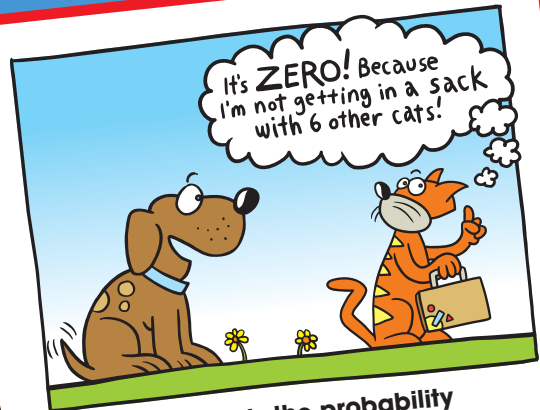


15 Probability and Statistics

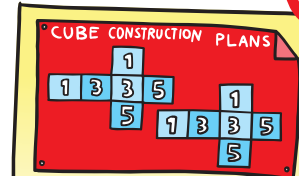
- 15.1 Outcomes and Events
- 15.2 Probability
- 15.3 Experimental and Theoretical Probability
- 15.4 Compound Events
- 15.5 Independent and Dependent Events
- 15.6 Samples and Populations
- 15.7 Comparing Populations



"If there are 7 cats in a sack and I draw one at random,..."



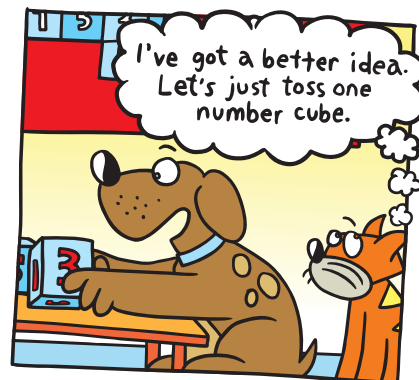
"... what is the probability that I will draw you?"



"I'm just about finished making my two number cubes."



"Now, here's how the game works. You toss the two cubes."



"If the sum is even, I win. If it's odd, you win."

What You Learned Before

● Writing Ratios (6.RP.1)

Example 1 There are 32 football players and 16 cheerleaders at your school. Write the ratio of cheerleaders to football players.

$$\begin{array}{l} \text{cheerleaders} \rightarrow \frac{16}{32} = \frac{1}{2} \\ \text{football players} \rightarrow \end{array} \quad \text{Write in simplest form.}$$

❖ So, the ratio of cheerleaders to football players is $\frac{1}{2}$.

Example 2

a. Write the ratio of girls to boys in Classroom A.

$$\frac{\text{Girls in Classroom A}}{\text{Boys in Classroom A}} = \frac{11}{14}$$

❖ So, the ratio of girls to boys in Classroom A is $\frac{11}{14}$.

| | Boys | Girls |
|-------------|------|-------|
| Classroom A | 14 | 11 |
| Classroom B | 12 | 8 |

b. Write the ratio of boys in Classroom B to the total number of students in both classes.

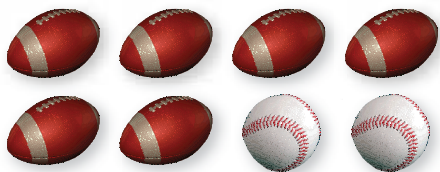
$$\frac{\text{Boys in Classroom B}}{\text{Total number of students}} = \frac{12}{14 + 11 + 12 + 8} = \frac{12}{45} = \frac{4}{15}$$

Write in simplest form.

❖ So, the ratio of boys in Classroom B to the total number of students is $\frac{4}{15}$.

Try It Yourself

Write the ratio in simplest form.

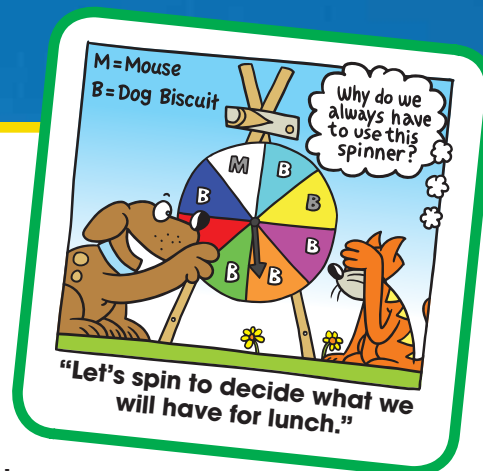
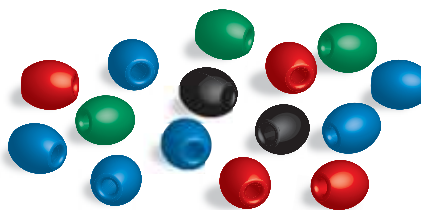


- baseballs to footballs
- footballs to total pieces of equipment



- sneakers to ballet slippers
- sneakers to total number of shoes

- green beads to blue beads
- red beads : green beads
- green beads : total number of beads



15.1 Outcomes and Events

Essential Question In an experiment, how can you determine the number of possible results?

An *experiment* is an investigation or a procedure that has varying results. Flipping a coin, rolling a number cube, and spinning a spinner are all examples of experiments.

1 ACTIVITY: Conducting Experiments

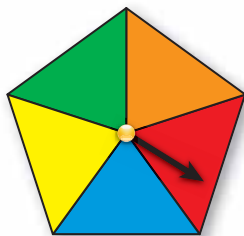
Work with a partner.

- a. You flip a dime.

There are possible results.

Out of 20 flips, you think you will flip heads times.

Flip a dime 20 times. Tally your results in a table. How close was your guess?



- b. You spin the spinner shown.

There are possible results.

Out of 20 spins, you think you will spin orange times.

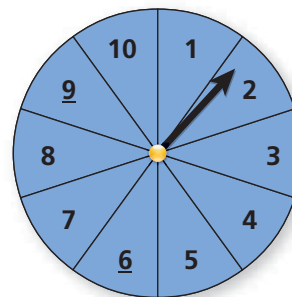
Spin the spinner 20 times. Tally your results in a table. How close was your guess?

- c. You spin the spinner shown.

There are possible results.

Out of 20 spins, you think you will spin a 4 times.

Spin the spinner 20 times. Tally your results in a table. How close was your guess?



COMMON CORE

Probability and Statistics

In this lesson, you will

- identify and count the outcomes of experiments.

Preparing for Standard 7.SP.5

2 ACTIVITY: Comparing Different Results

Work with a partner. Use the spinner in Activity 1(c).

- a. Do you have a better chance of spinning an even number or a multiple of 4? Explain your reasoning.
- b. Do you have a better chance of spinning an even number or an odd number? Explain your reasoning.

3 ACTIVITY: Rock Paper Scissors

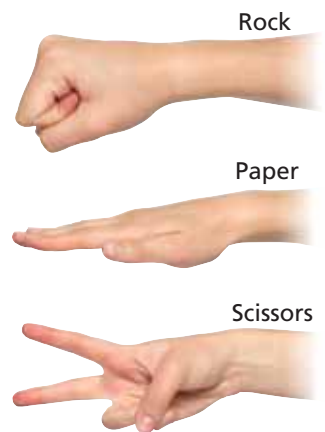
Math Practice 1

Interpret a Solution







How do your results compare to the possible results? Explain.

Work with a partner.

- Play Rock Paper Scissors 30 times. Tally your results in the table.
- How many possible results are there?
- Of the possible results, in how many ways can Player A win? Player B win? the players tie?
- Does one of the players have a better chance of winning than the other player? Explain your reasoning.



Rock *breaks* scissors.
Paper *covers* rock.
Scissors *cut* paper.

| | | Player A | | |
|----------|---|---|--|---|
| | |  |  |  |
| Player B |  | | | |
| |  | | | |
| |  | | | |

What Is Your Answer?

- IN YOUR OWN WORDS** In an experiment, how can you determine the number of possible results?

Practice

Use what you learned about experiments to complete Exercises 3 and 4 on page 636.

Key Vocabulary

experiment, p. 634
outcomes, p. 634
event, p. 634
favorable outcomes,
p. 634

Reading

When an experiment is performed *at random* or *randomly*, all of the possible outcomes are equally likely.

Key Ideas

Outcomes and Events

An **experiment** is an investigation or a procedure that has varying results. The possible results of an experiment are called **outcomes**. A collection of one or more outcomes is an **event**. The outcomes of a specific event are called **favorable outcomes**.

For example, randomly selecting a marble from a group of marbles is an experiment. Each marble in the group is an outcome. Selecting a green marble from the group is an event.

Possible outcomes



Event: Choosing a green marble

Number of favorable outcomes: 2



EXAMPLE 1 Identifying Outcomes



You roll the number cube.

a. What are the possible outcomes?

- The six possible outcomes are rolling a 1, 2, 3, 4, 5, and 6.

b. What are the favorable outcomes of rolling an even number?

| even | not even |
|---------|----------|
| 2, 4, 6 | 1, 3, 5 |

- The favorable outcomes of the event are rolling a 2, 4, and 6.

c. What are the favorable outcomes of rolling a number greater than 5?

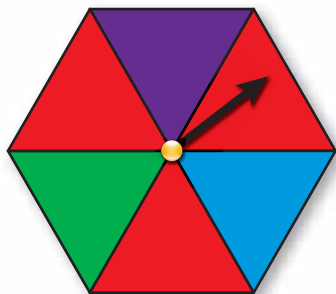
| greater than 5 | not greater than 5 |
|----------------|--------------------|
| 6 | 1, 2, 3, 4, 5 |

- The favorable outcome of the event is rolling a 6.

On Your Own

- You randomly choose a letter from a hat that contains the letters A through K.
 - What are the possible outcomes?
 - What are the favorable outcomes of choosing a vowel?

EXAMPLE 2 Counting Outcomes



You spin the spinner.

- How many possible outcomes are there?

The spinner has 6 sections. So, there are 6 possible outcomes.

- In how many ways can spinning red occur?

The spinner has 3 red sections. So, spinning red can occur in 3 ways.

- In how many ways can spinning *not* purple occur? What are the favorable outcomes of spinning *not* purple?

The spinner has 5 sections that are *not* purple. So, spinning *not* purple can occur in 5 ways.

| purple | <i>not</i> purple |
|--------|----------------------------|
| purple | red, red, red, green, blue |

The favorable outcomes of the event are red, red, red, green, and blue.

On Your Own

- You randomly choose a marble.



- How many possible outcomes are there?
- In how many ways can choosing blue occur?
- In how many ways can choosing *not* yellow occur? What are the favorable outcomes of choosing *not* yellow?

Vocabulary and Concept Check

- VOCABULARY** Is rolling an even number on a number cube an *outcome* or an *event*? Explain.
- WRITING** Describe how an outcome and a favorable outcome are different.

Practice and Problem Solving

You spin the spinner shown.



- How many possible results are there?
- Of the possible results, in how many ways can you spin an even number? an odd number?

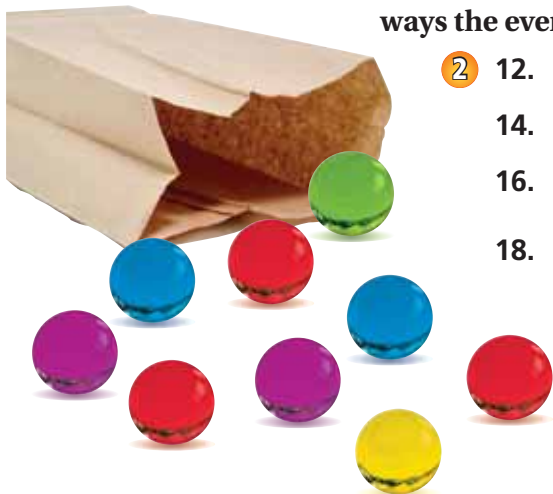
- 1 **TILES** What are the possible outcomes of randomly choosing one of the tiles shown?



You randomly choose one of the tiles shown above. Find the favorable outcomes of the event.

- | | |
|-------------------------------------|---------------------------------------|
| 6. Choosing a 6 | 7. Choosing an odd number |
| 8. Choosing a number greater than 5 | 9. Choosing an odd number less than 5 |
| 10. Choosing a number less than 3 | 11. Choosing a number divisible by 3 |

You randomly choose one marble from the bag. (a) Find the number of ways the event can occur. (b) Find the favorable outcomes of the event.



- | | | |
|---|---|------------------------------|
| 2 | 12. Choosing blue | 13. Choosing green |
| | 14. Choosing purple | 15. Choosing yellow |
| | 16. Choosing <i>not</i> red | 17. Choosing <i>not</i> blue |
| | 18. ERROR ANALYSIS Describe and correct the error in finding the number of ways that choosing <i>not</i> purple can occur. | |



| purple | <i>not</i> purple |
|--------|--------------------------|
| purple | red, blue, green, yellow |

Choosing *not* purple can occur in 4 ways.

19. **COINS** You have 10 coins in your pocket. Five are Susan B. Anthony dollars, two are Kennedy half-dollars, and three are presidential dollars. You randomly choose a coin. In how many ways can choosing *not* a presidential dollar occur?



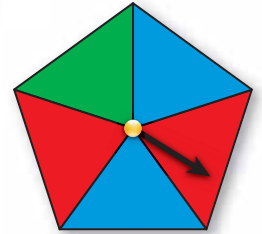
Susan B. Anthony dollar

Kennedy half-dollar



Presidential dollar

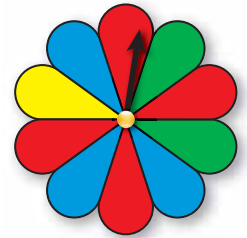
Spinner A



Tell whether the statement is *true* or *false*. If it is false, change the italicized word to make the statement true.

20. Spinning blue and spinning *green* have the same number of favorable outcomes on Spinner A.
21. Spinning blue has one *more* favorable outcome than spinning green on Spinner B.
22. There are *three* possible outcomes of spinning Spinner A.
23. Spinning *red* can occur in four ways on Spinner B.
24. Spinning not green can occur in *three* ways on Spinner B.

Spinner B



25. **MUSIC** A bargain bin contains classical and rock CDs. There are 60 CDs in the bin. Choosing a rock CD and *not* choosing a rock CD have the same number of favorable outcomes. How many rock CDs are in the bin?
26. **Precision** You randomly choose one of the cards and set it aside. Then you randomly choose a second card. Describe how the number of possible outcomes changes after the first card is chosen.



Fair Game Review What you learned in previous grades & lessons

Solve the proportion. (*Skills Review Handbook*)

27. $\frac{x}{10} = \frac{1}{5}$

28. $\frac{60}{n} = \frac{20}{7}$

29. $\frac{1}{3} = \frac{w}{36}$

30. $\frac{25}{17} = \frac{100}{b}$

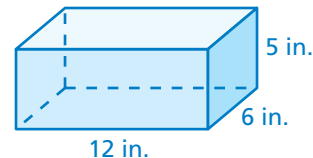
31. **MULTIPLE CHOICE** What is the surface area of the rectangular prism? (*Section 14.1*)

(A) 162 in.²

(B) 264 in.²

(C) 324 in.²

(D) 360 in.²



15.2 Probability

Essential Question How can you describe the likelihood of an event?

1 ACTIVITY: Black-and-White Spinner Game

Work with a partner. You work for a game company. You need to create a game that uses the spinner below.

- Write rules for a game that uses the spinner. Then play it.
- After playing the game, do you want to revise the rules? Explain.



COMMON
CORE

Probability and Statistics

In this lesson, you will

- understand the concept of probability and the relationship between probability and likelihood.
- find probabilities of events.

Learning Standards

7.SP.5

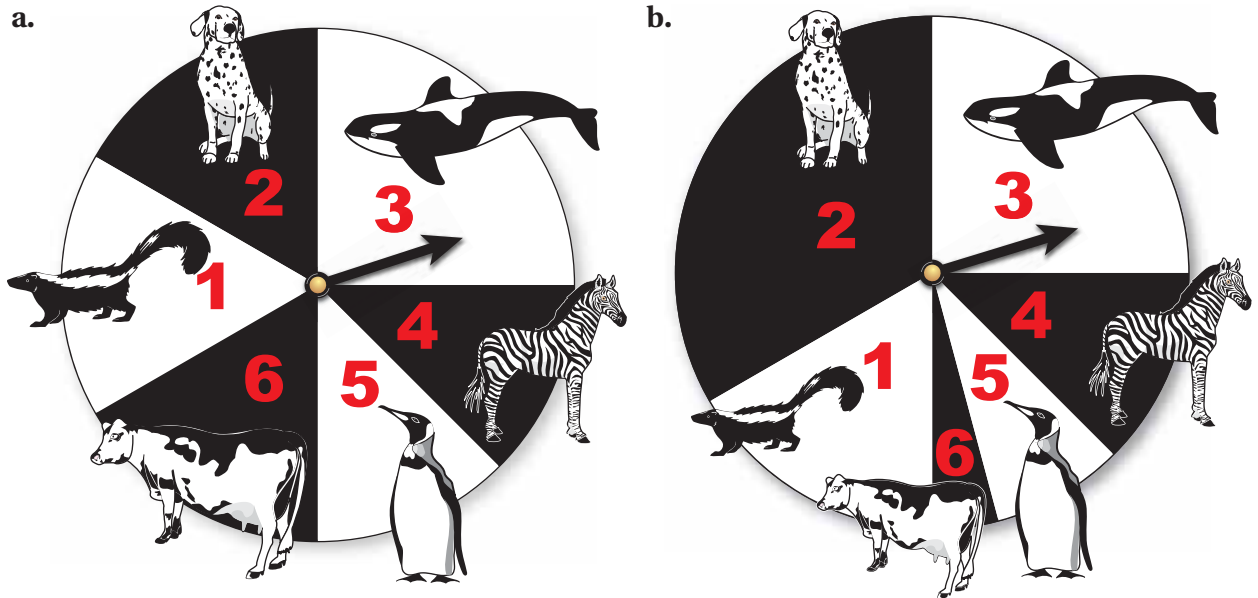
7.SP.7a

- CHOOSE TOOLS** Using the center of the spinner as the vertex, measure the angle of each pie-shaped section. Is each section the same size? How do you think this affects the likelihood of spinning a given number?
- Your friend is about to spin the spinner and wants to know how likely it is to spin a 3. How would you describe the likelihood of this event to your friend?

2 ACTIVITY: Changing the Spinner

Work with a partner. For each spinner, do the following.

- Measure the angle of each pie-shaped section.
- Tell whether you are more likely to spin a particular number. Explain your reasoning.
- Tell whether your rules from Activity 1 make sense for these spinners. Explain your reasoning.



3 ACTIVITY: Is This Game Fair?

Math Practice 3

Use Prior Results

How can you use the results of the previous activities to determine whether the game is fair?

Work with a partner. Apply the following rules to each spinner in Activities 1 and 2. Is the game fair? Why or why not? If not, who has the better chance of winning?

- Take turns spinning the spinner.
- If you spin an odd number, Player 1 wins.
- If you spin an even number, Player 2 wins.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you describe the likelihood of an event?
5. Describe the likelihood of spinning an 8 in Activity 1.
6. Describe a career in which it is important to know the likelihood of an event.

Practice

Use what you learned about the likelihood of an event to complete Exercises 4 and 5 on page 642.

Key Vocabulary

probability, p. 640

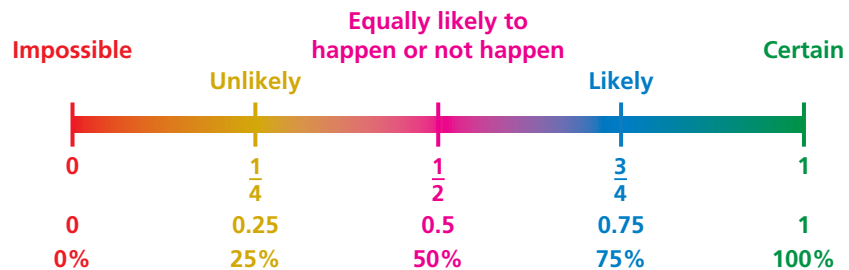
Study Tip

Probabilities can be written as fractions, decimals, or percents.

Key Idea

Probability

The **probability** of an event is a number that measures the likelihood that the event will occur. Probabilities are between 0 and 1, including 0 and 1. The diagram relates likelihoods (above the diagram) and probabilities (below the diagram).



EXAMPLE 1 Describing the Likelihood of an Event



There is an 80% chance of thunderstorms tomorrow. Describe the likelihood of the event.

The probability of thunderstorms tomorrow is 80%.

- Because 80% is close to 75%, it is *likely* that there will be thunderstorms tomorrow.

On Your Own

Describe the likelihood of the event given its probability.

- The probability that you land a jump on a snowboard is $\frac{1}{2}$.
- There is a 100% chance that the temperature will be less than 120°F tomorrow.

Now You're Ready
Exercises 6–9

Key Idea

Finding the Probability of an Event

When all possible outcomes are equally likely, the probability of an event is the ratio of the number of favorable outcomes to the number of possible outcomes. The probability of an event is written as $P(\text{event})$.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

EXAMPLE 2 Finding a Probability

You roll the number cube. What is the probability of rolling an odd number?



$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{odd}) = \frac{3}{6}$$

There are 3 odd numbers (1, 3, and 5).

There is a total of 6 numbers.

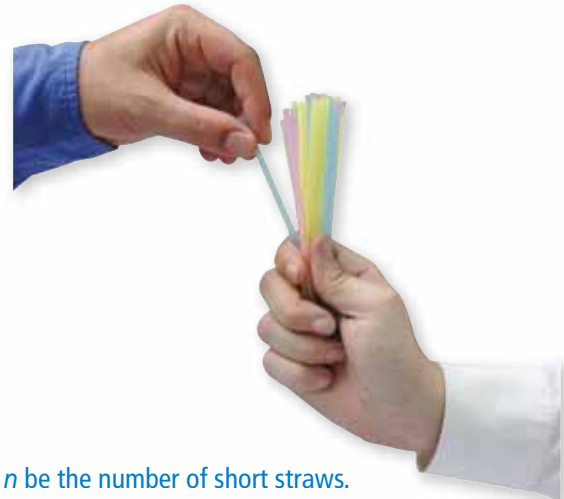
$$= \frac{1}{2} \quad \text{Simplify.}$$

∴ The probability of rolling an odd number is $\frac{1}{2}$, or 50%.

EXAMPLE 3 Using a Probability

The probability that you randomly draw a short straw from a group of 40 straws is $\frac{3}{20}$. How many are short straws?

- (A) 4 (B) 6
(C) 15 (D) 34



$$P(\text{short}) = \frac{\text{number of short straws}}{\text{total number of straws}}$$

$$\frac{3}{20} = \frac{n}{40}$$

Substitute. Let n be the number of short straws.

$$6 = n \quad \text{Solve for } n.$$

There are 6 short straws.

∴ So, the correct answer is (B).

On Your Own

Now You're Ready
Exercises 11–15

- In Example 2, what is the probability of rolling a number greater than 2?
- In Example 2, what is the probability of rolling a 7?
- The probability that you randomly draw a short straw from a group of 75 straws is $\frac{1}{15}$. How many are short straws?

15.2 Exercises



Vocabulary and Concept Check

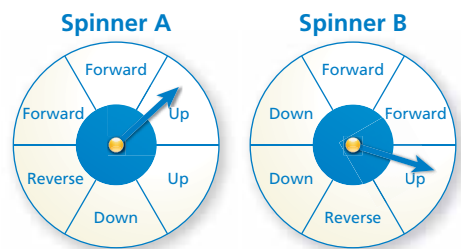
- VOCABULARY** Explain how to find the probability of an event.
- REASONING** Can the probability of an event be 1.5? Explain.
- OPEN-ENDED** Give a real-life example of an event that is impossible. Give a real-life example of an event that is certain.



Practice and Problem Solving

You are playing a game using the spinners shown.

- You want to move down. On which spinner are you more likely to spin “Down”? Explain.
- You want to move forward. Which spinner would you spin? Explain.



Describe the likelihood of the event given its probability.

- Your soccer team wins $\frac{3}{4}$ of the time.
- There is a 0% chance that you will grow 12 more feet.
- The probability that the sun rises tomorrow is 1.
- It rains on $\frac{1}{5}$ of the days in July.
- VIOLIN** You have a 50% chance of playing the correct note on a violin. Describe the likelihood of playing the correct note.



You randomly choose one shirt from the shelves.
Find the probability of the event.

- Choosing a red shirt
- Choosing a green shirt
- Not choosing a white shirt
- Not choosing a black shirt
- Choosing an orange shirt
- ERROR ANALYSIS** Describe and correct the error in finding the probability of *not* choosing a blue shirt from the shelves above.



$$P(\text{not blue}) = \frac{4}{10} = \frac{2}{5}$$

17. **CONTEST** The rules of a contest say that there is a 5% chance of winning a prize. Four hundred people enter the contest. Predict how many people will win a prize.

18. **RUBBER DUCKS** At a carnival, the probability that you choose a winning rubber duck from 25 ducks is 0.24.

- How many are *not* winning ducks?
- Describe the likelihood of *not* choosing a winning duck.



19. **DODECAHEDRON** A dodecahedron has twelve sides numbered 1 through 12. Find the probability and describe the likelihood of each event.

- Rolling a number less than 9
- Rolling a multiple of 3
- Rolling a number greater than 6

A Punnett square is a grid used to show possible gene combinations for the offspring of two parents. In the Punnett square shown, a boy is represented by XY. A girl is represented by XX.

| | | Mother's Genes | |
|----------------|---|----------------|---|
| | | X | X |
| Father's Genes | X | XX | |
| | Y | | |

20. Complete the Punnett square.

21. Explain why the probability of two parents having a boy or having a girl is equally likely.

22. **Critical Thinking** Two parents each have the gene combination Cs. The gene C is for curly hair. The gene s is for straight hair.

- Make a Punnett square for the two parents. When all outcomes are equally likely, what is the probability of a child having the gene combination CC?
- Any gene combination that includes a C results in curly hair. When all outcomes are equally likely, what is the probability of a child having curly hair?



Fair Game Review what you learned in previous grades & lessons

Solve the inequality. Graph the solution. (Section 11.2 and Section 11.3)

23. $x + 5 < 9$

24. $b - 2 \geq -7$

25. $1 > -\frac{w}{3}$

26. $6 \leq -2g$

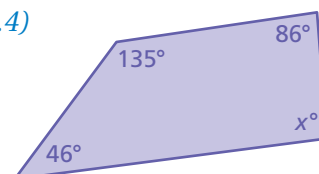
27. **MULTIPLE CHOICE** Find the value of x. (Section 12.4)

(A) 85

(B) 90

(C) 93

(D) 102



Essential Question How can you use relative frequencies to find probabilities?

When you conduct an experiment, the **relative frequency** of an event is the fraction or percent of the time that the event occurs.

$$\text{relative frequency} = \frac{\text{number of times the event occurs}}{\text{total number of times you conduct the experiment}}$$

1 ACTIVITY: Finding Relative Frequencies

Work with a partner.

- a. Flip a quarter 20 times and record your results. Then complete the table. Are the relative frequencies the same as the probability of flipping heads or tails? Explain.

| | Flipping Heads | Flipping Tails |
|--------------------|----------------|----------------|
| Relative Frequency | | |

- b. Compare your results with those of other students in your class. Are the relative frequencies the same? If not, why do you think they differ?
- c. Combine all of the results in your class. Then complete the table again. Did the relative frequencies change? What do you notice? Explain.
- d. Suppose everyone in your school conducts this experiment and you combine the results. How do you think the relative frequencies will change?



2 ACTIVITY: Using Relative Frequencies

Work with a partner. You have a bag of colored chips. You randomly select a chip from the bag and replace it. The table shows the number of times you select each color.

| Red | Blue | Green | Yellow |
|-----|------|-------|--------|
| 24 | 12 | 15 | 9 |

- a. There are 20 chips in the bag. Can you use the table to find the exact number of each color in the bag? Explain.
- b. You randomly select a chip from the bag and replace it. You do this 50 times, then 100 times, and you calculate the relative frequencies after each experiment. Which experiment do you think gives a better approximation of the exact number of each color in the bag? Explain.



COMMON
CORE

Probability and Statistics

In this lesson, you will

- find relative frequencies.
- use experimental probabilities to make predictions.
- use theoretical probabilities to find quantities.
- compare experimental and theoretical probabilities.

Learning Standards

- 7.SP.5
- 7.SP.6
- 7.SP.7a
- 7.SP.7b

3 ACTIVITY: Conducting an Experiment

Work with a partner. You toss a thumbtack onto a table. There are two ways the thumbtack can land.



Point up



On its side

Math Practice 4

Analyze Relationships

How can you use the results of your experiment to determine whether this is a uniform probability model?

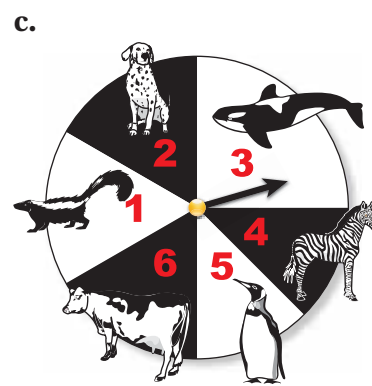
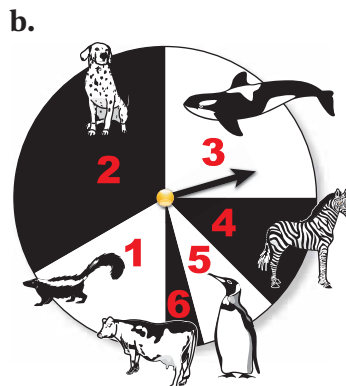
- Your friend says that because there are two outcomes, the probability of the thumbtack landing point up must be $\frac{1}{2}$. Do you think this conclusion is true? Explain.
- Toss a thumbtack onto a table 50 times and record your results. In a *uniform probability model*, each outcome is equally likely to occur. Do you think this experiment represents a uniform probability model? Explain. Use the relative frequencies to complete the following.

$$P(\text{point up}) = \text{■}$$

$$P(\text{on its side}) = \text{■}$$

What Is Your Answer?

- IN YOUR OWN WORDS** How can you use relative frequencies to find probabilities? Give an example.
- Your friend rolls a number cube 500 times. How many times do you think your friend will roll an odd number? Explain your reasoning.
- In Activity 2, your friend says, “There are no orange-colored chips in the bag.” Do you think this conclusion is true? Explain.
- Give an example of an experiment that represents a uniform probability model.
- Tell whether you can use each spinner to represent a uniform probability model. Explain your reasoning.



Practice

Use what you learned about relative frequencies to complete Exercises 6 and 7 on page 649.

Key Vocabulary

relative frequency,
p. 644
experimental
probability, p. 646
theoretical
probability, p. 647

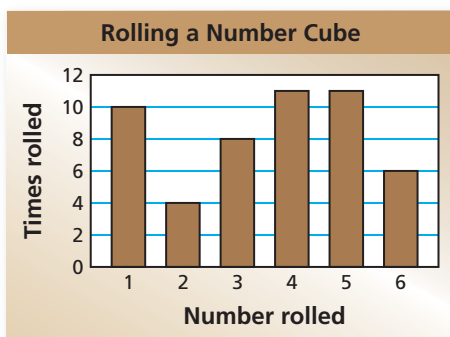
Key Idea

Experimental Probability

Probability that is based on repeated trials of an experiment is called **experimental probability**.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

EXAMPLE 1 Finding an Experimental Probability



The bar graph shows the results of rolling a number cube 50 times. What is the experimental probability of rolling an odd number?

The bar graph shows 10 ones, 8 threes, and 11 fives. So, an odd number was rolled $10 + 8 + 11 = 29$ times in a total of 50 rolls.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

$$P(\text{odd}) = \frac{29}{50}$$

An odd number was rolled 29 times.

There was a total of 50 rolls.

The experimental probability is $\frac{29}{50}$, 0.58, or 58%.

EXAMPLE 2 Making a Prediction



It rains 2 out of the last 12 days in March. If this trend continues, how many rainy days would you expect in April?

Find the experimental probability of a rainy day.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

$$P(\text{rain}) = \frac{2}{12} = \frac{1}{6}$$

It rains 2 days.

There is a total of 12 days.

"April showers bring May flowers." Old Proverb, 1557

To make a prediction, multiply the probability of a rainy day by the number of days in April.

$$\frac{1}{6} \cdot 30 = 5$$

So, you can predict that there will be 5 rainy days in April.

On Your Own

1. In Example 1, what is the experimental probability of rolling an even number?
2. At a clothing company, an inspector finds 5 defective pairs of jeans in a shipment of 200. If this trend continues, about how many pairs of jeans would you expect to be defective in a shipment of 5000?

Key Idea

Theoretical Probability

When all possible outcomes are equally likely, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the number of possible outcomes.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

EXAMPLE 3 Finding a Theoretical Probability



You randomly choose one of the letters shown. What is the theoretical probability of choosing a vowel?

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{vowel}) = \frac{3}{7}$$

There are 3 vowels.

There is a total of 7 letters.

∴ The probability of choosing a vowel is $\frac{3}{7}$, or about 43%.

EXAMPLE 4 Using a Theoretical Probability

The theoretical probability of winning a bobblehead when spinning a prize wheel is $\frac{1}{6}$. The wheel has 3 bobblehead sections. How many sections are on the wheel?

$$P(\text{bobblehead}) = \frac{\text{number of bobblehead sections}}{\text{total number of sections}}$$

$$\frac{1}{6} = \frac{3}{s}$$

Substitute. Let s be the total number of sections.

$$s = 18$$

Cross Products Property

∴ So, there are 18 sections on the wheel.

On Your Own

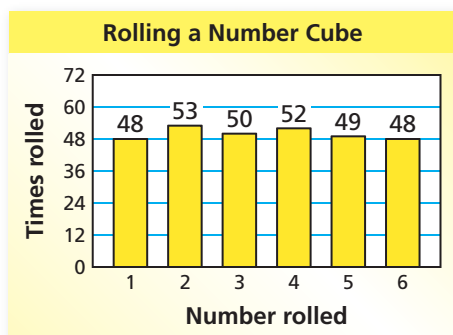
- In Example 3, what is the theoretical probability of choosing an X?
- The theoretical probability of spinning an odd number on a spinner is 0.6. The spinner has 10 sections. How many sections have odd numbers?
- The prize wheel in Example 4 was spun 540 times at a baseball game. About how many bobbleheads would you expect were won?



EXAMPLE 5 Comparing Experimental and Theoretical Probability

The bar graph shows the results of rolling a number cube 300 times.

- a. What is the experimental probability of rolling an odd number?



The bar graph shows 48 ones, 50 threes, and 49 fives. So, an odd number was rolled $48 + 50 + 49 = 147$ times in a total of 300 rolls.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

$$P(\text{odd}) = \frac{147}{300}$$

An odd number was rolled 147 times.

There was a total of 300 rolls.

$$= \frac{49}{100}, \text{ or } 49\%$$

- b. How does the experimental probability compare with the theoretical probability of rolling an odd number?

In Section 15.2, Example 2, you found that the theoretical probability of rolling an odd number is 50%. The experimental probability, 49%, is close to the theoretical probability.

- c. Compare the experimental probability in part (a) to the experimental probability in Example 1.

As the number of trials increased from 50 to 300, the experimental probability decreased from 58% to 49%. So, it became closer to the theoretical probability of 50%.

On Your Own

- Use the bar graph in Example 5 to find the experimental probability of rolling a number greater than 1. Compare the experimental probability to the theoretical probability of rolling a number greater than 1.

Vocabulary and Concept Check

- VOCABULARY** Describe how to find the experimental probability of an event.
- REASONING** You flip a coin 10 times and find the experimental probability of flipping tails to be 0.7. Does this seem reasonable? Explain.
- VOCABULARY** An event has a theoretical probability of 0.5. What does this mean?
- OPEN-ENDED** Describe an event that has a theoretical probability of $\frac{1}{4}$.
- LOGIC** A pollster surveys randomly selected individuals about an upcoming election. Do you think the pollster will use experimental probability or theoretical probability to make predictions? Explain.

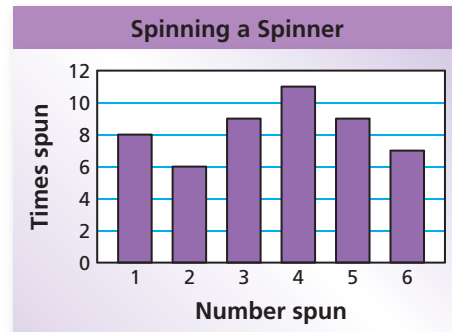
Practice and Problem Solving

Use the bar graph to find the relative frequency of the event.

- Spinning a 6
- Spinning an even number

Use the bar graph to find the experimental probability of the event.

- Spinning a number less than 3
- Not spinning a 1
- Spinning a 1 or a 3
- Spinning a 7



- EGGS** You check 20 cartons of eggs. Three of the cartons have at least one cracked egg. What is the experimental probability that a carton of eggs has at least one cracked egg?

- BOARD GAME** There are 105 lettered tiles in a board game. You choose the tiles shown. How many of the 105 tiles would you expect to be vowels?



- CARDS** You have a package of 20 assorted thank-you cards. You pick the four cards shown. How many of the 20 cards would you expect to have flowers on them?

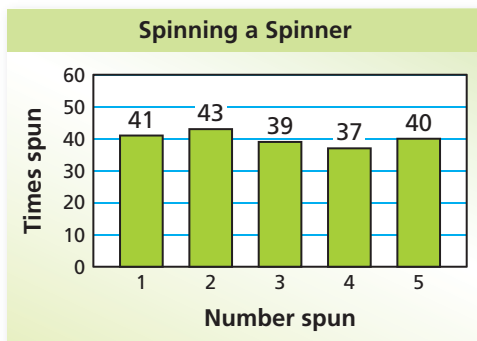
Use the spinner to find the theoretical probability of the event.



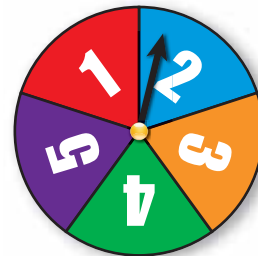
- 3 15. Spinning red
 16. Spinning a 1
 17. Spinning an odd number
 18. Spinning a multiple of 2
 19. Spinning a number less than 7
 20. Spinning a 9
 21. **LETTERS** Each letter of the alphabet is printed on an index card. What is the theoretical probability of randomly choosing any letter except Z?

- 4 22. **GAME SHOW** On a game show, a contestant randomly chooses a chip from a bag that contains numbers and strikes. The theoretical probability of choosing a strike is $\frac{3}{10}$. The bag contains 9 strikes. How many chips are in the bag?
 23. **MUSIC** The theoretical probability that a pop song plays on your MP3 player is 0.45. There are 80 songs on your MP3 player. How many of the songs are pop songs?
 24. **MODELING** There are 16 females and 20 males in a class.
 a. What is the theoretical probability that a randomly chosen student is female?
 b. One week later, there are 45 students in the class. The theoretical probability that a randomly chosen student is a female is the same as last week. How many males joined the class?

The bar graph shows the results of spinning the spinner 200 times. Compare the theoretical and experimental probabilities of the event.



- 5 25. Spinning a 4
 26. Spinning a 3
 27. Spinning a number greater than 4
 28. Should you use *theoretical* or *experimental* probability to predict the number of times you will spin a 3 in 10,000 spins?



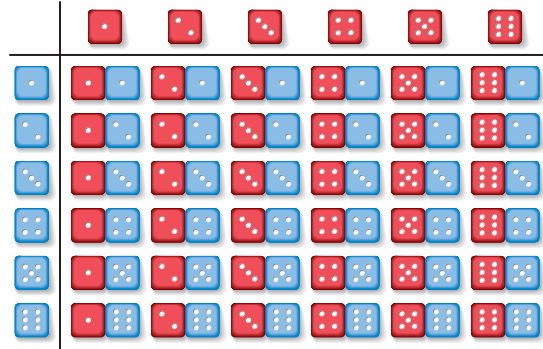
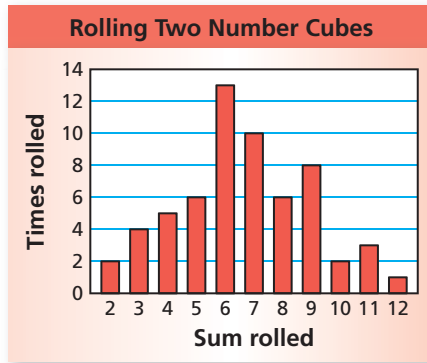
29. **NUMBER SENSE** The table at the right shows the results of flipping two coins 12 times each.

| HH | HT | TH | TT |
|----|----|----|----|
| 2 | 6 | 3 | 1 |

- a. What is the experimental probability of flipping two tails? Using this probability, how many times can you expect to flip two tails in 600 trials?
 b. The table at the left shows the results of flipping the same two coins 100 times each. What is the experimental probability of flipping two tails? Using this probability, how many times can you expect to flip two tails in 600 trials?
 c. Why is it important to use a large number of trials when using experimental probability to predict results?

| HH | HT | TH | TT |
|----|----|----|----|
| 23 | 29 | 26 | 22 |

You roll a pair of number cubes 60 times. You record your results in the bar graph shown.



30. Use the bar graph to find the experimental probability of rolling each sum. Is each sum equally likely? Explain. If not, which is most likely?
31. Use the table to find the theoretical probability of rolling each sum. Is each sum equally likely? Explain. If not, which is most likely?
32. **PROBABILITIES** Compare the probabilities you found in Exercises 30 and 31.
33. **REASONING** Consider the results of Exercises 30 and 31.
- Which sum would you expect to be most likely after 500 trials? 1000 trials? 10,000 trials?
 - Explain how experimental probability is related to theoretical probability as the number of trials increases.
34. **Project** When you toss a paper cup into the air, there are three ways for the cup to land: *open-end up*, *open-end down*, or *on its side*.
- Toss a paper cup 100 times and record your results. Do the outcomes for tossing the cup appear to be equally likely? Explain.
 - What is the probability of the cup landing open-end up? open-end down? on its side?
 - Use your results to predict the number of times the cup lands on its side in 1000 tosses.
 - Suppose you tape a quarter to the bottom of the cup. Do you think the cup will be *more likely* or *less likely* to land open-end up? Justify your answer.



Fair Game Review what you learned in previous grades & lessons

Find the annual interest rate. (*Skills Review Handbook*)

35. $I = \$16$, $P = \$200$, $t = 2$ years
36. $I = \$26.25$, $P = \$500$, $t = 18$ months
37. **MULTIPLE CHOICE** The volume of a prism is 9 cubic yards. What is its volume in cubic feet? (*Section 14.4*)
- (A) 3 ft^3 (B) 27 ft^3 (C) 81 ft^3 (D) 243 ft^3

15.4 Compound Events

Essential Question How can you find the number of possible outcomes of one or more events?

1 ACTIVITY: Comparing Combination Locks

Work with a partner. You are buying a combination lock. You have three choices.

- a. This lock has 3 wheels. Each wheel is numbered from 0 to 9.

The least three-digit combination possible is .

The greatest three-digit combination possible is .

How many possible combinations are there?

- b. Use the lock in part (a).

There are possible outcomes for the first wheel.

There are possible outcomes for the second wheel.

There are possible outcomes for the third wheel.

How can you use multiplication to determine the number of possible combinations?

- c. This lock is numbered from 0 to 39. Each combination uses three numbers in a right, left, right pattern. How many possible combinations are there?

- d. This lock has 4 wheels.

Wheel 1: 0–9

Wheel 2: A–J

Wheel 3: K–T

Wheel 4: 0–9

How many possible combinations are there?

- e. For which lock is it most difficult to guess the combination? Why?



COMMON
CORE

Probability and Statistics

In this lesson, you will

- use tree diagrams, tables, or a formula to find the number of possible outcomes.
- find probabilities of compound events.

Learning Standards

7.SP.8a

7.SP.8b

2 ACTIVITY: Comparing Password Security

Work with a partner. Which password requirement is most secure? Explain your reasoning. Include the number of different passwords that are possible for each requirement.

- a. The password must have four digits.

Username:

Password:

- b. The password must have five digits.

Username:

Password:

- c. The password must have six letters.

Username:

Password:

- d. The password must have eight digits or letters.

Username:

Password:

Math Practice 7

View as Components

What is the number of possible outcomes for each character of the password? Explain.

What Is Your Answer?

- IN YOUR OWN WORDS** How can you find the number of possible outcomes of one or more events?
- SECURITY** A hacker uses a software program to guess the passwords in Activity 2. The program checks 600 passwords per minute. What is the greatest amount of time it will take the program to guess each of the four types of passwords?

Practice

Use what you learned about the total number of possible outcomes of one or more events to complete Exercise 5 on page 657.

Key Vocabulary

sample space, p. 654
Fundamental Counting Principle, p. 654
compound event, p. 656

The set of all possible outcomes of one or more events is called the **sample space**.

You can use tables and tree diagrams to find the sample space of two or more events.

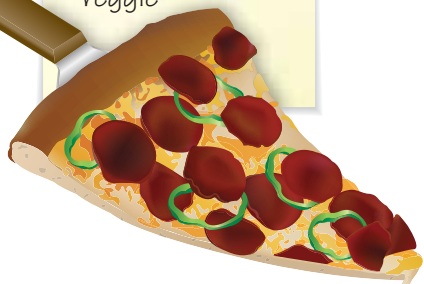
EXAMPLE 1 Finding a Sample Space

Crust

- Thin Crust
- Stuffed Crust

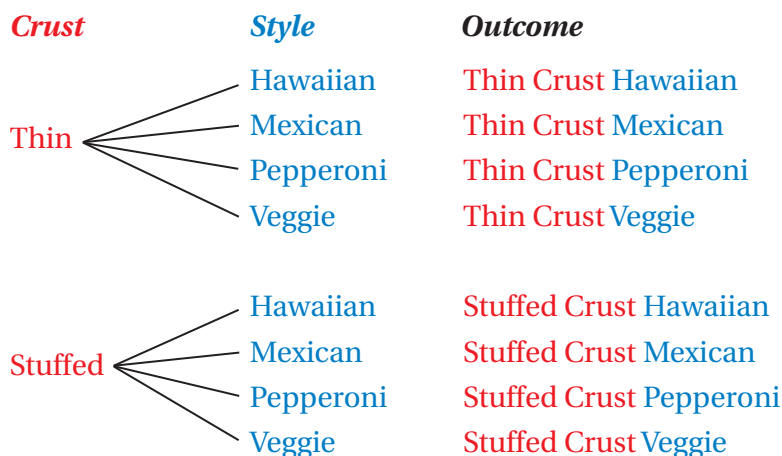
Style

- Hawaiian
- Mexican
- Pepperoni
- Veggie



You randomly choose a crust and style of pizza. Find the sample space. How many different pizzas are possible?

Use a tree diagram to find the sample space.



- There are 8 different outcomes in the sample space. So, there are 8 different pizzas possible.

On Your Own

Now You're Ready
Exercises 6 and 7

- WHAT IF?** The pizza shop adds a deep dish crust. Find the sample space. How many pizzas are possible?

Another way to find the total number of possible outcomes is to use the **Fundamental Counting Principle**.

Key Idea

Fundamental Counting Principle

An event M has m possible outcomes. An event N has n possible outcomes. The total number of outcomes of event M followed by event N is $m \times n$.

Study Tip



The Fundamental Counting Principle can be extended to more than two events.

EXAMPLE 2 Finding the Total Number of Possible Outcomes



Find the total number of possible outcomes of rolling a number cube and flipping a coin.

Method 1: Use a table to find the sample space. Let H = heads and T = tails.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|----|----|----|----|----|----|
|  | 1H | 2H | 3H | 4H | 5H | 6H |
|  | 1T | 2T | 3T | 4T | 5T | 6T |

∴ There are 12 possible outcomes.

Method 2: Use the Fundamental Counting Principle. Identify the number of possible outcomes of each event.

Event 1: Rolling a number cube has 6 possible outcomes.

Event 2: Flipping a coin has 2 possible outcomes.

$$6 \times 2 = 12 \quad \text{Fundamental Counting Principle}$$

∴ There are 12 possible outcomes.

EXAMPLE 3 Finding the Total Number of Possible Outcomes



How many different outfits can you make from the T-shirts, jeans, and shoes in the closet?

Use the Fundamental Counting Principle. Identify the number of possible outcomes for each event.

Event 1: Choosing a T-shirt has 7 possible outcomes.

Event 2: Choosing jeans has 4 possible outcomes.

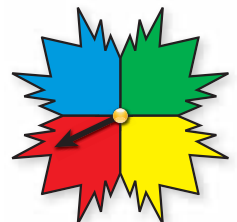
Event 3: Choosing shoes has 3 possible outcomes.

$$7 \times 4 \times 3 = 84 \quad \text{Fundamental Counting Principle}$$

∴ So, you can make 84 different outfits.

On Your Own

- Find the total number of possible outcomes of spinning the spinner and choosing a number from 1 to 5.
- How many different outfits can you make from 4 T-shirts, 5 pairs of jeans, and 5 pairs of shoes?



Now You're Ready
Exercises 8–11

A **compound event** consists of two or more events. As with a single event, the probability of a compound event is the ratio of the number of favorable outcomes to the number of possible outcomes.

EXAMPLE 4 Finding the Probability of a Compound Event

In Example 2, what is the probability of rolling a number greater than 4 and flipping tails?

There are two favorable outcomes in the sample space for rolling a number greater than 4 and flipping tails: 5T and 6T.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{greater than 4 and tails}) = \frac{2}{12} \quad \text{Substitute.}$$

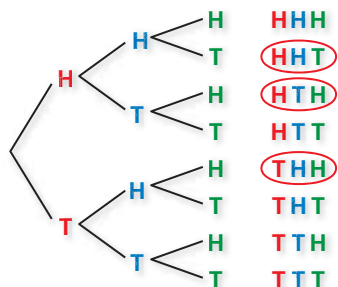
$$= \frac{1}{6} \quad \text{Simplify.}$$

∴ The probability is $\frac{1}{6}$, or $16\frac{2}{3}\%$.

EXAMPLE 5 Finding the Probability of a Compound Event

You flip three nickels. What is the probability of flipping two heads and one tails?

Use a tree diagram to find the sample space. Let H = heads and T = tails.



There are three favorable outcomes in the sample space for flipping two heads and one tails: HHT, HTH, and THH.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(2 \text{ heads and } 1 \text{ tails}) = \frac{3}{8} \quad \text{Substitute.}$$

∴ The probability is $\frac{3}{8}$, or 37.5%.

On Your Own

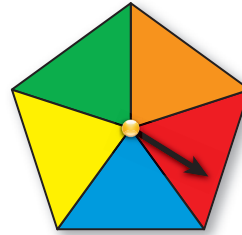
Now You're Ready
Exercises 15–24

- In Example 2, what is the probability of rolling at most 4 and flipping heads?
- In Example 5, what is the probability of flipping at least two tails?
- You roll two number cubes. What is the probability of rolling double threes?
- In Example 1, what is the probability of choosing a stuffed crust Hawaiian pizza?

15.4 Exercises

Vocabulary and Concept Check

- VOCABULARY** What is the sample space of an event? How can you find the sample space of two or more events?
- WRITING** Explain how to use the Fundamental Counting Principle.
- WRITING** Describe two ways to find the total number of possible outcomes of spinning the spinner and rolling the number cube.
- OPEN-ENDED** Give a real-life example of a compound event.



Practice and Problem Solving

- COMBINATIONS** The lock is numbered from 0 to 49. Each combination uses three numbers in a right, left, right pattern. Find the total number of possible combinations for the lock.



Use a tree diagram to find the sample space and the total number of possible outcomes.

1 6.

| Birthday Party | |
|----------------|---|
| Event | Miniature golf, Laser tag, Roller skating |
| Time | 1:00 P.M.–3:00 P.M., 6:00 P.M.–8:00 P.M. |

7.

| New School Mascot | |
|-------------------|--------------------------|
| Type | Lion, Bear, Hawk, Dragon |
| Style | Realistic, Cartoon |

Use the Fundamental Counting Principle to find the total number of possible outcomes.

2 8.

| Beverage | |
|----------|--|
| Size | Small, Medium, Large |
| Flavor | Milk, Orange juice, Apple juice, Iced tea, Lemonade, Water, Coffee |

9.

| MP3 Player | |
|------------|----------------------------------|
| Memory | 2 GB, 4 GB, 8 GB, 16 GB |
| Color | Silver, Green, Blue, Pink, Black |

3 10.

| Clown | |
|--------|--|
| Suit | Dots, Stripes, Checkers board |
| Wig | One color, Multicolor |
| Talent | Balloon animals, Juggling, Unicycle, Magic |

11.

| Meal | |
|-----------|--|
| Appetizer | Nachos, Soup, Spinach dip, Salad, Applesauce |
| Entrée | Chicken, Beef, Spaghetti, Fish |
| Dessert | Yogurt, Fruit, Ice cream |

12. **NOTE CARDS** A store sells three types of note cards. There are three sizes of each type. Show two ways to find the total number of note cards the store sells.

13. **ERROR ANALYSIS** A true-false quiz has five questions. Describe and correct the error in using the Fundamental Counting Principle to find the total number of ways that you can answer the quiz.



$$2 + 2 + 2 + 2 + 2 = 10$$

You can answer the quiz in 10 different ways.



14. **CHOOSE TOOLS** You randomly choose one of the marbles. Without replacing the first marble, you choose a second marble.

- Name two ways you can find the total number of possible outcomes.
- Find the total number of possible outcomes.

You spin the spinner and flip a coin. Find the probability of the compound event.

- Spinning a 1 and flipping heads
- Spinning an even number and flipping heads
- Spinning a number less than 3 and flipping tails
- Spinning a 6 and flipping tails
- Not spinning a 5 and flipping heads
- Spinning a prime number and *not* flipping heads



You spin the spinner, flip a coin, then spin the spinner again. Find the probability of the compound event.



- Spinning blue, flipping heads, then spinning a 1
- Spinning an odd number, flipping heads, then spinning yellow
- Spinning an even number, flipping tails, then spinning an odd number
- Not spinning red, flipping tails, then *not* spinning an even number

25. **TAKING A TEST** You randomly guess the answers to two questions on a multiple-choice test. Each question has three choices: A, B, and C.

- What is the probability that you guess the correct answers to both questions?
- Suppose you can eliminate one of the choices for each question. How does this change the probability that your guesses are correct?

26. **PASSWORD** You forget the last two digits of your password for a website.
- What is the probability that you randomly choose the correct digits?
 - Suppose you remember that both digits are even. How does this change the probability that your choices are correct?

27. **COMBINATION LOCK** The combination lock has 3 wheels, each numbered from 0 to 9.



- What is the probability that someone randomly guesses the correct combination in one attempt?
- Explain how to find the probability that someone randomly guesses the correct combination in five attempts.

28. **TRAINS** Your model train has one engine and eight train cars. Find the total number of ways you can arrange the train. (The engine must be first.)



29. **REPEATED REASONING** You have been assigned a 9-digit identification number.

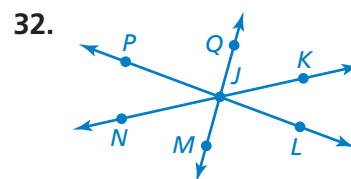
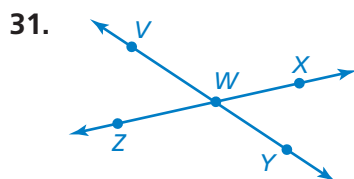
- Why should you use the Fundamental Counting Principle instead of a tree diagram to find the total number of possible identification numbers?
- How many identification numbers are possible?
- RESEARCH** Use the Internet to find out why the possible number of Social Security numbers is not the same as your answer to part (b).

30. **Problem Solving** From a group of 5 candidates, a committee of 3 people is selected. In how many different ways can the committee be selected?



Fair Game Review What you learned in previous grades & lessons

Name two pairs of adjacent angles and two pairs of vertical angles in the figure. (Section 12.1)



33. **MULTIPLE CHOICE** A drawing has a scale of 1 cm : 1 m. What is the scale factor of the drawing? (Section 12.5)

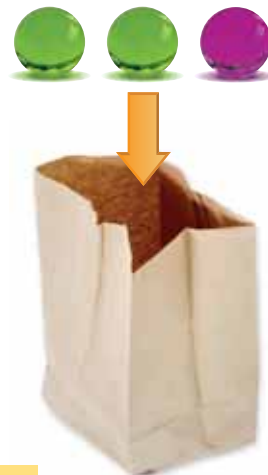
- (A) 1 : 1 (B) 1 : 100 (C) 10 : 1 (D) 100 : 1

15.5 Independent and Dependent Events

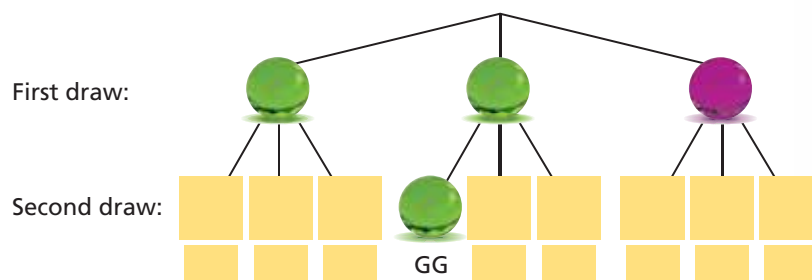
Essential Question What is the difference between dependent and independent events?

1 ACTIVITY: Drawing Marbles from a Bag (With Replacement)

Work with a partner. You have three marbles in a bag. There are two green marbles and one purple marble. Randomly draw a marble from the bag. Then put the marble back in the bag and draw a second marble.



- a. Complete the tree diagram. Let G = green and P = purple. Find the probability that both marbles are green.

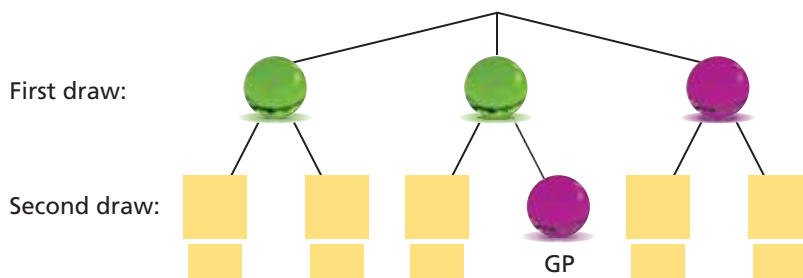


- b. Does the probability of getting a green marble on the second draw *depend* on the color of the first marble? Explain.

2 ACTIVITY: Drawing Marbles from a Bag (Without Replacement)

Work with a partner. Using the same marbles from Activity 1, randomly draw two marbles from the bag.

- a. Complete the tree diagram. Let G = green and P = purple. Find the probability that both marbles are green.



Is this event more likely than the event in Activity 1? Explain.

- b. Does the probability of getting a green marble on the second draw *depend* on the color of the first marble? Explain.



COMMON CORE

Probability and Statistics

In this lesson, you will

- identify independent and dependent events.
- use formulas to find probabilities of independent and dependent events.

Applying Standards

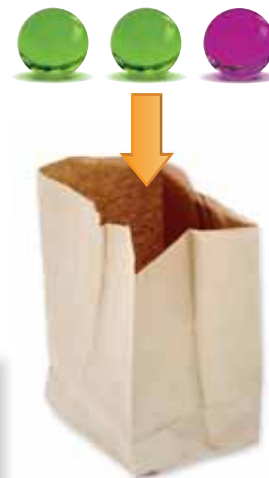
7.SP.8a

7.SP.8b

3 ACTIVITY: Conducting an Experiment

Work with a partner. Conduct two experiments.

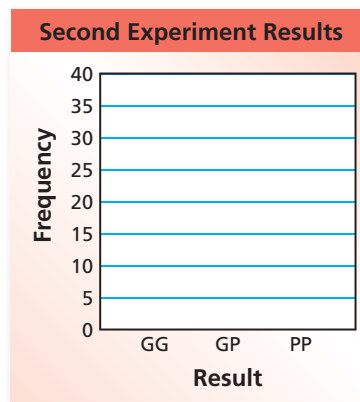
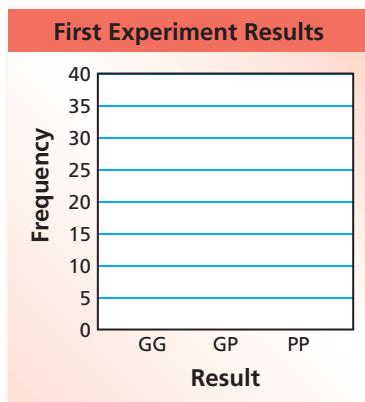
- In the first experiment, randomly draw one marble from the bag. Put it back. Draw a second marble. Repeat this 36 times. Record each result. Make a bar graph of your results.
- In the second experiment, randomly draw two marbles from the bag 36 times. Record each result. Make a bar graph of your results.



Math Practice 3

Use Definitions

In what other mathematical context have you seen the terms *independent* and *dependent*? How does knowing these definitions help you answer the questions in part (d)?



- For each experiment, estimate the probability of drawing two green marbles.
- Which experiment do you think represents *dependent events*? Which represents *independent events*? Explain your reasoning.

What Is Your Answer?

- IN YOUR OWN WORDS** What is the difference between dependent and independent events? Describe a real-life example of each.

In Questions 5–7, tell whether the events are *independent* or *dependent*. Explain your reasoning.

- You roll a 5 on a number cube and spin blue on a spinner.
- Your teacher chooses one student to lead a group, and then chooses another student to lead another group.
- You spin red on one spinner and green on another spinner.
- In Activities 1 and 2, what is the probability of drawing a green marble on the first draw? on the second draw? How do you think you can use these two probabilities to find the probability of drawing two green marbles?

Practice

Use what you learned about independent and dependent events to complete Exercises 3 and 4 on page 665.

Key Vocabulary

independent events,
p. 662
dependent events,
p. 663

Compound events may be *independent events* or *dependent events*. Events are **independent events** if the occurrence of one event *does not* affect the likelihood that the other event(s) will occur.

Key Idea
Probability of Independent Events

Words The probability of two or more independent events is the product of the probabilities of the events.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$
 $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$

EXAMPLE 1 Finding the Probability of Independent Events

You spin the spinner and flip the coin. What is the probability of spinning a prime number and flipping tails?

The outcome of spinning the spinner does not affect the outcome of flipping the coin. So, the events are independent.

$$P(\text{prime}) = \frac{3}{5}$$

← There are 3 prime numbers (2, 3, and 5).
 ← There is a total of 5 numbers.

$$P(\text{tails}) = \frac{1}{2}$$

← There is 1 tails side.
 ← There is a total of 2 sides.

Use the formula for the probability of independent events.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) \\ P(\text{prime and tails}) &= P(\text{prime}) \cdot P(\text{tails}) \\ &= \frac{3}{5} \cdot \frac{1}{2} && \text{Substitute.} \\ &= \frac{3}{10} && \text{Multiply.} \end{aligned}$$

∴ The probability of spinning a prime number and flipping tails is $\frac{3}{10}$, or 30%.

On Your Own

1. What is the probability of spinning a multiple of 2 and flipping heads?

Now You're Ready
Exercises 5–8

Events are **dependent events** if the occurrence of one event *does* affect the likelihood that the other event(s) will occur.

Key Idea

Probability of Dependent Events

Words The probability of two dependent events A and B is the probability of A times the probability of B after A occurs.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$

EXAMPLE 2 Finding the Probability of Dependent Events



People are randomly chosen to be game show contestants from an audience of 100 people. You are with 5 of your relatives and 6 other friends. What is the probability that one of your relatives is chosen first, and then one of your friends is chosen second?

Choosing an audience member changes the number of audience members left. So, the events are dependent.

$$P(\text{relative}) = \frac{5}{100} = \frac{1}{20}$$

There are 5 relatives.

There is a total of 100 audience members.

$$P(\text{friend}) = \frac{6}{99} = \frac{2}{33}$$

There are 6 friends.

There is a total of 99 audience members left.

Use the formula for the probability of dependent events.

$$P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$$

$$P(\text{relative and friend}) = P(\text{relative}) \cdot P(\text{friend after relative})$$

$$= \frac{1}{20} \cdot \frac{2}{33} \quad \text{Substitute.}$$

$$= \frac{1}{330} \quad \text{Simplify.}$$

∴ The probability is $\frac{1}{330}$, or about 0.3%.

On Your Own

- What is the probability that you, your relatives, and your friends are *not* chosen to be either of the first two contestants?

 Now You're Ready
Exercises 9–12

EXAMPLE 3 Finding the Probability of a Compound Event

A student randomly guesses the answer for each of the multiple-choice questions. What is the probability of answering all three questions correctly?

1. In what year did the United States gain independence from Britain?
A. 1492 B. 1776 C. 1788 D. 1795 E. 2000
2. Which amendment to the Constitution grants citizenship to all persons born in the United States and guarantees them equal protection under the law?
A. 1st B. 5th C. 12th D. 13th E. 14th
3. In what year did the Boston Tea Party occur?
A. 1607 B. 1773 C. 1776 D. 1780 E. 1812

Choosing the answer for one question does not affect the choice for the other questions. So, the events are independent.

Method 1: Use the formula for the probability of independent events.



$$\begin{aligned} P(\#1 \text{ and } \#2 \text{ and } \#3 \text{ correct}) &= P(\#1 \text{ correct}) \cdot P(\#2 \text{ correct}) \cdot P(\#3 \text{ correct}) \\ &= \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \quad \text{Substitute.} \\ &= \frac{1}{125} \quad \text{Multiply.} \end{aligned}$$

∴ The probability of answering all three questions correctly is $\frac{1}{125}$, or 0.8%.

Method 2: Use the Fundamental Counting Principle.

There are 5 choices for each question, so there are $5 \cdot 5 \cdot 5 = 125$ possible outcomes. There is only 1 way to answer all three questions correctly.

$$P(\#1 \text{ and } \#2 \text{ and } \#3 \text{ correct}) = \frac{1}{125}$$

∴ The probability of answering all three questions correctly is $\frac{1}{125}$, or 0.8%.

On Your Own

Now You're Ready
Exercises 18–22

3. The student can eliminate Choice A for all three questions. What is the probability of answering all three questions correctly? Compare this probability with the probability in Example 3. What do you notice?



Vocabulary and Concept Check



- DIFFERENT WORDS, SAME QUESTION** You randomly choose one of the chips. Without replacing the first chip, you choose a second chip. Which question is different? Find “both” answers.

What is the probability of choosing a 1 and then a blue chip?

What is the probability of choosing a 1 and then an even number?

What is the probability of choosing a green chip and then a chip that is *not* red?

What is the probability of choosing a number less than 2 and then an even number?

- WRITING** How do you find the probability of two events A and B when A and B are independent? dependent?



Practice and Problem Solving

Tell whether the events are *independent* or *dependent*. Explain.

- You roll a 4 on a number cube. Then you roll an even number on a different number cube.
- You randomly draw a lane number for a 100-meter race. Then your friend randomly draws a lane number for the same race.

You spin the spinner and flip a coin. Find the probability of the compound event.

- Spinning a 3 and flipping heads
- Spinning an even number and flipping tails
- Spinning a number greater than 1 and flipping tails
- Not* spinning a 2 and flipping heads



You randomly choose one of the tiles. Without replacing the first tile, you choose a second tile. Find the probability of the compound event.

- Choosing a 5 and then a 6
- Choosing an odd number and then a 20
- Choosing a number less than 7 and then a multiple of 4
- Choosing two even numbers

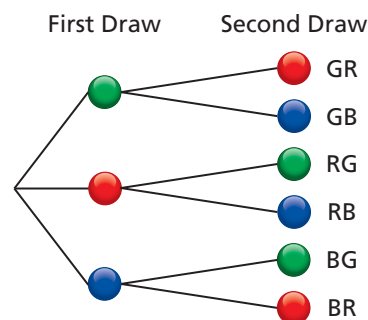
13. **ERROR ANALYSIS** Describe and correct the error in finding the probability.



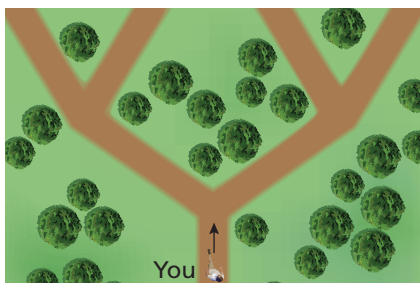
You randomly choose one of the marbles. Without replacing the first marble, you choose a second marble. What is the probability of choosing red and then green?

$$P(\text{red and green}) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

14. **LOGIC** A bag contains three marbles. Does the tree diagram show the outcomes for *independent* or *dependent* events? Explain.

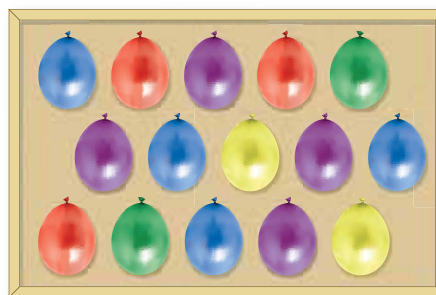


15. **EARRINGS** A jewelry box contains two gold hoop earrings and two silver hoop earrings. You randomly choose two earrings. What is the probability that both are silver hoop earrings?



16. **HIKING** You are hiking to a ranger station. There is one correct path. You come to a fork and randomly take the path on the left. You come to another fork and randomly take the path on the right. What is the probability that you are still on the correct path?

17. **CARNIVAL** At a carnival game, you randomly throw two darts at the board and break two balloons. What is the probability that both of the balloons you break are purple?



You spin the spinner, flip a coin, then spin the spinner again. Find the probability of the compound event.

- 3 18. Spinning a 4, flipping heads, then spinning a 7
19. Spinning an odd number, flipping heads, then spinning a 3
20. Spinning an even number, flipping tails, then spinning an odd number
21. *Not* spinning a 5, flipping heads, then spinning a 1
22. Spinning an odd number, *not* flipping heads, then *not* spinning a 6



23. **LANGUAGES** There are 16 students in your Spanish class. Your teacher randomly chooses one student at a time to take a verbal exam. What is the probability that you are *not* one of the first four students chosen?



24. **SHOES** Twenty percent of the shoes in a factory are black. One shoe is chosen and replaced. A second shoe is chosen and replaced. Then a third shoe is chosen. What is the probability that *none* of the shoes are black?
25. **PROBLEM SOLVING** Your teacher divides your class into two groups, and then randomly chooses a leader for each group. The probability that you are chosen to be a leader is $\frac{1}{12}$. The probability that both you and your best friend are chosen is $\frac{1}{132}$.
- Is your best friend in your group? Explain.
 - What is the probability that your best friend is chosen as a group leader?
 - How many students are in the class?
26. **Structure** After ruling out some of the answer choices, you randomly guess the answer for each of the story questions below.

1. Who was the oldest?
 A. Ned B. Yvonne C. Sun Li D. Angel E. Dusty
2. What city was Stacey from?
 A. Raleigh B. New York C. Roanoke D. Dallas E. San Diego

- How can the probability of getting both answers correct be 25%?
- How can the probability of getting both answers correct be $8\frac{1}{3}\%$?




Fair Game Review what you learned in previous grades & lessons

Draw a triangle with the given angle measures. Then classify the triangle. (Section 12.3)

27. $30^\circ, 60^\circ, 90^\circ$ 28. $20^\circ, 50^\circ, 110^\circ$ 29. $50^\circ, 50^\circ, 80^\circ$

30. **MULTIPLE CHOICE** Which set of numbers is in order from least to greatest? (Skills Review Handbook)

- (A) $\frac{2}{3}, 0.6, 67\%$ (B) $44.5\%, \frac{4}{9}, 0.4\bar{6}$
- (C) $0.269, 27\%, \frac{3}{11}$ (D) $2\frac{1}{7}, 214\%, 2.\bar{14}$

Key Vocabulary 
simulation, p. 668

A **simulation** is an experiment that is designed to reproduce the conditions of a situation or process. Simulations allow you to study situations that are impractical to create in real life.

EXAMPLE 1 Simulating Outcomes That Are Equally Likely

| | |
|-----|-----|
| HTH | HTT |
| HTT | HTH |
| HTT | TTT |
| HHH | HTT |
| HTT | TTT |
| HTT | HTH |
| HTH | HHH |
| HTT | HTT |
| TTT | HTH |
| HTH | HTT |

A couple plans on having three children. The gender of each child is equally likely. (a) Design a simulation involving 20 trials that you can use to model the genders of the children. (b) Use your simulation to find the experimental probability that all three children are boys.

- Choose an experiment that has two equally likely outcomes for each event (gender), such as tossing three coins. Let heads (H) represent a boy and tails (T) represent a girl.
- To find the experimental probability, you need repeated trials of the simulation. The table shows 20 trials.

$$P(\text{three boys}) = \frac{2}{20} = \frac{1}{10}$$

HHH occurred 2 times.

There is a total of 20 trials.

❖ The experimental probability is $\frac{1}{10}$, 0.1, or 10%.

EXAMPLE 2 Simulating Outcomes That Are Not Equally Likely

Study Tip 

In Example 2, the digits 1 through 6 represent 60% of the possible digits (0 through 9) in the tens place. Likewise, the digits 1 and 2 represent 20% of the possible digits in the ones place.

There is a 60% chance of rain on Monday and a 20% chance of rain on Tuesday. Design and use a simulation involving 50 randomly generated numbers to find the experimental probability that it will rain on both days.

Use the random number generator on a graphing calculator. Randomly generate 50 numbers from 0 to 99. The table below shows the results.

Let the digits 1 through 6 in the tens place represent rain on Monday. Let digits 1 and 2 in the ones place represent rain on Tuesday. Any number that meets these criteria represents rain on both days.

```
randInt(0,99,50)
(52 66 73 68 75...
```

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 52 | 66 | 73 | 68 | 75 | 28 | 35 | 47 | 48 | 2 |
| 16 | 68 | 49 | 3 | 77 | 35 | 92 | 78 | 6 | 6 |
| 58 | 18 | 89 | 39 | 24 | 80 | 32 | 41 | 77 | 21 |
| 32 | 40 | 96 | 59 | 86 | 1 | 12 | 0 | 94 | 73 |
| 40 | 71 | 28 | 61 | 1 | 24 | 37 | 25 | 3 | 25 |

$$P(\text{rain both days}) = \frac{7}{50}$$

7 numbers meet the criteria.

There is a total of 50 trials.

❖ The experimental probability is $\frac{7}{50}$, 0.14, or 14%.

EXAMPLE 3 Using a Spreadsheet to Simulate Outcomes



COMMON CORE

Probability and Statistics

In this extension, you will

- use simulations to find experimental probabilities.

Learning Standard
7.SP.8c

Each school year, there is a 50% chance that weather causes one or more days of school to be canceled. Design and use a simulation involving 50 randomly generated numbers to find the experimental probability that weather will cause school to be canceled in at least three of the next four school years.

Use a random number table in a spreadsheet. Randomly generate 50 four-digit whole numbers. The spreadsheet below shows the results.

Let the digits 1 through 5 represent school years with a cancellation. The numbers in the spreadsheet that contain at least three digits from 1 through 5 represent four school years in which at least three of the years have a cancellation.

Study Tip

To create a four-digit random number table in a spreadsheet, follow these steps.

1. Highlight the group of cells to use for your table.
2. Format the cells to display four-digit whole numbers.
3. Enter the formula $\text{RAND()}*10000$ into each cell.

| | A | B | C | D | E | F |
|----|------|------|------|------|------|---|
| 1 | 7584 | 3974 | 8614 | 2500 | 4629 | |
| 2 | 3762 | 3805 | 2725 | 7320 | 6487 | |
| 3 | 3024 | 1554 | 2708 | 1126 | 9395 | |
| 4 | 4547 | 6220 | 9497 | 7530 | 3036 | |
| 5 | 1719 | 0662 | 1814 | 6218 | 2766 | |
| 6 | 7938 | 9551 | 8552 | 4321 | 8043 | |
| 7 | 6951 | 0578 | 5560 | 0740 | 4479 | |
| 8 | 4714 | 4511 | 5115 | 6952 | 5609 | |
| 9 | 0797 | 3022 | 9067 | 2193 | 6553 | |
| 10 | 3300 | 5454 | 5351 | 6319 | 0387 | |
| 11 | | | | | | |

17 numbers contain at least three digits from 1 to 5.

$$P(\text{cancellation in at least three of the next four school years}) = \frac{17}{50}$$

There is a total of 50 trials.

∴ The experimental probability is $\frac{17}{50}$, 0.34, or 34%.

Practice

1. **QUIZ** You randomly guess the answers to four true-false questions. (a) Design a simulation that you can use to model the answers. (b) Use your simulation to find the experimental probability that you answer all four questions correctly.
2. **BASEBALL** A baseball team wins 70% of its games. Assuming this trend continues, design and use a simulation to find the experimental probability that the team wins the next three games.
3. **WHAT IF?** In Example 3, there is a 40% chance that weather causes one or more days of school to be canceled each school year. Find the experimental probability that weather will cause school to be canceled in at least three of the next four school years.
4. **REASONING** In Examples 1–3 and Exercises 1–3, try to find the theoretical probability of the event. What do you think happens to the experimental probability when you increase the number of trials in the simulation?

15 Study Help

You can use a **notetaking organizer** to write notes, vocabulary, and questions about a topic. Here is an example of a notetaking organizer for probability.

| | | | |
|---|--|--|---|
| Write important vocabulary or formulas in this space. | <p>If $P(\text{event}) = 0$, the event is <i>impossible</i>.</p> <p>If $P(\text{event}) = 0.25$, the event is <i>unlikely</i>.</p> <p>If $P(\text{event}) = 0.5$, the event is <i>equally likely to happen or not happen</i>.</p> <p>If $P(\text{event}) = 0.75$, the event is <i>likely</i>.</p> <p>If $P(\text{event}) = 1$, the event is <i>certain</i>.</p> | <h3>Probability</h3> <p>A number that measures the likelihood that an event will occur</p> <p>Can be written as a fraction, decimal, or percent</p> <p>Always between 0 and 1, inclusive</p> | Write your notes about the topic in this space. |
| | | How do you find the probability of two or more events? | |

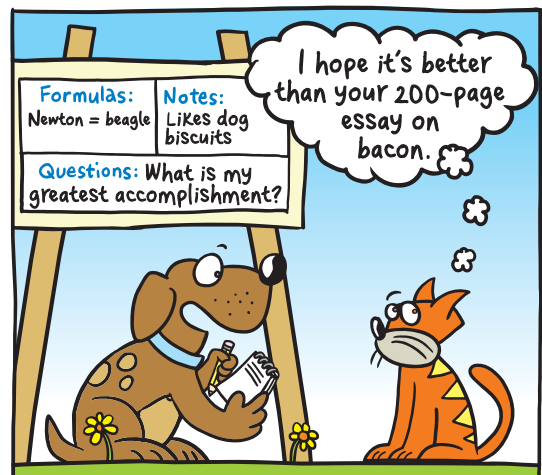
On Your Own

Make notetaking organizers to help you study these topics.

1. experimental probability
2. theoretical probability
3. Fundamental Counting Principle
4. independent events
5. dependent events

After you complete this chapter, make notetaking organizers for the following topics.

6. sample
7. population

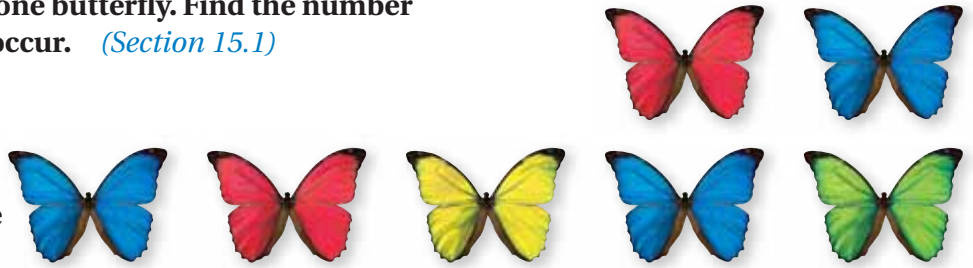


"I am using a **notetaking organizer** to plan my autobiography."

15.1–15.5 Quiz

You randomly choose one butterfly. Find the number of ways the event can occur. (Section 15.1)

1. Choosing red
2. Choosing brown
3. Choosing *not* blue



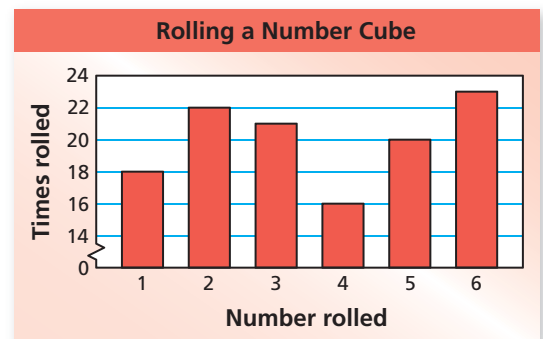
- 6 Green
- 3 White
- 4 Red
- 2 Blue
- 5 Yellow

You randomly choose one paper clip from the jar. Find the probability of the event. (Section 15.2)

4. Choosing a green paper clip
5. Choosing a yellow paper clip
6. *Not* choosing a yellow paper clip
7. Choosing a purple paper clip

Use the bar graph to find the experimental probability of the event. (Section 15.3)

8. Rolling a 4
9. Rolling a multiple of 3
10. Rolling a 2 or a 3
11. Rolling a number less than 7



Use the Fundamental Counting Principle to find the total number of possible outcomes. (Section 15.4)

12.

| Calculator | |
|------------|--|
| Type | Basic display, Scientific, Graphing, Financial |
| Color | Black, White, Silver |

13.

| Vacation | |
|-------------|---------------------------------|
| Destination | Florida, Italy, Mexico, England |
| Length | 1 week, 2 weeks |

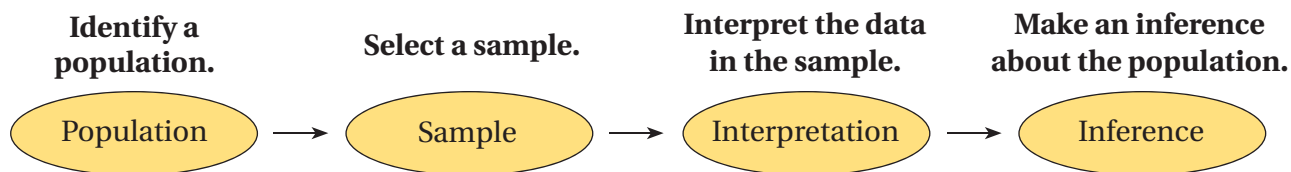


14. **BLACK PENS** You randomly choose one of the pens shown. What is the theoretical probability of choosing a black pen? (Section 15.3)
15. **BLUE PENS** You randomly choose one of the five pens shown. Your friend randomly chooses one of the remaining pens. What is the probability that you and your friend both choose a blue pen? (Section 15.5)

15.6 Samples and Populations

Essential Question How can you determine whether a sample accurately represents a population?

A **population** is an entire group of people or objects. A **sample** is a part of the population. You can use a sample to make an *inference*, or conclusion, about a population.



1 ACTIVITY: Identifying Populations and Samples

Work with a partner. Identify the population and the sample.



The students in a school



The students in a math class



The grizzly bears with GPS collars in a park



The grizzly bears in a park



150 quarters



All quarters in circulation



All books in a library



10 fiction books in a library



COMMON CORE

Probability and Statistics

In this lesson, you will

- determine when samples are representative of populations.
- use data from random samples to make predictions about populations.

Learning Standards

7.SP.1
7.SP.2

2 ACTIVITY: Identifying Random Samples

Work with a partner. When a sample is selected at random, each member of the population is equally likely to be selected. You want to know the favorite extracurricular activity of students at your school. Determine whether each method will result in a random sample. Explain your reasoning.

- You ask members of the school band.
- You publish a survey in the school newspaper.
- You ask every eighth student who enters the school in the morning.
- You ask students in your class.

There are many different ways to select a sample from a population. To make valid inferences about a population, you must choose a random sample very carefully so that it accurately represents the population.

3 ACTIVITY: Identifying Representative Samples

Work with a partner. A new power plant is being built outside a town. In each situation below, residents of the town are asked how they feel about the new power plant. Determine whether each conclusion is valid. Explain your reasoning.

- a. A local radio show takes calls from 500 residents. The table shows the results. The radio station concludes that most of the residents of the town oppose the new power plant.

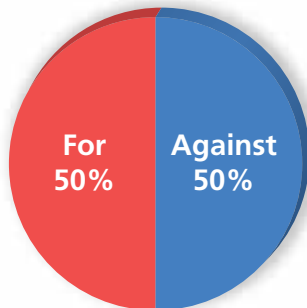
| New Power Plant | |
|-----------------|-----|
| For | 70 |
| Against | 425 |
| Don't know | 5 |

Math Practice 2

Understand Quantities

Can the size of a sample affect the validity of a conclusion about a population?

New Power Plant



- b. A news reporter randomly surveys 2 residents outside a supermarket. The graph shows the results. The reporter concludes that the residents of the town are evenly divided on the new power plant.

- c. You randomly survey 250 residents at a shopping mall. The table shows the results. You conclude that there are about twice as many residents of the town against the new power plant than for the new power plant.

| New Power Plant | |
|-----------------|-----|
| For | 32% |
| Against | 62% |
| Don't know | 6% |

What Is Your Answer?

- IN YOUR OWN WORDS** How can you determine whether a sample accurately represents a population?
- RESEARCH** Choose a topic that you would like to ask people's opinions about, and then write a survey question. How would you choose people to survey so that your sample is random? How many people would you survey? Conduct your survey and display your results. Would you change any part of your survey to make it more accurate? Explain.
- Does increasing the size of a sample necessarily make the sample representative of a population? Give an example to support your explanation.

Practice

Use what you learned about populations and samples to complete Exercises 3 and 4 on page 676.

Key Vocabulary

population, p. 672

sample, p. 672

unbiased sample,
p. 674

biased sample, p. 674

An **unbiased sample** is representative of a population. It is selected at random and is large enough to provide accurate data.

A **biased sample** is not representative of a population. One or more parts of the population are favored over others.

EXAMPLE 1 Identifying an Unbiased Sample

You want to estimate the number of students in a high school who ride the school bus. Which sample is unbiased?

- (A) 4 students in the hallway
- (B) all students in the marching band
- (C) 50 seniors at random
- (D) 100 students at random during lunch



Choice A is not large enough to provide accurate data.

Choice B is not selected at random.

Choice C is not representative of the population because seniors are more likely to drive to school than other students.

Choice D is representative of the population, selected at random, and large enough to provide accurate data.

∴ So, the correct answer is (D).

On Your Own

Now You're Ready
Exercises 5–7

- WHAT IF?** You want to estimate the number of seniors in a high school who ride the school bus. Which sample is unbiased? Explain.
- You want to estimate the number of eighth-grade students in your school who consider it relaxing to listen to music. You randomly survey 15 members of the band. Your friend surveys every fifth student whose name appears on an alphabetical list of eighth graders. Which sample is unbiased? Explain.

The results of an unbiased sample are proportional to the results of the population. So, you can use unbiased samples to make predictions about the population.

Biased samples are not representative of the population. So, you should not use them to make predictions about the population because the predictions may not be valid.

EXAMPLE 2 Determining Whether Conclusions Are Valid

You want to know how the residents of your town feel about adding a new stop sign. Determine whether each conclusion is valid.

- a. You survey the 20 residents who live closest to the new sign. Fifteen support the sign, and five do not. So, you conclude that 75% of the residents of your town support the new sign.

The sample is not representative of the population because residents who live close to the sign are more likely to support it.

∴ So, the sample is biased, and the conclusion is not valid.

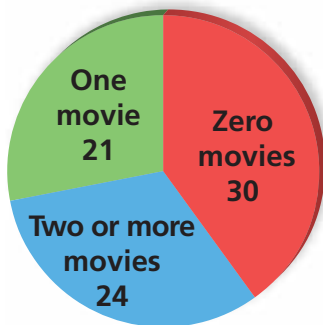
- b. You survey 100 residents at random. Forty support the new sign, and sixty do not. So, you conclude that 40% of the residents of your town support the new sign.

The sample is representative of the population, selected at random, and large enough to provide accurate data.

∴ So, the sample is unbiased, and the conclusion is valid.

EXAMPLE 3 Making Predictions

Movies per Week



You ask 75 randomly chosen students how many movies they watch each week. There are 1200 students in the school. Predict the number n of students in the school who watch one movie each week.

The sample is representative of the population, selected at random, and large enough to provide accurate data. So, the sample is unbiased, and you can use it to make a prediction about the population.

Write and solve a proportion to find n .

| Sample | Population |
|---|---|
| $\frac{\text{students in survey (one movie)}}{\text{number of students in survey}}$ | $= \frac{\text{students in school (one movie)}}{\text{number of students in school}}$ |
| $\frac{21}{75}$ | $= \frac{n}{1200}$ <i>Substitute.</i> |
| $336 = n$ | <i>Solve for n.</i> |

∴ So, about 336 students in the school watch one movie each week.

On Your Own

Now You're Ready
Exercises 8, 9,
and 12

- In Example 2, each of 25 randomly chosen firefighters supports the new sign. So, you conclude that 100% of the residents of your town support the new sign. Is the conclusion valid? Explain.
- In Example 3, predict the number of students in the school who watch two or more movies each week.

Vocabulary and Concept Check

- VOCABULARY** Why would you survey a sample instead of a population?
- CRITICAL THINKING** What should you consider when conducting a survey?

Practice and Problem Solving

Identify the population and the sample.

3. Residents of New Jersey  Residents of Ocean County 

4.  
4 cards All cards in a deck

Determine whether the sample is *biased* or *unbiased*. Explain.

5. You want to estimate the number of students in your school who play a musical instrument. You survey the first 15 students who arrive at a band class.
6. You want to estimate the number of books students in your school read over the summer. You survey every fourth student who enters the school.
7. You want to estimate the number of people in a town who think that a park needs to be remodeled. You survey every 10th person who enters the park.



Determine whether the conclusion is valid. Explain.

8. You want to determine the number of students in your school who have visited a science museum. You survey 50 students at random. Twenty have visited a science museum, and thirty have not. So, you conclude that 40% of the students in your school have visited a science museum.
9. You want to know how the residents of your town feel about building a new baseball stadium. You randomly survey 100 people who enter the current stadium. Eighty support building a new stadium, and twenty do not. So, you conclude that 80% of the residents of your town support building a new baseball stadium.

Which sample is better for making a prediction? Explain.

10. **Predict the number of students in a school who like gym class.**

| | |
|----------|--|
| Sample A | A random sample of 8 students from the yearbook |
| Sample B | A random sample of 80 students from the yearbook |

11. **Predict the number of defective pencils produced per day.**

| | |
|----------|---|
| Sample A | A random sample of 500 pencils from 20 machines |
| Sample B | A random sample of 500 pencils from 1 machine |

12. **FOOD** You ask 125 randomly chosen students to name their favorite food. There are 1500 students in the school. Predict the number of students in the school whose favorite food is pizza.

| Favorite Food | |
|---------------|----|
| Pizza | 58 |
| Hamburger | 36 |
| Pasta | 14 |
| Other | 17 |

Determine whether you would survey the population or a sample. Explain.

13. You want to know the average height of seventh graders in the United States.
 14. You want to know the favorite types of music of students in your homeroom.
 15. You want to know the number of students in your state who have summer jobs.

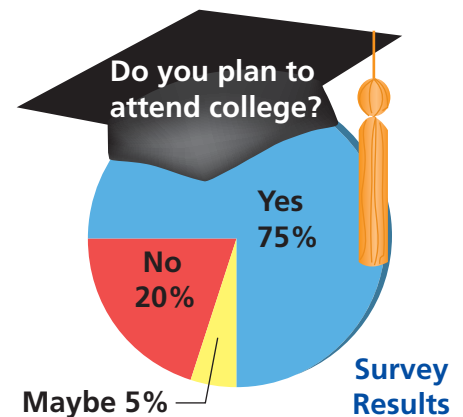
| Theater Ticket Sales | |
|----------------------|----------|
| Adults | Students |
| 522 | 210 |

16. **THEATER** You survey 72 randomly chosen students about whether they are going to attend the school play. Twelve say yes. Predict the number of students who attend the school.

17. **CRITICAL THINKING** Explain why 200 people with email addresses may not be a random sample. When might it be a random sample?
 18. **LOGIC** A person surveys residents of a town to determine whether a skateboarding ban should be overturned.

- a. Describe how the person could conduct the survey so that the sample is biased toward overturning the ban.
 b. Describe how the person could conduct the survey so that the sample is biased toward keeping the ban.

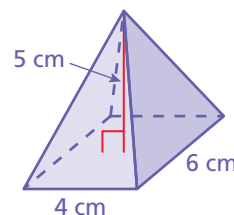
19. **Reasoning** A guidance counselor surveys a random sample of 60 out of 900 high school students. Using the survey results, the counselor predicts that approximately 720 students plan to attend college. Do you agree with her prediction? Explain.



Fair Game Review what you learned in previous grades & lessons

Write and solve a proportion to answer the question. (*Skills Review Handbook*)

20. What percent of 60 is 18?
 21. 70% of what number is 98?
 22. 30 is 15% of what number?
 23. What number is 0.6% of 500?
 24. **MULTIPLE CHOICE** What is the volume of the pyramid? (*Section 14.5*)
- (A) 40 cm^3 (B) 50 cm^3
 (C) 100 cm^3 (D) 120 cm^3



You have already used unbiased samples to make inferences about a population. In some cases, making an inference about a population from only one sample is not as precise as using multiple samples.

1 ACTIVITY: Using Multiple Random Samples

Work with a partner. You and a group of friends want to know how many students in your school listen to pop music. There are 840 students in your school. Each person in the group randomly surveys 20 students.

Step 1: The table shows your results. Make an inference about the number of students in your school who prefer pop music.

| Favorite Type of Music | | | |
|------------------------|-----|------|-----|
| Country | Pop | Rock | Rap |
| 4 | 10 | 5 | 1 |

Step 2: The table shows Kevin's results. Use these results to make another inference about the number of students in your school who prefer pop music.

| Favorite Type of Music | | | |
|------------------------|-----|------|-----|
| Country | Pop | Rock | Rap |
| 2 | 13 | 4 | 1 |

Compare the results of Steps 1 and 2.

Step 3: The table shows the results of three other friends. Use these results to make three more inferences about the number of students in your school who prefer pop music.

| | Favorite Type of Music | | | |
|--------------|------------------------|-----|------|-----|
| | Country | Pop | Rock | Rap |
| Steve | 3 | 8 | 7 | 2 |
| Laura | 5 | 10 | 4 | 1 |
| Ming | 5 | 9 | 3 | 3 |

Step 4: Describe the variation of the five inferences. Which one would you use to describe the number of students in your school who prefer pop music? Explain your reasoning.

Step 5: Show how you can use all five samples to make an inference.



COMMON CORE

Probability and Statistics

In this extension, you will

- use multiple samples to make predictions about populations.

Learning Standard
7.SP.2

Practice

- PACKING PEANUTS** Work with a partner. Mark 24 packing peanuts with either a red or a black marker. Put the peanuts into a paper bag. Trade bags with other students in the class.
 - Generate a sample by choosing a peanut from your bag six times, replacing the peanut each time. Record the number of times you choose each color. Repeat this process to generate four more samples. Organize the results in a table.
 - Use each sample to make an inference about the number of red peanuts in the bag. Then describe the variation of the five inferences. Make inferences about the numbers of red and black peanuts in the bag based on all the samples.
 - Take the peanuts out of the bag. How do your inferences compare to the population? Do you think you can make a more accurate prediction? If so, explain how.

2 ACTIVITY: Using Measures from Multiple Random Samples

Hours Worked Each Week

1: 6, 8, 6, 6, 7, 4, 10, 8, 7, 8
2: 10, 4, 4, 6, 8, 6, 7, 12, 8, 8
3: 10, 9, 8, 6, 5, 8, 6, 6, 9, 10
4: 4, 8, 4, 4, 5, 4, 4, 6, 5, 6
5: 6, 8, 8, 6, 12, 4, 10, 8, 6, 12
6: 10, 10, 8, 9, 16, 8, 7, 12, 16, 14
7: 4, 5, 6, 6, 4, 5, 6, 6, 4, 4
8: 16, 20, 8, 12, 10, 8, 8, 14, 16, 8

Work with a partner. You want to know the mean number of hours students with part-time jobs work each week. You go to 8 different schools. At each school, you randomly survey 10 students with part-time jobs. Your results are shown at the left.

Step 1: Find the mean of each sample.

Step 2: Make a box-and-whisker plot of the sample means.

Step 3: Use the box-and-whisker plot to estimate the actual mean number of hours students with part-time jobs work each week.

How does your estimate compare to the mean of the entire data set?

3 ACTIVITY: Using a Simulation

Work with a partner. Another way to generate multiple samples of data is to use a simulation. Suppose 70% of all seventh graders watch reality shows on television.

Step 1: Design a simulation involving 50 packing peanuts by marking 70% of the peanuts with a certain color. Put the peanuts into a paper bag.

Step 2: Simulate choosing a sample of 30 students by choosing peanuts from the bag, replacing the peanut each time. Record the results. Repeat this process to generate eight more samples. How much variation do you expect among the samples? Explain.

Step 3: Display your results.



Practice

2. **SPORTS DRINKS** You want to know whether student-athletes prefer water or sports drinks during games. You go to 10 different schools. At each school, you randomly survey 10 student-athletes. The percents of student-athletes who prefer water are shown.

60% 70% 60% 50% 80% 70% 30% 70% 80% 40%

- Make a box-and-whisker plot of the data.
 - Use the box-and-whisker plot to estimate the actual percent of student-athletes who prefer water. How does your estimate compare to the mean of the data?
3. **PART-TIME JOBS** Repeat Activity 2 using the medians of the samples.
4. **TELEVISION** In Activity 3, how do the percents in your samples compare to the given percent of seventh graders who watch reality shows on television?
5. **REASONING** Why is it better to make inferences about a population based on multiple samples instead of only one sample? What additional information do you gain by taking multiple random samples? Explain.

15.7 Comparing Populations

Essential Question How can you compare data sets that represent two populations?

1 ACTIVITY: Comparing Two Data Distributions

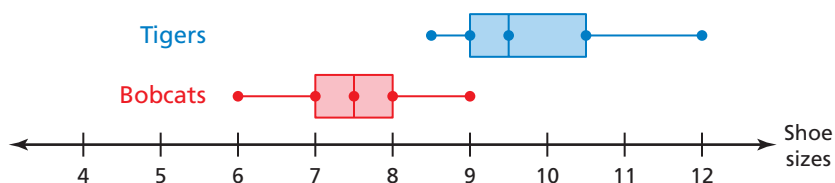
Work with a partner. You want to compare the shoe sizes of male students in two classes. You collect the data shown in the table.

| Male Students in Eighth-Grade Class | | | | | | | | | | | | | | |
|-------------------------------------|----------------|---|----------------|----------------|----------------|---|----------------|---|----------------|----------------|---|----------------|----------------|----------------|
| 7 | 9 | 8 | $7\frac{1}{2}$ | $8\frac{1}{2}$ | 10 | 6 | $6\frac{1}{2}$ | 8 | 8 | $8\frac{1}{2}$ | 9 | 11 | $7\frac{1}{2}$ | $8\frac{1}{2}$ |
| Male Students in Sixth-Grade Class | | | | | | | | | | | | | | |
| 6 | $5\frac{1}{2}$ | 6 | $6\frac{1}{2}$ | $7\frac{1}{2}$ | $8\frac{1}{2}$ | 7 | $5\frac{1}{2}$ | 5 | $5\frac{1}{2}$ | $6\frac{1}{2}$ | 7 | $4\frac{1}{2}$ | 6 | 6 |

- How can you display both data sets so that you can visually compare the measures of center and variation? Make the data display you chose.
- Describe the shape of each distribution.
- Complete the table.

| | Mean | Median | Mode | Range | Interquartile Range (IQR) | Mean Absolute Deviation (MAD) |
|-------------------------------------|------|--------|------|-------|---------------------------|-------------------------------|
| Male Students in Eighth-Grade Class | | | | | | |
| Male Students in Sixth-Grade Class | | | | | | |

- Compare the measures of center for the data sets.
- Compare the measures of variation for the data sets. Does one data set show more variation than the other? Explain.
- Do the distributions overlap? How can you tell using the data display you chose in part (a)?
- The double box-and-whisker plot below shows the shoe sizes of the members of two girls basketball teams. Can you conclude that at least one girl from each team has the same shoe size? Can you conclude that at least one girl from the Bobcats has a larger shoe size than one of the girls from the Tigers? Explain your reasoning.



COMMON CORE Probability and Statistics

In this lesson, you will

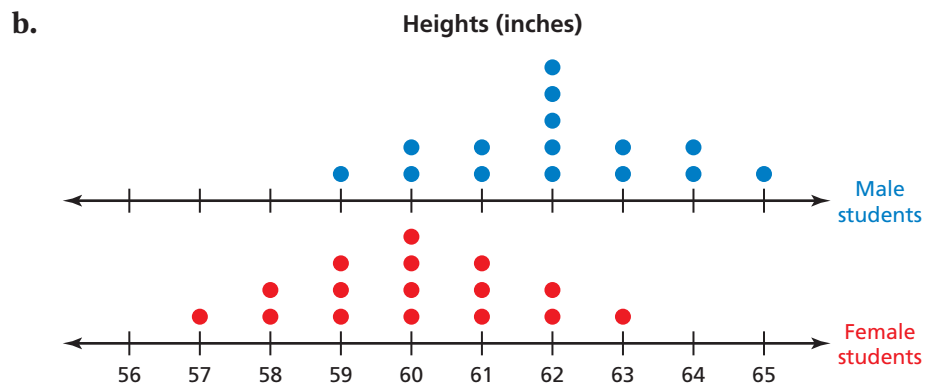
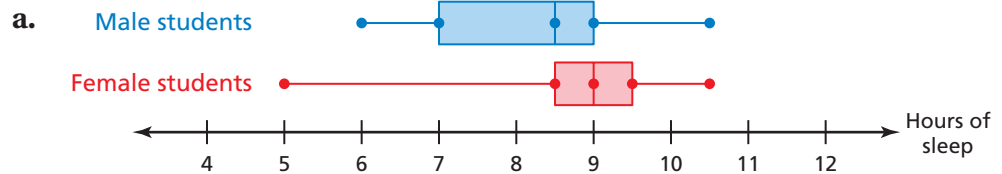
- use measures of center and variation to compare populations.
- use random samples to compare populations.

Learning Standards

7.SP.3
7.SP.4

2 ACTIVITY: Comparing Two Data Distributions

Work with a partner. Compare the shapes of the distributions. Do the two data sets overlap? Explain. If so, use measures of center and the least and the greatest values to describe the overlap between the two data sets.



Math Practice 5

Recognize Usefulness of Tools

How is each type of data display useful? Which do you prefer? Explain.

c. Ages of People in Two Exercise Classes

| 10:00 A.M. Class | 8:00 P.M. Class |
|------------------|-----------------|
| | 1 8 9 |
| | 2 1 2 2 7 9 9 |
| | 3 0 3 4 5 7 |
| 9 7 3 2 2 2 | 4 0 |
| 7 5 4 3 1 | 5 |
| 7 0 0 | 6 |
| 0 | 7 |

Key: 1 | 8 = 18

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you compare data sets that represent two populations?

Practice

Use what you learned about comparing data sets to complete Exercise 3 on page 684.

Recall that you use the mean and the mean absolute deviation (MAD) to describe symmetric distributions of data. You use the median and the interquartile range (IQR) to describe skewed distributions of data.

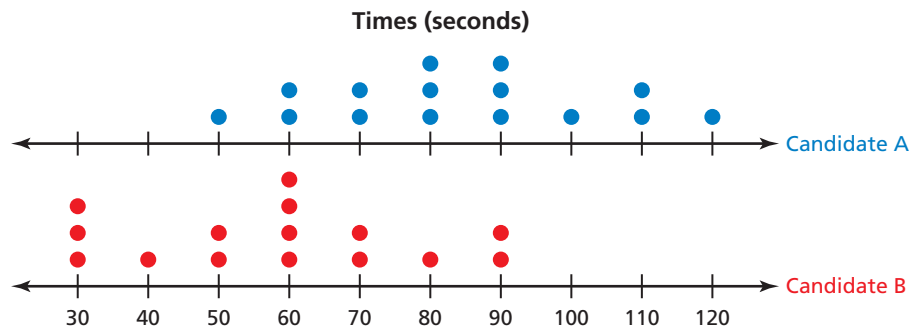
To compare two populations, use the mean and the MAD when both distributions are symmetric. Use the median and the IQR when either one or both distributions are skewed.

EXAMPLE 1 Comparing Populations

The double dot plot shows the time that each candidate in a debate spent answering each of 15 questions.

Study Tip

You can more easily see the visual overlap of dot plots that are aligned vertically.



- a. Compare the populations using measures of center and variation.

Both distributions are approximately symmetric, so use the mean and the MAD.

$$\begin{aligned} \text{Candidate A} \\ \text{Mean} &= \frac{1260}{15} = 84 \\ \text{MAD} &= \frac{244}{15} \approx 16 \end{aligned}$$

$$\begin{aligned} \text{Candidate B} \\ \text{Mean} &= \frac{870}{15} = 58 \\ \text{MAD} &= \frac{236}{15} \approx 16 \end{aligned}$$

- ∴ So, the variation in the times was about the same, but Candidate A had a greater mean time.

- b. Express the difference in the measures of center as a multiple of the measure of variation.

$$\frac{\text{mean for Candidate A} - \text{mean for Candidate B}}{\text{MAD}} = \frac{26}{16} \approx 1.6$$

- ∴ So, the difference in the means is about 1.6 times the MAD.

Study Tip

When two populations have similar variabilities, the value in part (b) describes the visual overlap between the data. In general, the greater the value, the less the overlap.

On Your Own

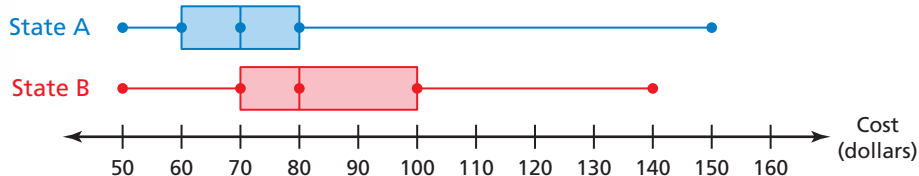
1. **WHAT IF?** Each value in the dot plot for Candidate A increases by 30 seconds. How does this affect the answers in Example 1? Explain.

You do not need to have all the data from two populations to make comparisons. You can use random samples to make comparisons.

EXAMPLE 2 Using Random Samples to Compare Populations

You want to compare the costs of speeding tickets in two states.

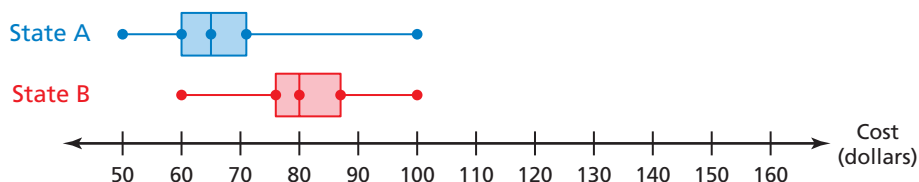
- a. The double box-and-whisker plot shows a random sample of 10 speeding tickets issued in two states. Compare the samples using measures of center and variation. Can you use this to make a valid comparison about speeding tickets in the two states? Explain.



Both distributions are skewed right, so use the median and the IQR.

- The median and the IQR for State A, 70 and 20, are less than the median and the IQR for State B, 80 and 30. However, the sample size is too small and the variability is too great to conclude that speeding tickets generally cost more in State B.

- b. The double box-and-whisker plot shows the medians of 100 random samples of 10 speeding tickets for each state. Compare the variability of the sample medians to the variability of the sample costs in part (a).



The IQR of the sample medians for each state is about 10.

- So, the sample medians vary much less than the sample costs.

- c. Make a conclusion about the costs of speeding tickets in the two states.

The sample medians show less variability. Most of the sample medians for State B are greater than the sample medians for State A.

- So, speeding tickets generally cost more in State B than in State A.

On Your Own

Now You're Ready
Exercise 8

2. **WHAT IF?** A random sample of 8 speeding tickets issued in State C has a median of \$120. Can you conclude that a speeding ticket in State C costs more than in States A and B? Explain.



Vocabulary and Concept Check

- REASONING** When comparing two populations, when should you use the mean and the MAD? the median and the IQR?
- WRITING** Two data sets have similar variabilities. Suppose the measures of center of the data sets differ by 4 times the measure of variation. Describe the visual overlap of the data.



Practice and Problem Solving

- SNAKES** The tables show the lengths of two types of snakes at an animal store.

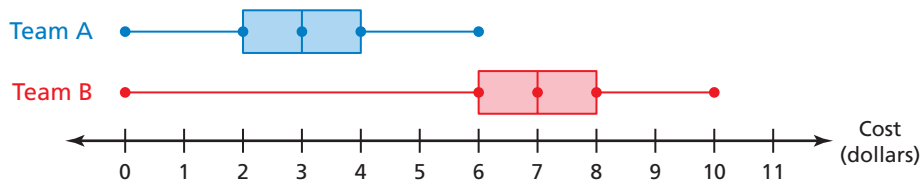
| Garter Snake Lengths (inches) | | | | | |
|-------------------------------|----|----|----|----|----|
| 26 | 30 | 22 | 15 | 21 | 24 |
| 28 | 32 | 24 | 25 | 18 | 35 |

| Water Snake Lengths (inches) | | | | | |
|------------------------------|----|----|----|----|----|
| 34 | 25 | 24 | 35 | 40 | 32 |
| 41 | 27 | 37 | 32 | 21 | 30 |

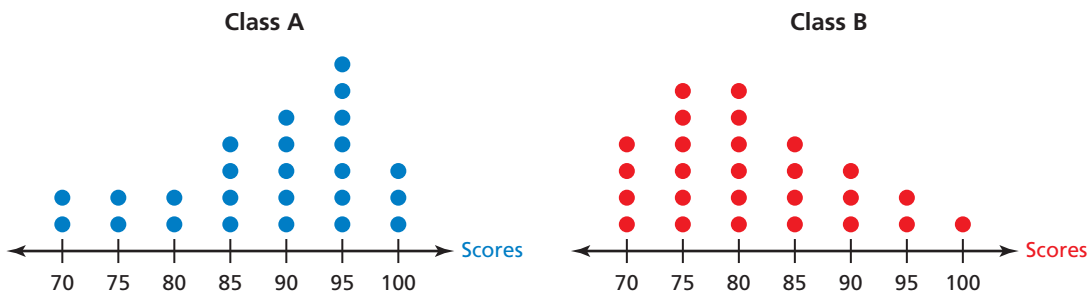
- Find the mean, median, mode, range, interquartile range, and mean absolute deviation for each data set.
- Compare the data sets.

1

- HOCKEY** The double box-and-whisker plot shows the goals scored per game by two hockey teams during a 20-game season.



- Compare the populations using measures of center and variation.
 - Express the difference in the measures of center as a multiple of the measure of variation.
- TEST SCORES** The dot plots show the test scores for two classes taught by the same teacher.



- Compare the populations using measures of center and variation.
- Express the difference in the measures of center as a multiple of each measure of variation.

6. **ATTENDANCE** The tables show the attendances at volleyball games and basketball games at a school during the year.

| Volleyball Game Attendance | | | | | | |
|----------------------------|----|----|-----|----|-----|----|
| 112 | 95 | 84 | 106 | 62 | 68 | 53 |
| 75 | 88 | 93 | 127 | 98 | 117 | 60 |
| 49 | 54 | 85 | 74 | 88 | 132 | |

| Basketball Game Attendance | | | | | | |
|----------------------------|-----|-----|-----|-----|-----|-----|
| 202 | 190 | 173 | 155 | 169 | 188 | 195 |
| 176 | 141 | 152 | 181 | 198 | 214 | 179 |
| 163 | 186 | 184 | 207 | 219 | 228 | |

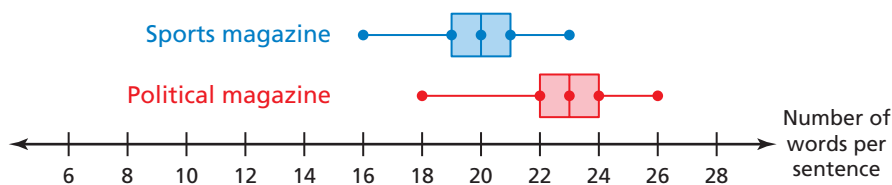
- a. Compare the populations using measures of center and variation.
 b. Express the difference in the measures of center as a multiple of each measure of variation.
7. **NUMBER SENSE** Compare the answers to Exercises 4(b), 5(b), and 6(b). Which value is the greatest? What does this mean?
- 2 8. **MAGAZINES** You want to compare the number of words per sentence in a sports magazine to the number of words per sentence in a political magazine.

- a. The data represent random samples of 10 sentences in each magazine. Compare the samples using measures of center and variation. Can you use this to make a valid comparison about the magazines? Explain.

Sports magazine: 9, 21, 15, 14, 25, 26, 9, 19, 22, 30

Political magazine: 31, 22, 17, 5, 23, 15, 10, 20, 20, 17

- b. The double box-and-whisker plot shows the means of 200 random samples of 20 sentences. Compare the variability of the sample means to the variability of the sample numbers of words in part (a).



- c. Make a conclusion about the numbers of words per sentence in the magazines.
9. **Project** You want to compare the average amounts of time students in sixth, seventh, and eighth grade spend on homework each week.
- a. Design an experiment involving random sampling that can help you make a comparison.
 b. Perform the experiment. Can you make a conclusion about which students spend the most time on homework? Explain your reasoning.



Fair Game Review What you learned in previous grades & lessons

Graph the inequality on a number line. (Section 11.1)

10. $x > 5$

11. $b \leq -3$

12. $n < -1.6$

13. $p \geq 2.5$

14. **MULTIPLE CHOICE** The number of students in the marching band increased from 100 to 125. What is the percent of increase? (Skills Review Handbook)

(A) 20%

(B) 25%

(C) 80%

(D) 500%

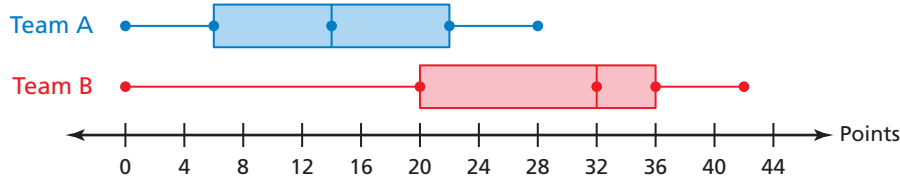
15.6–15.7 Quiz

1. Which sample is better for making a prediction? Explain. (Section 15.6)

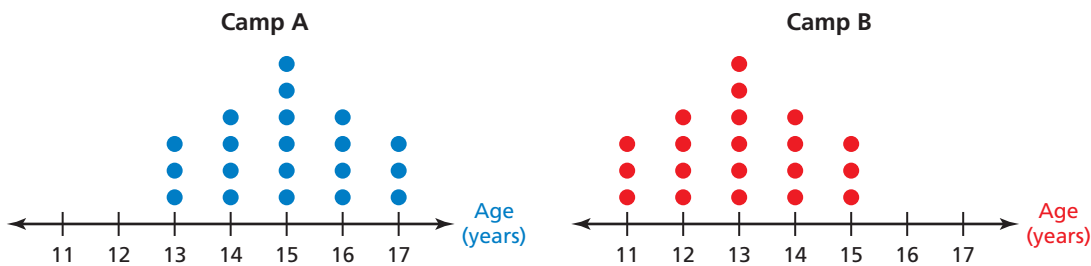
Predict the number of students in your school who play at least one sport.

| | |
|----------|---|
| Sample A | A random sample of 10 students from the school student roster |
| Sample B | A random sample of 80 students from the school student roster |

2. **GYMNASIUM** You want to estimate the number of students in your school who think the gymnasium should be remodeled. You survey 12 students on the basketball team. Determine whether the sample is *biased* or *unbiased*. Explain. (Section 15.6)
3. **TOWN COUNCIL** You want to know how the residents of your town feel about a recent town council decision. You survey 100 residents at random. Sixty-five support the decision, and thirty-five do not. So, you conclude that 65% of the residents of your town support the decision. Determine whether the conclusion is valid. Explain. (Section 15.6)
4. **FIELD TRIP** Of 60 randomly chosen students surveyed, 16 chose the aquarium as their favorite field trip. There are 720 students in the school. Predict the number of students in the school who would choose the aquarium as their favorite field trip. (Section 15.6)
5. **FOOTBALL** The double box-and-whisker plot shows the points scored per game by two football teams during the regular season. (Section 15.7)



- a. Compare the populations using measures of center and variation.
- b. Express the difference in the measures of center as a multiple of the measure of variation.
6. **SUMMER CAMP** The dot plots show the ages of campers at two summer camps. (Section 15.7)



- a. Compare the populations using measures of center and variation.
- b. Express the difference in the measures of center as a multiple of the measure of variation.

Review Key Vocabulary

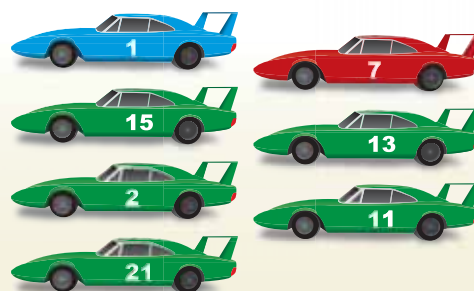
| | | |
|----------------------------|--|----------------------------|
| experiment, p. 634 | experimental probability, p. 646 | independent events, p. 662 |
| outcomes, p. 634 | theoretical probability, p. 647 | dependent events, p. 663 |
| event, p. 634 | sample space, p. 654 | simulation, p. 668 |
| favorable outcomes, p. 634 | Fundamental Counting Principle, p. 654 | population, p. 672 |
| probability, p. 640 | compound event, p. 656 | sample, p. 672 |
| relative frequency, p. 644 | | unbiased sample, p. 674 |
| | | biased sample, p. 674 |

Review Examples and Exercises

15.1 Outcomes and Events (pp. 632–637)

You randomly choose one toy race car.

- In how many ways can choosing a green car occur?
- In how many ways can choosing a car that is *not* green occur? What are the favorable outcomes of choosing a car that is *not* green?



- There are 5 green cars. So, choosing a green car can occur in 5 ways.
- There are 2 cars that are *not* green. So, choosing a car that is *not* green can occur in 2 ways.

| green | not green |
|-----------------------------------|-----------|
| green, green, green, green, green | blue, red |

∴ The favorable outcomes of the event are blue and red.

Exercises

You spin the spinner. (a) Find the number of ways the event can occur. (b) Find the favorable outcomes of the event.

- Spinning a 1
- Spinning a 3
- Spinning an odd number
- Spinning an even number
- Spinning a number greater than 0
- Spinning a number less than 3



15.2 Probability (pp. 638–643)

You flip a coin. What is the probability of flipping tails?

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{tails}) = \frac{1}{2}$$

There is 1 tails.

There is a total of 2 sides.

∴ The probability of flipping tails is $\frac{1}{2}$, or 50%.

Exercises

7. You roll a number cube. Find the probability of rolling an even number.

15.3 Experimental and Theoretical Probability (pp. 644–651)



- a. The bar graph shows the results of spinning the spinner 70 times. What is the experimental probability of spinning a 2?

The bar graph shows 12 twos. So, the spinner landed on two 12 times in a total of 70 spins.

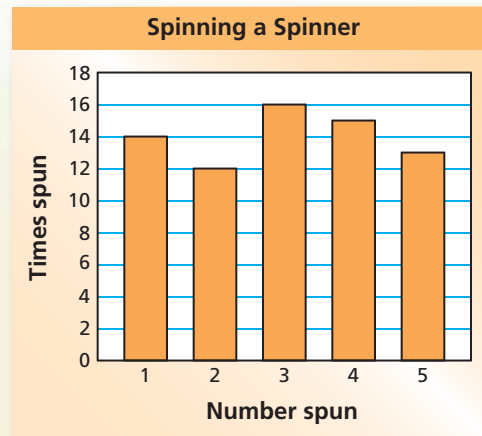
$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

Two was landed on 12 times.

$$P(2) = \frac{12}{70} = \frac{6}{35}$$

There was a total of 70 spins.

∴ The experimental probability is $\frac{6}{35}$, or about 17%.



- b. The theoretical probability of choosing a purple grape from a bag is $\frac{2}{9}$. There are 8 purple grapes in the bag. How many grapes are in the bag?

$$P(\text{purple}) = \frac{\text{number of purple grapes}}{\text{total number of grapes}}$$

$$\frac{2}{9} = \frac{8}{g}$$

Substitute. Let g be the total number of grapes.

$$g = 36$$

Solve for g .

∴ So, there are 36 grapes in the bag.

Exercises

Use the bar graph on page 456 to find the experimental probability of the event.

8. Spinning a 3
9. Spinning an odd number
10. *Not* spinning a 5
11. Spinning a number greater than 3

Use the spinner to find the theoretical probability of the event.

12. Spinning blue
13. Spinning a 1
14. Spinning an even number
15. Spinning a 4
16. The theoretical probability of spinning an even number on a spinner is $\frac{2}{3}$. The spinner has 8 even-numbered sections. How many sections are on the spinner?



15.4 Compound Events (pp. 652–659)

- a. How many different home theater systems can you make from 6 DVD players, 8 TVs, and 3 brands of speakers?

$$6 \times 8 \times 3 = 144 \quad \text{Fundamental Counting Principle}$$

So, you can make 144 different home theater systems.

- b. You flip two pennies. What is the probability of flipping two heads?

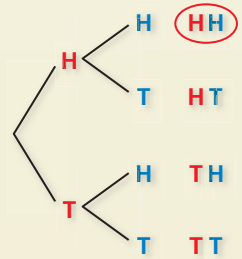
Use a tree diagram to find the probability. Let H = heads and T = tails.

There is one favorable outcome in the sample space for flipping two heads: HH.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(2 \text{ heads}) = \frac{1}{4} \quad \text{Substitute.}$$

The probability is $\frac{1}{4}$, or 25%.



Exercises

17. You have 6 bracelets and 15 necklaces. Find the number of ways you can wear one bracelet and one necklace.
18. You flip two coins and roll a number cube. What is the probability of flipping two tails and rolling an even number?



15.5 Independent and Dependent Events (pp. 660–669)

You randomly choose one of the tiles and flip the coin. What is the probability of choosing a vowel and flipping heads?

Choosing one of the tiles does not affect the outcome of flipping the coin. So, the events are independent.

$$P(\text{vowel}) = \frac{2}{7}$$

There are 2 vowels (A and E).

There is a total of 7 tiles.

$$P(\text{tails}) = \frac{1}{2}$$

There is 1 tails side.

There is a total of 2 sides.



Use the formula for the probability of independent events.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) \\ &= \frac{2}{7} \cdot \frac{1}{2} = \frac{1}{7} \end{aligned}$$

∴ The probability of choosing a vowel and flipping heads is $\frac{1}{7}$, or about 14%.

Exercises

You randomly choose one of the tiles above and flip the coin. Find the probability of the compound event.

19. Choosing a blue tile and flipping tails
20. Choosing the letter G and flipping tails

You randomly choose one of the tiles above. Without replacing the first tile, you randomly choose a second tile. Find the probability of the compound event.

21. Choosing a green tile and then a blue tile
22. Choosing a red tile and then a vowel

15.6 Samples and Populations (pp. 672–679)

You want to estimate the number of students in your school whose favorite subject is math. You survey every third student who leaves the school. Determine whether the sample is *biased* or *unbiased*.

The sample is representative of the population, selected at random, and large enough to provide accurate data.

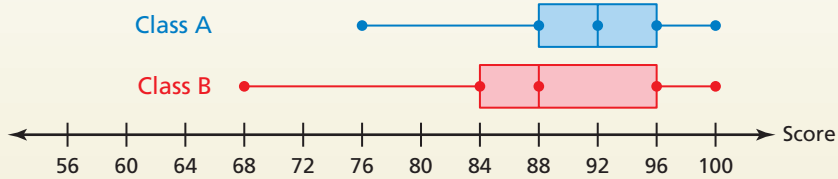
∴ So, the sample is unbiased.

Exercises

23. You want to estimate the number of students in your school whose favorite subject is biology. You survey the first 10 students who arrive at biology club. Determine whether the sample is *biased* or *unbiased*. Explain.

15.7 Comparing Populations (pp. 680–685)

The double box-and-whisker plot shows the test scores for two French classes taught by the same teacher.



- a. Compare the populations using measures of center and variation.

Both distributions are skewed left, so use the median and the IQR.

- The median for Class A, 92, is greater than the median for Class B, 88. The IQR for Class B, 12, is greater than the IQR for Class A, 8. The scores in Class A are generally greater and have less variability than the scores in Class B.

- b. Express the difference in the measures of center as a multiple of each measure of variation.

$$\frac{\text{median for Class A} - \text{median for Class B}}{\text{IQR for Class A}} = \frac{4}{8} = 0.5$$

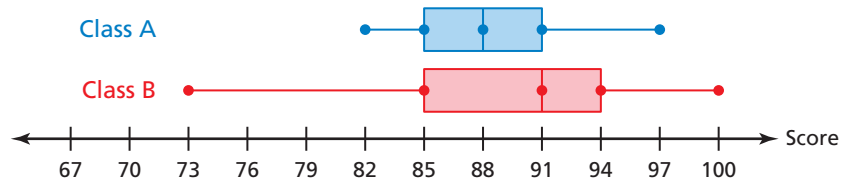
$$\frac{\text{median for Class A} - \text{median for Class B}}{\text{IQR for Class B}} = \frac{4}{12} = 0.3$$

- So, the difference in the medians is about 0.3 to 0.5 times the IQR.

Exercises

24. **SPANISH TEST** The double box-and-whisker plot shows the test scores of two Spanish classes taught by the same teacher.

- a. Compare the populations using measures of center and variation.
- b. Express the difference in the measures of center as a multiple of each measure of variation.



15 Chapter Test

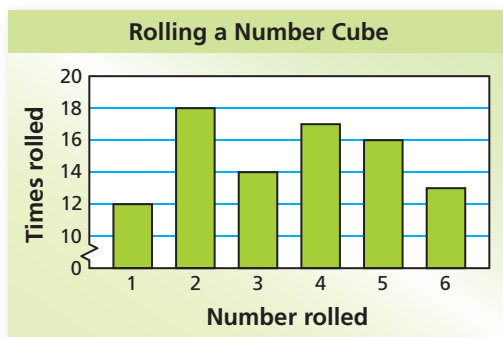
You randomly choose one game piece. (a) Find the number of ways the event can occur. (b) Find the favorable outcomes of the event.

- Choosing green
- Choosing *not* yellow



- Use the Fundamental Counting Principle to find the total number of different sunscreens possible.

| Sunscreen | |
|-----------|--------------------|
| SPF | 10, 15, 30, 45, 50 |
| Type | Lotion, Spray, Gel |



Use the bar graph to find the experimental probability of the event.

- Rolling a 1 or a 2
- Rolling an odd number
- Not rolling a 5

Use the spinner to find the theoretical probability of the event(s).

- Spinning an even number
- Spinning a 1 and then a 2

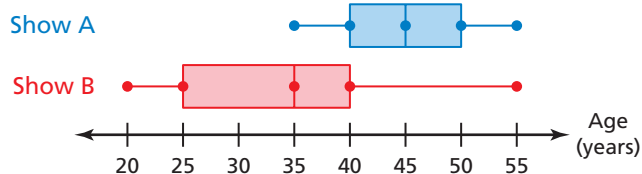


You randomly choose one chess piece. Without replacing the first piece, you randomly choose a second piece. Find the probability of choosing the first piece, then the second piece.

- Bishop and bishop
- King and queen

- LUNCH** You want to estimate the number of students in your school who prefer to bring a lunch from home rather than buy one at school. You survey five students who are standing in the lunch line. Determine whether the sample is *biased* or *unbiased*. Explain.

- AGES** The double box-and-whisker plot shows the ages of the viewers of two television shows in a small town.



- Compare the populations using measures of center and variation.
- Express the difference in the measures of center as a multiple of each measure of variation.

15 Standards Assessment

1. A school athletic director asked each athletic team member to name his or her favorite professional sports team. The results are below:

- D.C. United: 3
- Florida Panthers: 8
- Jacksonville Jaguars: 26
- Jacksonville Sharks: 7
- Miami Dolphins: 22
- Miami Heat: 15
- Miami Marlins: 20
- Minnesota Lynx: 4
- New York Knicks: 5
- Orlando Magic: 18
- Tampa Bay Buccaneers: 17
- Tampa Bay Lightning: 12
- Tampa Bay Rays: 28
- Other: 6

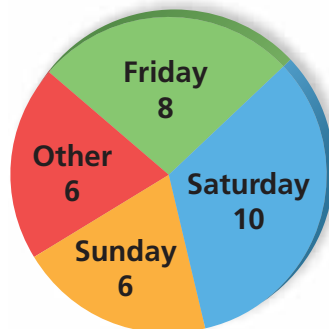
One athletic team member is picked at random. What is the likelihood that this team member's favorite professional sports team is *not* located in Florida? (7.SP.5)

- A. certain
- B. likely, but not certain
- C. unlikely, but not impossible
- D. impossible

2. Each student in your class voted for his or her favorite day of the week. Their votes are shown below:



Favorite Day of the Week



A student from your class is picked at random. What is the probability that this student's favorite day of the week is Sunday? (7.SP.7b)

Test-Taking Strategy
Use Intelligent Guessing

What's the probability of drawing 1 hyena out of a bag with 2 hyenas and 3 mice?
(A) -10% (B) 40% (C) 60% (D) 500%

40% < 60%
I'm hoping 40%.

"You know it can't be -10% or 500%.
So, you can **intelligently guess** between 40% and 60%."

3. How far, in millimeters, will the tip of the hour hand of the clock travel in 2 hours? (Use $\frac{22}{7}$ for π .) (7.G.4)



- F. 44 mm
 G. 88 mm
 H. 264 mm
 I. 528 mm
4. Nathaniel solved the proportion in the box below.

$$\frac{16}{40} = \frac{p}{27}$$

$$16 \cdot p = 40 \cdot 27$$

$$16p = 1080$$

$$\frac{16p}{16} = \frac{1080}{16}$$

$$p = 67.5$$

- What should Nathaniel do to correct the error that he made? (7.RP.2c)
- A. Add 40 to 16 and 27 to p .
 B. Subtract 16 from 40 and 27 from p .
 C. Multiply 16 by 27 and p by 40.
 D. Divide 16 by 27 and p by 40.
5. A North American hockey rink contains 5 face-off circles. Each of these circles has a radius of 15 feet. What is the total area, in square feet, of all the face-off circles? (Use 3.14 for π .) (7.G.4)
- F. 706.5 ft²
 G. 2826 ft²
 H. 3532.5 ft²
 I. 14,130 ft²

