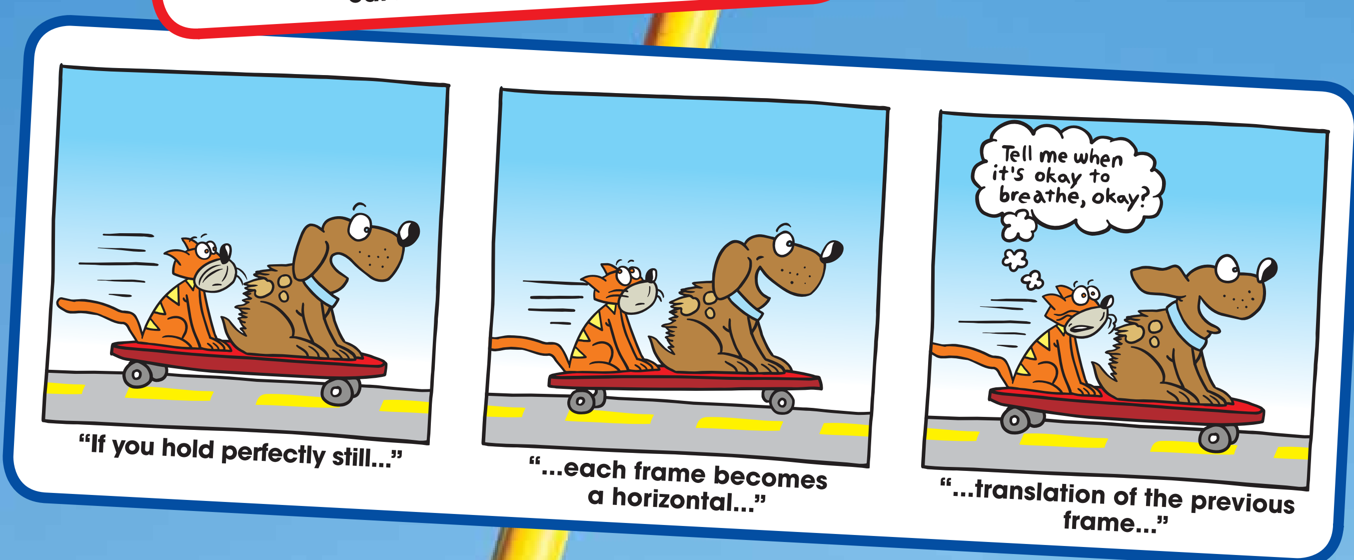
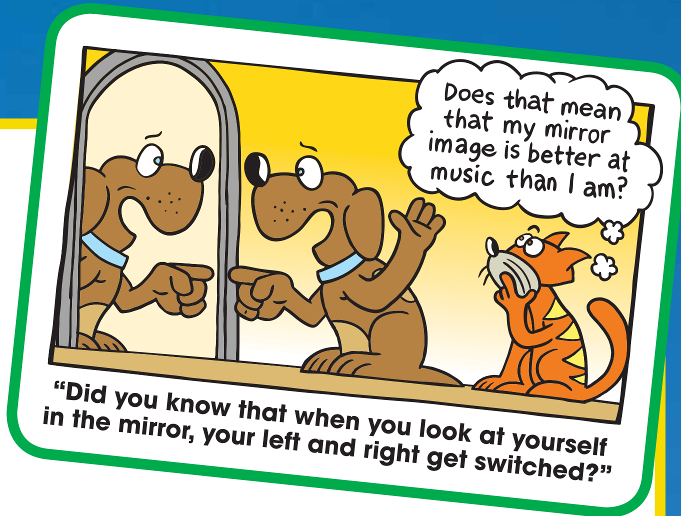
2 Transformations

"Just 2 more minutes. I'm almost done with my 'cat tessellation' painting."



"If you hold perfectly still..." "...each frame becomes a horizontal..." "...translation of the previous frame..."

What You Learned Before

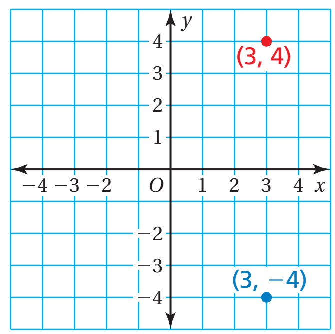


Reflecting Points (6.NS.6b)

Example 1 **Reflect (3, −4) in the *x*-axis.**

Plot (3, −4).

To reflect (3, −4) in the x-axis, use the same x-coordinate, 3, and take the opposite of the y-coordinate. The opposite of −4 is 4.



So, the reflection of (3, −4) in the x-axis is (3, 4).

Try It Yourself

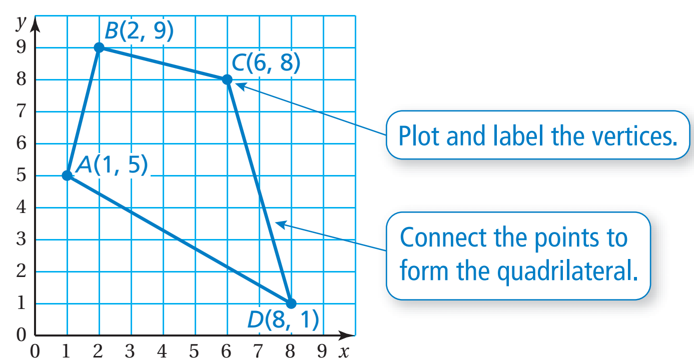
Reflect the point in (a) the *x*-axis and (b) the *y*-axis.

1. (7, 3)   **2.** (−4, 6)   **3.** (5, −5)   **4.** (−8, −3)

5. (0, 1)   **6.** (−5, 0)   **7.** (4, −6.5)   **8.** $\left(-3\frac{1}{2}, -4\right)$

Drawing a Polygon in a Coordinate Plane (6.G.3)

Example 2 **The vertices of a quadrilateral are *A*(1, 5), *B*(2, 9), *C*(6, 8), and *D*(8, 1). Draw the quadrilateral in a coordinate plane.**



Try It Yourself

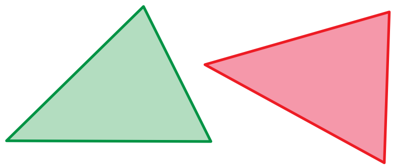
Draw the polygon with the given vertices in a coordinate plane.

9. $J(1, 1)$, $K(5, 6)$, $M(9, 3)$ **10.** $Q(2, 3)$, $R(2, 8)$, $S(7, 8)$, $T(7, 3)$

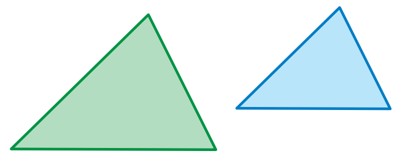
2.1 Congruent Figures

Essential Question How can you identify congruent triangles?

Two figures are congruent when they have the same size and the same shape.



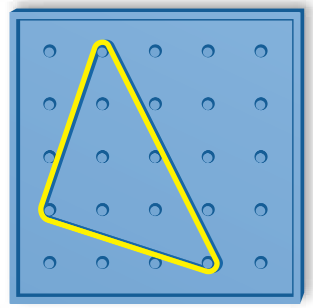
Congruent
Same size *and* shape



Not Congruent
Same shape, but not same size

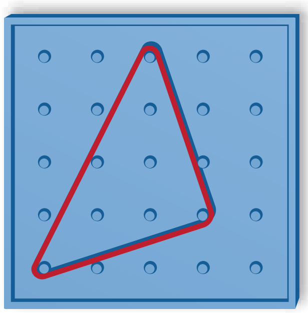
1 ACTIVITY: Identifying Congruent Triangles

Work with a partner.

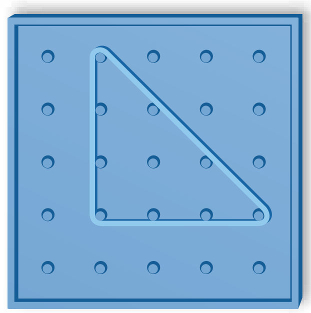


- **Which of the geoboard triangles below are congruent to the geoboard triangle at the right?**
- **Form each triangle on a geoboard.**
- **Measure each side with a ruler. Record your results in a table.**
- **Write a conclusion about the side lengths of triangles that are congruent.**

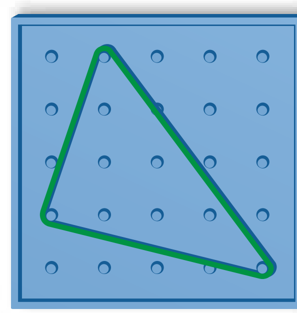
a.



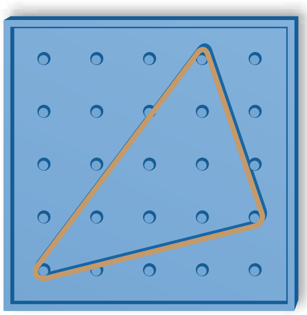
b.



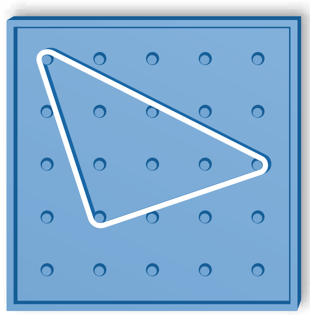
c.



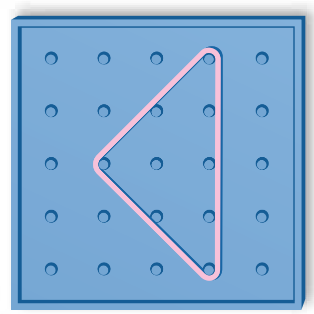
d.



e.



f.



Common Core
Geometry
In this lesson, you will
- name corresponding angles and corresponding sides of congruent figures.
- identify congruent figures.

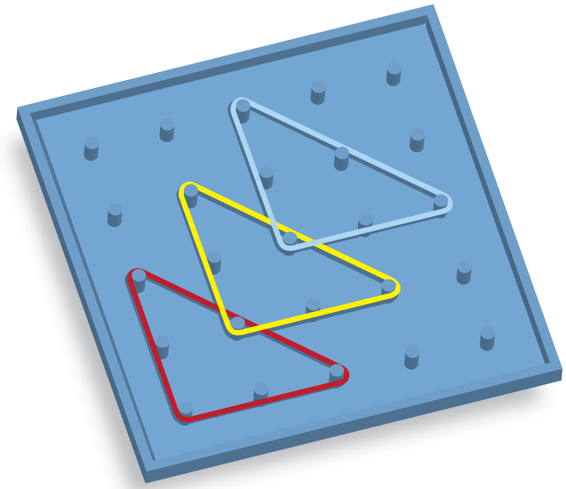
Preparing for Standard 8.G.2

Math Practice 5

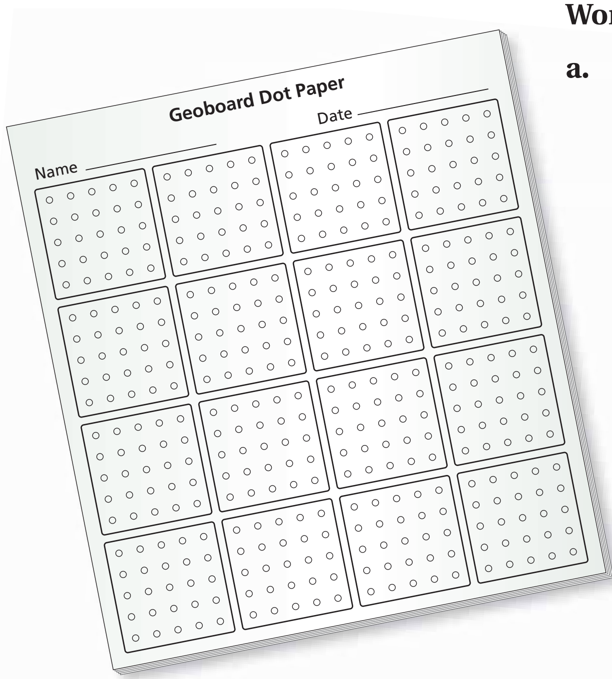
Recognize Usefulness of Tools

What are some advantages and disadvantages of using a geoboard to construct congruent triangles?

The geoboard at the right shows three congruent triangles.



2 ACTIVITY: Forming Congruent Triangles



Work with a partner.

a. Form the yellow triangle in Activity 1 on your geoboard. Record the triangle on geoboard dot paper.

b. Move each vertex of the triangle one peg to the right. Is the new triangle congruent to the original triangle? How can you tell?

c. On a 5-by-5 geoboard, make as many different triangles as possible, each of which is congruent to the yellow triangle in Activity 1. Record each triangle on geoboard dot paper.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you identify congruent triangles? Use the conclusion you wrote in Activity 1 as part of your answer.

4. Can you form a triangle on your geoboard whose side lengths are 3, 4, and 5 units? If so, draw such a triangle on geoboard dot paper.

Practice Use what you learned about congruent triangles to complete Exercises 4 and 5 on page 46.

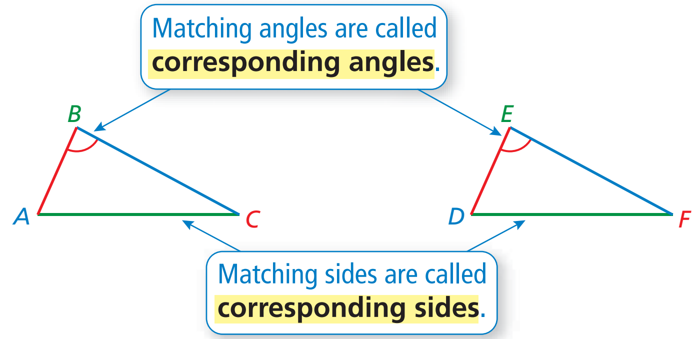
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
congruent figures, *p. 44*
corresponding angles, *p. 44*
corresponding sides, *p. 44*

Key Idea

Congruent Figures

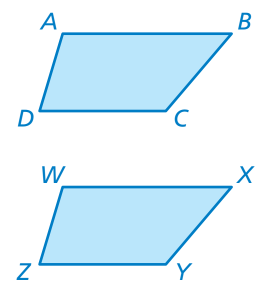
Figures that have the same size and the same shape are called **congruent figures**. The triangles below are congruent.



EXAMPLE 1 Naming Corresponding Parts

The figures are congruent. Name the corresponding angles and the corresponding sides.

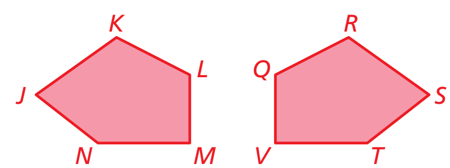
Corresponding Angles	*Corresponding Sides*
$\angle A$ and $\angle W$	Side AB and Side WX
$\angle B$ and $\angle X$	Side BC and Side XY
$\angle C$ and $\angle Y$	Side CD and Side YZ
$\angle D$ and $\angle Z$	Side AD and Side WZ



On Your Own

Now You're Ready
Exercises 6 and 7

1. The figures are congruent. Name the corresponding angles and the corresponding sides.



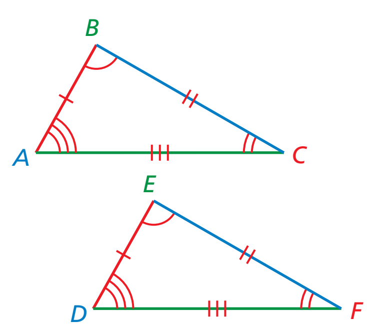
Key Idea

Identifying Congruent Figures

Two figures are congruent when corresponding angles and corresponding sides are congruent.

Triangle ABC is congruent to Triangle DEF.

$\triangle ABC \cong \triangle DEF$



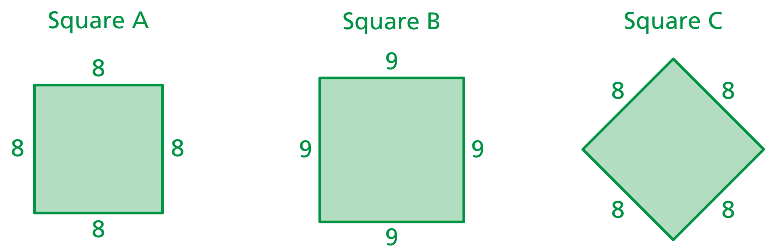
Reading

The symbol $\cong$ means *is congruent to*.

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Identifying Congruent Figures

Which square is congruent to Square A?



Each square has four right angles. So, corresponding angles are congruent. Check to see if corresponding sides are congruent.

Square A and Square B

Each side length of Square A is 8, and each side length of Square B is 9. So, corresponding sides are not congruent.

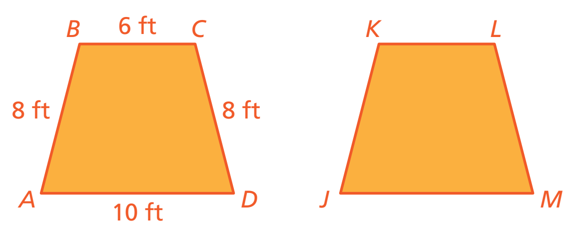
Square A and Square C

Each side length of Square A and Square C is 8. So, corresponding sides are congruent.

So, Square C is congruent to Square A.

EXAMPLE 3 Using Congruent Figures

Trapezoids *ABCD* and *JKLM* are congruent.



a. What is the length of side *JM*?

Side *JM* corresponds to side *AD*.

So, the length of side *JM* is 10 feet.

b. What is the perimeter of *JKLM*?

The perimeter of *ABCD* is $10 + 8 + 6 + 8 = 32$ feet. Because the trapezoids are congruent, their corresponding sides are congruent.

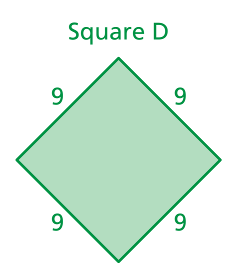
So, the perimeter of *JKLM* is also 32 feet.

On Your Own

Now You're Ready
Exercises 8, 9, and 12

2. Which square in Example 2 is congruent to Square D?

3. In Example 3, which angle of *JKLM* corresponds to $\angle C$? What is the length of side *KJ*?



2.1 Exercises

Check It Out
Help with Homework
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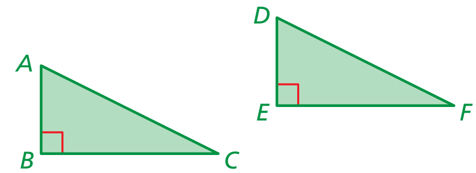
Vocabulary and Concept Check

1. **VOCABULARY** $\triangle ABC$ is congruent to $\triangle DEF$.

   a. Identify the corresponding angles.

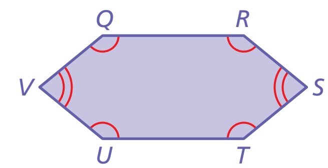
   b. Identify the corresponding sides.



2. **VOCABULARY** Explain how you can tell that two figures are congruent.

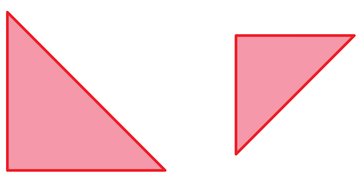
3. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

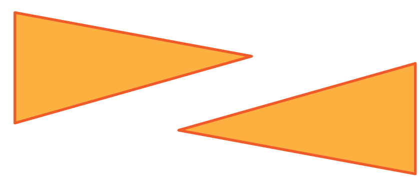
$\angle R$	$\angle U$	$\angle V$	$\angle Q$



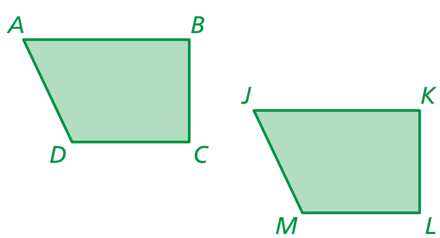
Practice and Problem Solving

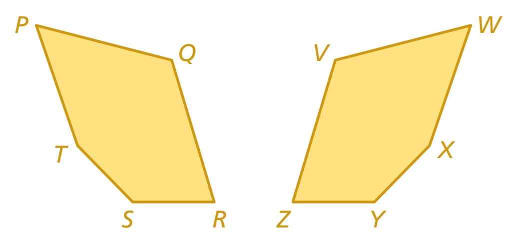
Tell whether the triangles are *congruent* or *not congruent*.

4. 

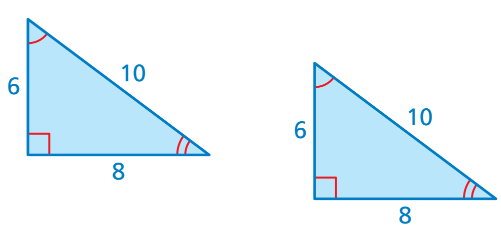
5. 

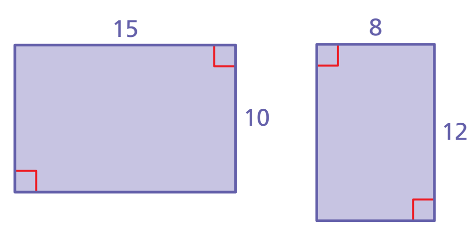
The figures are congruent. Name the corresponding angles and the corresponding sides.

1 6. 

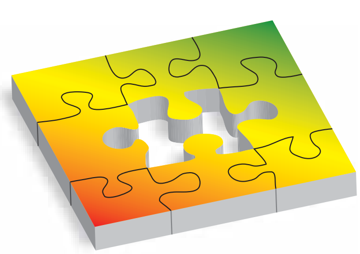
7. 

Tell whether the two figures are congruent. Explain your reasoning.

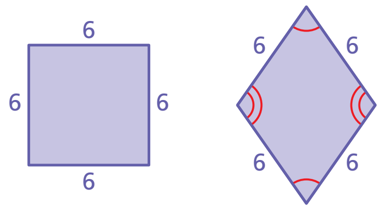
2 8. 

9. 

10. **PUZZLE** Describe the relationship between the unfinished puzzle and the missing piece.

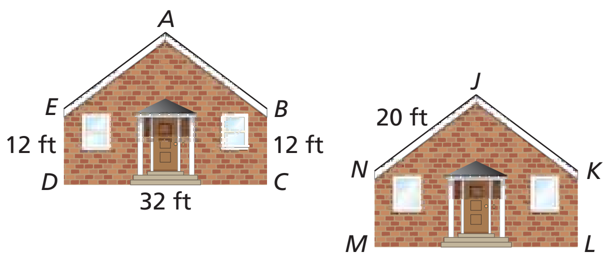


11. **ERROR ANALYSIS** Describe and correct the error in telling whether the two figures are congruent.



> ✗ Both figures have four sides, and the corresponding side lengths are equal. So, they are congruent.

3 12. **HOUSES** The fronts of the houses are identical.



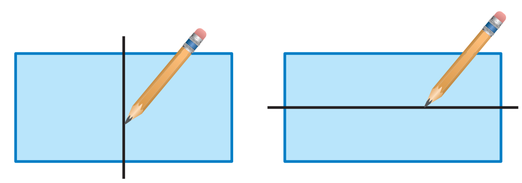
a. What is the length of side LM?

b. Which angle of $JKLMN$ corresponds to $\angle D$?

c. Side AB is congruent to side AE. What is the length of side AB?

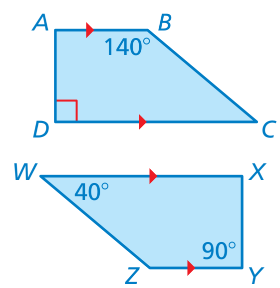
d. What is the perimeter of $ABCDE$?

13. **REASONING** Here are two ways to draw *one* line to divide a rectangle into two congruent figures. Draw three other ways.



14. **CRITICAL THINKING** Are the areas of two congruent figures equal? Explain. Draw a diagram to support your answer.

15. **True or False?** The trapezoids are congruent. Determine whether the statement is *true* or *false*. Explain your reasoning.



a. Side AB is congruent to side YZ.

b. $\angle A$ is congruent to $\angle X$.

c. $\angle A$ corresponds to $\angle X$.

d. The sum of the angle measures of $ABCD$ is 360°.

Fair Game Review What you learned in previous grades & lessons

Plot and label the ordered pair in a coordinate plane. *(Skills Review Handbook)*

16. $A(5, 3)$ 17. $B(4, -1)$ 18. $C(-2, 6)$ 19. $D(-4, -2)$

20. **MULTIPLE CHOICE** You have 2 quarters and 5 dimes in your pocket. Write the ratio of quarters to the total number of coins. *(Skills Review Handbook)*

Ⓐ $\frac{2}{5}$ Ⓑ 2 : 7 Ⓒ 5 to 7 Ⓓ $\frac{7}{2}$

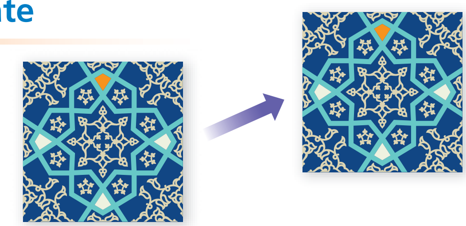
2.2 Translations

Essential Question How can you arrange tiles to make a tessellation?

The Meaning of a Word ● Translate

When you **translate** a tile, you slide it from one place to another.

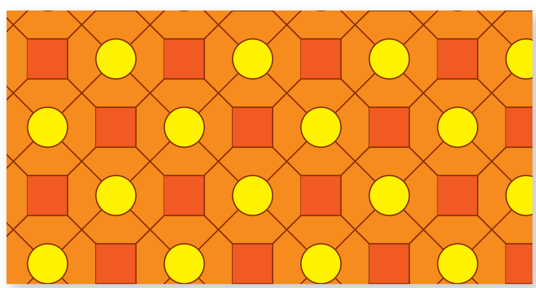
When tiles cover a floor with no empty spaces, the collection of tiles is called a *tessellation*.

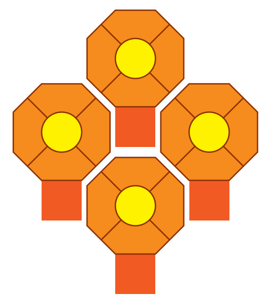


1 ACTIVITY: Describing Tessellations

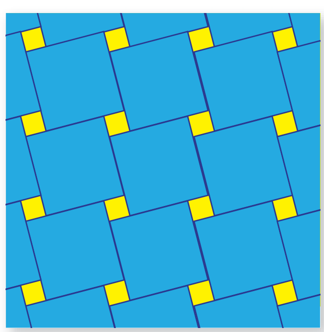
Work with a partner. Can you make the tessellation by translating single tiles that are all of the same shape and design? If so, show how.

a. Sample: Tile Pattern / Single Tiles

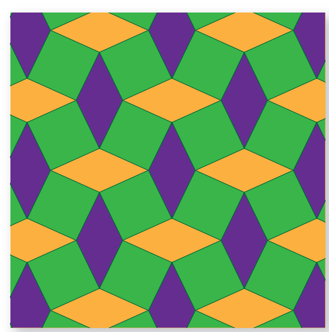




b.



c.



COMMON CORE

Geometry

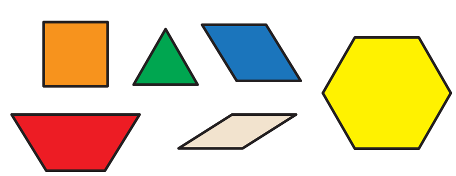
In this lesson, you will

- identify translations.
- translate figures in the coordinate plane.

Learning Standards
8.G.1
8.G.2
8.G.3

2 ACTIVITY: Tessellations and Basic Shapes

Work with a partner.



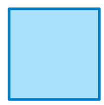
a. Which pattern blocks can you use to make a tessellation? For each one that works, draw the tessellation.

b. Can you make the tessellation by translating? Or do you have to rotate or flip the pattern blocks?

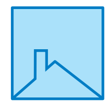
3 ACTIVITY: Designing Tessellations

Work with a partner. Design your own tessellation. Use one of the basic shapes from Activity 2.

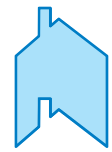
Sample:



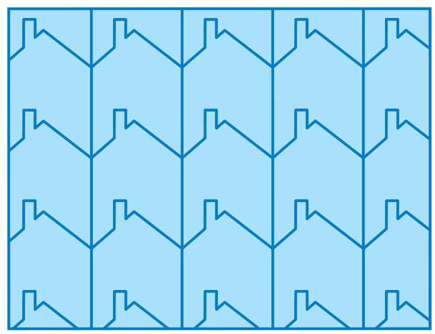
Step 1: Start with a square.



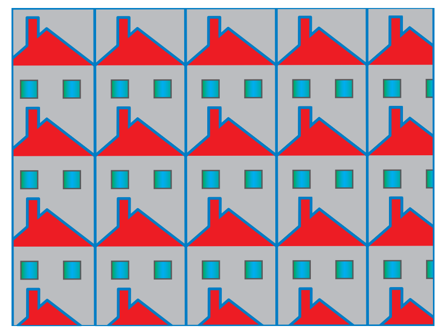
Step 2: Cut a design out of one side.



Step 3: Tape it to the other side to make your pattern.



Step 4: Translate the pattern to make your tessellation.



Step 5: Color the tessellation.

4 ACTIVITY: Translating in the Coordinate Plane

Math Practice 3

Justify Conclusions

What information do you need to conclude that two figures are congruent?

Work with a partner.

a. Draw a rectangle in a coordinate plane. Find the dimensions of the rectangle.

b. Move each vertex 3 units right and 4 units up. Draw the new figure. List the vertices.

c. Compare the dimensions and the angle measures of the new figure to those of the original rectangle.

d. Are the opposite sides of the new figure still parallel? Explain.

e. Can you conclude that the two figures are congruent? Explain.

f. Compare your results with those of other students in your class. Do you think the results are true for any type of figure?

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you arrange tiles to make a tessellation? Give an example.

6. **PRECISION** Explain why any parallelogram can be translated to make a tessellation.

Practice Use what you learned about translations to complete Exercises 4–6 on page 52.

2.2 Lesson

Check It Out
Lesson Tutorials
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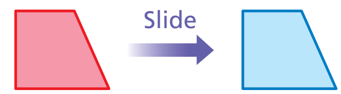
Key Vocabulary
transformation, *p. 50*
image, *p. 50*
translation, *p. 50*

A **transformation** changes a figure into another figure. The new figure is called the **image**.

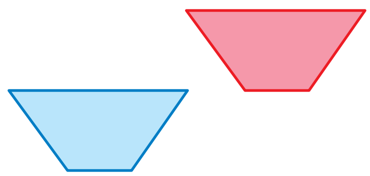
A **translation** is a transformation in which a figure *slides* but does not turn. Every point of the figure moves the same distance and in the same direction.



EXAMPLE 1 Identifying a Translation

Tell whether the blue figure is a translation of the red figure.

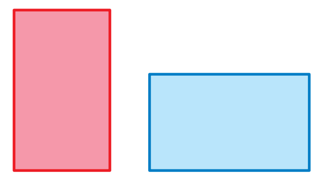
a.



The red figure *slides* to form the blue figure.

So, the blue figure is a translation of the red figure.

b.



The red figure *turns* to form the blue figure.

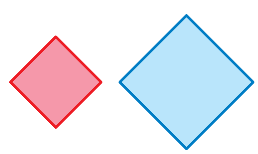
So, the blue figure is *not* a translation of the red figure.

On Your Own

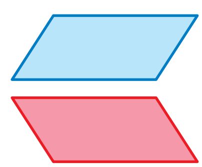
Now You're Ready
Exercises 4–9

Tell whether the blue figure is a translation of the red figure. Explain.

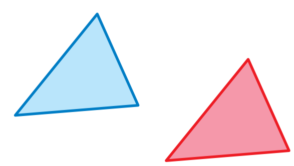
1.



2.



3.

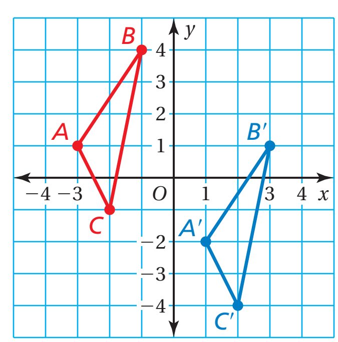


Key Idea

Translations in the Coordinate Plane

Words To translate a figure a units horizontally and b units vertically in a coordinate plane, add a to the x-coordinates and b to the y-coordinates of the vertices.

Positive values of a and b represent translations up and right. Negative values of a and b represent translations down and left.



Algebra $(x, y) \rightarrow (x + a, y + b)$

In a translation, the original figure and its image are congruent.

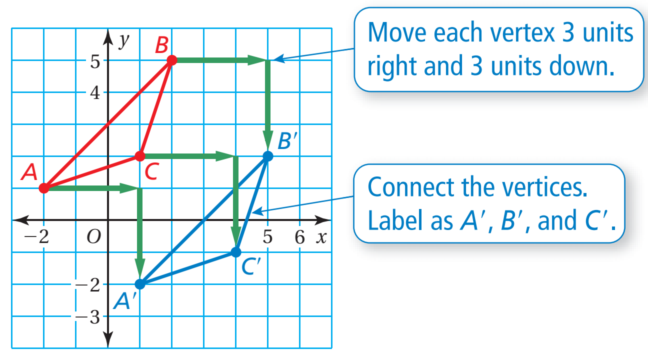
Reading

A' is read "A prime." Use *prime* symbols when naming an image.

$A \rightarrow A'$
$B \rightarrow B'$
$C \rightarrow C'$

EXAMPLE 2 Translating a Figure in the Coordinate Plane

Translate the red triangle 3 units right and 3 units down. What are the coordinates of the image?



∴ The coordinates of the image are $A'(1, -2)$, $B'(5, 2)$, and $C'(4, -1)$.

On Your Own

Now You're Ready
Exercises 10 and 11

4. **WHAT IF?** The red triangle is translated 4 units left and 2 units up. What are the coordinates of the image?

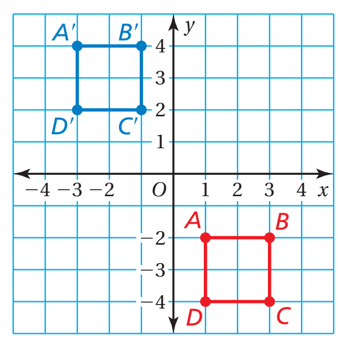
EXAMPLE 3 Translating a Figure Using Coordinates

The vertices of a square are $A(1, -2)$, $B(3, -2)$, $C(3, -4)$, and $D(1, -4)$. Draw the figure and its image after a translation 4 units left and 6 units up.

Add -4 to each x-coordinate. So, subtract 4 from each x-coordinate.

Add 6 to each y-coordinate.

Vertices of $ABCD$	$(x - 4, y + 6)$	Vertices of $A'B'C'D'$
$A(1, -2)$	$(1 - 4, -2 + 6)$	$A'(-3, 4)$
$B(3, -2)$	$(3 - 4, -2 + 6)$	$B'(-1, 4)$
$C(3, -4)$	$(3 - 4, -4 + 6)$	$C'(-1, 2)$
$D(1, -4)$	$(1 - 4, -4 + 6)$	$D'(-3, 2)$



∴ The figure and its image are shown at the above right.

On Your Own

Now You're Ready
Exercises 12–15

5. The vertices of a triangle are $A(-2, -2)$, $B(0, 2)$, and $C(3, 0)$. Draw the figure and its image after a translation 1 unit left and 2 units up.

2.2 Exercises

Check It Out
Help with Homework
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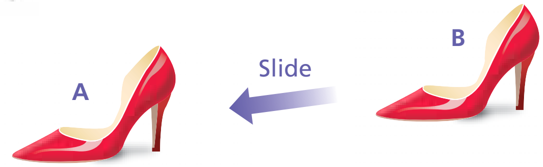
Vocabulary and Concept Check

1. **VOCABULARY** Which figure is the image?

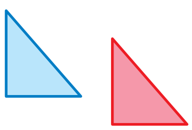
2. **VOCABULARY** How do you translate a figure in a coordinate plane?

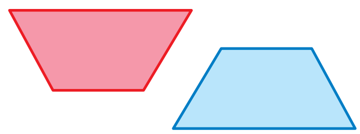


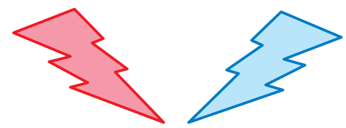
3. **WRITING** Can you translate the letters in the word TOKYO to form the word KYOTO? Explain.

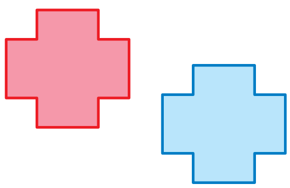
Practice and Problem Solving

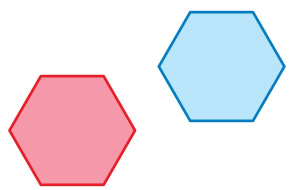
Tell whether the blue figure is a translation of the red figure.

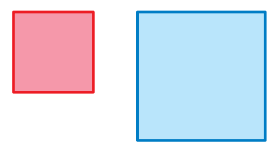
1 4.


5.


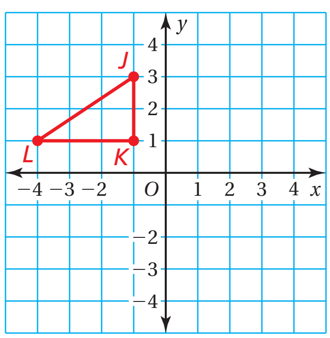
6.


7.


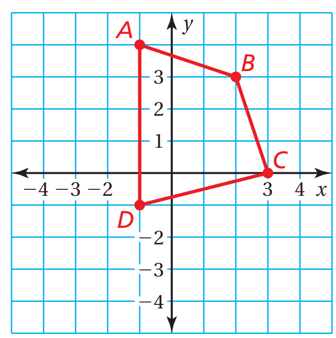
8.


9.


2 10. Translate the triangle 4 units right and 3 units down. What are the coordinates of the image?



11. Translate the figure 2 units left and 4 units down. What are the coordinates of the image?



The vertices of a triangle are $L(0, 1)$, $M(1, -2)$, and $N(-2, 1)$. Draw the figure and its image after the translation.

3 12. 1 unit left and 6 units up

13. 5 units right

14. $(x + 2, y + 3)$

15. $(x - 3, y - 4)$

16. **ICONS** You can click and drag an icon on a computer screen. Is this an example of a translation? Explain.

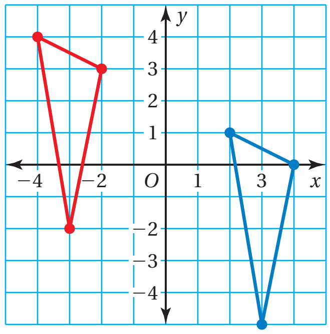
Describe the translation of the point to its image.

17. $(3, -2) \longrightarrow (1, 0)$

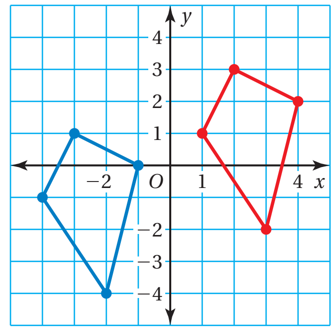
18. $(-8, -4) \longrightarrow (-3, 5)$

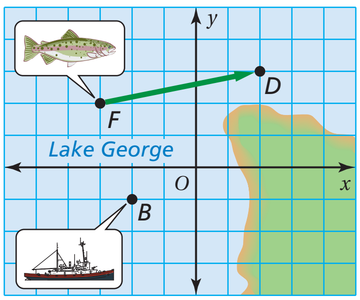
Describe the translation from the red figure to the blue figure.

19.



20.





21. **FISHING** A school of fish translates from point *F* to point *D*.

a. Describe the translation of the school of fish.

b. Can the fishing boat make the same translation? Explain.

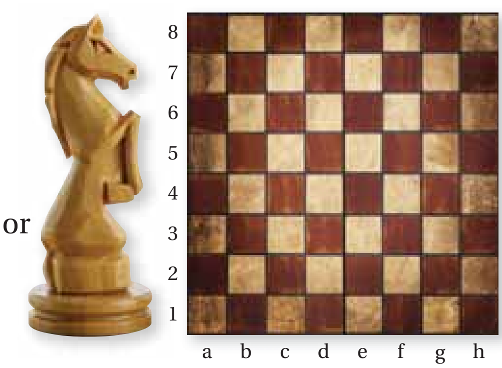
c. Describe a translation the fishing boat could make to get to point *D*.

22. **REASONING** The vertices of a triangle are $A(0, -3)$, $B(2, -1)$, and $C(3, -3)$. You translate the triangle 5 units right and 2 units down. Then you translate the image 3 units left and 8 units down. Is the original triangle congruent to the final image? If so, give two ways to show that they are congruent.

23. Problem Solving In chess, a knight can move only in an L-shaped pattern:

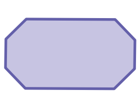
- *two* vertical squares, then *one* horizontal square;
- *two* horizontal squares, then *one* vertical square;
- *one* vertical square, then *two* horizontal squares; or
- *one* horizontal square, then *two* vertical squares.

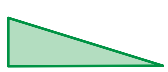
Write a series of translations to move the knight from g8 to g5.



Fair Game Review What you learned in previous grades & lessons

Tell whether you can fold the figure in half so that one side matches the other. *(Skills Review Handbook)*

24. 

25. 

26. 

27. 

28. **MULTIPLE CHOICE** You put \$550 in an account that earns 4.4% simple interest per year. How much interest do you earn in 6 months? *(Skills Review Handbook)*

Ⓐ \$1.21 Ⓑ \$12.10 Ⓒ \$121.00 Ⓓ \$145.20

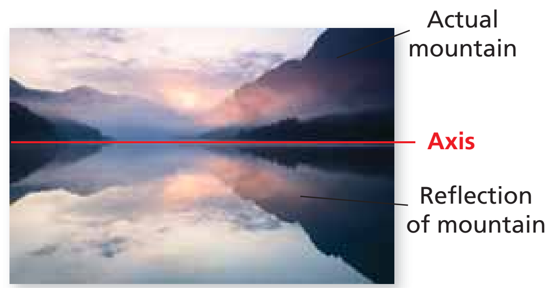
2.3 Reflections

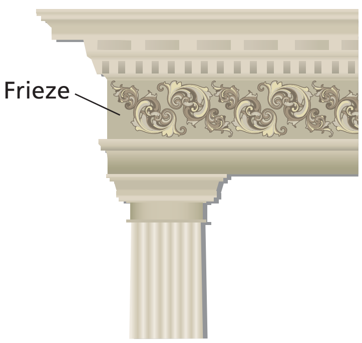
Essential Question How can you use reflections to classify a frieze pattern?

The Meaning of a Word ● Reflection

When you look at a mountain by a lake, you can see the **reflection**, or mirror image, of the mountain in the lake.

If you fold the photo on its axis, the mountain and its reflection will align.



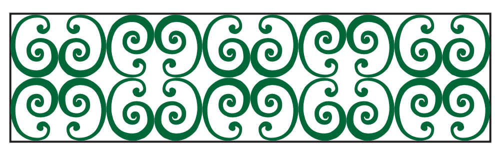


A *frieze* is a horizontal band that runs at the top of a building. A frieze is often decorated with a design that repeats.

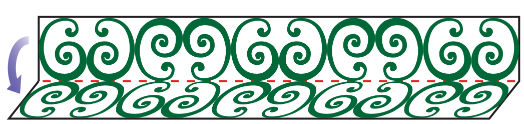
- All frieze patterns are translations of themselves.
- Some frieze patterns are reflections of themselves.

1 ACTIVITY: Frieze Patterns and Reflections

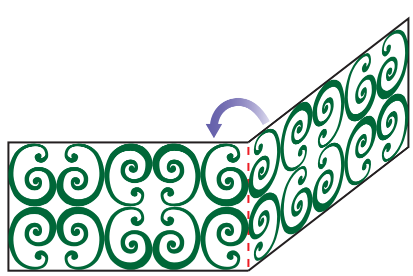
Work with a partner. Consider the frieze pattern shown.



a. Is the frieze pattern a reflection of itself when folded horizontally? Explain.



b. Is the frieze pattern a reflection of itself when folded vertically? Explain.



COMMON CORE

Geometry

In this lesson, you will

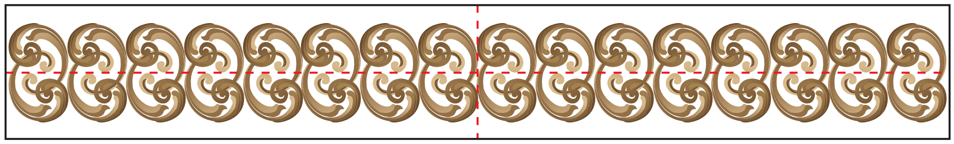
- identify reflections.
- reflect figures in the *x*-axis or the *y*-axis of the coordinate plane.

Learning Standards
8.G.1
8.G.2
8.G.3

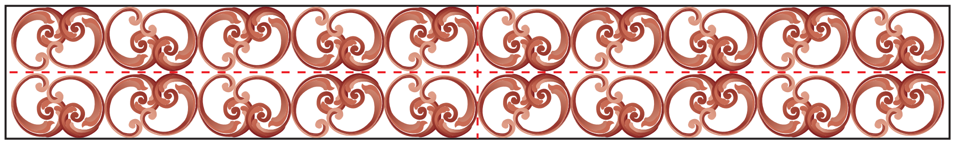
2 ACTIVITY: Frieze Patterns and Reflections

Work with a partner. Is the frieze pattern a reflection of itself when folded *horizontally*, *vertically*, or *neither*?

a.



b.



3 ACTIVITY: Reflecting in the Coordinate Plane

Work with a partner.

Math Practice 7

Look for Patterns

What do you notice about the vertices of the original figure and the image? How does this help you determine whether the figures are congruent?

a. Draw a rectangle in Quadrant I of a coordinate plane. Find the dimensions of the rectangle.

b. Copy the axes and the rectangle onto a piece of transparent paper.

Flip the transparent paper once so that the rectangle is in Quadrant IV. Then align the origin and the axes with the coordinate plane.

Draw the new figure in the coordinate plane. List the vertices.

c. Compare the dimensions and the angle measures of the new figure to those of the original rectangle.

d. Are the opposite sides of the new figure still parallel? Explain.

e. Can you conclude that the two figures are congruent? Explain.

f. Flip the transparent paper so that the original rectangle is in Quadrant II. Draw the new figure in the coordinate plane. List the vertices. Then repeat parts (c) – (e).

g. Compare your results with those of other students in your class. Do you think the results are true for any type of figure?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use reflections to classify a frieze pattern?

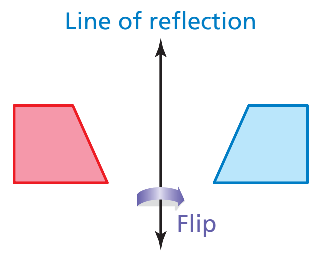
Practice Use what you learned about reflections to complete Exercises 4–6 on page 58.

2.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
reflection, *p. 56*
line of reflection, *p. 56*

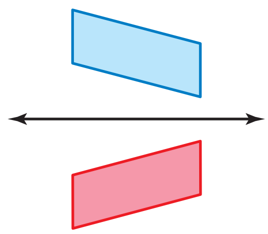
A **reflection**, or *flip*, is a transformation in which a figure is reflected in a line called the **line of reflection**. A reflection creates a mirror image of the original figure.



EXAMPLE 1 Identifying a Reflection

Tell whether the blue figure is a reflection of the red figure.

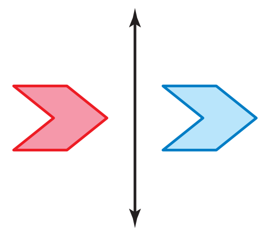
a.



The red figure can be *flipped* to form the blue figure.

So, the blue figure is a reflection of the red figure.

b.



If the red figure were *flipped*, it would point to the left.

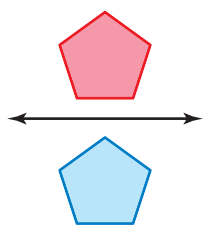
So, the blue figure is *not* a reflection of the red figure.

On Your Own

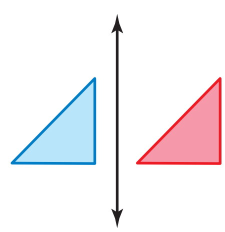
Now You're Ready
Exercises 4–9

Tell whether the blue figure is a reflection of the red figure. Explain.

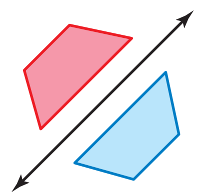
1.



2.



3.



Key Idea

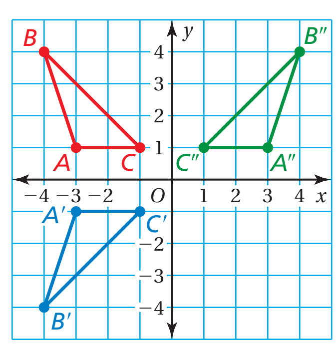
Reflections in the Coordinate Plane

Words To reflect a figure in the x-axis, take the opposite of the y-coordinate.

To reflect a figure in the y-axis, take the opposite of the x-coordinate.

Algebra Reflection in x-axis: $(x, y) \rightarrow (x, -y)$

Reflection in y-axis: $(x, y) \rightarrow (-x, y)$

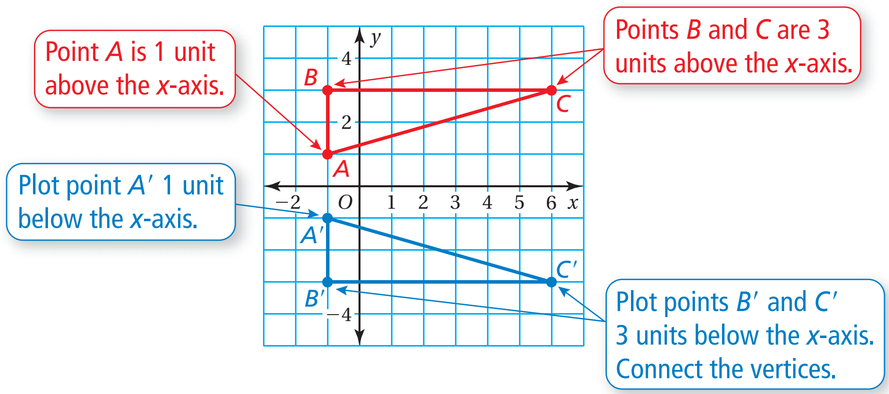


In a reflection, the original figure and its image are congruent.

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Reflecting a Figure in the *x*-axis

The vertices of a triangle are $A(-1, 1)$, $B(-1, 3)$, and $C(6, 3)$. Draw the figure and its reflection in the x-axis. What are the coordinates of the image?



⁘ The coordinates of the image are $A'(-1, -1)$, $B'(-1, -3)$, and $C'(6, -3)$.

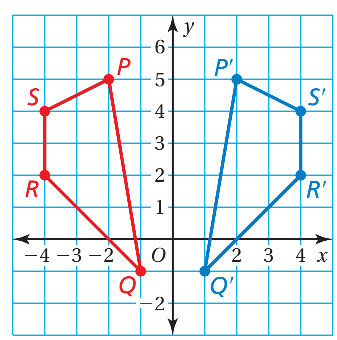
EXAMPLE 3 Reflecting a Figure in the *y*-axis

The vertices of a quadrilateral are $P(-2, 5)$, $Q(-1, -1)$, $R(-4, 2)$, and $S(-4, 4)$. Draw the figure and its reflection in the y-axis.

Take the opposite of the *x*-coordinate.

The *y*-coordinate does not change.

Vertices of *PQRS*	$(-x, y)$	Vertices of *P′Q′R′S′*
$P(-2, 5)$	$(-(-2), 5)$	$P'(2, 5)$
$Q(-1, -1)$	$(-(-1), -1)$	$Q'(1, -1)$
$R(-4, 2)$	$(-(-4), 2)$	$R'(4, 2)$
$S(-4, 4)$	$(-(-4), 4)$	$S'(4, 4)$



⁘ The figure and its image are shown at the above right.

On Your Own

Now You're Ready
Exercises 10–17

4. The vertices of a rectangle are $A(-4, -3)$, $B(-4, -1)$, $C(-1, -1)$, and $D(-1, -3)$.
   a. Draw the figure and its reflection in the x-axis.
   b. Draw the figure and its reflection in the y-axis.
   c. Are the images in parts (a) and (b) congruent? Explain.

2.3 Exercises

Check It Out
Help with Homework
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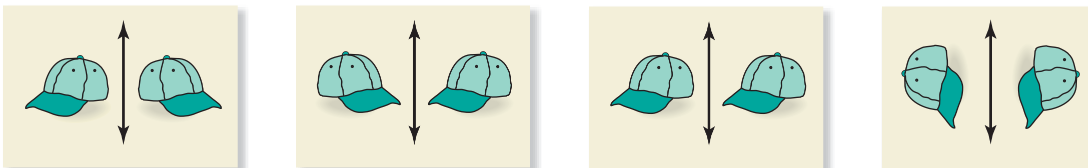
Vocabulary and Concept Check

1. **WHICH ONE DOESN'T BELONG?** Which transformation does *not* belong with the other three? Explain your reasoning.

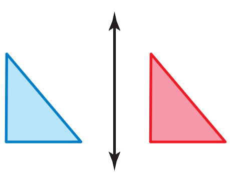


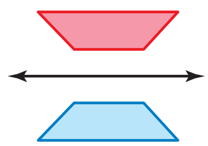
2. **WRITING** How can you tell when one figure is a reflection of another figure?

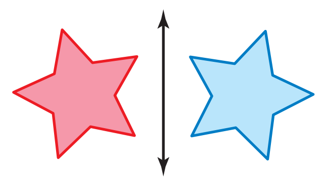
3. **REASONING** A figure lies entirely in Quadrant I. The figure is reflected in the x-axis. In which quadrant is the image?

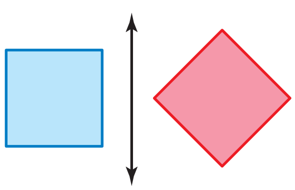
Practice and Problem Solving

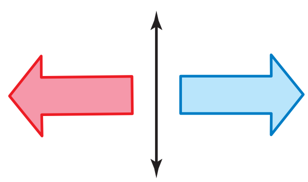
Tell whether the blue figure is a reflection of the red figure.

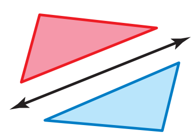
1 4. 

5. 

6. 

7. 

8. 

9. 

Draw the figure and its reflection in the x-axis. Identify the coordinates of the image.

2 10. $A(3, 2)$, $B(4, 4)$, $C(1, 3)$

11. $M(-2, 1)$, $N(0, 3)$, $P(2, 2)$

12. $H(2, -2)$, $J(4, -1)$, $K(6, -3)$, $L(5, -4)$

13. $D(-2, -1)$, $E(0, -1)$, $F(0, -5)$, $G(-2, -5)$

Draw the figure and its reflection in the y-axis. Identify the coordinates of the image.

3 14. $Q(-4, 2)$, $R(-2, 4)$, $S(-1, 1)$

15. $T(4, -2)$, $U(4, 2)$, $V(6, -2)$

16. $W(2, -1)$, $X(5, -2)$, $Y(5, -5)$, $Z(2, -4)$

17. $J(2, 2)$, $K(7, 4)$, $L(9, -2)$, $M(3, -1)$

18. **ALPHABET** Which letters look the same when reflected in the line?

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

The coordinates of a point and its image are given. Is the reflection in the *x-axis* or *y-axis*?

19. $(2, -2) \longrightarrow (2, 2)$

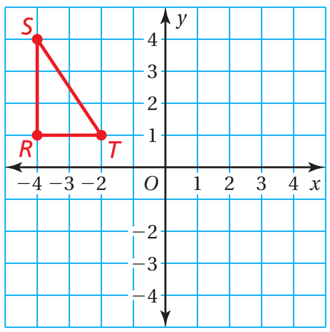
20. $(-4, 1) \longrightarrow (4, 1)$

21. $(-2, -5) \longrightarrow (2, -5)$

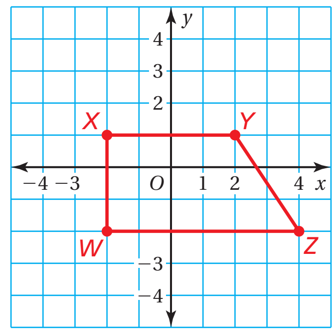
22. $(-3, -4) \longrightarrow (-3, 4)$

Find the coordinates of the figure after the transformations.

23. Translate the triangle 1 unit right and 5 units down. Then reflect the image in the y-axis.



24. Reflect the trapezoid in the x-axis. Then translate the trapezoid 2 units left and 3 units up.



25. **REASONING** In Exercises 23 and 24, is the original figure congruent to the final image? Explain.

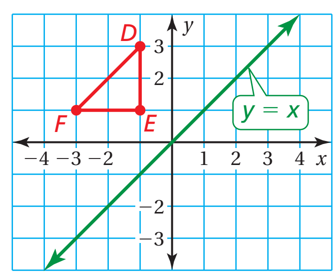
26. **NUMBER SENSE** You reflect a point (x, y) in the x-axis, and then in the y-axis. What are the coordinates of the final image?

27. **EMERGENCY VEHICLE** Hold a mirror to the left side of the photo of the vehicle.

a. What word do you see in the mirror?

b. Why do you think it is written that way on the front of the vehicle?

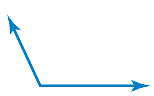
28. **Critical Thinking** Reflect the triangle in the line $y = x$. How are the x- and y-coordinates of the image related to the x- and y-coordinates of the original triangle?



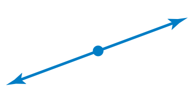
Fair Game Review What you learned in previous grades & lessons

Classify the angle as *acute, right, obtuse,* or *straight*. *(Skills Review Handbook)*

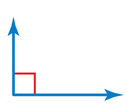
29.



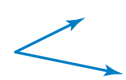
30.



31.



32.



33. **MULTIPLE CHOICE** 36 is 75% of what number? *(Skills Review Handbook)*

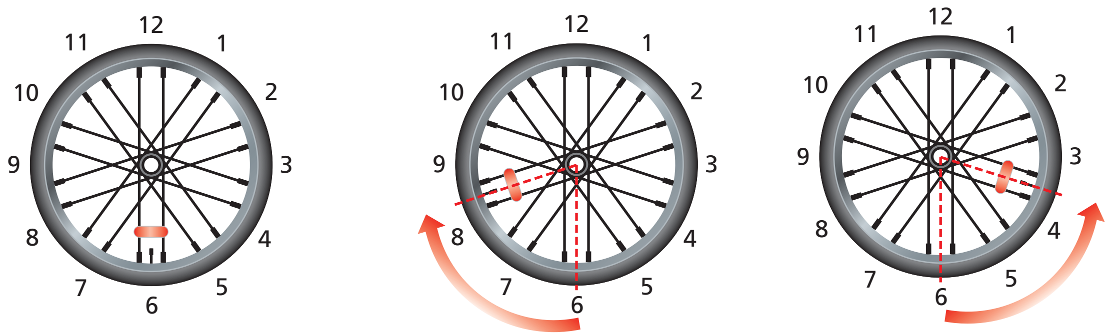
Ⓐ 27 Ⓑ 48 Ⓒ 54 Ⓓ 63

2.4 Rotations

Essential Question What are the three basic ways to move an object in a plane?

The Meaning of a Word ● Rotate

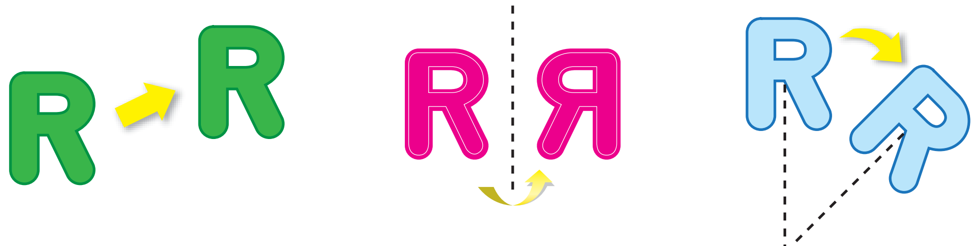
A bicycle wheel can **rotate** clockwise or counterclockwise.



1 ACTIVITY: Three Basic Ways to Move Things

There are three basic ways to move objects on a flat surface.

_____ the object. _____ the object. _____ the object.

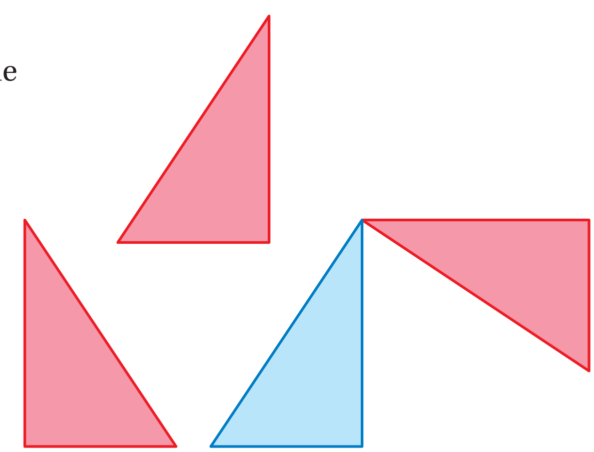


Work with a partner.

a. What type of triangle is the blue triangle? Is it congruent to the red triangles? Explain.

b. Decide how you can move the blue triangle to obtain each red triangle.

c. Is each move a *translation*, a *reflection*, or a *rotation*?



Common Core

Geometry

In this lesson, you will

- identify rotations.
- rotate figures in the coordinate plane.
- use more than one transformation to find images of figures.

Learning Standards
8.G.1
8.G.2
8.G.3

2 ACTIVITY: Rotating in the Coordinate Plane

Work with a partner.

a. Draw a rectangle in Quadrant II of a coordinate plane. Find the dimensions of the rectangle.

b. Copy the axes and the rectangle onto a piece of transparent paper.

Align the origin and the vertices of the rectangle on the transparent paper with the coordinate plane. Turn the transparent paper so that the rectangle is in Quadrant I and the axes align.

Draw the new figure in the coordinate plane. List the vertices.

c. Compare the dimensions and the angle measures of the new figure to those of the original rectangle.

d. Are the opposite sides of the new figure still parallel? Explain.

e. Can you conclude that the two figures are congruent? Explain.

f. Turn the transparent paper so that the original rectangle is in Quadrant IV. Draw the new figure in the coordinate plane. List the vertices. Then repeat parts (c)–(e).

g. Compare your results with those of other students in your class. Do you think the results are true for any type of figure?

Math Practice 6

Calculate Accurately

What must you do to rotate the figure correctly?

What Is Your Answer?

3. **IN YOUR OWN WORDS** What are the three basic ways to move an object in a plane? Draw an example of each.

4. **PRECISION** Use the results of Activity 2(b).

   a. Draw four angles using the conditions below.
   - The origin is the vertex of each angle.
   - One side of each angle passes through a vertex of the original rectangle.
   - The other side of each angle passes through the corresponding vertex of the rotated rectangle.

   b. Measure each angle in part (a). For each angle, measure the distances between the origin and the vertices of the rectangles. What do you notice?

   c. How can the results of part (b) help you rotate a figure?

5. **PRECISION** Repeat the procedure in Question 4 using the results of Activity 2(f).

Practice Use what you learned about transformations to complete Exercises 7–9 on page 65.

2.4 Lesson

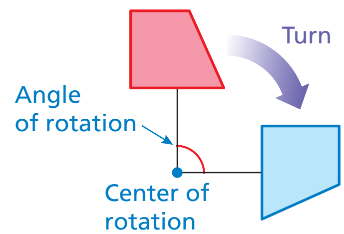
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
rotation, *p. 62*
center of rotation, *p. 62*
angle of rotation, *p. 62*

Key Idea

Rotations

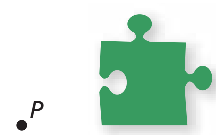
A **rotation**, or *turn*, is a transformation in which a figure is rotated about a point called the **center of rotation**. The number of degrees a figure rotates is the **angle of rotation**.



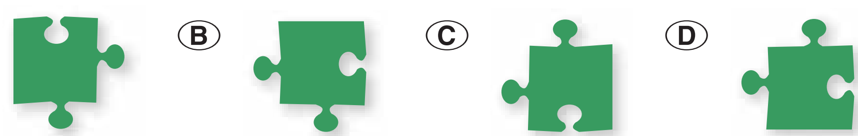
In a rotation, the original figure and its image are congruent.

EXAMPLE 1 Identifying a Rotation

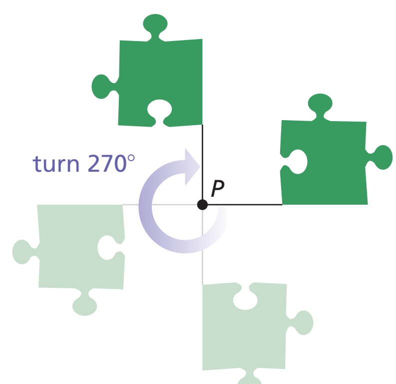
You must rotate the puzzle piece 270° clockwise about point *P* to fit it into a puzzle. Which piece fits in the puzzle as shown?



Ⓐ Ⓑ Ⓒ Ⓓ



Rotate the puzzle piece 270° clockwise about point *P*.



So, the correct answer is Ⓒ.

Study Tip
When rotating figures, it may help to sketch the rotation in several steps, as shown in Example 1.

On Your Own

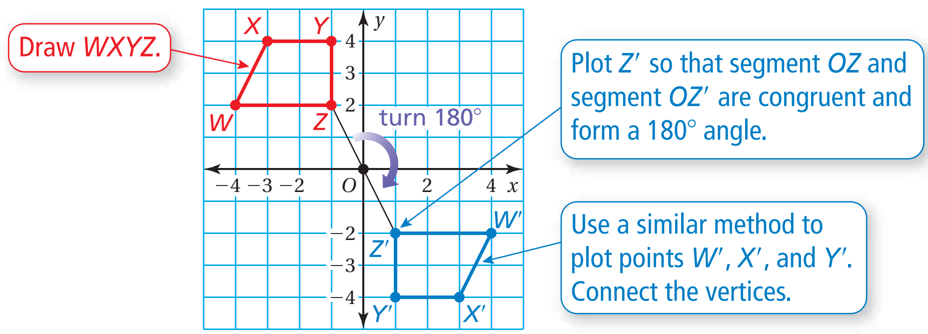
Now You're Ready
Exercises 10–12

1. Which piece is a 90° counterclockwise rotation about point *P*?
2. Is Choice D a rotation of the original puzzle piece? If not, what kind of transformation does the image show?

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Rotating a Figure

The vertices of a trapezoid are $W(-4, 2)$, $X(-3, 4)$, $Y(-1, 4)$, and $Z(-1, 2)$. Rotate the trapezoid 180° about the origin. What are the coordinates of the image?



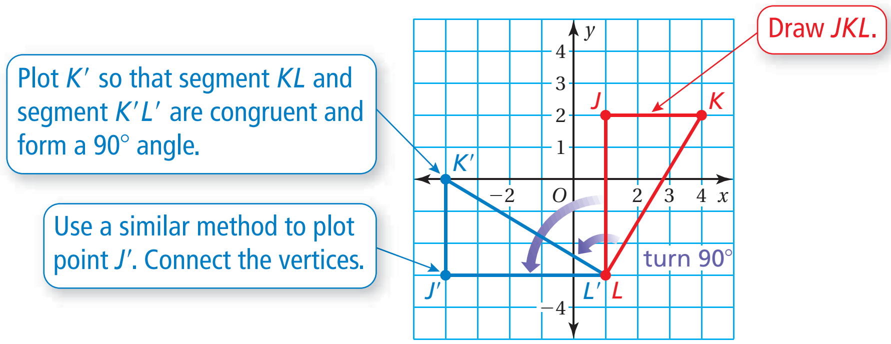
∴ The coordinates of the image are $W'(4, -2)$, $X'(3, -4)$, $Y'(1, -4)$, and $Z'(1, -2)$.

Study Tip

A 180° clockwise rotation and a 180° counterclockwise rotation have the same image. So, you do not need to specify direction when rotating a figure 180°.

EXAMPLE 3 Rotating a Figure

The vertices of a triangle are $J(1, 2)$, $K(4, 2)$, and $L(1, -3)$. Rotate the triangle 90° counterclockwise about vertex L. What are the coordinates of the image?



∴ The coordinates of the image are $J'(-4, -3)$, $K'(-4, 0)$, and $L'(1, -3)$.

Common Error

Be sure to pay attention to whether a rotation is clockwise or counterclockwise.

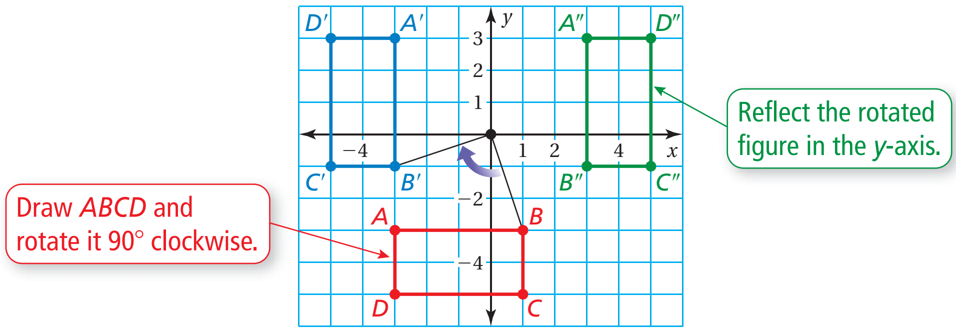
On Your Own

Now You're Ready
Exercises 13–18

3. A triangle has vertices $Q(4, 5)$, $R(4, 0)$, and $S(1, 0)$.
   a. Rotate the triangle 90° counterclockwise about the origin.
   b. Rotate the triangle 180° about vertex S.
   c. Are the images in parts (a) and (b) congruent? Explain.

EXAMPLE 4 Using More than One Transformation

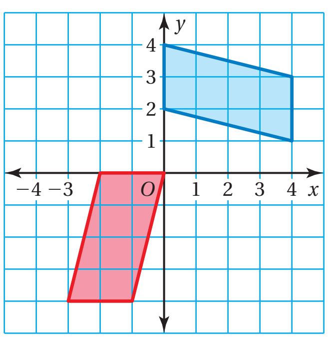
The vertices of a rectangle are $A(-3, -3)$, $B(1, -3)$, $C(1, -5)$, and $D(-3, -5)$. Rotate the rectangle 90° clockwise about the origin, and then reflect it in the y-axis. What are the coordinates of the image?



The coordinates of the image are $A''(3, 3)$, $B''(3, -1)$, $C''(5, -1)$ and $D''(5, 3)$.

The image of a translation, reflection, or rotation is congruent to the original figure. So, two figures are congruent when one can be obtained from the other by a sequence of translations, reflections, and rotations.

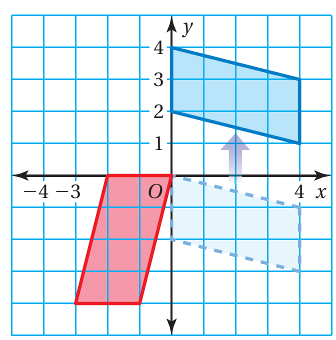
EXAMPLE 5 Describing a Sequence of Transformations



The red figure is congruent to the blue figure. Describe a sequence of transformations in which the blue figure is the image of the red figure.

You can turn the red figure 90° so that it has the same orientation as the blue figure. So, begin with a rotation.

After rotating, you need to slide the figure up.



So, one possible sequence of transformations is a 90° counterclockwise rotation about the origin followed by a translation 4 units up.

On Your Own

Now You're Ready Exercises 22–25

4. The vertices of a triangle are $P(-1, 2)$, $Q(-1, 0)$, and $R(2, 0)$. Rotate the triangle 180° about vertex R, and then reflect it in the x-axis. What are the coordinates of the image?

5. In Example 5, describe a different sequence of transformations in which the blue figure is the image of the red figure.

2.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** What are the coordinates of the center of rotation in Example 2? Example 3?

MENTAL MATH **A figure lies entirely in Quadrant II. In which quadrant will the figure lie after the given clockwise rotation about the origin?**

2. 90°
3. 180°
4. 270°
5. 360°

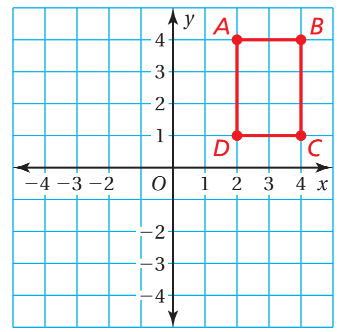
6. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

> What are the coordinates of the figure after a 90° clockwise rotation about the origin?

> What are the coordinates of the figure after a 270° clockwise rotation about the origin?

> What are the coordinates of the figure after turning the figure 90° to the right about the origin?

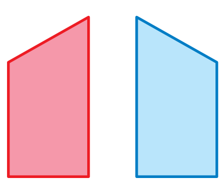
> What are the coordinates of the figure after a 270° counterclockwise rotation about the origin?



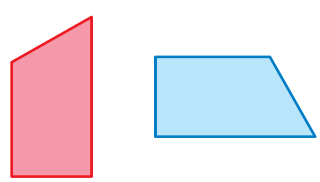
Practice and Problem Solving

Identify the transformation.

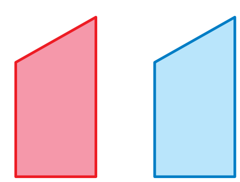
7.



8.

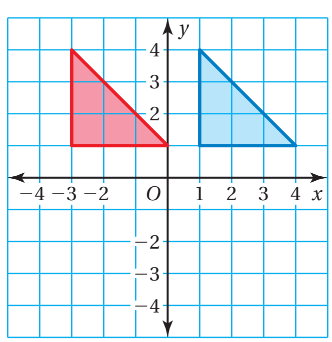


9.

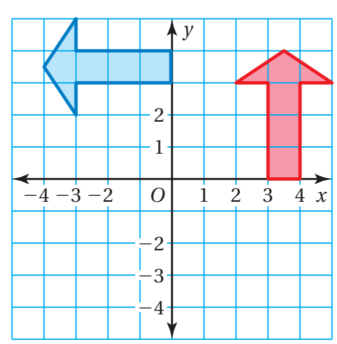


Tell whether the blue figure is a rotation of the red figure about the origin. If so, give the angle and direction of rotation.

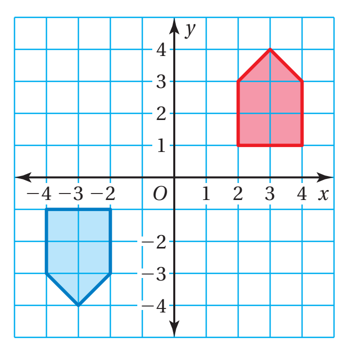
1 10.



11.



12.



The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.

2 3 **13.** $A(2, -2)$, $B(4, -1)$, $C(4, -3)$, $D(2, -4)$
90° counterclockwise about the origin

14. $F(1, 2)$, $G(3, 5)$, $H(3, 2)$
180° about the origin

15. $J(-4, 1)$, $K(-2, 1)$, $L(-4, -3)$
90° clockwise about vertex L

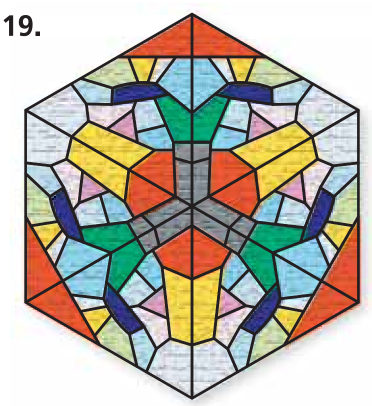
16. $P(-3, 4)$, $Q(-1, 4)$, $R(-2, 1)$, $S(-4, 1)$
180° about vertex R

17. $W(-6, -2)$, $X(-2, -2)$, $Y(-2, -6)$, $Z(-5, -6)$
270° counterclockwise about the origin

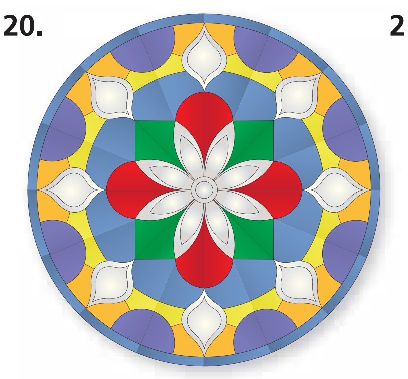
18. $A(1, -1)$, $B(5, -6)$, $C(1, -6)$
90° counterclockwise about vertex A

A figure has *rotational symmetry* if a rotation of 180° or less produces an image that fits exactly on the original figure. Explain why the figure has rotational symmetry.

19.



20.



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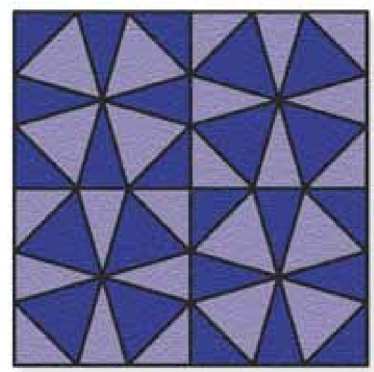


The vertices of a figure are given. Find the coordinates of the figure after the transformations given.

4 **22.** $R(-7, -5)$, $S(-1, -2)$, $T(-1, -5)$

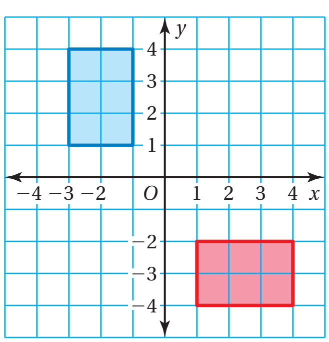
Rotate 90° counterclockwise about the origin. Then translate 3 units left and 8 units up.

23. $J(-4, 4)$, $K(-3, 4)$, $L(-1, 1)$, $M(-4, 1)$

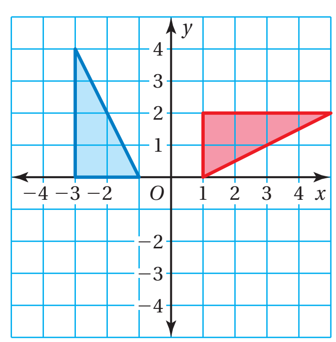
Reflect in the x-axis, and then rotate 180° about the origin.

The red figure is congruent to the blue figure. Describe two different sequences of transformations in which the blue figure is the image of the red figure.

5 **24.**



25.



26. **REASONING** A trapezoid has vertices $A(-6, -2)$, $B(-3, -2)$, $C(-1, -4)$, and $D(-6, -4)$.

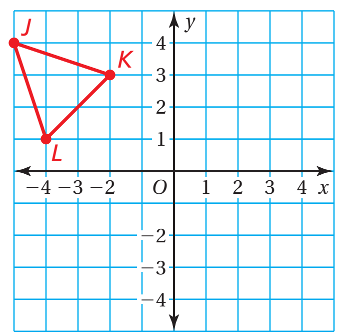
   a. Rotate the trapezoid 180° about the origin. What are the coordinates of the image?

   b. Describe a way to obtain the same image without using rotations.

27. **TREASURE MAP** You want to find the treasure located on the map at ✕. You are located at ●. The following transformations will lead you to the treasure, but they are not in the correct order. Find the correct order. Use each transformation exactly once.

- Rotate 180° about the origin.
- Reflect in the y-axis.
- Rotate 90° counterclockwise about the origin.
- Translate 1 unit right and 1 unit up.

28. **CRITICAL THINKING** Consider $\triangle JKL$.



   a. Rotate $\triangle JKL$ 90° clockwise about the origin. How are the x- and y-coordinates of $\triangle J'K'L'$ related to the x- and y-coordinates of $\triangle JKL$?

   b. Rotate $\triangle JKL$ 180° about the origin. How are the x- and y-coordinates of $\triangle J'K'L'$ related to the x- and y-coordinates of $\triangle JKL$?

   c. Do you think your answers to parts (a) and (b) hold true for any figure? Explain.

29. **Reasoning** You rotate a triangle 90° counterclockwise about the origin. Then you translate its image 1 unit left and 2 units down. The vertices of the final image are $(-5, 0)$, $(-2, 2)$, and $(-2, -1)$. What are the vertices of the original triangle?

Fair Game Review What you learned in previous grades & lessons

Tell whether the ratios form a proportion. *(Skills Review Handbook)*

30. $\frac{3}{5}, \frac{15}{20}$ 31. $\frac{2}{3}, \frac{12}{18}$ 32. $\frac{7}{28}, \frac{12}{48}$ 33. $\frac{54}{72}, \frac{36}{45}$

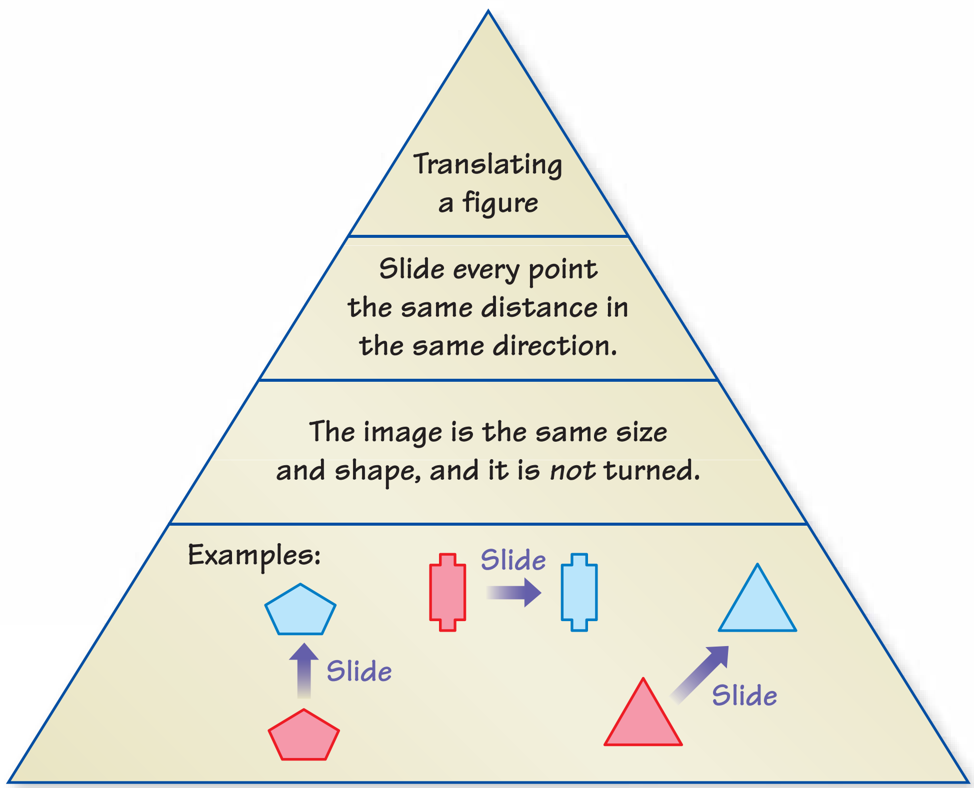
34. **MULTIPLE CHOICE** What is the solution of the equation $x + 6 \div 2 = 5$? *(Section 1.1)*

   Ⓐ $x = -16$ Ⓑ $x = 2$ Ⓒ $x = 4$ Ⓓ $x = 16$

Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use a **summary triangle** to explain a concept. Here is an example of a summary triangle for translating a figure.



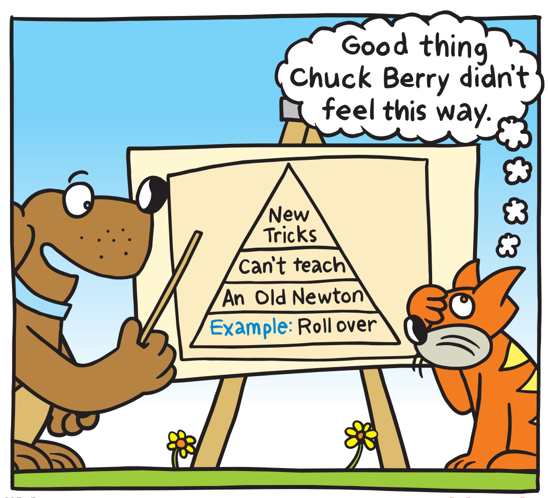
On Your Own

Make summary triangles to help you study these topics.

1. congruent figures
2. reflecting a figure
3. rotating a figure

After you complete this chapter, make summary triangles for the following topics.

4. similar figures
5. perimeters of similar figures
6. areas of similar figures
7. dilating a figure
8. transforming a figure



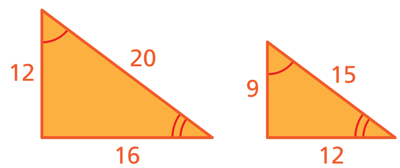
"I hope my owner sees my summary triangle. I just can't seem to learn 'roll over.' "

2.1–2.4 Quiz

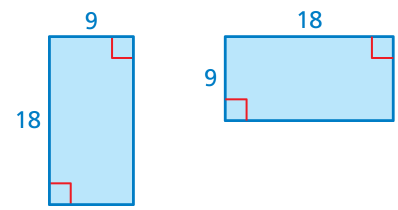
Check It Out
Progress Check
BigIdeasMath.com

Tell whether the two figures are congruent. Explain your reasoning. *(Section 2.1)*

1.

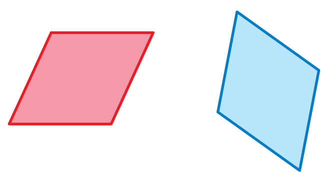


2.

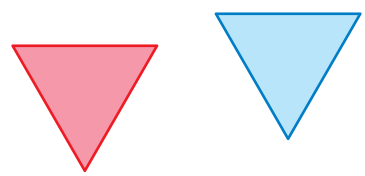


Tell whether the blue figure is a translation of the red figure. *(Section 2.2)*

3.

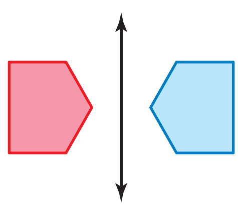


4.

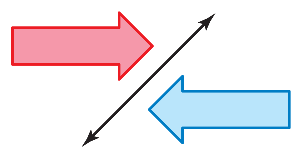


Tell whether the blue figure is a reflection of the red figure. *(Section 2.3)*

5.

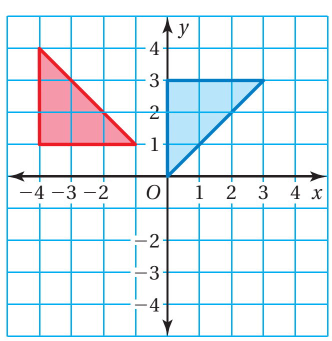


6.

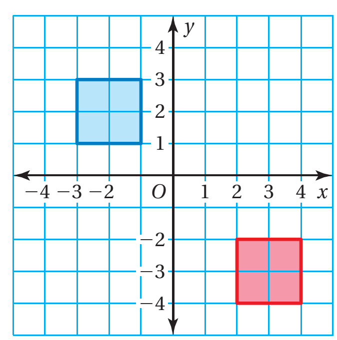


The red figure is congruent to the blue figure. Describe two different sequences of transformations in which the blue figure is the image of the red figure. *(Section 2.4)*

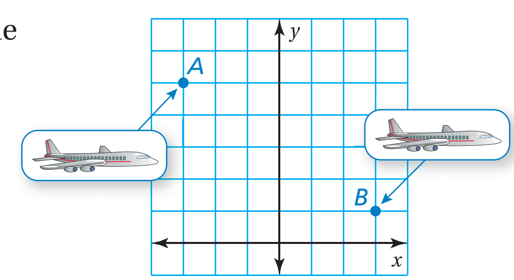
7.

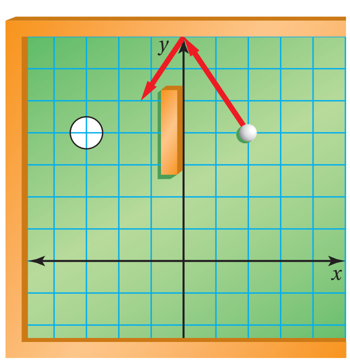


8.



9. **AIRPLANE** Describe a translation of the airplane from point *A* to point *B*. *(Section 2.2)*





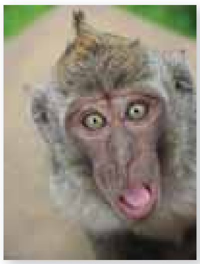
10. **MINIGOLF** You hit the golf ball along the red path so that its image will be a reflection in the *y*-axis. Does the golf ball land in the hole? Explain. *(Section 2.3)*

2.5 Similar Figures

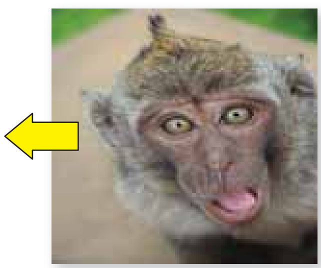
Essential Question How can you use proportions to help make decisions in art, design, and magazine layouts?

In a computer art program, when you click and drag on a side of a photograph, you distort it.

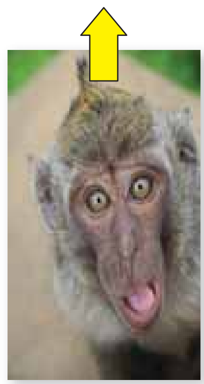
But when you click and drag on a corner of the photograph, the dimensions remain proportional to the original.



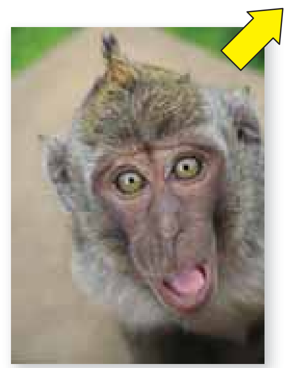
Original photograph



Distorted



Distorted

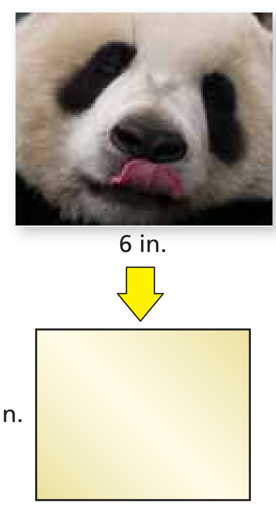


Proportional

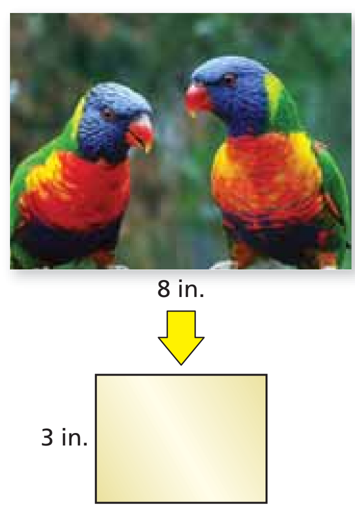
1 ACTIVITY: Reducing Photographs

Work with a partner. You are trying to reduce the photograph to the indicated size for a nature magazine. Can you reduce the photograph to the indicated size without distorting or cropping? Explain your reasoning.

a.



b.



Common Core

Geometry

In this lesson, you will

- name corresponding angles and corresponding sides of similar figures.
- identify similar figures.
- find unknown measures of similar figures.

Preparing for Standard 8.G.4

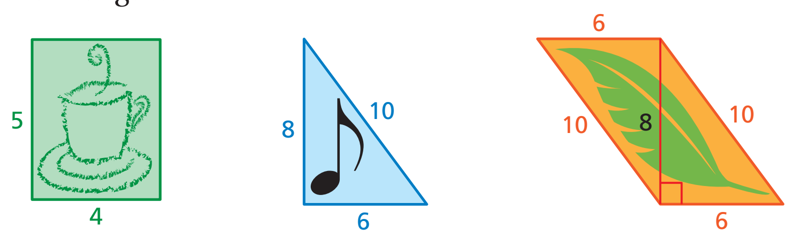
2 ACTIVITY: Creating Designs

Work with a partner.

a. Tell whether the dimensions of the new designs are proportional to the dimensions of the original design. Explain your reasoning.

Original **Design 1** **Design 2**

b. Draw two designs whose dimensions are proportional to the given design. Make one bigger and one smaller. Label the sides of the designs with their lengths.



Math Practice 4

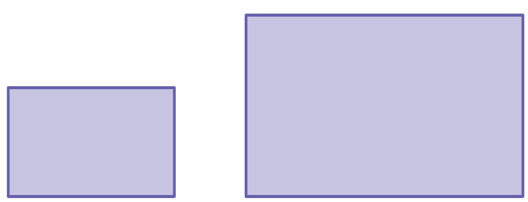
Analyze Relationships

How can you use mathematics to determine whether the dimensions are proportional?

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use proportions to help make decisions in art, design, and magazine layouts? Give two examples.

4. **a.** Use a computer art program to draw two rectangles whose dimensions are proportional to each other.



b. Print the two rectangles on the same piece of paper.

c. Use a centimeter ruler to measure the length and the width of each rectangle.

d. Find the following ratios. What can you conclude?

$$\frac{\text{Length of larger}}{\text{Length of smaller}} \qquad \frac{\text{Width of larger}}{\text{Width of smaller}}$$

"I love this statue. It seems similar to a big statue I saw in New York."

Practice Use what you learned about similar figures to complete Exercises 4 and 5 on page 74.

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Key Vocabulary
similar figures, *p. 72*

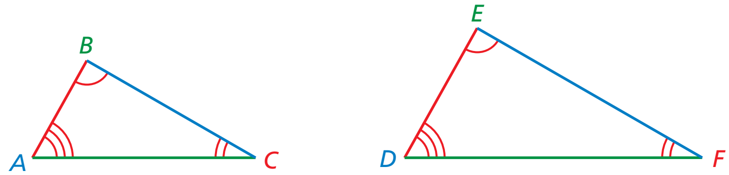
Reading
The symbol ~ means *is similar to*.

Common Error
When writing a similarity statement, make sure to list the vertices of the figures in the correct order.

Key Idea

Similar Figures

Figures that have the same shape but not necessarily the same size are called **similar figures**.



Triangle ABC is similar to Triangle DEF.

Words Two figures are similar when

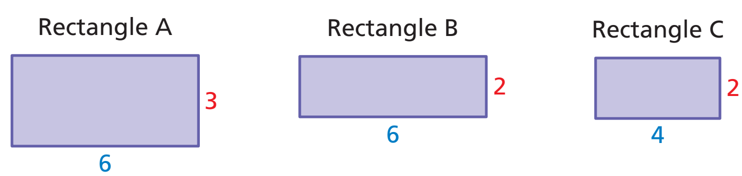
- corresponding side lengths are proportional and
- corresponding angles are congruent.

Symbols

Side Lengths	Angles	Figures
$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\triangle ABC \sim \triangle DEF$

EXAMPLE 1 Identifying Similar Figures

Which rectangle is similar to Rectangle A?



Each figure is a rectangle. So, corresponding angles are congruent. Check to see if corresponding side lengths are proportional.

Rectangle A and Rectangle B

$$\frac{\text{Length of A}}{\text{Length of B}} = \frac{6}{6} = 1 \qquad \frac{\text{Width of A}}{\text{Width of B}} = \frac{3}{2}$$

Not proportional

Rectangle A and Rectangle C

$$\frac{\text{Length of A}}{\text{Length of C}} = \frac{6}{4} = \frac{3}{2} \qquad \frac{\text{Width of A}}{\text{Width of C}} = \frac{3}{2}$$

Proportional

So, Rectangle C is similar to Rectangle A.

On Your Own

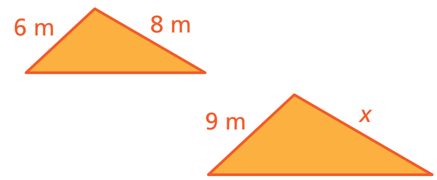
Now You're Ready
Exercises 4–7

1. Rectangle D is 3 units long and 1 unit wide. Which rectangle is similar to Rectangle D?

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Finding an Unknown Measure in Similar Figures

The triangles are similar. Find *x*.



Because the triangles are similar, corresponding side lengths are proportional. So, write and solve a proportion to find x.

$\frac{6}{9} = \frac{8}{x}$ Write a proportion.

$6x = 72$ Cross Products Property

$x = 12$ Divide each side by 6.

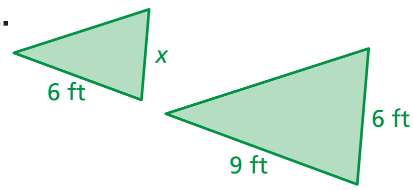
So, x is 12 meters.

On Your Own

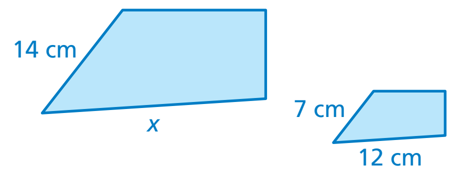
Now You're Ready
Exercises 8–11

The figures are similar. Find *x*.

2.

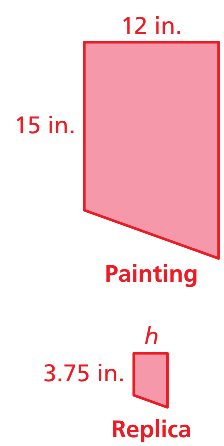


3.



EXAMPLE 3 Real-Life Application

An artist draws a replica of a painting that is on the Berlin Wall. The painting includes a red trapezoid. The shorter base of the similar trapezoid in the replica is 3.75 inches. What is the height *h* of the trapezoid in the replica?



Because the trapezoids are similar, corresponding side lengths are proportional. So, write and solve a proportion to find h.

$\frac{3.75}{15} = \frac{h}{12}$ Write a proportion.

$12 \cdot \frac{3.75}{15} = 12 \cdot \frac{h}{12}$ Multiplication Property of Equality

$3 = h$ Simplify.

So, the height of the trapezoid in the replica is 3 inches.

On Your Own

4. **WHAT IF?** The longer base in the replica is 4.5 inches. What is the length of the longer base in the painting?

2.5 Exercises

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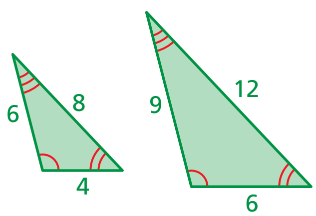
Vocabulary and Concept Check

1. **VOCABULARY** How are corresponding angles of two similar figures related?
2. **VOCABULARY** How are corresponding side lengths of two similar figures related?
3. **CRITICAL THINKING** Are two figures that have the same size and shape similar? Explain.

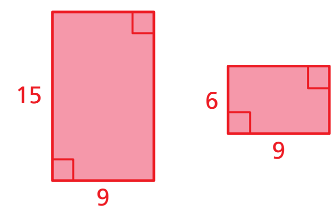
Practice and Problem Solving

Tell whether the two figures are similar. Explain your reasoning.

1 **4.**



5.



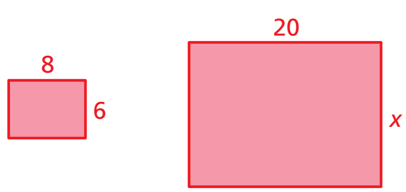
In a coordinate plane, draw the figures with the given vertices. Which figures are similar? Explain your reasoning.

6. Rectangle A: (0, 0), (4, 0), (4, 2), (0, 2)
Rectangle B: (0, 0), (−6, 0), (−6, 3), (0, 3)
Rectangle C: (0, 0), (4, 0), (4, 2), (0, 2)

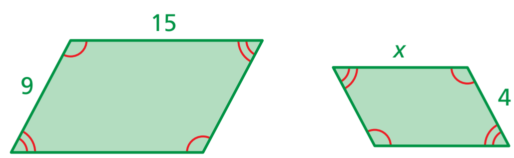
7. Figure A: (−4, 2), (−2, 2), (−2, 0), (−4, 0)
Figure B: (1, 4), (4, 4), (4, 1), (1, 1)
Figure C: (2, −1), (5, −1), (5, −3), (2, −3)

The figures are similar. Find *x*.

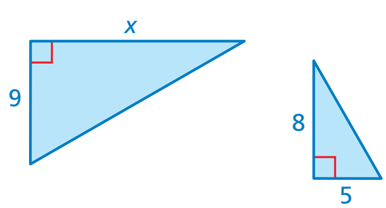
2 **8.**



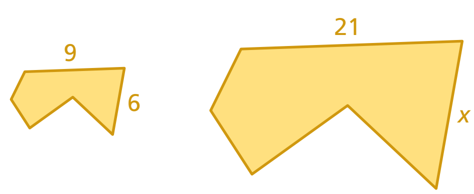
9.



10.



11.



12. **MEXICO** A Mexican flag is 63 inches long and 36 inches wide. Is the drawing at the right similar to the Mexican flag?

13. **DESKS** A student's rectangular desk is 30 inches long and 18 inches wide. The teacher's desk is similar to the student's desk and has a length of 50 inches. What is the width of the teacher's desk?

14. LOGIC Are the following figures *always*, *sometimes*, or *never* similar? Explain.

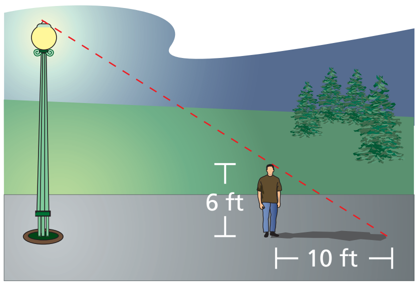
a. two triangles
b. two squares
c. two rectangles
d. a square and a triangle

15. CRITICAL THINKING Can you draw two quadrilaterals each having two 130° angles and two 50° angles that are *not* similar? Justify your answer.

16. SIGN All the angle measures in the sign are 90°.

a. You increase each side length by 20%. Is the new sign similar to the original?

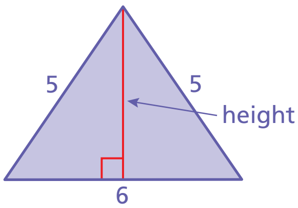
b. You increase each side length by 6 inches. Is the new sign similar to the original?



17. STREETLIGHT A person standing 20 feet from a streetlight casts a shadow as shown. How many times taller is the streetlight than the person? Assume the triangles are similar.

18. REASONING Is an object similar to a scale drawing of the object? Explain.

19. GEOMETRY Use a ruler to draw two different isosceles triangles similar to the one shown. Measure the heights of each triangle to the nearest centimeter.



a. Is the ratio of the corresponding heights proportional to the ratio of the corresponding side lengths?

b. Do you think this is true for all similar triangles? Explain.

20. **Critical Thinking** Given $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle JKL$, is $\triangle ABC \sim \triangle JKL$? Give an example or a non-example.

Fair Game Review What you learned in previous grades & lessons

Simplify. *(Skills Review Handbook)*

21. $\left(\frac{4}{9}\right)^2$ **22.** $\left(\frac{3}{8}\right)^2$ **23.** $\left(\frac{7}{4}\right)^2$ **24.** $\left(\frac{6.5}{2}\right)^2$

25. MULTIPLE CHOICE You solve the equation $S = \ell w + 2wh$ for w. Which equation is correct? *(Section 1.4)*

Ⓐ $w = \dfrac{S - \ell}{2h}$ Ⓑ $w = \dfrac{S - 2h}{\ell}$ Ⓒ $w = \dfrac{S}{\ell + 2h}$ Ⓓ $w = S - \ell - 2h$

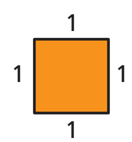
2.6 Perimeters and Areas of Similar Figures

Essential Question How do changes in dimensions of similar geometric figures affect the perimeters and the areas of the figures?

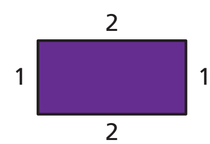
1 ACTIVITY: Creating Similar Figures

Work with a partner. Use pattern blocks to make a figure whose dimensions are 2, 3, and 4 times greater than those of the original figure.

a. Square

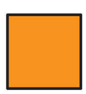


b. Rectangle



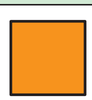
2 ACTIVITY: Finding Patterns for Perimeters

Work with a partner. Copy and complete the table for the perimeter *P* of each figure in Activity 1. Describe the pattern.

Figure	Original Side Lengths	Double Side Lengths	Triple Side Lengths	Quadruple Side Lengths
	$P =$			
	$P =$			

3 ACTIVITY: Finding Patterns for Areas

Work with a partner. Copy and complete the table for the area *A* of each figure in Activity 1. Describe the pattern.

Figure	Original Side Lengths	Double Side Lengths	Triple Side Lengths	Quadruple Side Lengths
	$A =$			
	$A =$			

COMMON CORE

Geometry

In this lesson, you will

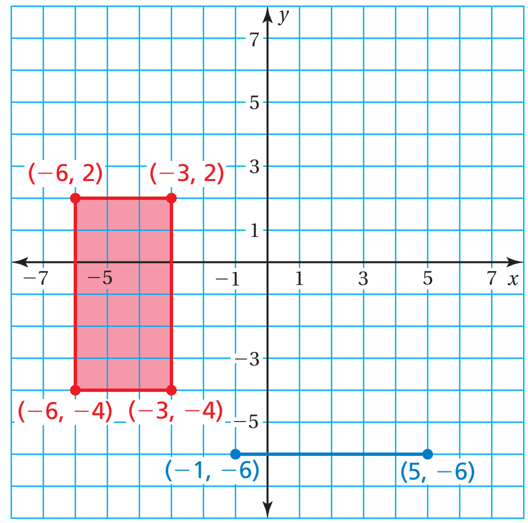
- understand the relationship between perimeters of similar figures.
- understand the relationship between areas of similar figures.
- find ratios of perimeters and areas for similar figures.

Preparing for Standard 8.G.4

4 ACTIVITY: Drawing and Labeling Similar Figures

Work with a partner.

a. Find a blue rectangle that is similar to the red rectangle and has one side from $(-1, -6)$ to $(5, -6)$. Label the vertices.



Check that the two rectangles are similar by showing that the ratios of corresponding sides are equal.

$$\frac{\text{Red Length}}{\text{Blue Length}} \stackrel{?}{=} \frac{\text{Red Width}}{\text{Blue Width}}$$

$$\frac{\text{change in } y}{\text{change in } y} \stackrel{?}{=} \frac{\text{change in } x}{\text{change in } x}$$

$$\frac{\square}{\square} \stackrel{?}{=} \frac{\square}{\square}$$

$$\frac{\square}{\square} \stackrel{?}{=} \frac{\square}{\square}$$

The ratios are equal. So, the rectangles are similar.

Math Practice 1

Analyze Givens

What values should you use to fill in the proportion? Does it matter where each value goes? Explain.

b. Compare the perimeters and the areas of the figures. Are the results the same as your results from Activities 2 and 3? Explain.

c. There are three other blue rectangles that are similar to the red rectangle and have the given side.

- Draw each one. Label the vertices of each.
- Show that each is similar to the original red rectangle.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How do changes in dimensions of similar geometric figures affect the perimeters and the areas of the figures?

6. What information do you need to know to find the dimensions of a figure that is similar to another figure? Give examples to support your explanation.

Practice Use what you learned about perimeters and areas of similar figures to complete Exercises 8 and 9 on page 80.

2.6 Lesson

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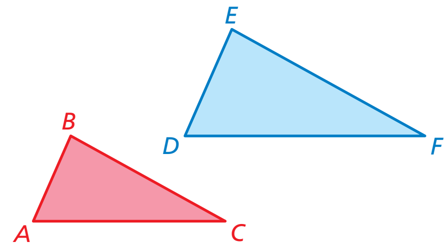
Key Idea

Perimeters of Similar Figures

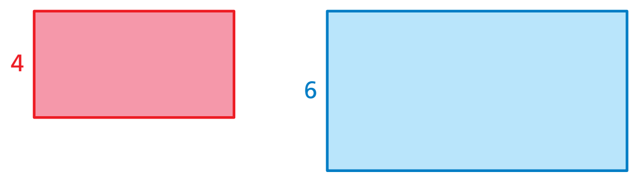
When two figures are similar, the ratio of their perimeters is equal to the ratio of their corresponding side lengths.



$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

EXAMPLE 1 Finding Ratios of Perimeters

Find the ratio (red to blue) of the perimeters of the similar rectangles.



$$\frac{\text{Perimeter of red rectangle}}{\text{Perimeter of blue rectangle}} = \frac{4}{6} = \frac{2}{3}$$

∴ The ratio of the perimeters is $\frac{2}{3}$.

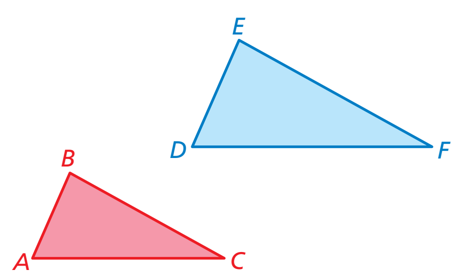
On Your Own

1. The height of Figure A is 9 feet. The height of a similar Figure B is 15 feet. What is the ratio of the perimeter of A to the perimeter of B?

Key Idea

Areas of Similar Figures

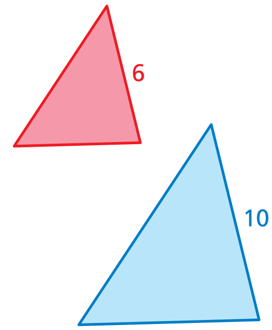
When two figures are similar, the ratio of their areas is equal to the *square* of the ratio of their corresponding side lengths.



$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

EXAMPLE 2 Finding Ratios of Areas

Find the ratio (red to blue) of the areas of the similar triangles.



$$\frac{\text{Area of red triangle}}{\text{Area of blue triangle}} = \left(\frac{6}{10}\right)^2$$

$$= \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

The ratio of the areas is $\frac{9}{25}$.

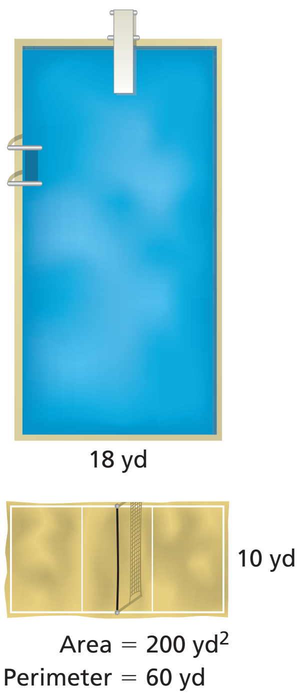
On Your Own

Now You're Ready
Exercises 4–7

2. The base of Triangle P is 8 meters. The base of a similar Triangle Q is 7 meters. What is the ratio of the area of P to the area of Q?

EXAMPLE 3 Using Proportions to Find Perimeters and Areas

A swimming pool is similar in shape to a volleyball court. Find the perimeter *P* and the area *A* of the pool.



The rectangular pool and the court are similar. So, use the ratio of corresponding side lengths to write and solve proportions to find the perimeter and the area of the pool.

Perimeter

$$\frac{\text{Perimeter of court}}{\text{Perimeter of pool}} = \frac{\text{Width of court}}{\text{Width of pool}}$$

$$\frac{60}{P} = \frac{10}{18}$$

$$1080 = 10P$$

$$108 = P$$

Area

$$\frac{\text{Area of court}}{\text{Area of pool}} = \left(\frac{\text{Width of court}}{\text{Width of pool}}\right)^2$$

$$\frac{200}{A} = \left(\frac{10}{18}\right)^2$$

$$\frac{200}{A} = \frac{100}{324}$$

$$64{,}800 = 100A$$

$$648 = A$$

So, the perimeter of the pool is 108 yards, and the area is 648 square yards.

On Your Own

3. **WHAT IF?** The width of the pool is 16 yards. Find the perimeter *P* and the area *A* of the pool.

2.6 Exercises

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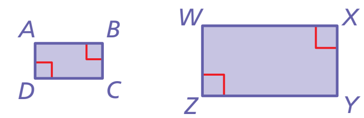
Vocabulary and Concept Check

1. **WRITING** How are the perimeters of two similar figures related?

2. **WRITING** How are the areas of two similar figures related?

3. **NUMBER SENSE** Rectangle *ABCD* is similar to Rectangle *WXYZ*. The area of *ABCD* is 30 square inches. Explain how to find the area of *WXYZ*.

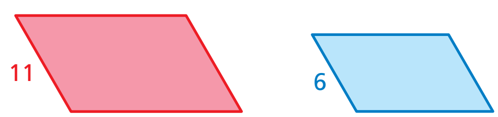


$\frac{AD}{WZ} = \frac{1}{2}$ $\qquad$ $\frac{AB}{WX} = \frac{1}{2}$

Practice and Problem Solving

The two figures are similar. Find the ratios (red to blue) of the perimeters and of the areas.

1 2 4.



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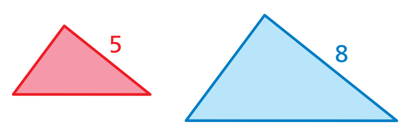

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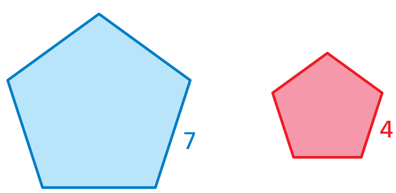

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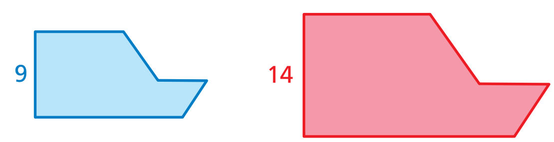


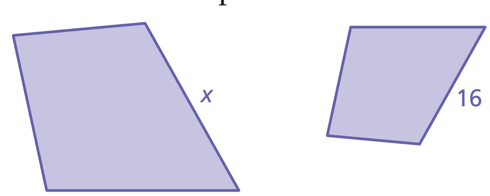
8. **PERIMETER** How does doubling the side lengths of a right triangle affect its perimeter?

9. **AREA** How does tripling the side lengths of a right triangle affect its area?

The figures are similar. Find *x*.

10. The ratio of the perimeters is 7 : 10.

11. The ratio of the perimeters is 8 : 5.



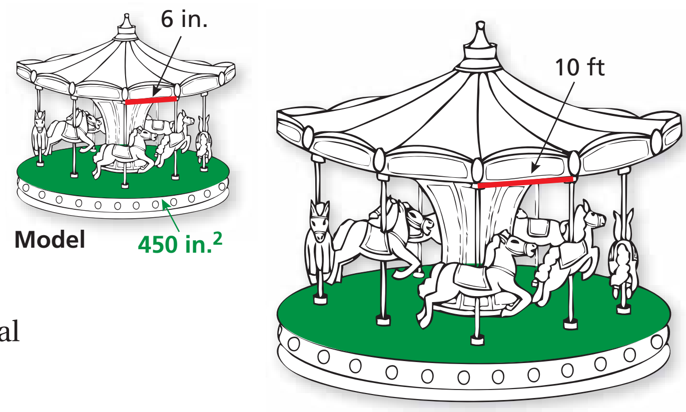
12. **FOOSBALL** The playing surfaces of two foosball tables are similar. The ratio of the corresponding side lengths is 10 : 7. What is the ratio of the areas?

13. **CHEERLEADING** A rectangular school banner has a length of 44 inches, a perimeter of 156 inches, and an area of 1496 square inches. The cheerleaders make signs similar to the banner. The length of a sign is 11 inches. What is its perimeter and its area?

14. REASONING The vertices of two rectangles are $A(-5, -1)$, $B(-1, -1)$, $C(-1, -4)$, $D(-5, -4)$ and $W(1, 6)$, $X(7, 6)$, $Y(7, -2)$, $Z(1, -2)$. Compare the perimeters and the areas of the rectangles. Are the rectangles similar? Explain.

15. SQUARE The ratio of the side length of Square A to the side length of Square B is 4 : 9. The side length of Square A is 12 yards. What is the perimeter of Square B?

16. FABRIC The cost of the fabric is \$1.31. What would you expect to pay for a similar piece of fabric that is 18 inches by 42 inches?



17. AMUSEMENT PARK A scale model of a merry-go-round and the actual merry-go-round are similar.

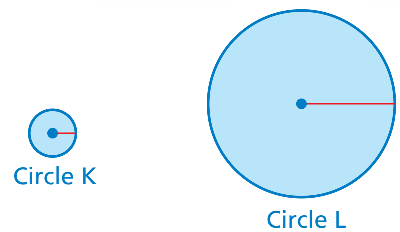
a. How many times greater is the base area of the actual merry-go-round than the base area of the scale model? Explain.

b. What is the base area of the actual merry-go-round in square feet?

18. STRUCTURE The circumference of Circle K is π. The circumference of Circle L is 4π.

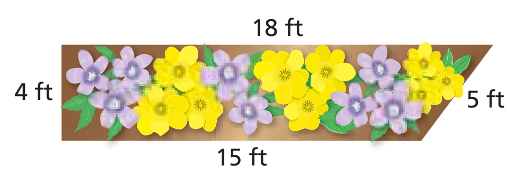
a. What is the ratio of their circumferences? of their radii? of their areas?

b. What do you notice?



19. GEOMETRY A triangle with an area of 10 square meters has a base of 4 meters. A similar triangle has an area of 90 square meters. What is the *height* of the larger triangle?

20. Problem Solving You need two bottles of fertilizer to treat the flower garden shown. How many bottles do you need to treat a similar garden with a perimeter of 105 feet?



Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 1.3)*

21. $4x + 12 = -2x$ **22.** $2b + 6 = 7b - 2$ **23.** $8(4n + 13) = 6n$

24. MULTIPLE CHOICE Last week, you collected 20 pounds of cans for recycling. This week, you collect 25 pounds of cans for recycling. What is the percent of increase? *(Skills Review Handbook)*

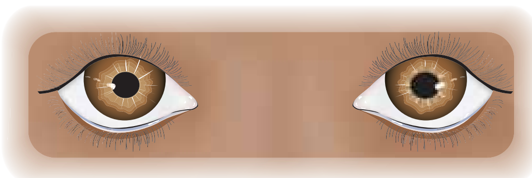
Ⓐ 20% Ⓑ 25% Ⓒ 80% Ⓓ 125%

2.7 Dilations

Essential Question How can you enlarge or reduce a figure in the coordinate plane?

The Meaning of a Word ● Dilate

When you have your eyes checked, the optometrist sometimes **dilates** one or both of the pupils of your eyes.

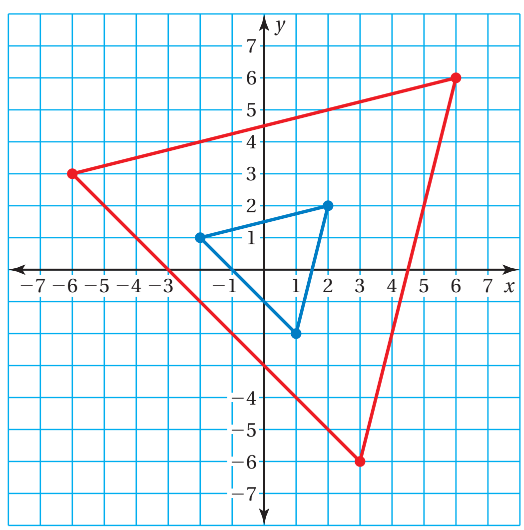


1 ACTIVITY: Comparing Triangles in a Coordinate Plane

Work with a partner. Write the coordinates of the vertices of the blue triangle. Then write the coordinates of the vertices of the red triangle.

a. How are the two sets of coordinates related?

b. How are the two triangles related? Explain your reasoning.



c. Draw a green triangle whose coordinates are twice the values of the corresponding coordinates of the blue triangle. How are the green and blue triangles related? Explain your reasoning.

d. How are the coordinates of the red and green triangles related? How are the two triangles related? Explain your reasoning.

Common Core

Geometry

In this lesson, you will

- identify dilations.
- dilate figures in the coordinate plane.
- use more than one transformation to find images of figures.

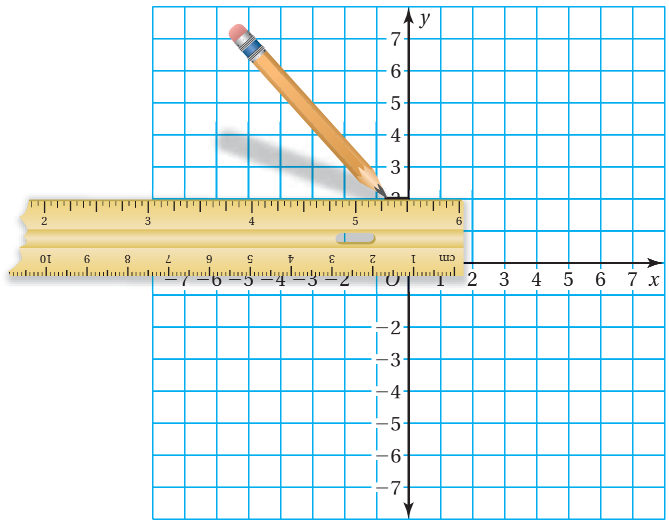
Learning Standards
8.G.3
8.G.4

2 ACTIVITY: Drawing Triangles in a Coordinate Plane

Work with a partner.

a. Draw the triangle whose vertices are $(0, 2)$, $(-2, 2)$, and $(1, -2)$.

b. Multiply each coordinate of the vertices by 2 to obtain three new vertices. Draw the triangle given by the three new vertices. How are the two triangles related?



c. Repeat part (b) by multiplying by 3 instead of 2.

Math Practice 3

Use Prior Results

What are the four types of transformations you studied in this chapter? What information can you use to fill in your table?

3 ACTIVITY: Summarizing Transformations

Work with a partner. Make a table that summarizes the relationships between the original figure and its image for the four types of transformations you studied in this chapter.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you enlarge or reduce a figure in the coordinate plane?

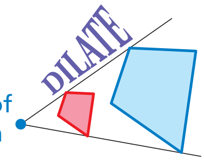
5. Describe how knowing how to enlarge or reduce figures in a technical drawing is important in a career such as drafting.

Practice Use what you learned about dilations to complete Exercises 4–6 on page 87.

2.7 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

A **dilation** is a transformation in which a figure is made larger or smaller with respect to a point called the **center of dilation**.



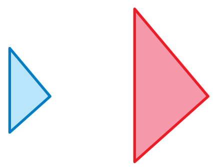
Center of dilation

EXAMPLE 1 Identifying a Dilation

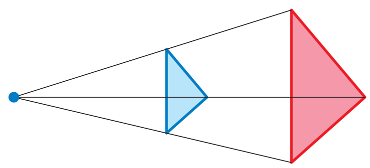
Key Vocabulary
dilation, *p. 84*
center of dilation, *p. 84*
scale factor, *p. 84*

Tell whether the blue figure is a dilation of the red figure.

a.

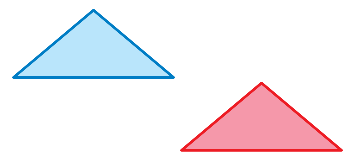


Lines connecting corresponding vertices meet at a point.



So, the blue figure is a dilation of the red figure.

b.



The figures have the same size and shape. The red figure *slides* to form the blue figure.

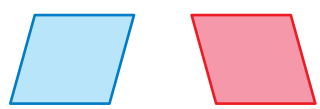
So, the blue figure is *not* a dilation of the red figure. It is a translation.

On Your Own

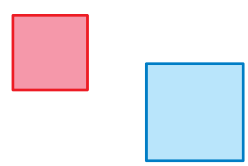
Now You're Ready
Exercises 7–12

Tell whether the blue figure is a dilation of the red figure. Explain.

1.



2.



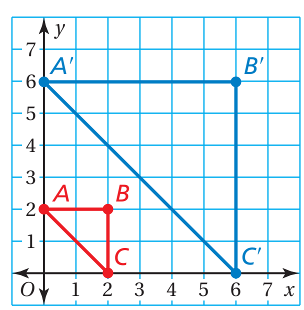
In a dilation, the original figure and its image are similar. The ratio of the side lengths of the image to the corresponding side lengths of the original figure is the **scale factor** of the dilation.

Key Idea

Dilations in the Coordinate Plane

Words To dilate a figure with respect to the origin, multiply the coordinates of each vertex by the scale factor k.

Algebra $(x, y) \rightarrow (kx, ky)$



- When $k > 1$, the dilation is an enlargement.
- When $k > 0$ and $k < 1$, the dilation is a reduction.

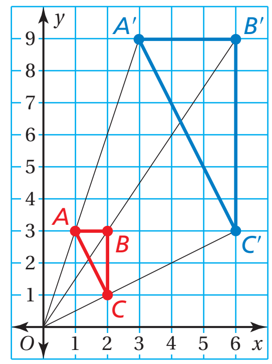
Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Dilating a Figure

Draw the image of Triangle *ABC* after a dilation with a scale factor of 3. Identify the type of dilation.

Multiply each x- and y-coordinate by the scale factor 3.

Vertices of ABC	$(3x, 3y)$	Vertices of $A'B'C'$
$A(1, 3)$	$(3 \cdot 1, 3 \cdot 3)$	$A'(3, 9)$
$B(2, 3)$	$(3 \cdot 2, 3 \cdot 3)$	$B'(6, 9)$
$C(2, 1)$	$(3 \cdot 2, 3 \cdot 1)$	$C'(6, 3)$



The image is shown at the right. The dilation is an *enlargement* because the scale factor is greater than 1.

Study Tip

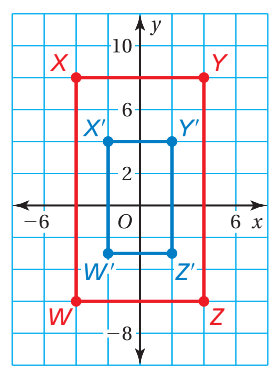
You can check your answer by drawing a line from the origin through each vertex of the original figure. The vertices of the image should lie on these lines.

EXAMPLE 3 Dilating a Figure

Draw the image of Rectangle *WXYZ* after a dilation with a scale factor of 0.5. Identify the type of dilation.

Multiply each x- and y-coordinate by the scale factor 0.5.

Vertices of $WXYZ$	$(0.5x, 0.5y)$	Vertices of $W'X'Y'Z'$
$W(-4, -6)$	$(0.5 \cdot (-4), 0.5 \cdot (-6))$	$W'(-2, -3)$
$X(-4, 8)$	$(0.5 \cdot (-4), 0.5 \cdot 8)$	$X'(-2, 4)$
$Y(4, 8)$	$(0.5 \cdot 4, 0.5 \cdot 8)$	$Y'(2, 4)$
$Z(4, -6)$	$(0.5 \cdot 4, 0.5 \cdot (-6))$	$Z'(2, -3)$



The image is shown at the right. The dilation is a *reduction* because the scale factor is greater than 0 and less than 1.

On Your Own

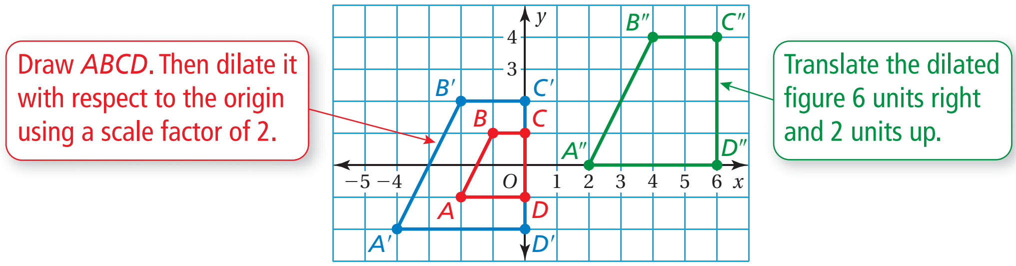
Now You're Ready
Exercises 13–18

3. **WHAT IF?** Triangle *ABC* in Example 2 is dilated by a scale factor of 2. What are the coordinates of the image?

4. **WHAT IF?** Rectangle *WXYZ* in Example 3 is dilated by a scale factor of $\frac{1}{4}$. What are the coordinates of the image?

EXAMPLE 4 Using More than One Transformation

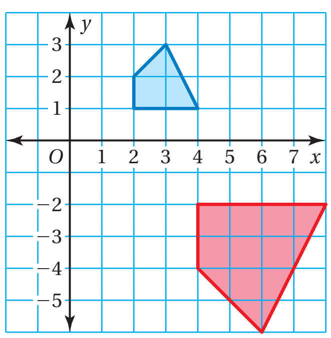
The vertices of a trapezoid are $A(-2, -1)$, $B(-1, 1)$, $C(0, 1)$, and $D(0, -1)$. Dilate the trapezoid with respect to the origin using a scale factor of 2. Then translate it 6 units right and 2 units up. What are the coordinates of the image?



The coordinates of the image are $A''(2, 0)$, $B''(4, 4)$, $C''(6, 4)$, and $D''(6, 0)$.

The image of a translation, reflection, or rotation is congruent to the original figure, and the image of a dilation is similar to the original figure. So, two figures are similar when one can be obtained from the other by a sequence of translations, reflections, rotations, and dilations.

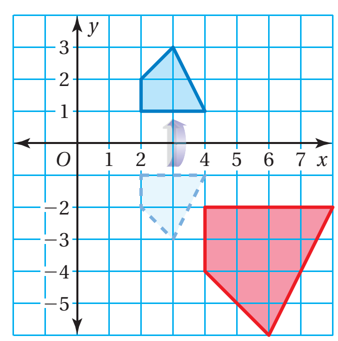
EXAMPLE 5 Describing a Sequence of Transformations



The red figure is similar to the blue figure. Describe a sequence of transformations in which the blue figure is the image of the red figure.

From the graph, you can see that the blue figure is one-half the size of the red figure. So, begin with a dilation with respect to the origin using a scale factor of $\frac{1}{2}$.

After dilating, you need to flip the figure in the x-axis.



So, one possible sequence of transformations is a dilation with respect to the origin using a scale factor of $\frac{1}{2}$ followed by a reflection in the x-axis.

On Your Own

Now You're Ready
Exercises 23–28

5. In Example 4, use a scale factor of 3 in the dilation. Then rotate the figure 180° about the image of vertex C. What are the coordinates of the image?

6. In Example 5, can you reflect the red figure first, and then perform the dilation to obtain the blue figure? Explain.

2.7 Exercises

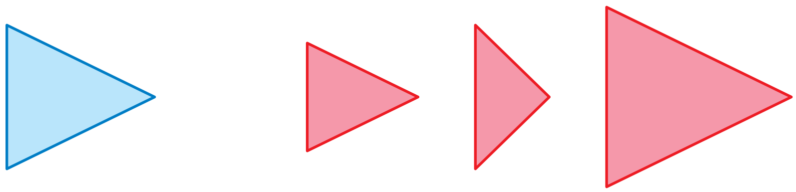
Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** How is a dilation different from other transformations?

2. **VOCABULARY** For what values of scale factor k is a dilation called an *enlargement*? a *reduction*?

3. **REASONING** Which figure is *not* a dilation of the blue figure? Explain.

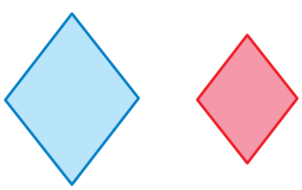


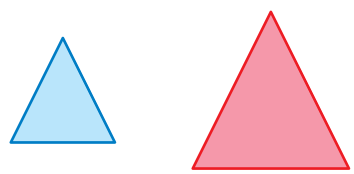
Practice and Problem Solving

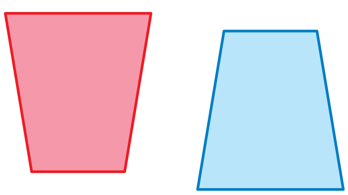
Draw the triangle with the given vertices. Multiply each coordinate of the vertices by 3, and then draw the new triangle. How are the two triangles related?

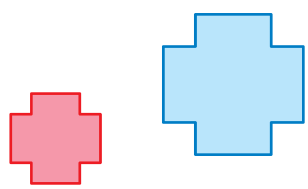
4. $(0, 2), (3, 2), (3, 0)$
5. $(-1, 1), (-1, -2), (2, -2)$
6. $(-3, 2), (1, 2), (1, -4)$

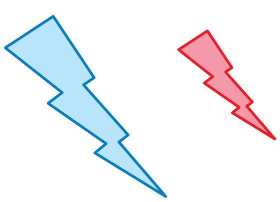
Tell whether the blue figure is a dilation of the red figure.

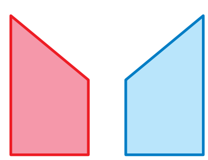
7. 

8. 

9. 

10. 

11. 

12. 

The vertices of a figure are given. Draw the figure and its image after a dilation with the given scale factor. Identify the type of dilation.

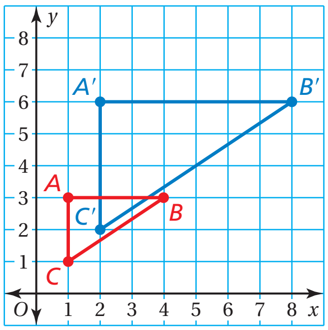
13. $A(1, 1), B(1, 4), C(3, 1); k = 4$
14. $D(0, 2), E(6, 2), F(6, 4); k = 0.5$
15. $G(-2, -2), H(-2, 6), J(2, 6); k = 0.25$
16. $M(2, 3), N(5, 3), P(5, 1); k = 3$
17. $Q(-3, 0), R(-3, 6), T(4, 6), U(4, 0); k = \frac{1}{3}$
18. $V(-2, -2), W(-2, 3), X(5, 3), Y(5, -2); k = 5$

19. ERROR ANALYSIS Describe and correct the error in listing the coordinates of the image after a dilation with a scale factor of $\frac{1}{2}$.

Vertices of ABC	$(2x, 2y)$	Vertices of $A'B'C'$
$A(2, 5)$	$(2 \cdot 2, 2 \cdot 5)$	$A'(4, 10)$
$B(2, 0)$	$(2 \cdot 2, 2 \cdot 0)$	$B'(4, 0)$
$C(4, 0)$	$(2 \cdot 4, 2 \cdot 0)$	$C'(8, 0)$

The blue figure is a dilation of the red figure. Identify the type of dilation and find the scale factor.

20.



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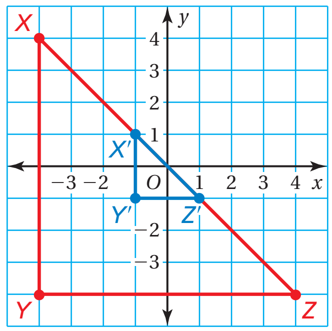

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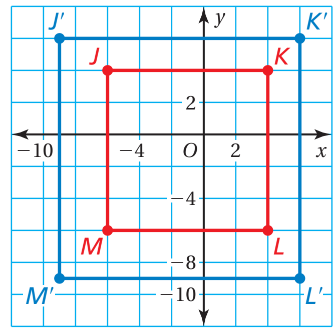


The vertices of a figure are given. Find the coordinates of the figure after the transformations given.

4 **23.** $A(-5, 3)$, $B(-2, 3)$, $C(-2, 1)$, $D(-5, 1)$

Reflect in the y-axis. Then dilate with respect to the origin using a scale factor of 2.

24. $F(-9, -9)$, $G(-3, -6)$, $H(-3, -9)$

Dilate with respect to the origin using a scale factor of $\frac{2}{3}$. Then translate 6 units up.

25. $J(1, 1)$, $K(3, 4)$, $L(5, 1)$

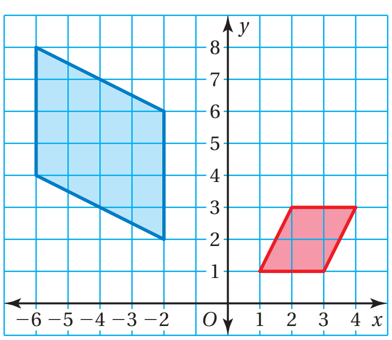
Rotate 90° clockwise about the origin. Then dilate with respect to the origin using a scale factor of 3.

26. $P(-2, 2)$, $Q(4, 2)$, $R(2, -6)$, $S(-4, -6)$

Dilate with respect to the origin using a scale factor of 5. Then dilate with respect to the origin using a scale factor of 0.5.

The red figure is similar to the blue figure. Describe a sequence of transformations in which the blue figure is the image of the red figure.

5 **27.**



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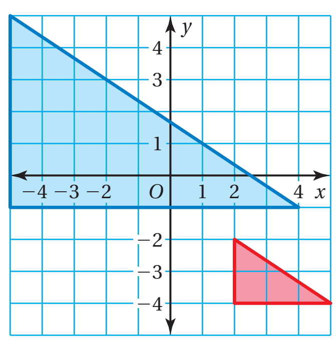


29. STRUCTURE In Exercises 27 and 28, is the blue figure still the image of the red figure when you perform the sequence in the opposite order? Explain.

30. **OPEN-ENDED** Draw a rectangle on a coordinate plane. Choose a scale factor of 2, 3, 4, or 5, and then dilate the rectangle. How many times greater is the area of the image than the area of the original rectangle?

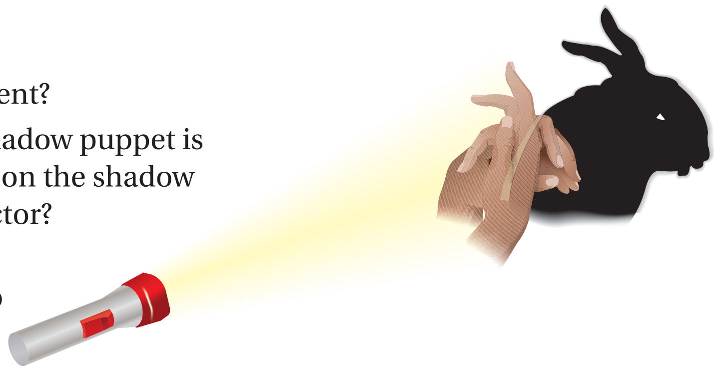
31. **SHADOW PUPPET** You can use a flashlight and a shadow puppet (your hands) to project shadows on the wall.

a. Identify the type of dilation.

b. What does the flashlight represent?

c. The length of the ears on the shadow puppet is 3 inches. The length of the ears on the shadow is 4 inches. What is the scale factor?

d. Describe what happens as the shadow puppet moves closer to the flashlight. How does this affect the scale factor?



32. **REASONING** A triangle is dilated using a scale factor of 3. The image is then dilated using a scale factor of $\frac{1}{2}$. What scale factor could you use to dilate the original triangle to get the final image? Explain.

CRITICAL THINKING The coordinate notation shows how the coordinates of a figure are related to the coordinates of its image after transformations. What are the transformations? Are the figure and its image similar or congruent? Explain.

33. $(x, y) \rightarrow (2x + 4, 2y - 3)$

34. $(x, y) \rightarrow (-x - 1, y - 2)$

35. $(x, y) \rightarrow \left(\frac{1}{3}x, -\frac{1}{3}y\right)$

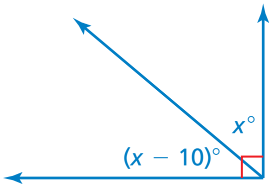
36. **STRUCTURE** How are the transformations $(2x + 3, 2y - 1)$ and $(2(x + 3), 2(y + 1))$ different?

37. **Problem Solving** The vertices of a trapezoid are $A(-2, 3)$, $B(2, 3)$, $C(5, -2)$, and $D(-2, -2)$. Dilate the trapezoid with respect to vertex A using a scale factor of 2. What are the coordinates of the image? Explain the method you used.

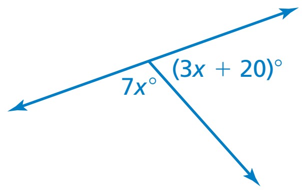
Fair Game Review What you learned in previous grades & lessons

Tell whether the angles are *complementary* or *supplementary*. Then find the value of *x*. *(Skills Review Handbook)*

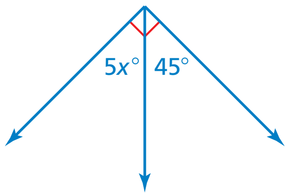
38.



39.



40.



41. **MULTIPLE CHOICE** Which quadrilateral is *not* a parallelogram? *(Skills Review Handbook)*

Ⓐ rhombus Ⓑ trapezoid Ⓒ square Ⓓ rectangle

2.5–2.7 Quiz

Check It Out
Progress Check
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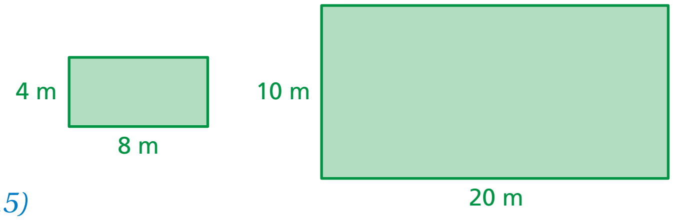
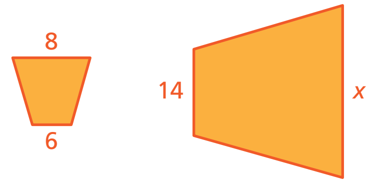
1. Tell whether the two rectangles are similar. Explain your reasoning. *(Section 2.5)*

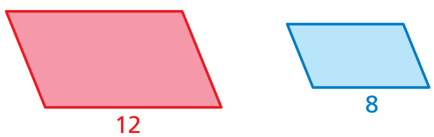


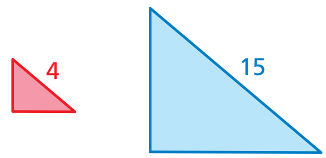
The figures are similar. Find *x*. *(Section 2.5)*

2. 

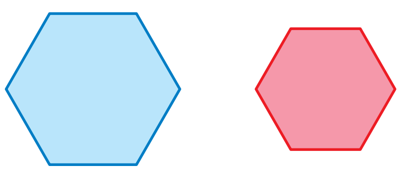
3. 

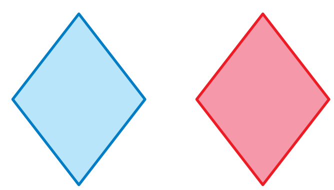
The two figures are similar. Find the ratios (red to blue) of the perimeters and of the areas. *(Section 2.6)*

4. 

5. 

Tell whether the blue figure is a dilation of the red figure. *(Section 2.7)*

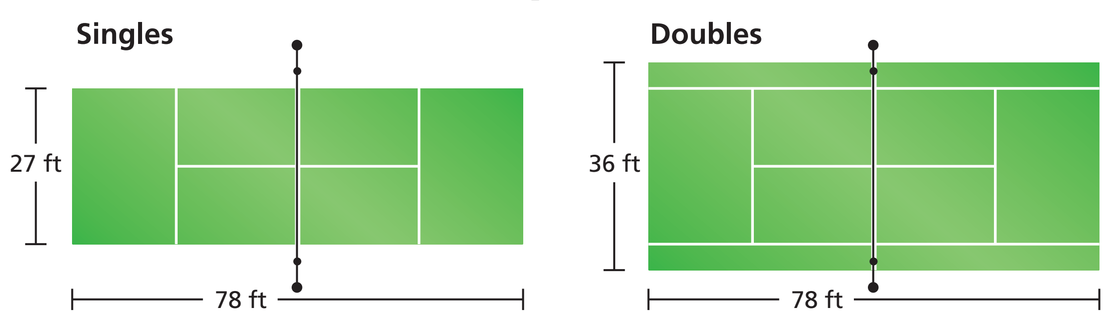
6. 

7. 

8. **SCREENS** The TV screen is similar to the computer screen. What is the area of the TV screen? *(Section 2.6)*

9. **GEOMETRY** The vertices of a rectangle are $A(2, 4)$, $B(5, 4)$, $C(5, -1)$, and $D(2, -1)$. Dilate the rectangle with respect to the origin using a scale factor of $\frac{1}{2}$. Then translate it 4 units left and 3 units down. What are the coordinates of the image? *(Section 2.7)*

10. **TENNIS COURT** The tennis courts for singles and doubles matches are different sizes. Are the courts similar? Explain. *(Section 2.5)*



2 Chapter Review

Check It Out
Vocabulary Help
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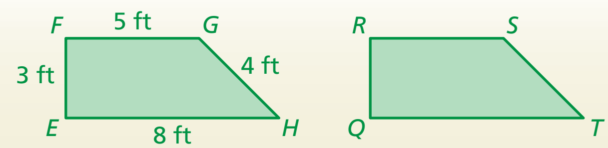
Review Key Vocabulary

congruent figures, *p. 44*
corresponding angles, *p. 44*
corresponding sides, *p. 44*
transformation, *p. 50*
image, *p. 50*
translation, *p. 50*
reflection, *p. 56*
line of reflection, *p. 56*
rotation, *p. 62*
center of rotation, *p. 62*
angle of rotation, *p. 62*
similar figures, *p. 72*
dilation, *p. 84*
center of dilation, *p. 84*
scale factor, *p. 84*

Review Examples and Exercises

2.1 Congruent Figures *(pp. 42–47)*

Trapezoids *EFGH* and *QRST* are congruent.



a. What is the length of side *QT*?

Side *QT* corresponds to side *EH*.

So, the length of side *QT* is 8 feet.

b. Which angle of *QRST* corresponds to $\angle H$?

$\angle T$ corresponds to $\angle H$.

Exercises

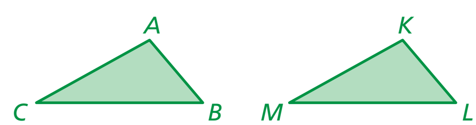
Use the figures above.

1. What is the length of side *QR*?

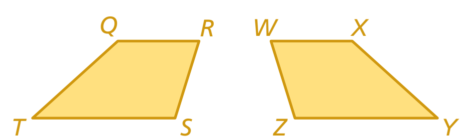
2. What is the perimeter of *QRST*?

The figures are congruent. Name the corresponding angles and the corresponding sides.

3.

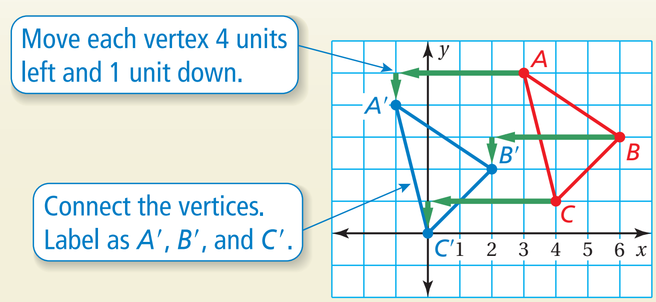


4.



2.2 Translations *(pp. 48–53)*

Translate the red triangle 4 units left and 1 unit down. What are the coordinates of the image?

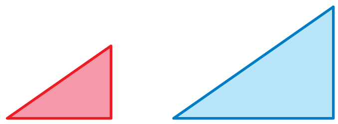


The coordinates of the image are $A'(-1, 4)$, $B'(2, 2)$, and $C'(0, 0)$.

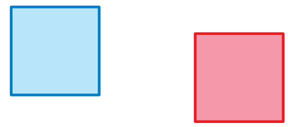
Exercises

Tell whether the blue figure is a translation of the red figure.

5.



6.

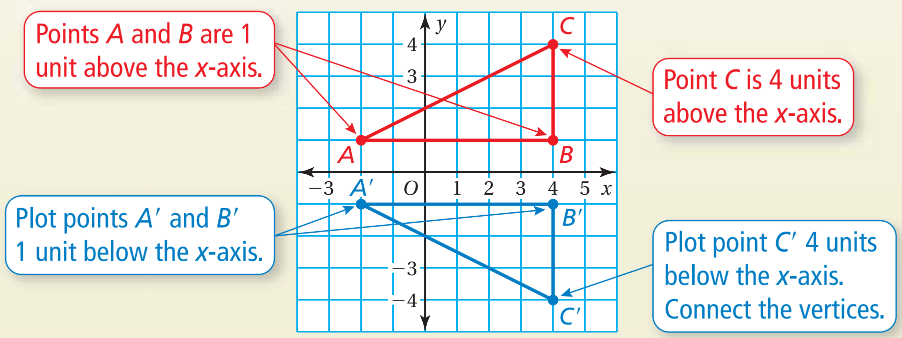


7. The vertices of a quadrilateral are $W(1, 2)$, $X(1, 4)$, $Y(4, 4)$, and $Z(4, 2)$. Draw the figure and its image after a translation 3 units left and 2 units down.

8. The vertices of a triangle are $A(-1, -2)$, $B(-2, 2)$, and $C(-3, 0)$. Draw the figure and its image after a translation 5 units right and 1 unit up.

2.3 Reflections *(pp. 54–59)*

The vertices of a triangle are $A(-2, 1)$, $B(4, 1)$, and $C(4, 4)$. Draw the figure and its reflection in the x-axis. What are the coordinates of the image?

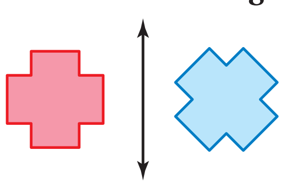


The coordinates of the image are $A'(-2, -1)$, $B'(4, -1)$, and $C'(4, -4)$.

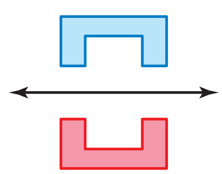
Exercises

Tell whether the blue figure is a reflection of the red figure.

9.



10.



Draw the figure and its reflection in (a) the x-axis and (b) the y-axis.

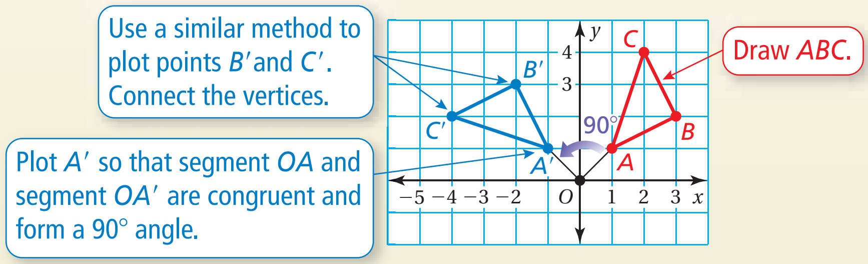
11. $A(2, 0)$, $B(1, 5)$, $C(4, 3)$

12. $D(-5, -5)$, $E(-5, -1)$, $F(-2, -2)$, $G(-2, -5)$

13. The vertices of a rectangle are $E(-1, 1)$, $F(-1, 3)$, $G(-5, 3)$, and $H(-5, 1)$. Find the coordinates of the figure after reflecting in the x-axis, and then translating 3 units right.

2.4 Rotations (pp. 60–67)

The vertices of a triangle are $A(1, 1)$, $B(3, 2)$, and $C(2, 4)$. Rotate the triangle 90° counterclockwise about the origin. What are the coordinates of the image?

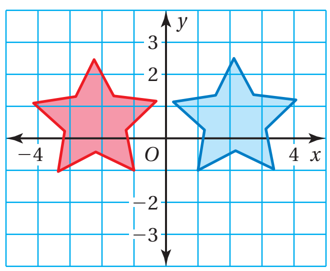


The coordinates of the image are $A'(-1, 1)$, $B'(-2, 3)$, and $C'(-4, 2)$.

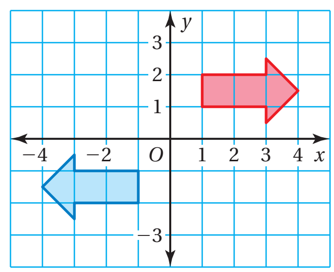
Exercises

Tell whether the blue figure is a rotation of the red figure about the origin. If so, give the angle and the direction of rotation.

14.



15.



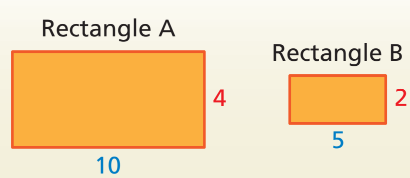
The vertices of a triangle are $A(-4, 2)$, $B(-2, 2)$, and $C(-3, 4)$. Rotate the triangle about the origin as described. Find the coordinates of the image.

16. 180°

17. 270° clockwise

2.5 Similar Figures (pp. 70–75)

a. Is Rectangle A similar to Rectangle B?



Each figure is a rectangle. So, corresponding angles are congruent. Check to see if corresponding side lengths are proportional.

$$\frac{\text{Length of A}}{\text{Length of B}} = \frac{10}{5} = 2 \qquad \frac{\text{Width of A}}{\text{Width of B}} = \frac{4}{2} = 2$$

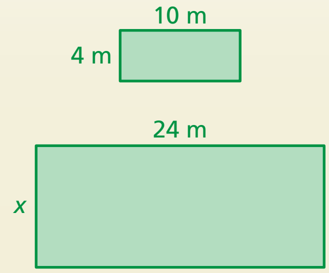
Proportional

So, Rectangle A is similar to Rectangle B.

b. The two rectangles are similar. Find x.

Because the rectangles are similar, corresponding side lengths are proportional. So, write and solve a proportion to find x.

$\dfrac{10}{24} = \dfrac{4}{x}$	Write a proportion.
$10x = 96$	Cross Products Property
$x = 9.6$	Divide each side by 10.

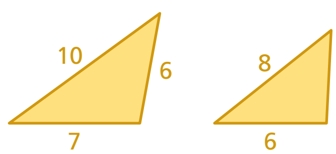


So, x is 9.6 meters.

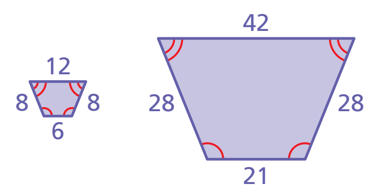
Exercises

Tell whether the two figures are similar. Explain your reasoning.

18.

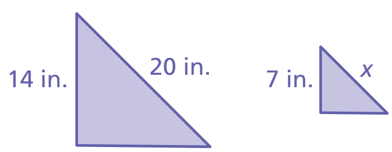


19.

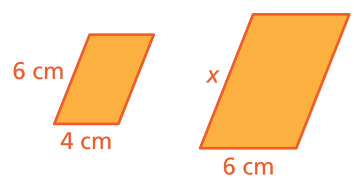


The figures are similar. Find x.

20.



21.

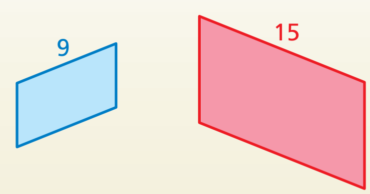


2.6 Perimeters and Areas of Similar Figures *(pp. 76–81)*

a. Find the ratio (red to blue) of the perimeters of the similar parallelograms.

$$\frac{\text{Perimeter of red parallelogram}}{\text{Perimeter of blue parallelogram}} = \frac{15}{9}$$

$$= \frac{5}{3}$$

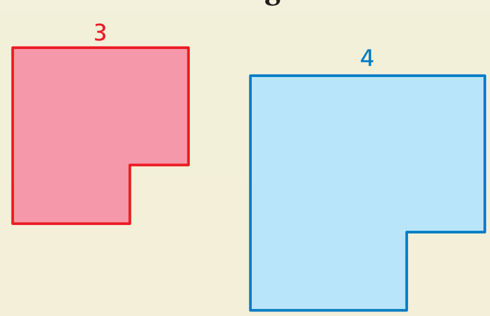


The ratio of the perimeters is $\frac{5}{3}$.

b. Find the ratio (red to blue) of the areas of the similar figures.

$$\frac{\text{Area of red figure}}{\text{Area of blue figure}} = \left(\frac{3}{4}\right)^2$$

$$= \frac{9}{16}$$

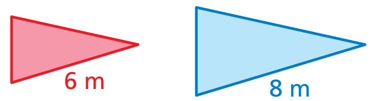


The ratio of the areas is $\frac{9}{16}$.

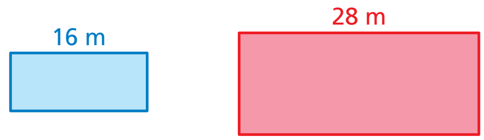
Exercises

The two figures are similar. Find the ratios (red to blue) of the perimeters and of the areas.

22.



23.



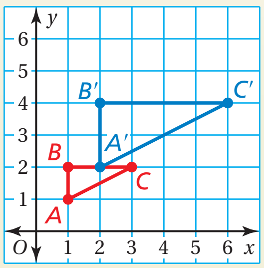
24. **PHOTOS** Two photos are similar. The ratio of the corresponding side lengths is 3 : 4. What is the ratio of the areas?

2.7 Dilations *(pp. 82–89)*

Draw the image of Triangle *ABC* after a dilation with a scale factor of 2. Identify the type of dilation.

Multiply each *x*- and *y*-coordinate by the scale factor 2.

Vertices of *ABC*	$(2x, 2y)$	Vertices of *A′B′C′*
$A(1, 1)$	$(2 \cdot 1, 2 \cdot 1)$	$A'(2, 2)$
$B(1, 2)$	$(2 \cdot 1, 2 \cdot 2)$	$B'(2, 4)$
$C(3, 2)$	$(2 \cdot 3, 2 \cdot 2)$	$C'(6, 4)$

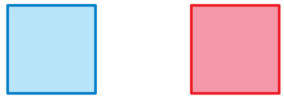


The image is shown at the above right. The dilation is an *enlargement* because the scale factor is greater than 1.

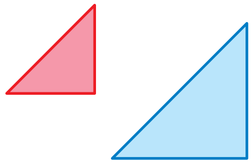
Exercises

Tell whether the blue figure is a dilation of the red figure.

25.



26.



The vertices of a figure are given. Draw the figure and its image after a dilation with the given scale factor. Identify the type of dilation.

27. $P(-3, -2)$, $Q(-3, 0)$, $R(0, 0)$; $k = 4$

28. $B(3, 3)$, $C(3, 6)$, $D(6, 6)$, $E(6, 3)$; $k = \frac{1}{3}$

29. The vertices of a rectangle are $Q(-6, 2)$, $R(6, 2)$, $S(6, -4)$, and $T(-6, -4)$. Dilate the rectangle with respect to the origin using a scale factor of $\frac{3}{2}$. Then translate it 5 units right and 1 unit down. What are the coordinates of the image?

2 Chapter Test

Check It Out
Test Practice
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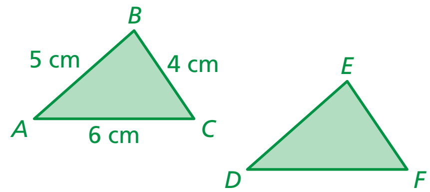
Triangles *ABC* and *DEF* are congruent.

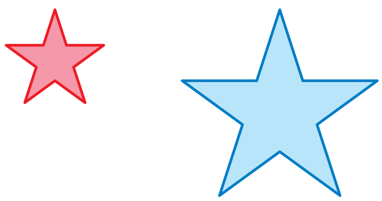
1. Which angle of *DEF* corresponds to $\angle C$?

2. What is the perimeter of *DEF*?

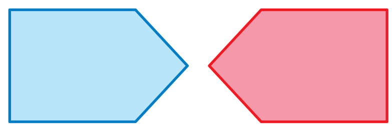


Tell whether the blue figure is a *translation*, *reflection*, *rotation*, or *dilation* of the red figure.

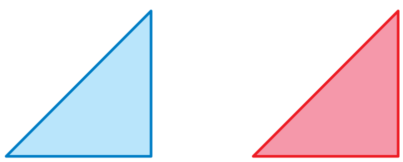
3.



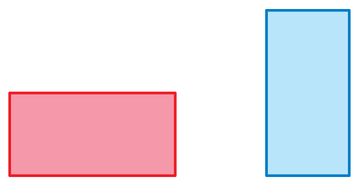
4.



5.



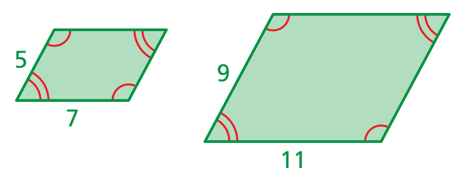
6.



7. The vertices of a triangle are $A(2, 5)$, $B(1, 2)$, and $C(3, 1)$. Reflect the triangle in the x-axis, and then rotate the triangle 90° counterclockwise about the origin. What are the coordinates of the image?

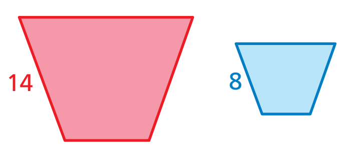
8. The vertices of a triangle are $A(2, 4)$, $B(2, 1)$, and $C(5, 1)$. Dilate the triangle with respect to the origin using a scale factor of 2. Then translate the triangle 2 units left and 1 unit up. What are the coordinates of the image?

9. Tell whether the parallelograms are similar. Explain your reasoning.

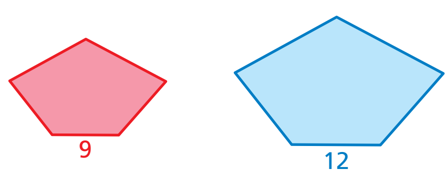


The two figures are similar. Find the ratios (red to blue) of the perimeters and of the areas.

10.

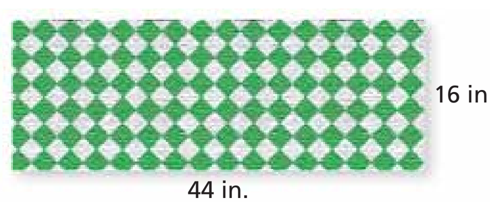


11.



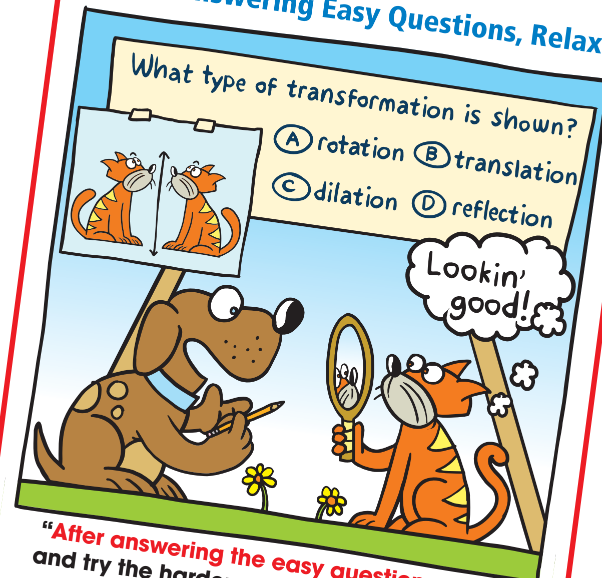
12. SCREENS A wide-screen television measures 36 inches by 54 inches. A movie theater screen measures 42 feet by 63 feet. Are the screens similar? Explain.

13. CURTAINS You want to use the rectangular piece of fabric shown to make a set of curtains for your window. Name the types of congruent shapes you can make with one straight cut. Draw an example of each type.



2 Standards Assessment

Test-Taking Strategy
After Answering Easy Questions, Relax



"After answering the easy questions, relax and try the harder ones. For this, the image is flipped. So, it's D."

1. A clockwise rotation of 90° is equivalent to a counterclockwise rotation of how many degrees? *(8.G.2)*

2. The formula $K = C + 273.15$ converts temperatures from Celsius C to Kelvin K. Which of the following formulas is *not* correct? *(8.EE.7a)*

A. $K - C = 273.15$

B. $C = K - 273.15$

C. $C - K = -273.15$

D. $C = K + 273.15$

3. Joe wants to solve the equation $-3(x + 2) = 12x$. What should he do first? *(8.EE.7a)*

F. Subtract 2 from each side.

G. Add 3 to each side.

H. Multiply each side by -3.

I. Divide each side by -3.

4. Which transformation *turns* a figure? *(8.G.1)*

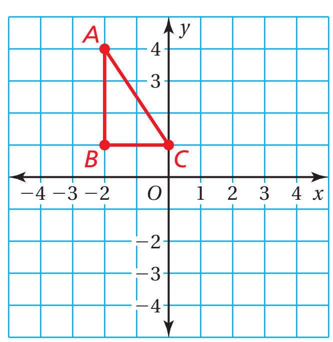
A. translation

B. reflection

C. rotation

D. dilation

5. A triangle is graphed in the coordinate plane below.



Translate the triangle 3 units right and 2 units down. What are the coordinates of the image? *(8.G.3)*

F. $A'(1, 4)$, $B'(1, 1)$, $C'(3, 1)$

G. $A'(1, 2)$, $B'(1, -1)$, $C'(3, -1)$

H. $A'(-2, 2)$, $B'(-2, -1)$, $C'(0, -1)$

I. $A'(0, 1)$, $B'(0, -2)$, $C'(2, -2)$

6. Dale solved the equation in the box shown. What should Dale do to correct the error that he made? *(8.EE.7b)*

$$-\frac{x}{3} + \frac{2}{5} = -\frac{7}{15}$$
$$-\frac{x}{3} + \frac{2}{5} - \frac{2}{5} = -\frac{7}{15} - \frac{2}{5}$$
$$-\frac{x}{3} = -\frac{13}{15}$$
$$3 \cdot \left(-\frac{x}{3}\right) = 3 \cdot \left(-\frac{13}{15}\right)$$
$$x = -2\frac{3}{5}$$

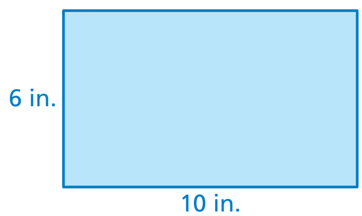
A. Add $\frac{2}{5}$ to each side to get $-\frac{x}{3} = -\frac{1}{15}$.

B. Multiply each side by -3 to get $x + \frac{2}{5} = \frac{7}{5}$.

C. Multiply each side by -3 to get $x = 2\frac{3}{5}$.

D. Subtract $\frac{2}{5}$ from each side to get $-\frac{x}{3} = -\frac{5}{10}$.

7. Jenny dilates the rectangle below using a scale factor of $\frac{1}{2}$.



What is the area of the dilated rectangle in square inches? *(8.G.4)*

8. The vertices of a rectangle are $A(-4, 2)$, $B(3, 2)$, $C(3, -5)$, and $D(-4, -5)$. If the rectangle is dilated by a scale factor of 3, what will be the coordinates of vertex C'? *(8.G.3)*

F. $(9, -15)$

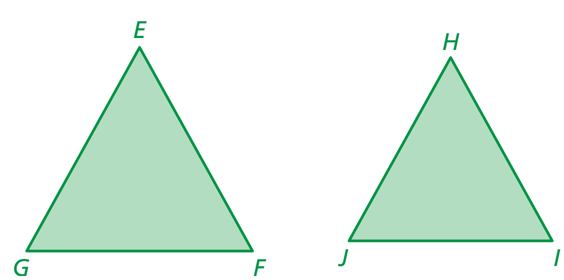
G. $(-12, 6)$

H. $(-12, -15)$

I. $(9, 6)$

9. In the figures, Triangle *EFG* is a dilation of Triangle *HIJ*.

Which proportion is *not* necessarily correct for Triangle *EFG* and Triangle *HIJ*? *(8.G.4)*



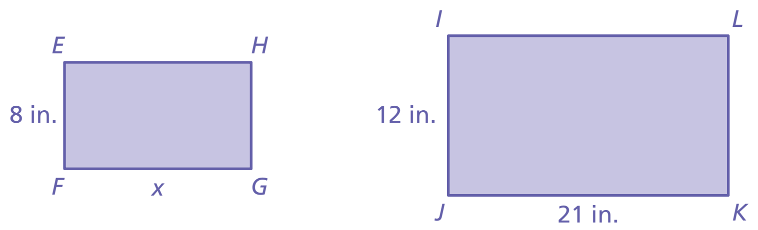
A. $\frac{EF}{FG} = \frac{HI}{IJ}$

B. $\frac{EG}{HI} = \frac{FG}{IJ}$

C. $\frac{GE}{EF} = \frac{JH}{HI}$

D. $\frac{EF}{HI} = \frac{GE}{JH}$

10. In the figures below, Rectangle *EFGH* is a dilation of Rectangle *IJKL*.



What is x? *(8.G.4)*

F. 14 in.

G. 15 in.

H. 16 in.

I. 17 in.

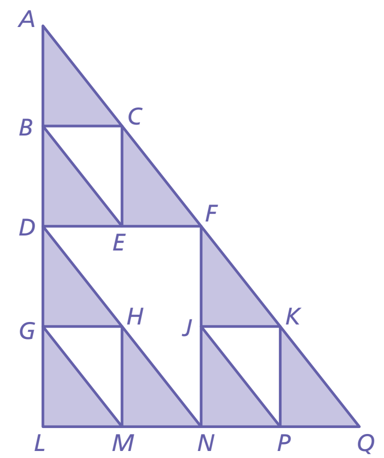
11. Several transformations are used to create the pattern. *(8.G.2, 8.G.4)*

Think Solve Explain

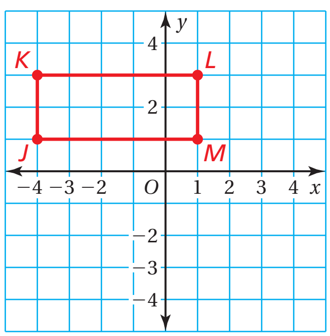
Part A Describe the transformation of Triangle *GLM* to Triangle *DGH*.

Part B Describe the transformation of Triangle *ALQ* to Triangle *GLM*.

Part C Triangle *DFN* is a dilation of Triangle *GHM*. Find the scale factor.



12. A rectangle is graphed in the coordinate plane below.



Rotate the triangle 180° about the origin. What are the coordinates of the image? *(8.G.3)*

A. $J'(4, -1)$, $K'(4, -3)$, $L'(-1, -3)$, $M'(-1, -1)$

B. $J'(-4, -1)$, $K'(-4, -3)$, $L'(1, -3)$, $M'(1, -1)$

C. $J'(1, 4)$, $K'(3, 4)$, $L'(3, -1)$, $M'(1, -1)$

D. $J'(-4, 1)$, $K'(-4, 3)$, $L'(1, 3)$, $M'(1, 1)$