9.
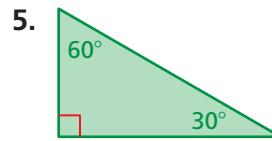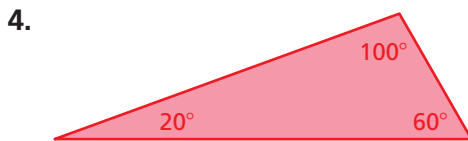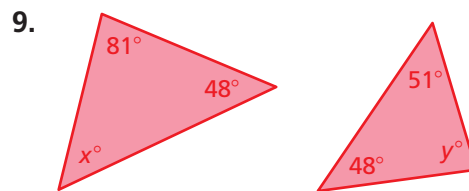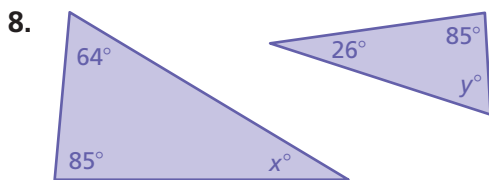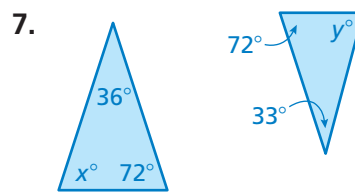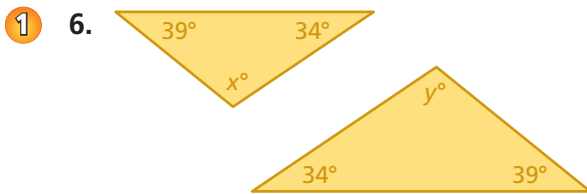
10. **RULERS** Which of the rulers are similar in shape? Explain.

Tell whether the triangles are similar. Explain.

11.

12.
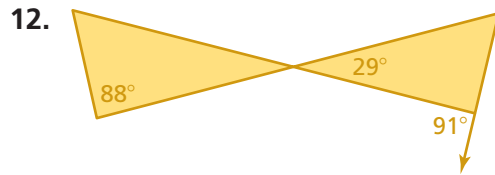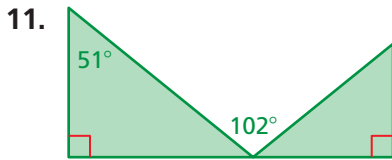
❷ 13. TREASURE The map shows the number of steps you must take to get to the treasure. However, the map is old, and the last dimension is unreadable. Explain why the triangles are similar. How many steps do you take from the pyramids to the treasure?
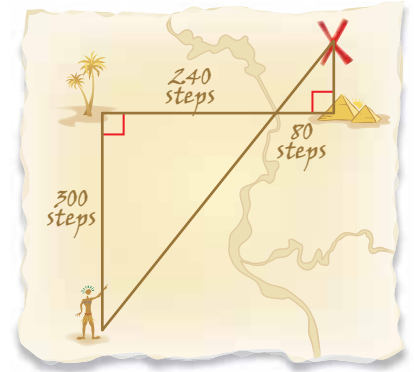
14. CRITICAL THINKING The side lengths of a triangle are increased by 50% to make a similar triangle. Does the area increase by 50% as well? Explain.
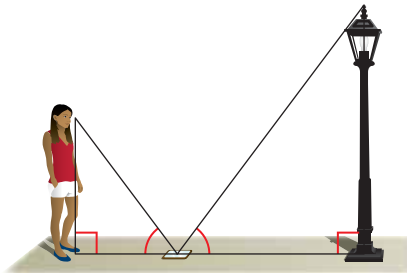

15. PINE TREE A person who is 6 feet tall casts a 3-foot-long shadow. A nearby pine tree casts a 15-foot-long shadow. What is the height h of the pine tree?

16. OPEN-ENDED You place a mirror on the ground 6 feet from the lamppost. You move back 3 feet and see the top of the lamppost in the mirror. What is the height of the lamppost?

17. REASONING In each of two right triangles, one angle measure is two times another angle measure. Are the triangles similar? Explain your reasoning.

18. Geometry In the diagram, segments *BG*, *CF*, and *DE* are parallel. The length of segment *BD* is 6.32 feet, and the length of segment *DE* is 6 feet. Name all pairs of similar triangles in the diagram. Then find the lengths of segments *BG* and *CF*.
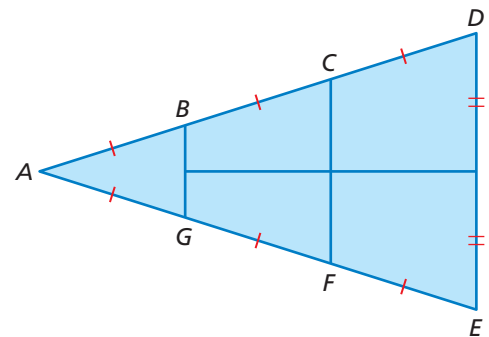
 Fair Game Review *What you learned in previous grades & lessons*

Solve the equation for y. *(Section 1.4)*

19. $y - 5x = 3$

20. $4x + 6y = 12$

21. $2x - \frac{1}{4}y = 1$

22. MULTIPLE CHOICE What is the value of x? *(Section 3.2)*

  Ⓐ 17

  Ⓑ 62

  Ⓒ 118

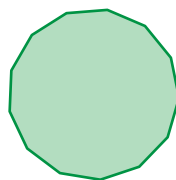 **Ⓓ** 152

Check It Out
Progress Check
BigIdeasMath.com
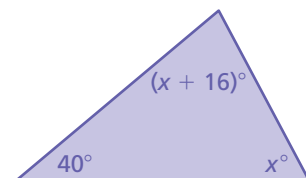
Find the sum of the interior angle measures of the polygon. *(Section 3.3)*
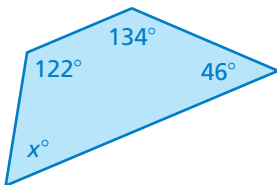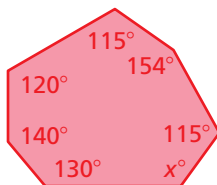
1.

2.

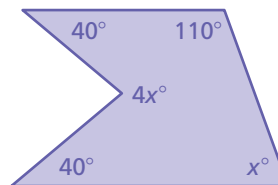**Find the measures of the interior angles of the polygon.** *(Section 3.3)*

3.

$134°$
$122°$ $46°$
$x°$

4.

$115°$
$154°$
$120°$
$140°$ $115°$
$130°$ $x°$

5.

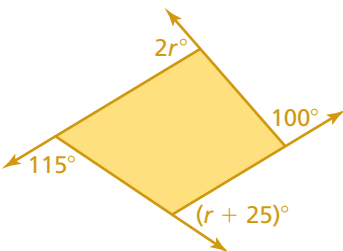$40°$ $110°$
$4x°$
$40°$ $x°$

Find the measures of the exterior angles of the polygon. *(Section 3.3)*

6.

$2r°$
$100°$
$115°$
$(r + 25)°$

7.

$x°$
$2x°$
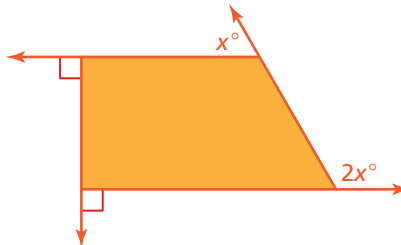
Tell whether the triangles are similar. Explain. *(Section 3.4)*

8.

$95°$ $x°$
$46°$
$46°$
$39°$ $y°$

9.

$51°$
$40°$ $79°$
$x°$ $40°$ $y°$
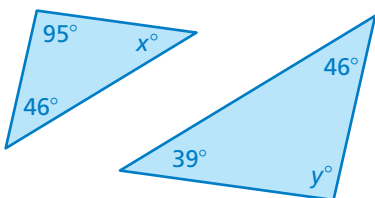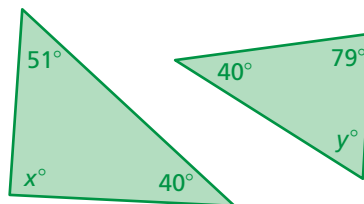
10. **REASONING** The sum of the interior angle measures of a polygon is 4140°. How many sides does the polygon have? *(Section 3.3)*

11. **SWAMP** You are trying to find the distance ℓ across a patch of swamp water. *(Section 3.4)*

    a. Explain why $\triangle VWX$ and $\triangle YZX$ are similar.

    b. What is the distance across the patch of swamp water?

V
100 ft
30 ft Z
W 60 ft X
ℓ
Y

3 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com
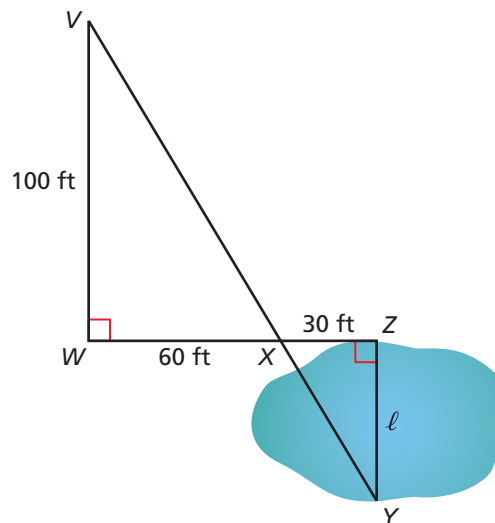
Review Key Vocabulary

transversal, *p. 104*
interior angles, *p. 105*
exterior angles, *p. 105*

interior angles of a polygon, *p. 112*
exterior angles of a polygon, *p. 112*

convex polygon, *p. 119*
concave polygon, *p. 119*
regular polygon, *p. 121*
indirect measurement, *p. 129*
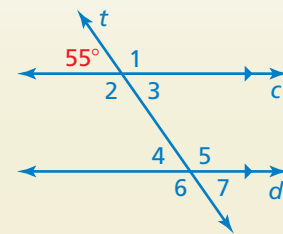
Review Examples and Exercises

3.1 Parallel Lines and Transversals *(pp. 102–109)*

Use the figure to find the measure of ∠6.

∠2 and the 55° angle are supplementary.
So, the measure of ∠2 is $180° - 55° = 125°$.

∠2 and ∠6 are corresponding angles.
They are congruent.

∴ So, the measure of ∠6 is 125°.

Exercises

Use the figure to find the measure of the angle. Explain your reasoning.

1. ∠8
2. ∠5
3. ∠7
4. ∠2
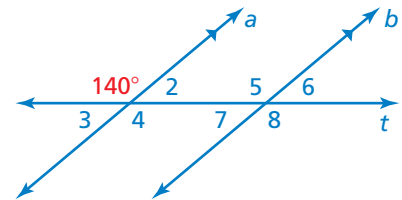
3.2 Angles of Triangles *(pp. 110–115)*

a. Find the value of x.
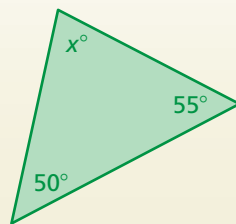
$$x + 50 + 55 = 180$$
$$x + 105 = 180$$
$$x = 75$$

∴ The value of x is 75.

b. Find the measure of the exterior angle.
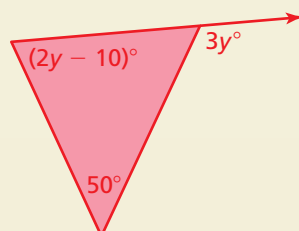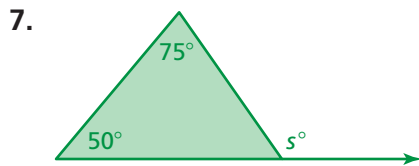
$$3y = (2y - 10) + 50$$
$$3y = 2y + 40$$
$$y = 40$$

∴ So, the measure of the exterior angle is $3(40)° = 120°$.

Exercises

Find the measures of the interior angles.

5.

6.

Find the measure of the exterior angle.
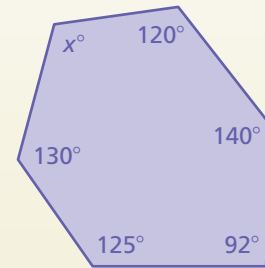
7.

8.
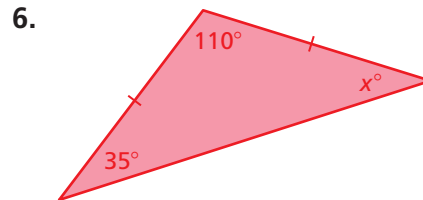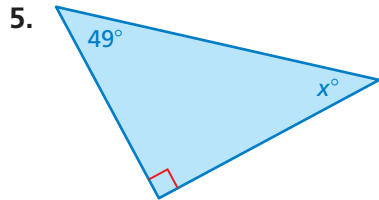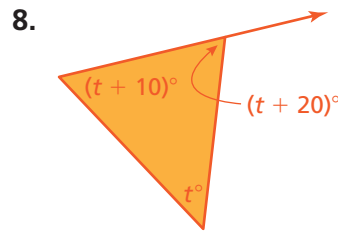
3.3 **Angles of Polygons** *(pp. 118–125)*

a. **Find the value of x.**

Step 1: The polygon has 6 sides. Find the sum of the interior angle measures.

$$S = (n - 2) \cdot 180°$$ Write the formula.

$$= (6 - 2) \cdot 180°$$ Substitute 6 for n.

$$= 720$$ Simplify. The sum of the interior angle measures is 720°.

Step 2: Write and solve an equation.

$$130 + 125 + 92 + 140 + 120 + x = 720$$

$$607 + x = 720$$

$$x = 113$$

∴ The value of x is 113.

b. **Find the measures of the exterior angles of the polygon.**
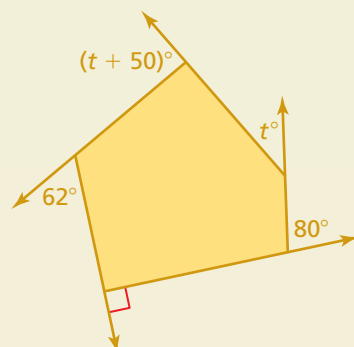
Write and solve an equation for t.

$$t + 80 + 90 + 62 + (t + 50) = 360$$

$$2t + 282 = 360$$

$$2t = 78$$

$$t = 39$$

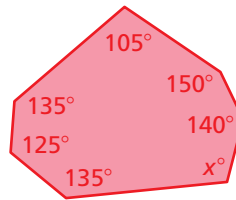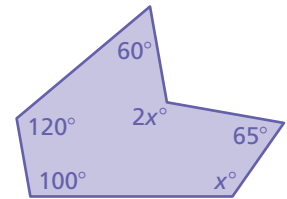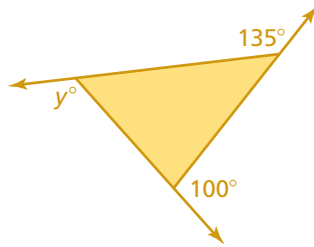∴ So, the measures of the exterior angles are 39°, 80°, 90°, 62°, and $(39 + 50)° = 89°$.

Exercises

Find the measures of the interior angles of the polygon.

9.

128° 95°

60° $x°$

10.

105°
150°
135°
140°
125°
135° $x°$
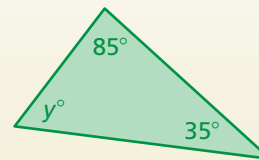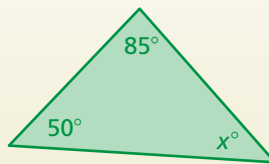
11.

60°
120° $2x°$
65°
100° $x°$
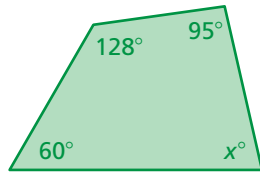
Find the measures of the exterior angles of the polygon.

12.

135°
$y°$
100°

13.

$z°$
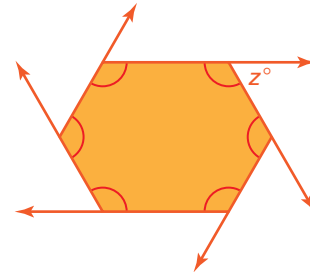
3.4 **Using Similar Triangles** *(pp. 126–131)*

Tell whether the triangles are similar. Explain.

85°
50° $x°$

85°
$y°$ 35°

Write and solve an equation to find x.

$$50 + 85 + x = 180$$
$$135 + x = 180$$
$$x = 45$$

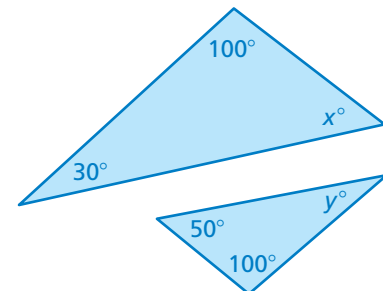∴ The triangles do not have two pairs of congruent angles. So, the triangles are not similar.

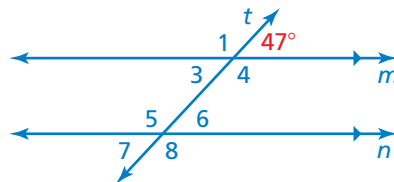## Exercises

Tell whether the triangles are similar. Explain.

14.

68°
$x°$
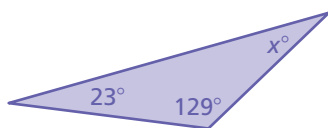22°
$y°$

15.

100°
$x°$
30°
50°
$y°$
100°
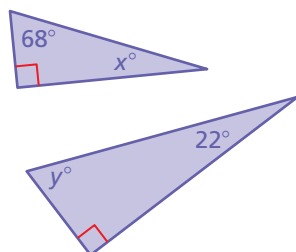
Use the figure to find the measure of the angle. Explain your reasoning.

   1. $\angle 1$ **2.** $\angle 8$
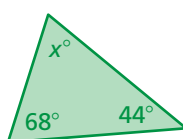
   3. $\angle 4$ **4.** $\angle 5$
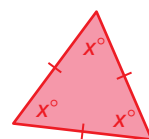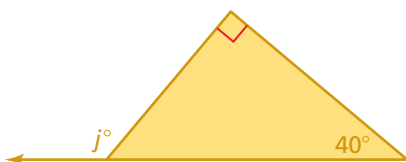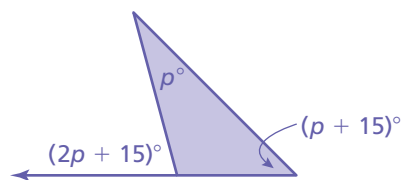
Find the measures of the interior angles.

  5.   **6.**   **7.**

Find the measure of the exterior angle.
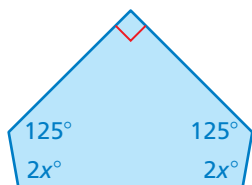
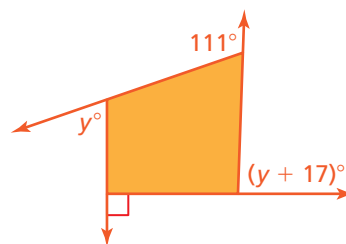 **8.**   **9.**

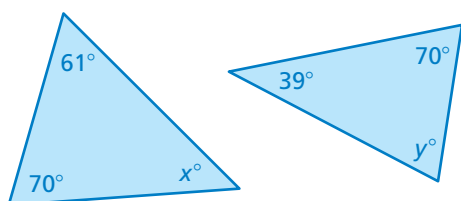**10.** Find the measures of the interior angles of the polygon.

11. Find the measures of the exterior angles of the polygon.

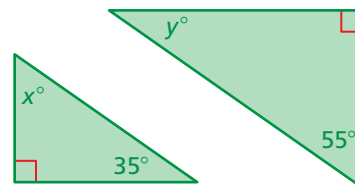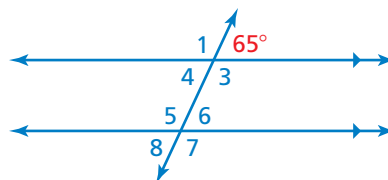**Tell whether the triangles are similar. Explain.**
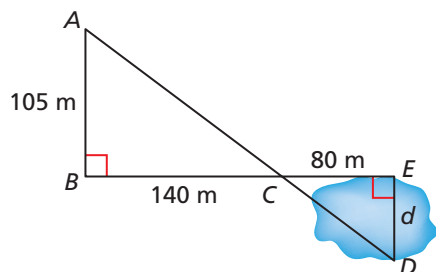
 12.   **13.**

14. **WRITING** Describe two ways you can find the measure of $\angle 5$.

15. **POND** Use the given measurements to find the distance d across the pond.

1. The border of a Canadian one-dollar coin is shaped like an 11-sided regular polygon. The shape was chosen to help visually impaired people identify the coin. How many degrees are in each angle along the border? Round your answer to the nearest degree. *(8.G.5)*

2. A public utility charges its residential customers for natural gas based on the number of therms used each month. The formula below shows how the monthly cost C in dollars is related to the number t of therms used.

$$C = 11 + 1.6t$$

Solve this formula for t. *(8.EE.7b)*

A. $t = \dfrac{C}{12.6}$

B. $t = \dfrac{C - 11}{1.6}$

C. $t = \dfrac{C}{1.6} - 11$

D. $t = C - 12.6$

3. What is the value of x? *(8.EE.7b)*
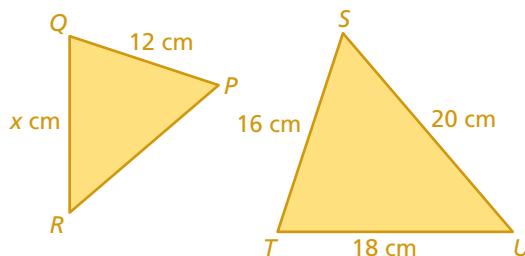
$$5(x - 4) = 3x$$

F. -10

G. 2

H. $2\dfrac{1}{2}$

I. 10

4. In the figures below, $\triangle PQR$ is a dilation of $\triangle STU$.

What is the value of x? *(8.G.4)*

A. 9.6

B. $10\dfrac{2}{3}$

C. 13.5

D. 15

5. What is the value of x?   *(8.G.5)*

**Hiking Trails**

Oak Trail   125°

Pine Trail $x°$

Ash Trail

6. Olga was solving an equation in the box shown.

$$-\frac{2}{5}(10x - 15) = -30$$

$$10x - 15 = -30\left(-\frac{2}{5}\right)$$

$$10x - 15 = 12$$

$$10x - 15 + 15 = 12 + 15$$

$$10x = 27$$

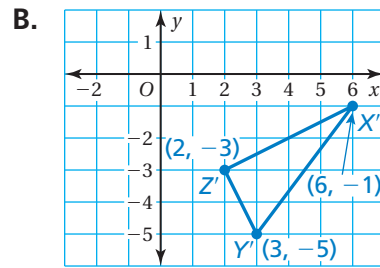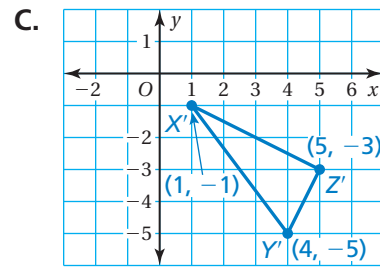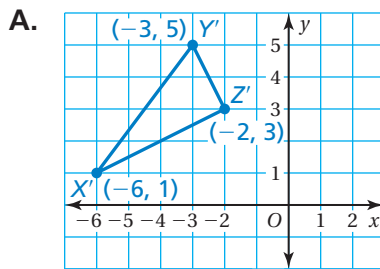$$\frac{10x}{10} = \frac{27}{10}$$

$$x = \frac{27}{10}$$

What should Olga do to correct the error that she made?   *(8.EE.7b)*

F. Multiply both sides by $-\frac{5}{2}$ instead of $-\frac{2}{5}$.

G. Multiply both sides by $\frac{2}{5}$ instead of $-\frac{2}{5}$.

H. Distribute $-\frac{2}{5}$ to get $-4x - 6$.

I. Add 15 to -30.

7. In the coordinate plane below, $\triangle XYZ$ is plotted and its vertices are labeled.
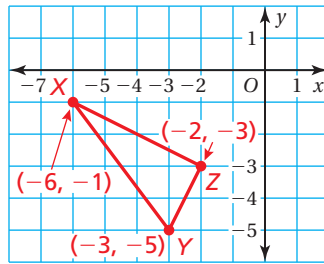
Which of the following shows $\triangle X'Y'Z'$, the image of $\triangle XYZ$ after it is reflected in the y-axis?   *(8.G.3)*

A.

C.

B.

D.
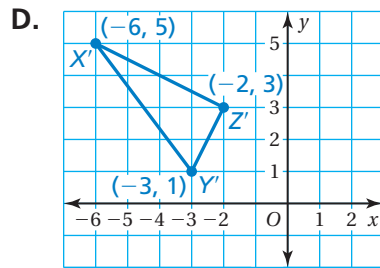
8. The sum S of the interior angle measures of a polygon with n sides can be found byusing a formula.   *(8.G.5)*

Think
Solve
Explain

Part A Write the formula.

Part B A quadrilateral has angles measuring 100°, 90°, and 90°. Find the measure of its fourth angle. Show your work and explain your reasoning.

Part C The sum of the measures of the angles of the pentagon shown is 540°. Divide the pentagon into triangles to show why this must be true. Show your work and explain your reasoning.