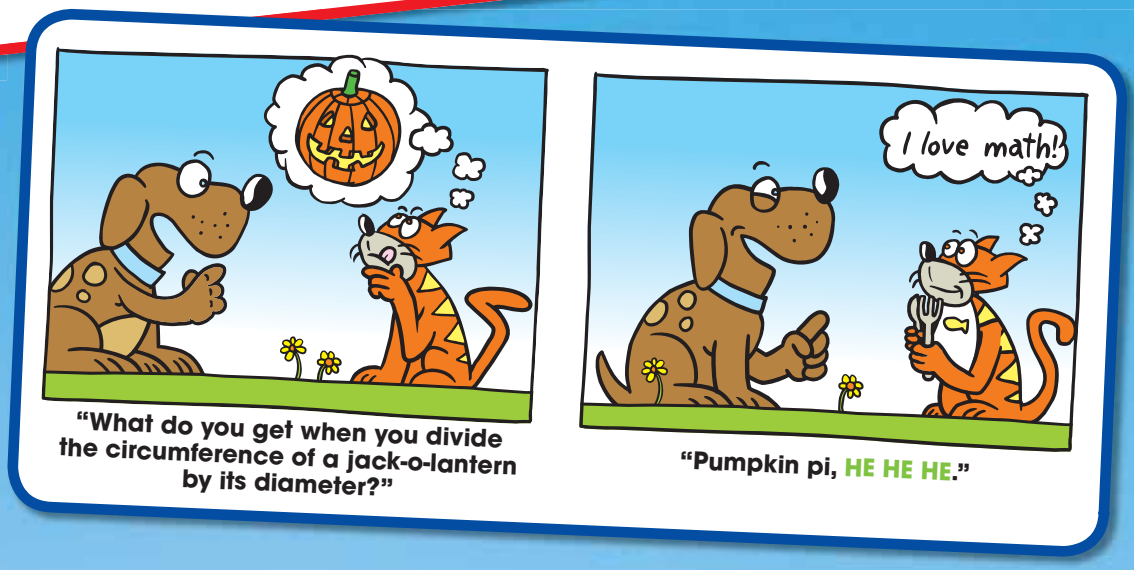
13 Circles and Area

13.1 Circles and Circumference

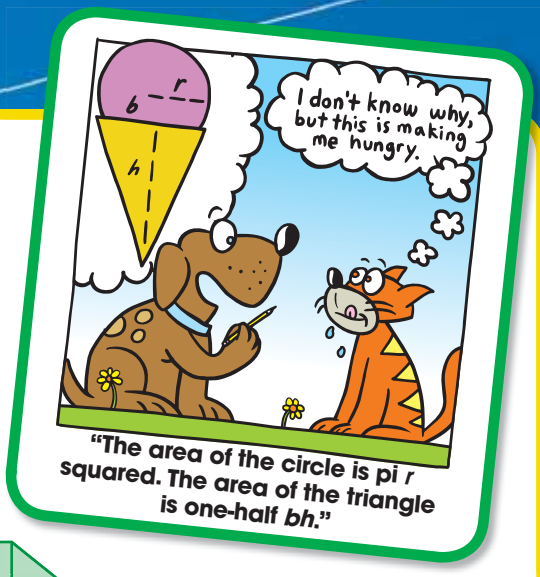
13.2 Perimeters of Composite Figures

13.3 Areas of Circles

13.4 Areas of Composite Figures



What You Learned Before



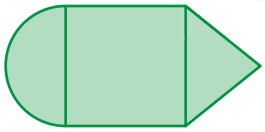
Classifying Figures (4.G.2)

Identify the basic shapes in the figure.

Example 1

Rectangle, right triangle

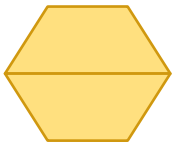
Example 2

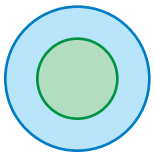


Semicircle, square, and triangle

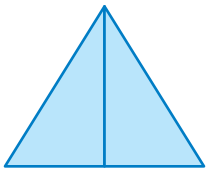
Try It Yourself

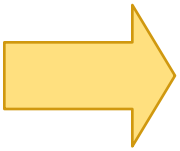
Identify the basic shapes in the figure.

1. 

2. 

3. 

4. 

5. 

6. 

Squaring Numbers and Using Order of Operations (6.EE.1)

Example 3 **Evaluate 4^2.**

$$4^2 = 4 \cdot 4 = 16$$

4^2 means to multiply 4 by itself.

Example 4 **Evaluate $3 \cdot 6^2$.**

$$3 \cdot 6^2 = 3 \cdot (6 \cdot 6) = 3 \cdot 36 = 108$$

Use order of operations. Evaluate the exponent, and then multiply.

Try It Yourself

Evaluate the expression.

7. 5^2
8. 12^2
9. $3 \cdot 2^2$
10. $4 \cdot 7^2$
11. $3(1 + 8)^2$
12. $2(3 + 7)^2 - 3 \cdot 4$

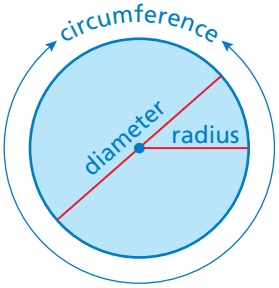
13.1 Circles and Circumference

Essential Question How can you find the circumference of a circle?

Archimedes was a Greek mathematician, physicist, engineer, and astronomer.

Archimedes discovered that in any circle the ratio of circumference to diameter is always the same. Archimedes called this ratio pi, or π (a letter from the Greek alphabet).

$$\pi = \frac{\text{circumference}}{\text{diameter}}$$

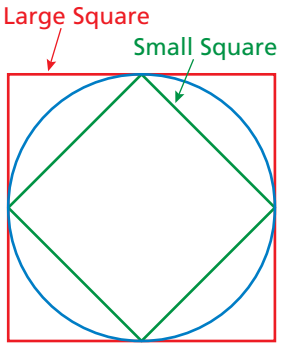


In Activities 1 and 2, you will use the same strategy Archimedes used to approximate π.

1 ACTIVITY: Approximating Pi

Work with a partner. Copy the table. Record your results in the table.

- **Measure the perimeter of the large square in millimeters.**
- **Measure the diameter of the circle in millimeters.**
- **Measure the perimeter of the small square in millimeters.**
- **Calculate the ratios of the two perimeters to the diameter.**
- **The average of these two ratios is an approximation of π.**



Common Core

Geometry

In this lesson, you will

- describe a circle in terms of radius and diameter.
- understand the concept of pi.
- find circumferences of circles and perimeters of semicircles.

Learning Standard 7.G.4

Sides	Large Perimeter	Diameter of Circle	Small Perimeter	$\frac{\text{Large Perimeter}}{\text{Diameter}}$	$\frac{\text{Small Perimeter}}{\text{Diameter}}$	Average of Ratios
4						
6						
8						
10						

A page from *Sir Cumference and the First Round Table* by Cindy Neuschwander

2 ACTIVITY: Approximating Pi

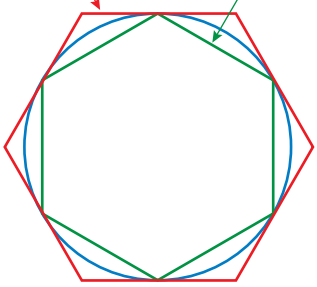
Math Practice 3

Make Conjectures

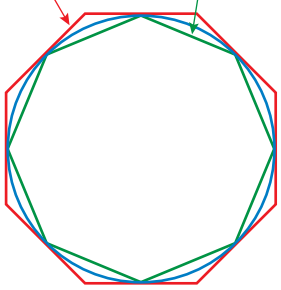
How can you use the results of the activity to find an approximation of pi?

Continue your approximation of pi. Complete the table from Activity 1 using a hexagon (6 sides), an octagon (8 sides), and a decagon (10 sides).

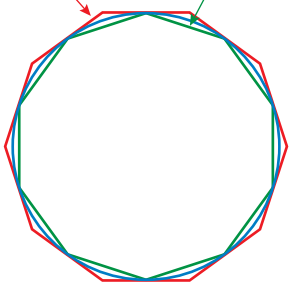
a. Large Hexagon, Small Hexagon



b. Large Octagon, Small Octagon



c. Large Decagon, Small Decagon



d. From the table, what can you conclude about the value of π? Explain your reasoning.

e. Archimedes calculated the value of π using polygons with 96 sides. Do you think his calculations were more or less accurate than yours?

What Is Your Answer?

3. IN YOUR OWN WORDS Now that you know an approximation for pi, explain how you can use it to find the circumference of a circle. Write a formula for the circumference C of a circle whose diameter is d.

4. CONSTRUCTION Use a compass to draw three circles. Use your formula from Question 3 to find the circumference of each circle.

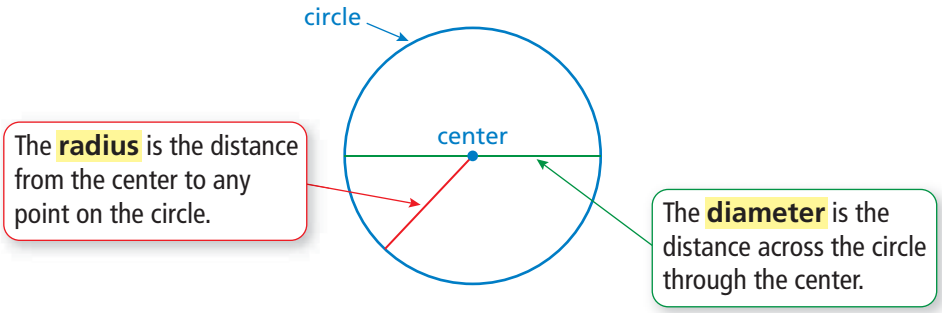
Practice Use what you learned about circles and circumference to complete Exercises 9–11 on page 553.

13.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
circle, *p. 550*
center, *p. 550*
radius, *p. 550*
diameter, *p. 550*
circumference, *p. 551*
pi, *p. 551*
semicircle, *p. 552*

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**.



Key Idea

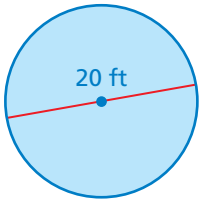
Radius and Diameter

Words The diameter d of a circle is twice the radius r. The radius r of a circle is one-half the diameter d.

Algebra **Diameter:** $d = 2r$ **Radius:** $r = \frac{d}{2}$

EXAMPLE 1 Finding a Radius and a Diameter

a. The diameter of a circle is 20 feet. Find the radius.



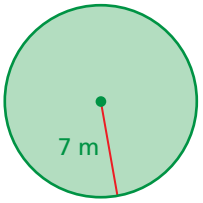
$r = \frac{d}{2}$ Radius of a circle

$= \frac{20}{2}$ Substitute 20 for d.

$= 10$ Divide.

∴ The radius is 10 feet.

b. The radius of a circle is 7 meters. Find the diameter.



$d = 2r$ Diameter of a circle

$= 2(7)$ Substitute 7 for r.

$= 14$ Multiply.

∴ The diameter is 14 meters.

On Your Own

Now You're Ready
Exercises 3–8

1. The diameter of a circle is 16 centimeters. Find the radius.
2. The radius of a circle is 9 yards. Find the diameter.

The distance around a circle is called the **circumference**. The ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for *every* circle and is represented by the Greek letter π, called **pi**. The value of π can be approximated as 3.14 or $\frac{22}{7}$.

Study Tip

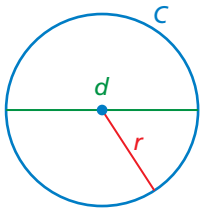
When the radius or diameter is a multiple of 7, it is easier to use $\frac{22}{7}$ as the estimate of π.

Key Idea

Circumference of a Circle

Words The circumference C of a circle is equal to π times the diameter d or π times twice the radius r.

Algebra $C = \pi d$ or $C = 2\pi r$



EXAMPLE 2 Finding Circumferences of Circles

a. Find the circumference of the flying disc. Use 3.14 for π.

$C = 2\pi r$ Write formula for circumference.

$\approx 2 \cdot 3.14 \cdot 5$ Substitute 3.14 for π and 5 for r.

$= 31.4$ Multiply.

∴ The circumference is about 31.4 inches.

b. Find the circumference of the watch face. Use $\frac{22}{7}$ for π.

$C = \pi d$ Write formula for circumference.

$\approx \frac{22}{7} \cdot 28$ Substitute $\frac{22}{7}$ for π and 28 for d.

$= 88$ Multiply.

∴ The circumference is about 88 millimeters.

On Your Own

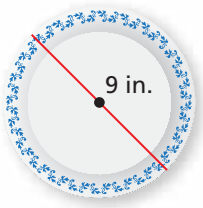
Now You're Ready
Exercises 9–11

Find the circumference of the object. Use 3.14 or $\frac{22}{7}$ for π.

3.

4.

5.



EXAMPLE 3 Estimating a Diameter

$C = 31.4$ in.

The circumference of the roll of caution tape decreases 10.5 inches after a construction worker uses some of the tape. Which is the best estimate of the diameter of the roll after the decrease?

Ⓐ 5 inches Ⓑ 7 inches Ⓒ 10 inches Ⓓ 12 inches

After the decrease, the circumference of the roll is $31.4 - 10.5 = 20.9$ inches.

$C = \pi d$ Write formula for circumference.

$20.9 \approx 3.14 \cdot d$ Substitute 20.9 for C and 3.14 for π.

$21 \approx 3d$ Round 20.9 up to 21. Round 3.14 down to 3.

$7 = d$ Divide each side by 3.

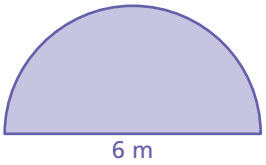
∴ The correct answer is Ⓑ.

On Your Own

6. **WHAT IF?** The circumference of the roll of tape decreases 5.25 inches. Estimate the diameter of the roll after the decrease.

EXAMPLE 4 Finding the Perimeter of a Semicircular Region

A semicircle is one-half of a circle. Find the perimeter of the semicircular region.



The straight side is 6 meters long. The distance around the curved part is one-half the circumference of a circle with a diameter of 6 meters.

$\frac{C}{2} = \frac{\pi d}{2}$ Divide the circumference by 2.

$\approx \frac{3.14 \cdot 6}{2}$ Substitute 3.14 for π and 6 for d.

$= 9.42$ Simplify.

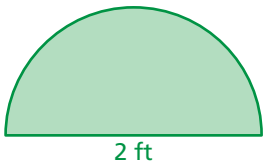
∴ So, the perimeter is about $6 + 9.42 = 15.42$ meters.

On Your Own

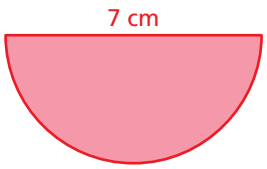
Now You're Ready
Exercises 15 and 16

Find the perimeter of the semicircular region.

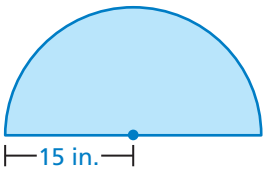
7.



8.



9.



13.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** What is the relationship between the radius and the diameter of a circle?

2. **WHICH ONE DOESN'T BELONG?** Which phrase does *not* belong with the other three? Explain your reasoning.

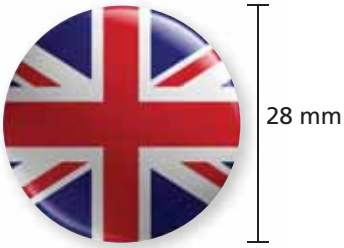
the distance around a circle	π times twice the radius
π times the diameter	the distance from the center to any point on the circle

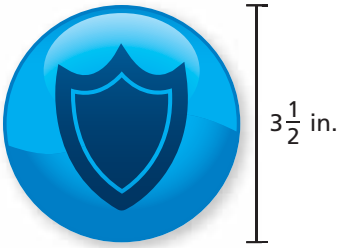
Practice and Problem Solving

Find the radius of the button.

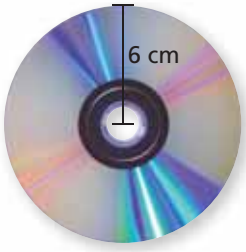
1

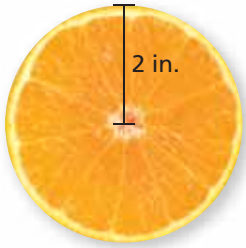
3. 

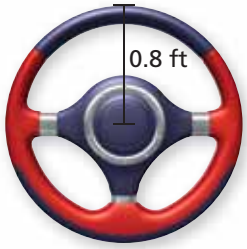
4. 

5. 

Find the diameter of the object.

6. 

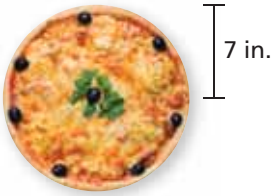
7. 

8. 

Find the circumference of the pizza. Use 3.14 or $\frac{22}{7}$ for π.

2

9. 

10. 

11. 

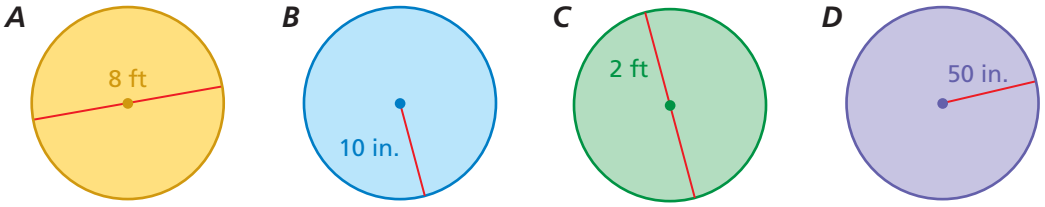
12. **CHOOSE TOOLS** Choose a real-life circular object. Explain why you might need to know its circumference. Then find the circumference.

13. SINKHOLE A circular sinkhole has a circumference of 75.36 meters. A week later, it has a circumference of 150.42 meters.

a. Estimate the diameter of the sinkhole each week.

b. How many times greater is the diameter of the sinkhole now compared to the previous week?

14. REASONING Consider the circles *A*, *B*, *C*, and *D*.

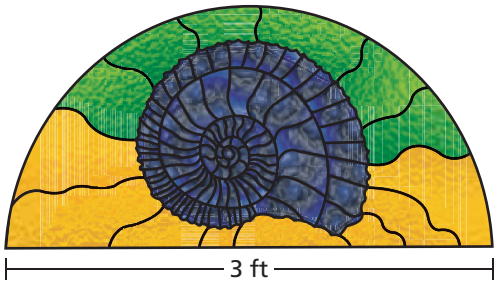


a. Without calculating, which circle has the greatest circumference?

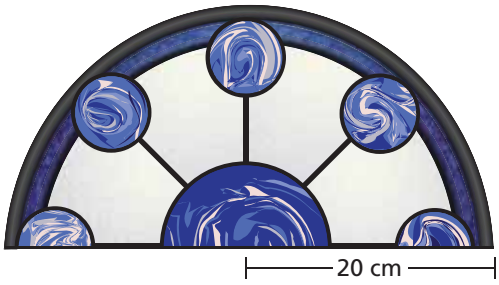
b. Without calculating, which circle has the least circumference?

Find the perimeter of the window.

15.

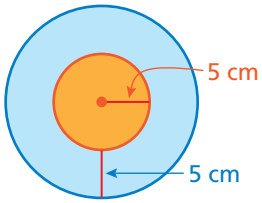


16.

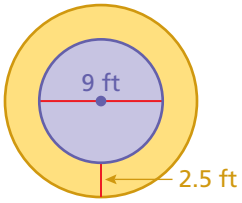


Find the circumferences of both circles.

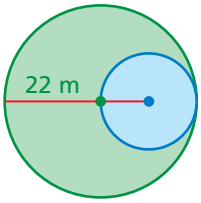
17.



18.

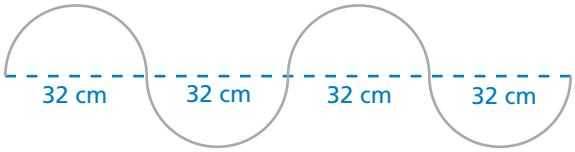


19.



20. STRUCTURE Because the ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for every circle, is the ratio $\frac{\text{circumference}}{\text{radius}}$ the same for every circle? Explain.

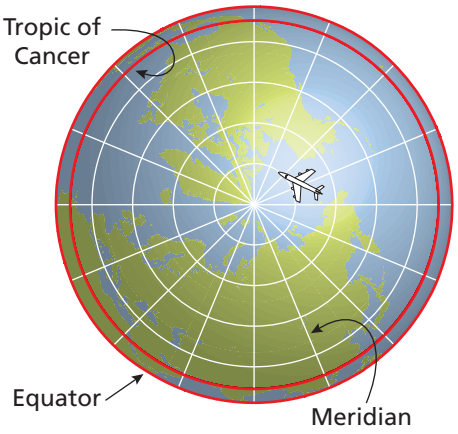
21. WIRE A wire is bent to form four semicircles. How long is the wire?



22. CRITICAL THINKING Explain how to draw a circle with a circumference of π^2 inches. Then draw the circle.

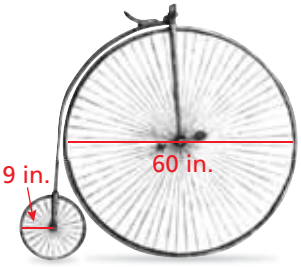
23. **AROUND THE WORLD** "Lines" of latitude on Earth are actually circles. The Tropic of Cancer is the northernmost line of latitude at which the Sun appears directly overhead at noon. The Tropic of Cancer has a radius of 5854 kilometers.

To qualify for an around-the-world speed record, a pilot must cover a distance no less than the circumference of the Tropic of Cancer, cross all meridians, and land on the same airfield where he started.



a. What is the minimum distance that a pilot must fly to qualify for an around-the-world speed record?

b. **RESEARCH** Estimate the time it would take for a pilot to qualify for the speed record.



24. **PROBLEM SOLVING** Bicycles in the late 1800s looked very different than they do today.

a. How many rotations does each tire make after traveling 600 feet? Round your answers to the nearest whole number.

b. Would you rather ride a bicycle made with two large wheels or two small wheels? Explain.

25. **Logic** The length of the minute hand is 150% of the length of the hour hand.

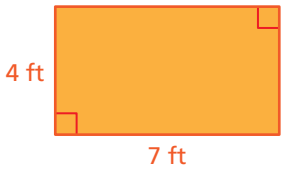
a. What distance will the tip of the minute hand move in 45 minutes? Explain how you found your answer.

b. In 1 hour, how much farther does the tip of the minute hand move than the tip of the hour hand? Explain how you found your answer.

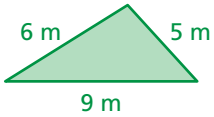
Fair Game Review What you learned in previous grades & lessons

Find the perimeter of the polygon. *(Skills Review Handbook)*

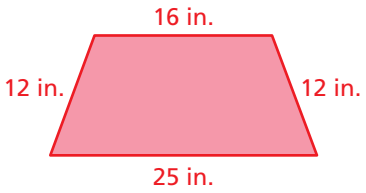
26.



27.



28.



29. **MULTIPLE CHOICE** What is the median of the data set? *(Skills Review Handbook)*

12, 25, 16, 9, 5, 22, 27, 20

Ⓐ 7 Ⓑ 16 Ⓒ 17 Ⓓ 18

13.2 Perimeters of Composite Figures

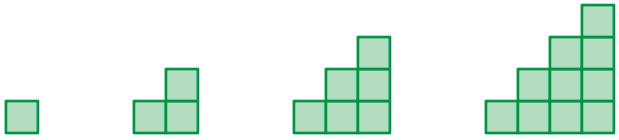
Essential Question How can you find the perimeter of a composite figure?

1 ACTIVITY: Finding a Pattern

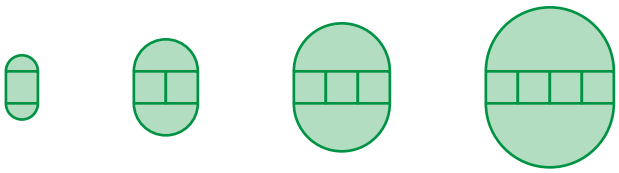
Work with a partner. Describe the pattern of the perimeters. Use your pattern to find the perimeter of the tenth figure in the sequence. (Each small square has a perimeter of 4.)

a.

b.



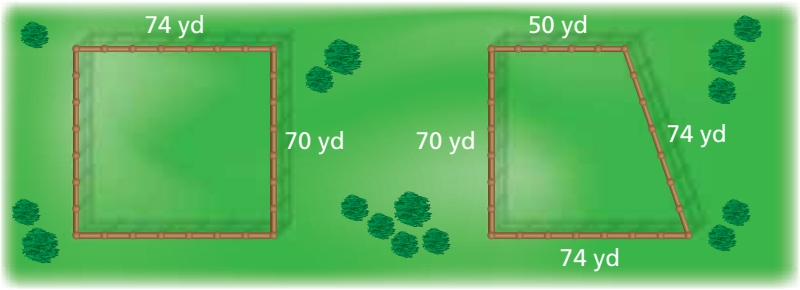
c.



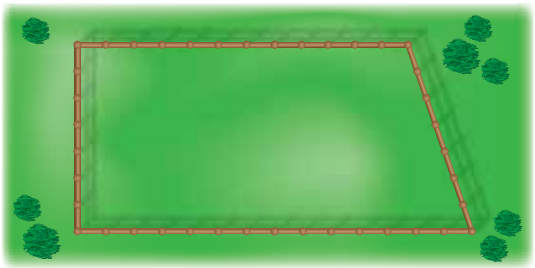
2 ACTIVITY: Combining Figures

Work with a partner.

a. A rancher is constructing a rectangular corral and a trapezoidal corral, as shown. How much fencing does the rancher need to construct both corrals?



b. Another rancher is constructing one corral by combining the two corrals above, as shown. Does this rancher need more or less fencing? Explain your reasoning.



c. How can the rancher in part (b) combine the two corrals to use even less fencing?

Common Core

Geometry

In this lesson, you will

- find perimeters of composite figures.

Applying Standard 7.G.4

3 ACTIVITY: Submitting a Bid

Work with a partner. You want to bid on a tiling contract. You will be supplying and installing the brown tile that borders the swimming pool. In the figure, each grid square represents 1 square foot.

- **Your cost for the tile is $4 per linear foot.**
- **It takes about 15 minutes to prepare, install, and clean each foot of tile.**

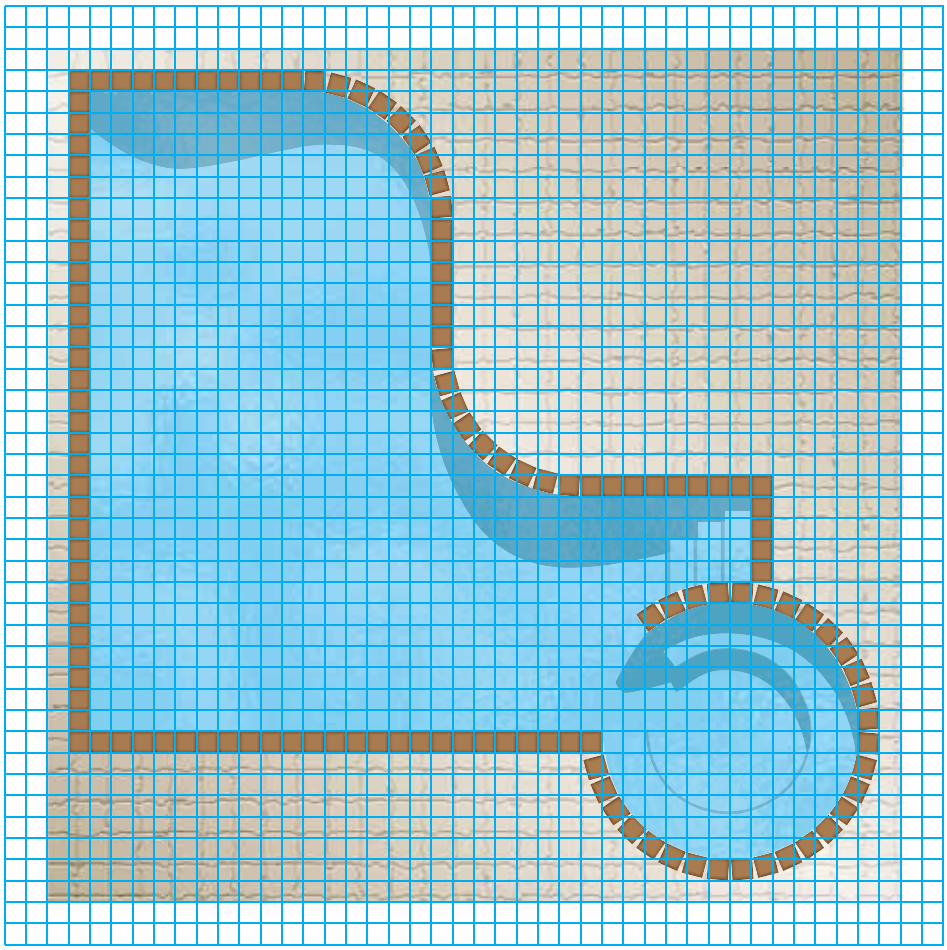
a. How many brown tiles do you need for the border?

b. Write a bid for how much you will charge to supply and install the tile. Include what you want to charge as an hourly wage. Estimate what you think your profit will be.

Math Practice 6

Communicate Precisely

What do you need to include to create an accurate bid? Explain.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the perimeter of a composite figure? Use a semicircle, a triangle, and a parallelogram to draw a composite figure. Label the dimensions. Find the perimeter of the figure.

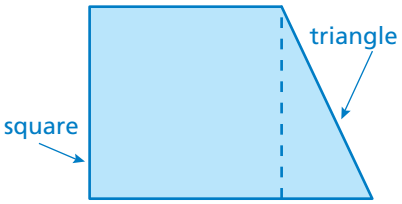
Practice Use what you learned about perimeters of composite figures to complete Exercises 3–5 on page 560.

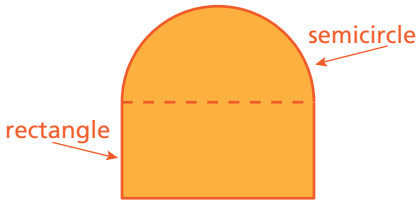
13.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
composite figure, *p. 558*

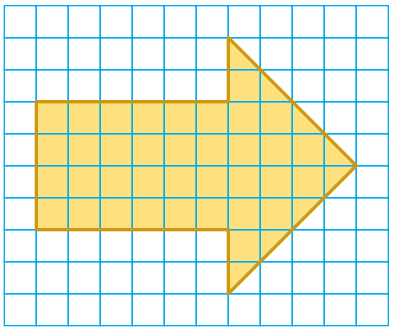
A **composite figure** is made up of triangles, squares, rectangles, semicircles, and other two-dimensional figures. Here are two examples.



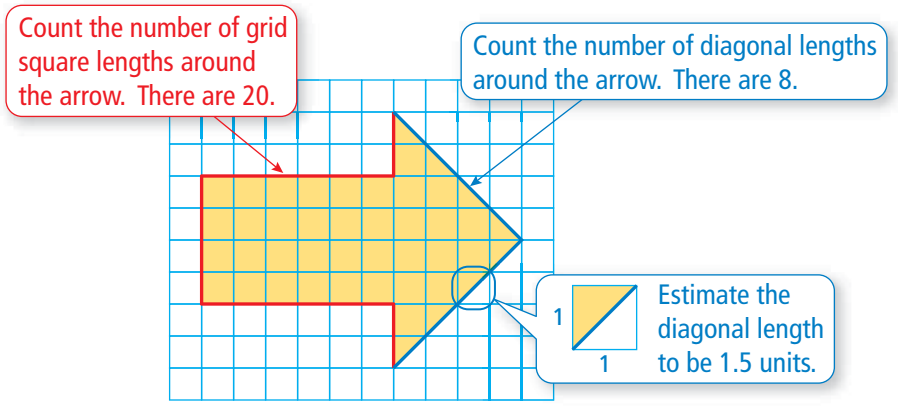


To find the perimeter of a composite figure, find the distance around the figure.

EXAMPLE 1 Estimating a Perimeter Using Grid Paper



Estimate the perimeter of the arrow.



Length of 20 grid square lengths: $20 \times 1 = 20$ units

Length of 8 diagonal lengths: $8 \times 1.5 = 12$ units

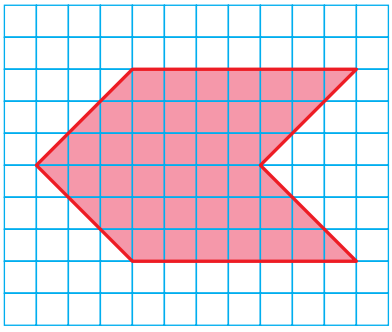
∴ So, the perimeter is about $20 + 12 = 32$ units.

On Your Own

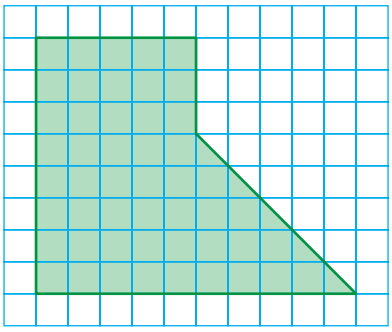
Now You're Ready
Exercises 3–8

Estimate the perimeter of the figure.

1.



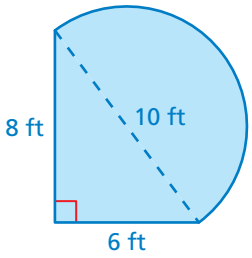
2.



Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Finding a Perimeter

The figure is made up of a semicircle and a triangle. Find the perimeter.



The distance around the triangular part of the figure is $6 + 8 = 14$ feet.

The distance around the semicircle is one-half the circumference of a circle with a diameter of 10 feet.

$$\frac{C}{2} = \frac{\pi d}{2}$$ Divide the circumference by 2.

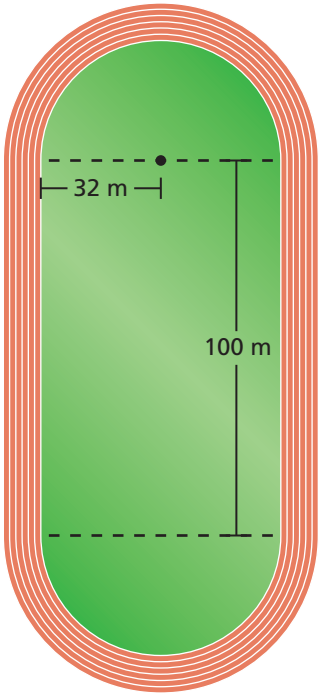
$$\approx \frac{3.14 \cdot 10}{2}$$ Substitute 3.14 for π and 10 for d.

$$= 15.7$$ Simplify.

∴ So, the perimeter is about $14 + 15.7 = 29.7$ feet.

EXAMPLE 3 Finding a Perimeter

The running track is made up of a rectangle and two semicircles. Find the perimeter.



The semicircular ends of the track form a circle with a radius of 32 meters. Find its circumference.

$$C = 2\pi r$$ Write formula for circumference.

$$\approx 2 \cdot 3.14 \cdot 32$$ Substitute 3.14 for π and 32 for r.

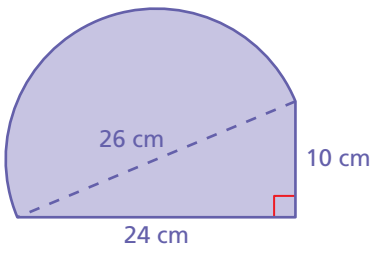
$$= 200.96$$ Multiply.

∴ So, the perimeter is about $100 + 100 + 200.96 = 400.96$ meters.

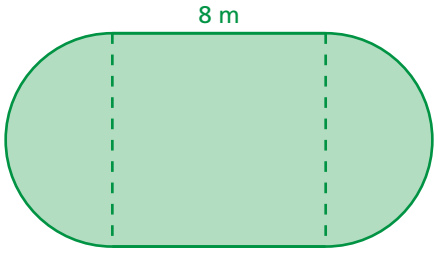
Now You're Ready
Exercises 9–11

On Your Own

3. The figure is made up of a semicircle and a triangle. Find the perimeter.



4. The figure is made up of a square and two semicircles. Find the perimeter.

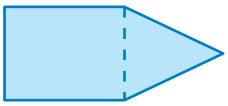


13.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** Is the perimeter of the composite figure equal to the sum of the perimeters of the individual figures? Explain.



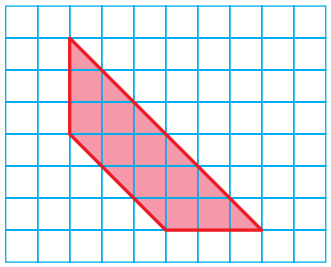
2. **OPEN-ENDED** Draw a composite figure formed by a parallelogram and a trapezoid.

Practice and Problem Solving

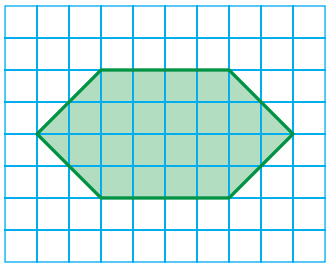
Estimate the perimeter of the figure.

1

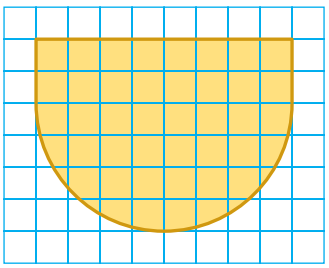
3.



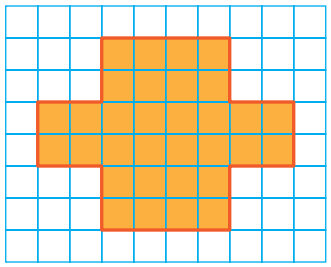
4.



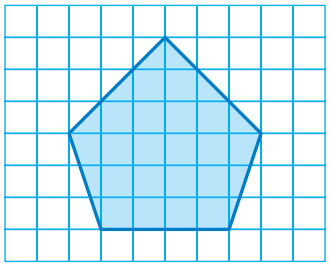
5.



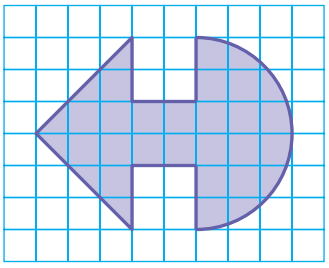
6.



7.



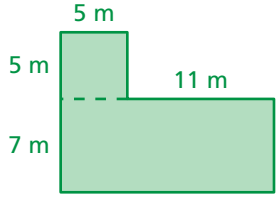
8.



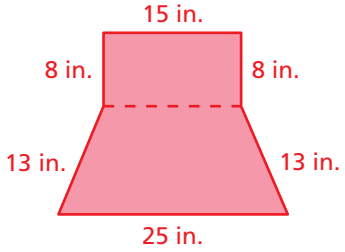
Find the perimeter of the figure.

2

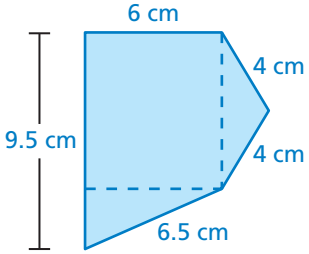
9.



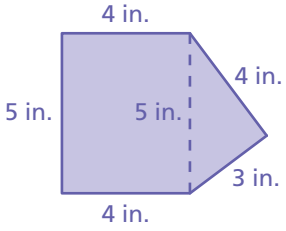
10.



11.



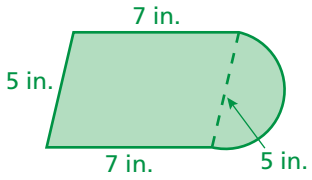
12. **ERROR ANALYSIS** Describe and correct the error in finding the perimeter of the figure.



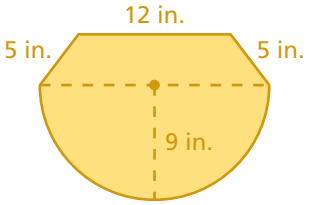
✗ Perimeter = 4 + 3 + 4 + 5 + 4 + 5
= 25 in.

Find the perimeter of the figure.

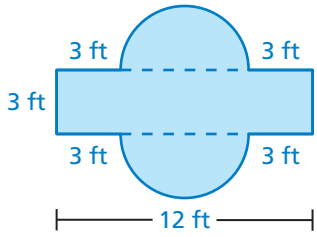
13.

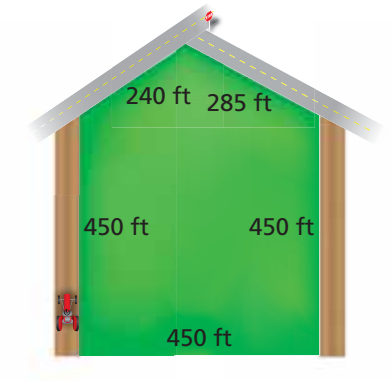


14.

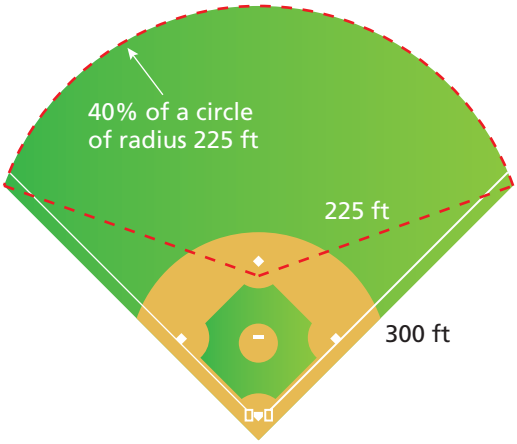


15.





16. **PASTURE** A farmer wants to fence a section of land for a horse pasture. Fencing costs $27 per yard. How much will it cost to fence the pasture?



17. **BASEBALL** You run around the perimeter of the baseball field at a rate of 9 feet per second. How long does it take you to run around the baseball field?

18. **TRACK** In Example 3, the running track has six lanes. Explain why the starting points for the six runners are staggered. Draw a diagram as part of your explanation.

19. Critical Thinking How can you add a figure to a composite figure without increasing its perimeter? Draw a diagram to support your answer.

Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Skills Review Handbook)*

20. $2.15(3)^2$ **21.** $4.37(8)^2$ **22.** $3.14(7)^2$ **23.** $8.2(5)^2$

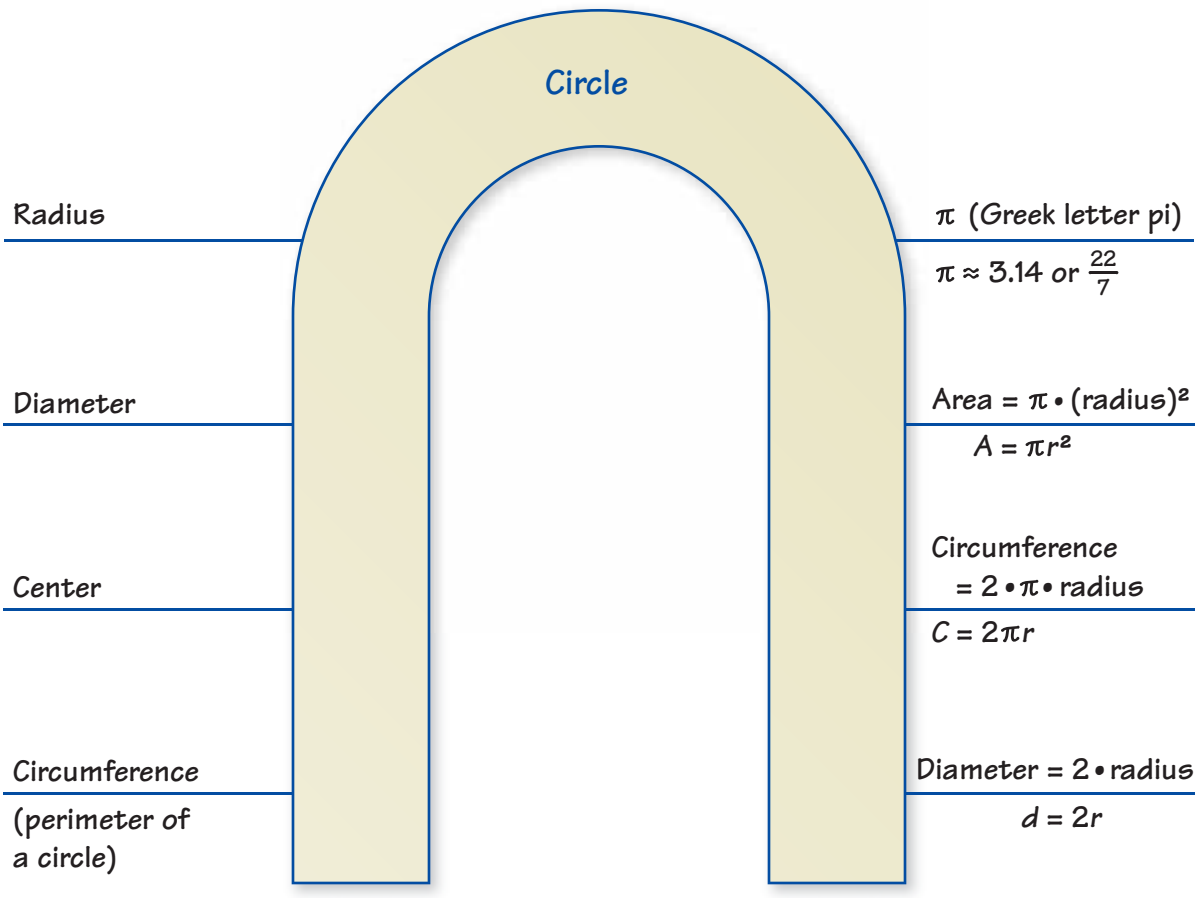
24. **MULTIPLE CHOICE** Which expression is equivalent to $(5y + 4) - 2(7 - 2y)$? *(Skills Review Handbook)*

(A) $y - 10$ (B) $9y + 18$ (C) $3y - 10$ (D) $9y - 10$

13 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use a **word magnet** to organize formulas or phrases that are associated with a vocabulary word or term. Here is an example of a word magnet for circle.



On Your Own

Make word magnets to help you study these topics.

1. semicircle
2. composite figure
3. perimeter

After you complete this chapter, make word magnets for the following topics.

4. area of a circle
5. area of a composite figure

"I'm trying to make a word magnet for happiness, but I can only think of two words."

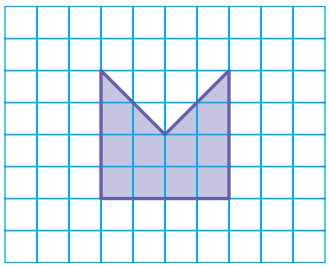
13.1–13.2 Quiz

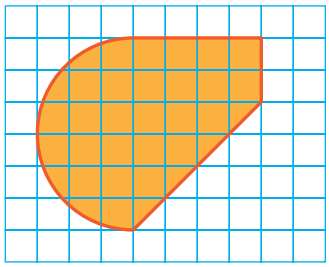
Check It Out
Progress Check
BigIdeasMath.com

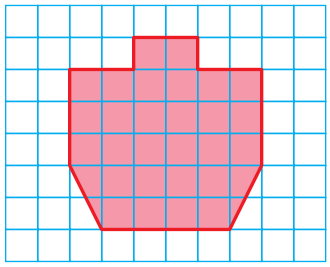
1. The diameter of a circle is 36 centimeters. Find the radius. *(Section 13.1)*

2. The radius of a circle is 11 inches. Find the diameter. *(Section 13.1)*

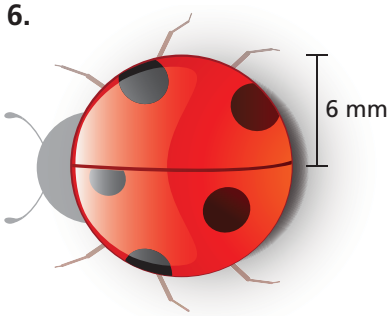
Estimate the perimeter of the figure. *(Section 13.2)*

3. 

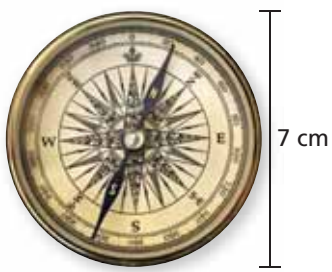
4. 

5. 

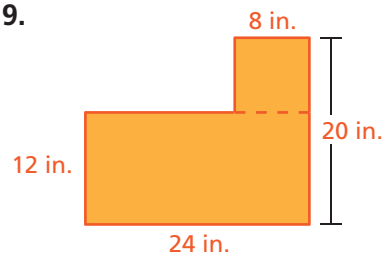
Find the circumference of the circle. Use 3.14 or $\frac{22}{7}$ for π. *(Section 13.1)*

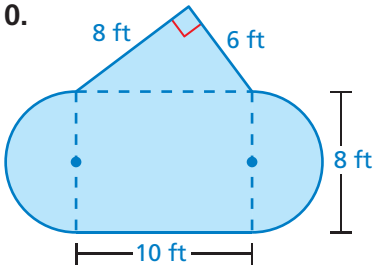
6. 

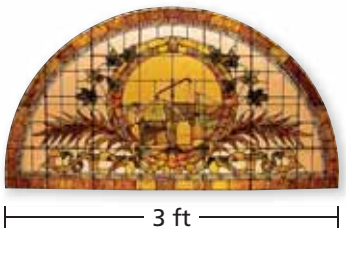
7. 

8. 

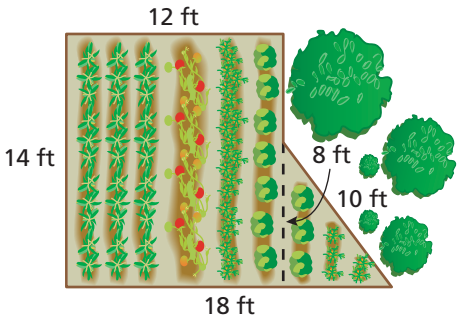
Find the perimeter of the figure. *(Section 13.1 and Section 13.2)*

9. 

10. 

11. 

12. **BUTTON** What is the circumference of a circular button with a diameter of 8 millimeters? *(Section 13.1)*



13. **GARDEN** You want to fence part of a yard to make a vegetable garden. How many feet of fencing do you need to surround the garden? *(Section 13.2)*

14. **BAKING** A baker is using two circular pans. The larger pan has a diameter of 12 inches. The smaller pan has a diameter of 7 inches. How much greater is the circumference of the larger pan than that of the smaller pan? *(Section 13.1)*

13.3 Areas of Circles

Essential Question How can you find the area of a circle?

1 ACTIVITY: Estimating the Area of a Circle

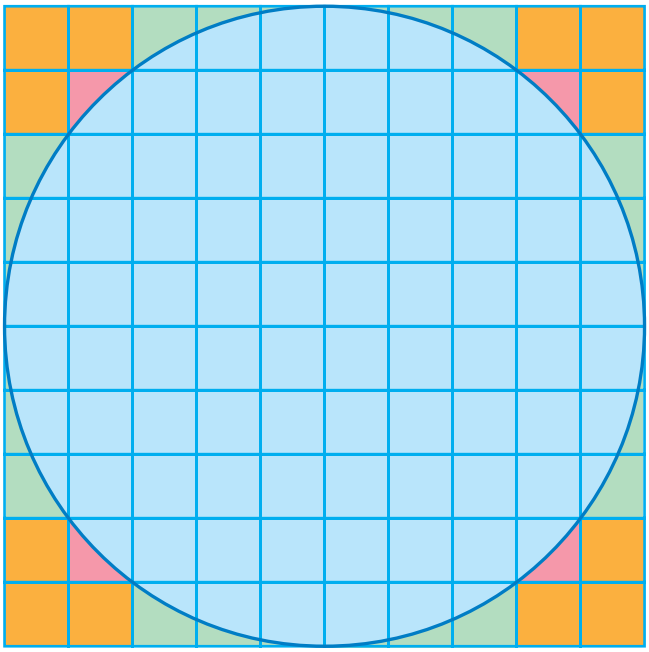
Work with a partner. Each square in the grid is 1 unit by 1 unit.

a. Find the area of the large 10-by-10 square.

b. Copy and complete the table.

Region			
Area (square units)			

c. Use your results to estimate the area of the circle. Explain your reasoning.



d. Fill in the blanks. Explain your reasoning.

Area of large square = ____ • 5^2 square units

Area of circle ≈ ____ • 5^2 square units

e. What dimension of the circle does 5 represent? What can you conclude?

Common Core

Geometry

In this lesson, you will

- find areas of circles and semicircles.

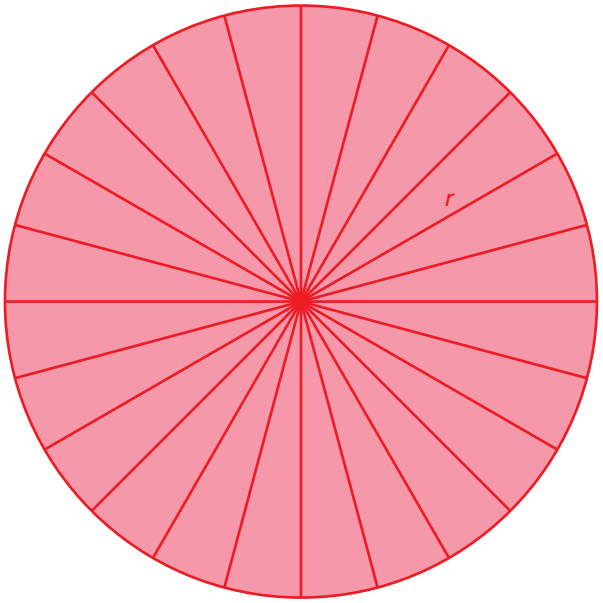
Learning Standard 7.G.4

2 ACTIVITY: Approximating the Area of a Circle

Work with a partner.

a. Draw a circle. Label the radius as r.

b. Divide the circle into 24 equal sections.

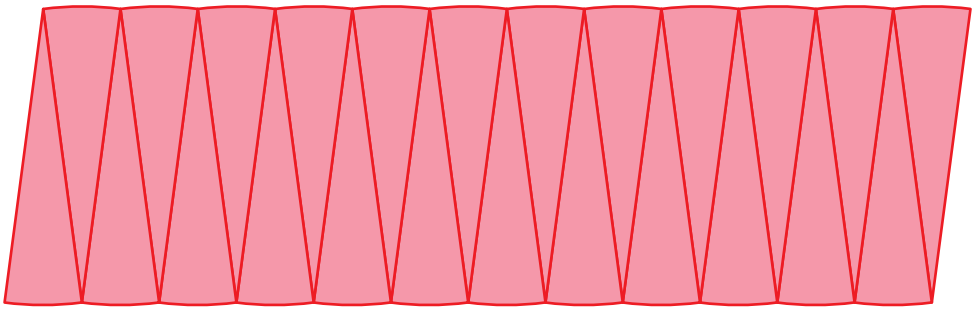


c. Cut the sections apart. Then arrange them to approximate a parallelogram.

Math Practice 1

Interpret a Solution

What does the area of the parallelogram represent? Explain.



d. What is the approximate height and base of the parallelogram?

e. Find the area of the parallelogram. What can you conclude?

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you find the area of a circle?

4. Write a formula for the area of a circle with radius r. Find an object that is circular. Use your formula to find the area.

Practice Use what you learned about areas of circles to complete Exercises 3–5 on page 568.

13.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

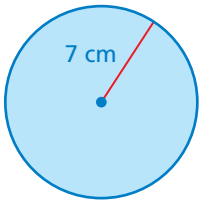
Key Idea

Area of a Circle

Words The area A of a circle is the product of π and the square of the radius.

Algebra $A = \pi r^2$

EXAMPLE 1 Finding Areas of Circles



a. Find the area of the circle. Use $\frac{22}{7}$ for π.

Estimate $3 \times 7^2 \approx 3 \times 50 = 150$

$A = \pi r^2$ Write formula for area.

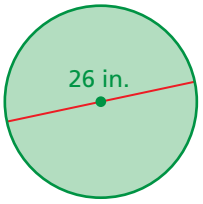
$\approx \frac{22}{7} \cdot 7^2$ Substitute $\frac{22}{7}$ for π and 7 for r.

$= \frac{22}{\cancel{7}_{1}} \cdot \cancel{49}^{7}$ Evaluate 7^2. Divide out the common factor.

$= 154$ Multiply.

∴ The area is about 154 square centimeters.

Reasonable? $154 \approx 150$ ✓



b. Find the area of the circle. Use 3.14 for π.

The radius is $26 \div 2 = 13$ inches.

Estimate $3 \times 13^2 \approx 3 \times 170 = 510$

$A = \pi r^2$ Write formula for area.

$\approx 3.14 \cdot 13^2$ Substitute 3.14 for π and 13 for r.

$= 3.14 \cdot 169$ Evaluate 13^2.

$= 530.66$ Multiply.

∴ The area is about 530.66 square inches.

Reasonable? $530.66 \approx 510$ ✓

On Your Own

Now You're Ready
Exercises 3–10

1. Find the area of a circle with a radius of 6 feet. Use 3.14 for π.
2. Find the area of a circle with a diameter of 28 meters. Use $\frac{22}{7}$ for π.

EXAMPLE 2 Describing a Distance

You want to find the distance the monster truck travels when the tires make one 360-degree rotation. Which best describes this distance?

(A) the radius of the tire
(B) the diameter of the tire
(C) the circumference of the tire
(D) the area of the tire

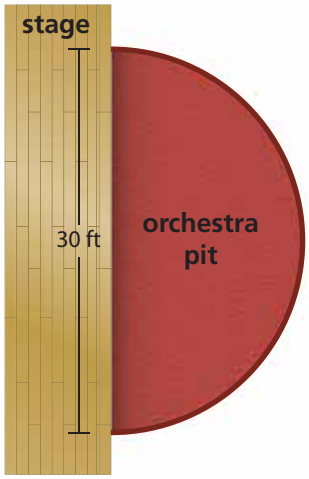
The distance the truck travels after one rotation is the same as the distance *around* the tire. So, the circumference of the tire best describes the distance in one rotation.

∴ The correct answer is (C).

On Your Own

3. You want to find the height of one of the tires. Which measurement would best describe the height?

EXAMPLE 3 Finding the Area of a Semicircle



Find the area of the semicircular orchestra pit.

The area of the orchestra pit is one-half the area of a circle with a diameter of 30 feet.

The radius of the circle is $30 \div 2 = 15$ feet.

$$\frac{A}{2} = \frac{\pi r^2}{2}$$ Divide the area by 2.

$$\approx \frac{3.14 \cdot 15^2}{2}$$ Substitute 3.14 for π and 15 for r.

$$= \frac{3.14 \cdot 225}{2}$$ Evaluate 15^2.

$$= 353.25$$ Simplify.

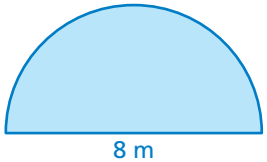
∴ So, the area of the orchestra pit is about 353.25 square feet.

On Your Own

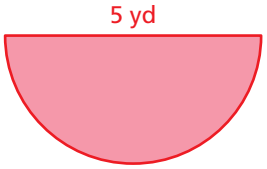
Now You're Ready
Exercises 13–15

Find the area of the semicircle.

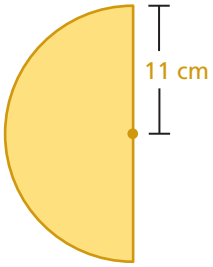
4.



5.



6.



13.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Explain how to find the area of a circle given its diameter.
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the area of a circle with a diameter of 1 m?	What is the area of a circle with a diameter of 100 cm?
What is the area of a circle with a radius of 100 cm?	What is the area of a circle with a radius of 500 mm?

Practice and Problem Solving

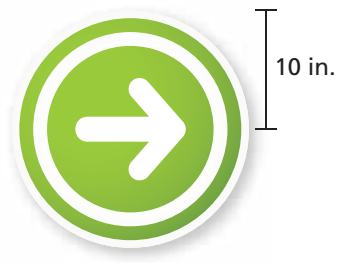
Find the area of the circle. Use 3.14 or $\frac{22}{7}$ for π.

1

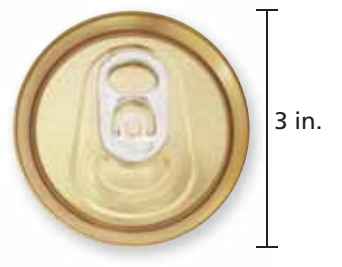
3.

4.

5.



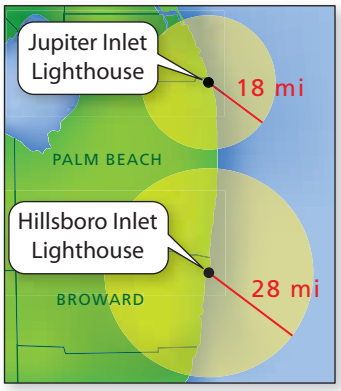
6.



7.

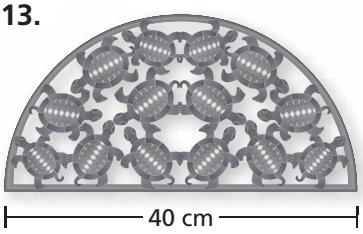
8.

9. Find the area of a circle with a diameter of 56 millimeters.
10. Find the area of a circle with a radius of 5 feet.
11. **TORTILLA** The diameter of a flour tortilla is 12 inches. What is the area?
12. **LIGHTHOUSE** The Hillsboro Inlet Lighthouse lights up how much more area than the Jupiter Inlet Lighthouse?

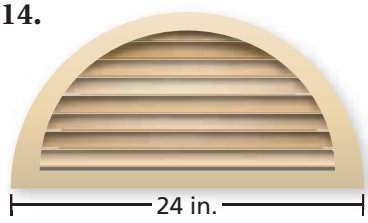


Find the area of the semicircle.

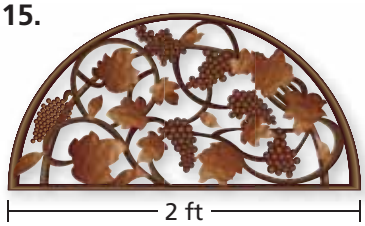
13.



14.



15.



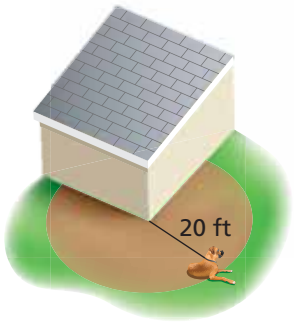
16. **REPEATED REASONING** Consider five circles with radii of 1, 2, 4, 8, and 16 inches.

a. Copy and complete the table. Write your answers in terms of π.

b. Compare the areas and circumferences. What happens to the circumference of a circle when you double the radius? What happens to the area?

c. What happens when you triple the radius?

Radius	Circumference	Area
1	2π in.	π in.^{2}
2		
4		
8		
16		

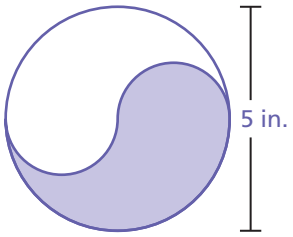


17. **DOG** A dog is leashed to the corner of a house. How much running area does the dog have? Explain how you found your answer.

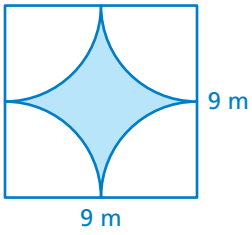
18. **CRITICAL THINKING** Is the area of a semicircle with a diameter of x *greater than*, *less than*, or *equal to* the area of a circle with a diameter of $\frac{1}{2}x$? Explain.

Reasoning **Find the area of the shaded region. Explain how you found your answer.**

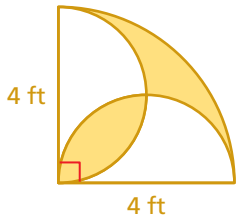
19.



20.



21.



Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Skills Review Handbook)*

22. $\frac{1}{2}(7)(4) + 6(5)$

23. $\frac{1}{2} \cdot 8^2 + 3(7)$

24. $12(6) + \frac{1}{4} \cdot 2^2$

25. **MULTIPLE CHOICE** What is the product of $-8\frac{1}{3}$ and $3\frac{2}{5}$? *(Skills Review Handbook)*

(A) $-28\frac{1}{3}$ (B) $-24\frac{2}{15}$ (C) $24\frac{2}{15}$ (D) $28\frac{1}{3}$

13.4 Areas of Composite Figures

Essential Question How can you find the area of a composite figure?

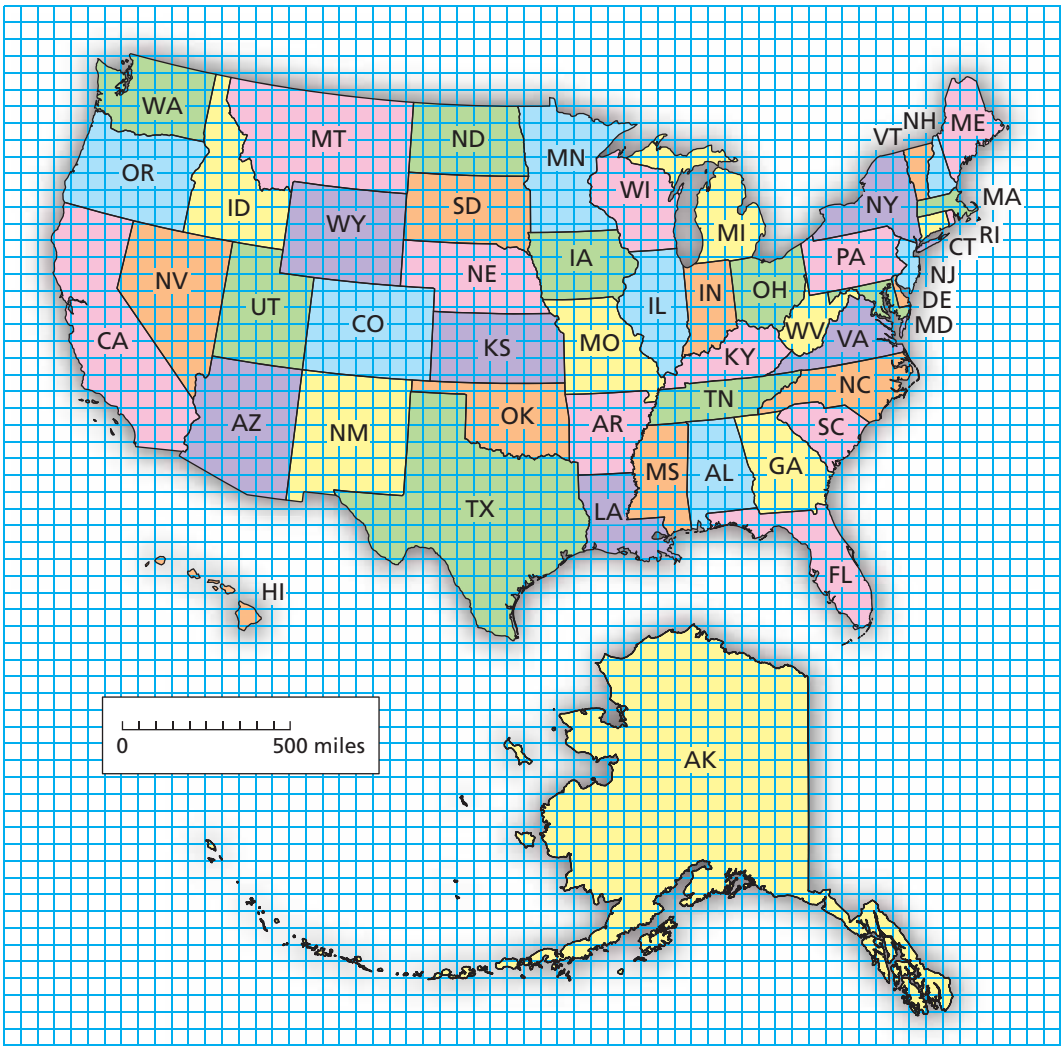
1 ACTIVITY: Estimating Area

Work with a partner.

a. Choose a state. On grid paper, draw a larger outline of the state.

b. Use your drawing to estimate the area (in square miles) of the state.

c. Which state areas are easy to find? Which are difficult? Why?



Common Core

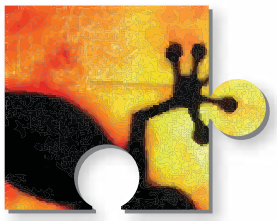
Geometry

In this lesson, you will

- find areas of composite figures by separating them into familiar figures.
- solve real-life problems.

Learning Standard 7.G.6

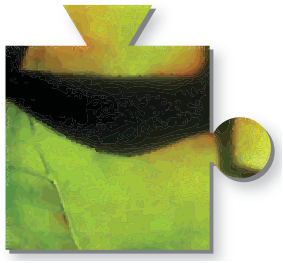
2 ACTIVITY: Estimating Areas

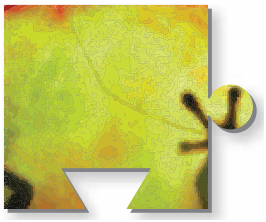


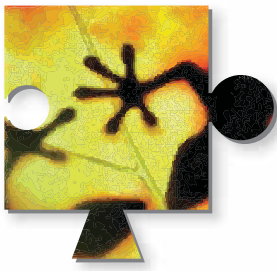
Work with a partner. The completed puzzle has an area of 150 square centimeters.

a. Estimate the area of each puzzle piece.

b. Check your work by adding the six areas. Why is this a check?







3 ACTIVITY: Filling a Square with Circles

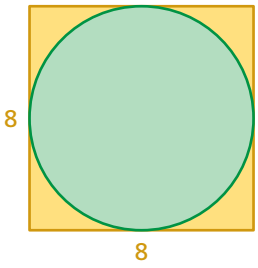
Math Practice 1

Make a Plan

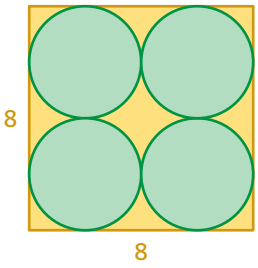
What steps will you use to solve this problem?

Work with a partner. Which pattern fills more of the square with circles? Explain.

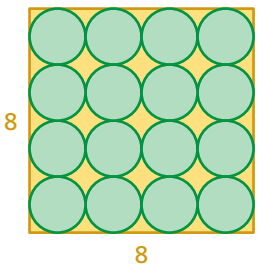
a.



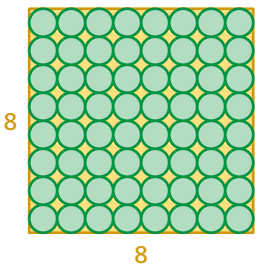
b.



c.



d.



What Is Your Answer?

4. IN YOUR OWN WORDS How can you find the area of a composite figure?

5. Summarize the area formulas for all the basic figures you have studied. Draw a single composite figure that has each type of basic figure. Label the dimensions and find the total area.

Practice Use what you learned about areas of composite figures to complete Exercises 3–5 on page 574.

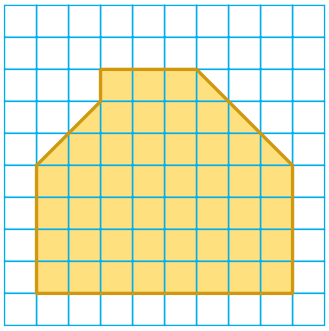
13.4 Lesson

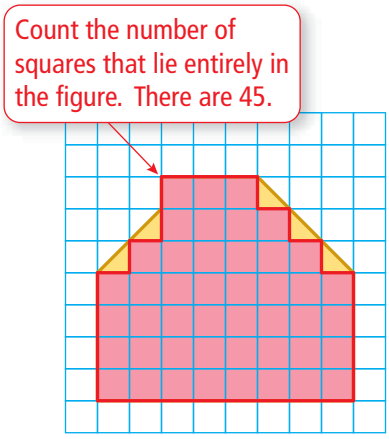
Check It Out
Lesson Tutorials
BigIdeasMath.com

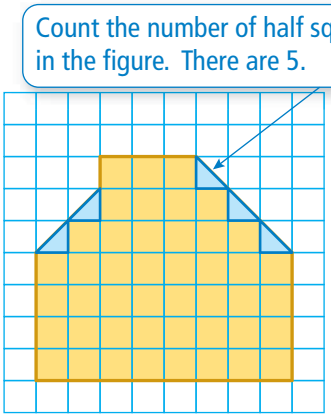
To find the area of a composite figure, separate it into figures with areas you know how to find. Then find the sum of the areas of those figures.

EXAMPLE 1 Finding an Area Using Grid Paper

Find the area of the yellow figure.







The area of a half square is $1 \div 2 = 0.5$ square unit.

Area of 45 squares: $45 \times 1 = 45$ square units

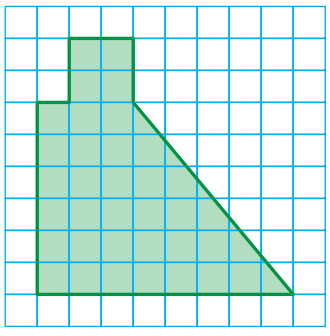
Area of 5 half squares: $5 \times 0.5 = 2.5$ square units

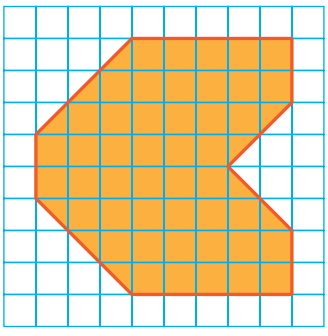
∴ So, the area is $45 + 2.5 = 47.5$ square units.

On Your Own

Now You're Ready
Exercises 3–8

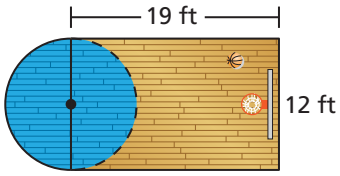
Find the area of the shaded figure.

1. 

2. 

EXAMPLE 2 Finding an Area

Find the area of the portion of the basketball court shown.



The figure is made up of a rectangle and a semicircle. Find the area of each figure.

Area of Rectangle

$$A = \ell w$$
$$= 19(12)$$
$$= 228$$

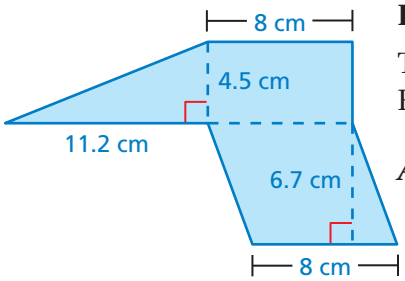
Area of Semicircle

$$A = \frac{\pi r^2}{2}$$
$$\approx \frac{3.14 \cdot 6^2}{2}$$
$$= 56.52$$

The semicircle has a radius of $\frac{12}{2} = 6$ feet.

∴ So, the area is about $228 + 56.52 = 284.52$ square feet.

EXAMPLE 3 Finding an Area



Find the area of the figure.

The figure is made up of a triangle, a rectangle, and a parallelogram. Find the area of each figure.

Area of Triangle

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(11.2)(4.5)$$
$$= 25.2$$

Area of Rectangle

$$A = \ell w$$
$$= 8(4.5)$$
$$= 36$$

Area of Parallelogram

$$A = bh$$
$$= 8(6.7)$$
$$= 53.6$$

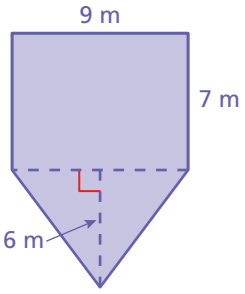
∴ So, the area is $25.2 + 36 + 53.6 = 114.8$ square centimeters.

On Your Own

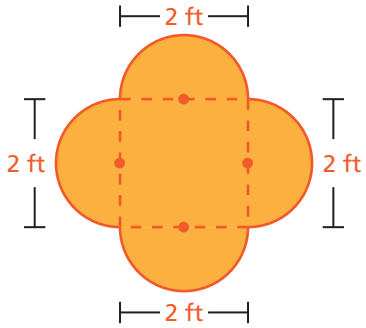
Now You're Ready
Exercises 9 and 10

Find the area of the figure.

3.



4.



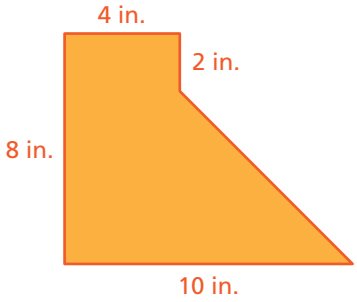
13.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** Describe two different ways to find the area of the figure. Name the types of figures you used and the dimensions of each.

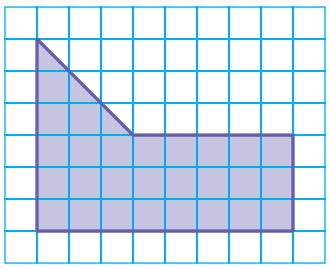
2. **REASONING** Draw a trapezoid. Explain how you can think of the trapezoid as a composite figure to find its area.



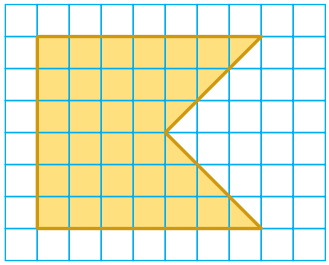
Practice and Problem Solving

Find the area of the figure.

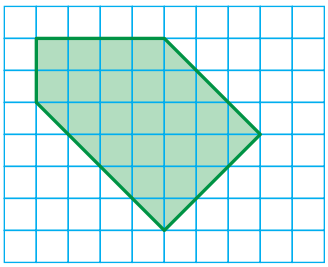
1 3.



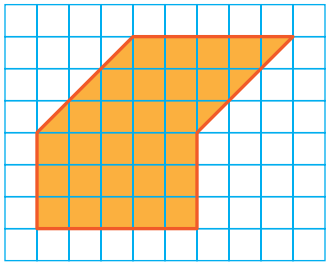
4.



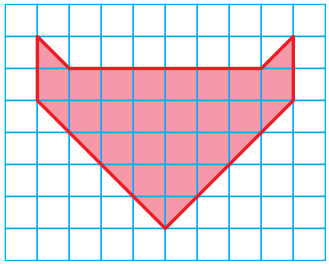
5.



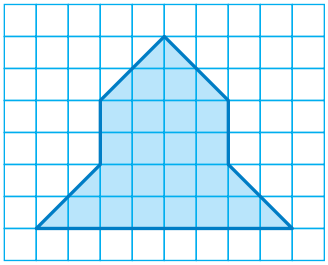
6.



7.

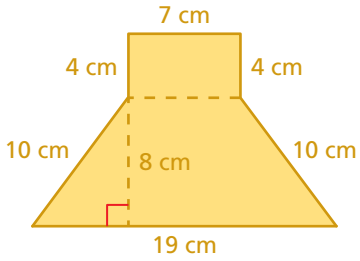


8.

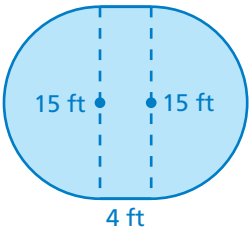


Find the area of the figure.

2 3 9.



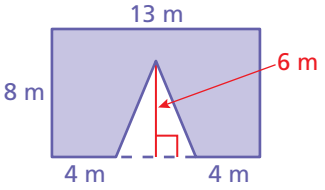
10.



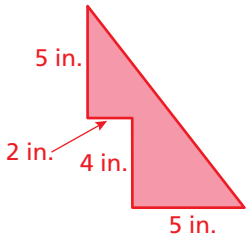
11. **OPEN-ENDED** Trace your hand and your foot on grid paper. Then estimate the area of each. Which one has the greater area?

Find the area of the figure.

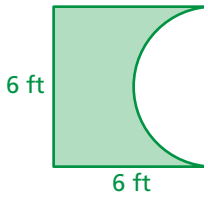
12.



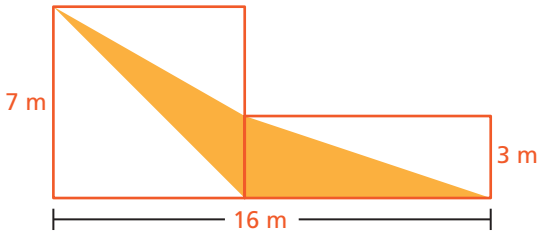
13.

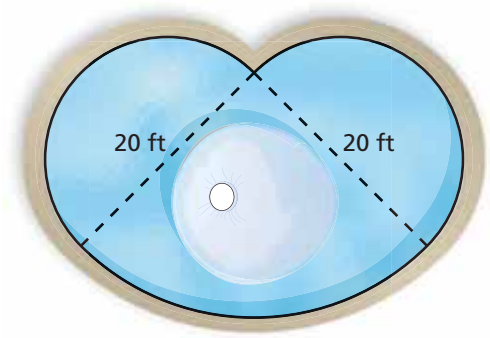


14.



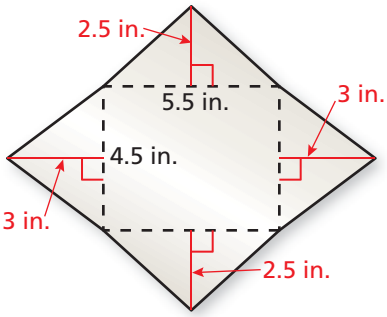
15. **STRUCTURE** The figure is made up of a square and a rectangle. Find the area of the shaded region.

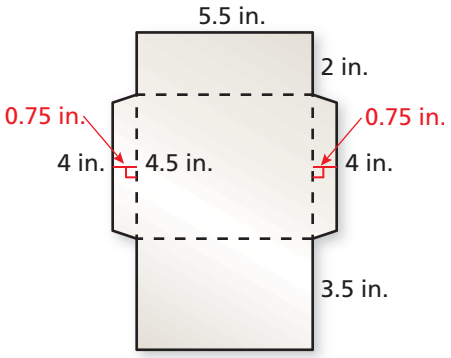




16. **FOUNTAIN** The fountain is made up of two semicircles and a quarter circle. Find the perimeter and the area of the fountain.

17. Critical Thinking You are deciding on two different designs for envelopes.





a. Which design has the greater area?

b. You make 500 envelopes using the design with the greater area. Using the same amount of paper, how many more envelopes can you make with the other design?

Fair Game Review What you learned in previous grades & lessons

Write the phrase as an expression. *(Skills Review Handbook)*

18. 12 less than a number x

19. a number y divided by 6

20. a number b increased by 3

21. the product of 7 and a number w

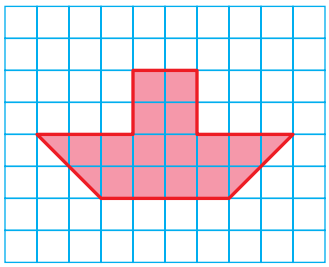
22. **MULTIPLE CHOICE** What number is 0.02% of 50? *(Skills Review Handbook)*

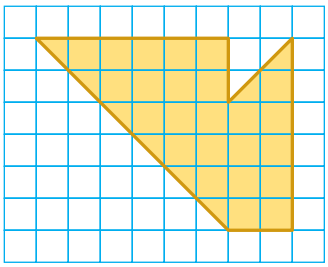
(A) 0.01 (B) 0.1 (C) 1 (D) 100

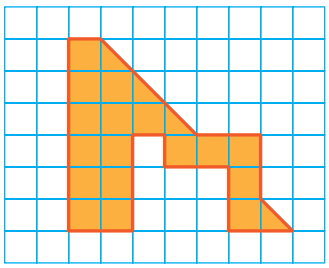
13.3–13.4 Quiz

Check It Out
Progress Check
BigIdeasMath.com

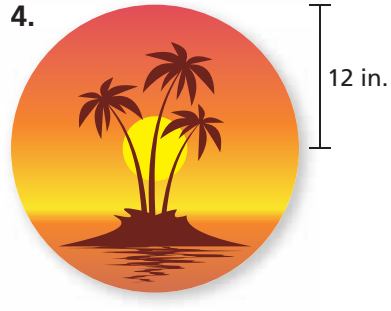
Find the area of the figure. *(Section 13.4)*

1. 

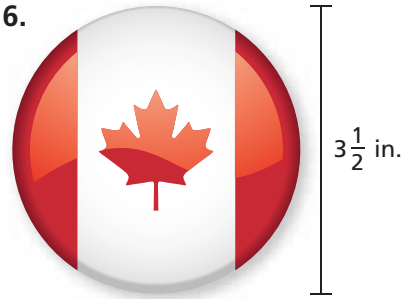
2. 

3. 

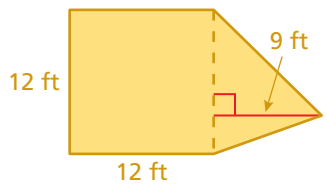
Find the area of the circle. Use 3.14 or $\frac{22}{7}$ for π. *(Section 13.3)*

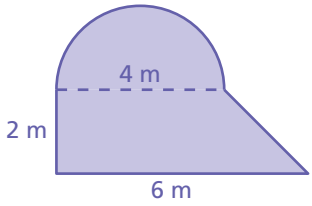
4. 

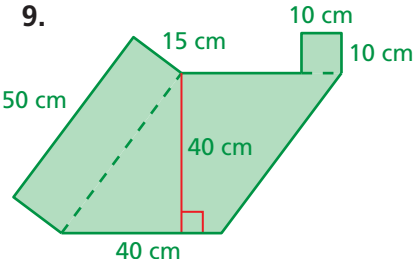
5. 

6. 

Find the area of the figure. *(Section 13.4)*

7. 

8. 

9. 

10. **POT HOLDER** A knitted pot holder is shaped like a circle. Its radius is 3.5 inches. What is its area? *(Section 13.3)*

11. **CARD** The heart-shaped card is made up of a square and two semicircles. What is the area of the card? *(Section 13.4)*

12. **DESK** A desktop is shaped like a semicircle with a diameter of 28 inches. What is the area of the desktop? *(Section 13.3)*

13. **RUG** The circular rug is placed on a square floor. The rug touches all four walls. How much of the floor space is *not* covered by the rug? *(Section 13.4)*

13 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

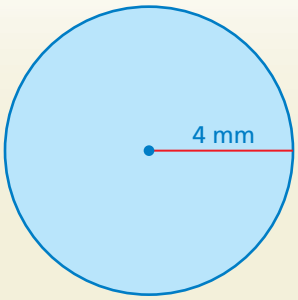
circle, *p. 550*
center, *p. 550*
radius, *p. 550*
diameter, *p. 550*
circumference, *p. 551*
pi, *p. 551*
semicircle, *p. 552*
composite figure, *p. 558*

Review Examples and Exercises

13.1 Circles and Circumference *(pp. 548–555)*

Find the circumference of the circle. Use 3.14 for π.

The radius is 4 millimeters.



$C = 2\pi r$ Write formula for circumference.

$\approx 2 \cdot 3.14 \cdot 4$ Substitute 3.14 for π and 4 for r.

$= 25.12$ Multiply.

∴ The circumference is about 25.12 millimeters.

Exercises

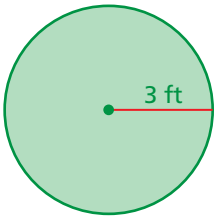
Find the radius of the circle with the given diameter.

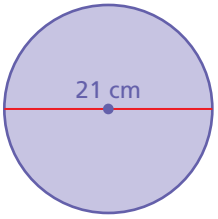
1. 8 inches
2. 60 millimeters
3. 100 meters
4. 3 yards

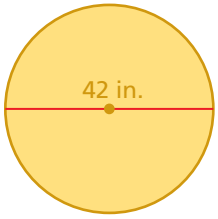
Find the diameter of the circle with the given radius.

5. 20 feet
6. 5 meters
7. 1 inch
8. 25 millimeters

Find the circumference of the circle. Use 3.14 or $\frac{22}{7}$ for π.

9.


10.


11.


13.2 Perimeters of Composite Figures *(pp. 556–561)*

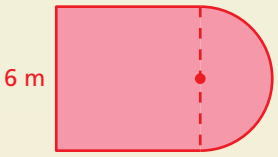
The figure is made up of a semicircle and a square. Find the perimeter.

The distance around the square part is $6 + 6 + 6 = 18$ meters. The distance around the semicircle is one-half the circumference of a circle with $d = 6$ meters.

$$\frac{C}{2} = \frac{\pi d}{2}$$ Divide the circumference by 2.

$$\approx \frac{3.14 \cdot 6}{2}$$ Substitute 3.14 for π and 6 for d.

$$= 9.42$$ Simplify.

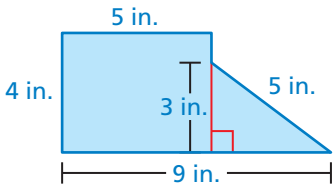


∴ So, the perimeter is about $18 + 9.42 = 27.42$ meters.

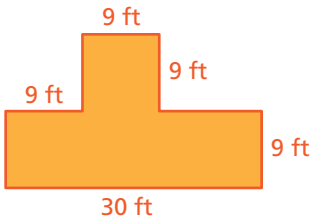
Exercises

Find the perimeter of the figure.

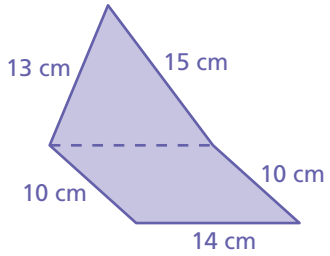
12.



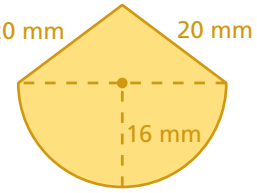
13.



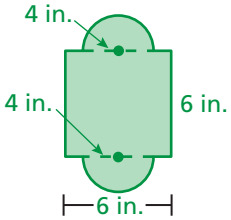
14.



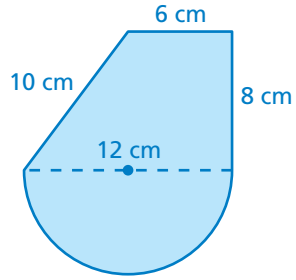
15.



16.



17.



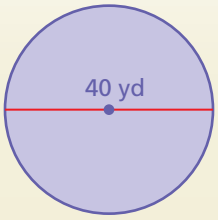
13.3 Areas of Circles *(pp. 564–569)*

Find the area of the circle. Use 3.14 for π.

$$A = \pi r^2$$ Write formula for area.

$$\approx 3.14 \cdot 20^2$$ Substitute 3.14 for π and 20 for r.

$$= 1256$$ Multiply.

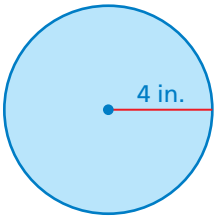


∴ The area is about 1256 square yards.

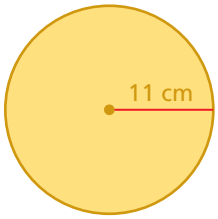
Exercises

Find the area of the circle. Use 3.14 or $\frac{22}{7}$ for π.

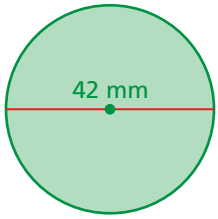
18.



19.

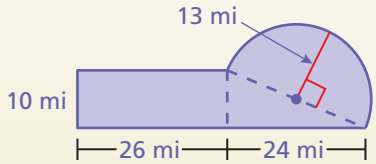


20.



13.4 Areas of Composite Figures *(pp. 570–575)*

Find the area of the figure.



The figure is made up of a rectangle, a triangle and a semicircle. Find the area of each figure.

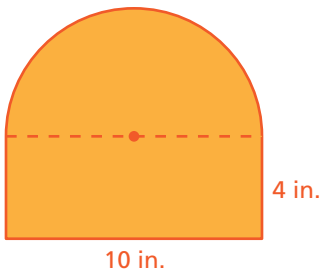
Area of Rectangle	*Area of Triangle*	*Area of Semicircle*
$A = \ell w$	$A = \frac{1}{2}bh$	$A = \frac{\pi r^2}{2}$
$= 26(10)$	$= \frac{1}{2}(10)(24)$	$\approx \frac{3.14 \cdot 13^2}{2}$
$= 260$	$= 120$	$= 265.33$

∴ So, the area is about $260 + 120 + 265.33 = 645.33$ square miles.

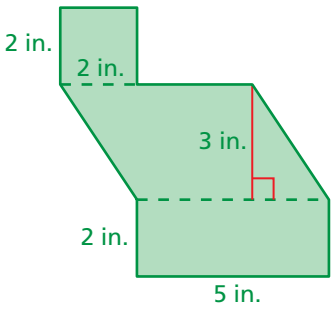
Exercises

Find the area of the figure.

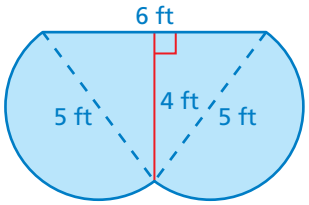
21.



22.



23.



13 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Find the radius of the circle with the given diameter.

1. 10 inches

2. 5 yards

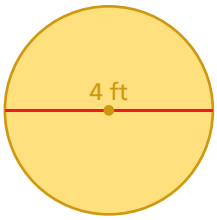
Find the diameter of the circle with the given radius.

3. 34 feet

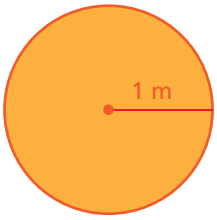
4. 19 meters

Find the circumference and the area of the circle. Use 3.14 or $\frac{22}{7}$ for π.

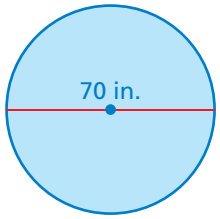
5.



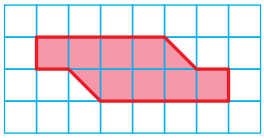
6.



7.

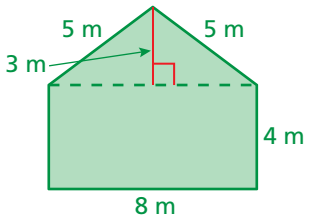


8. Estimate the perimeter of the figure. Then find the area.

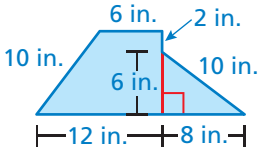


Find the perimeter and the area of the figure. Use 3.14 or $\frac{22}{7}$ for π.

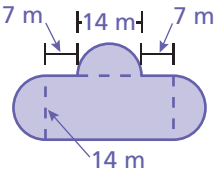
9.



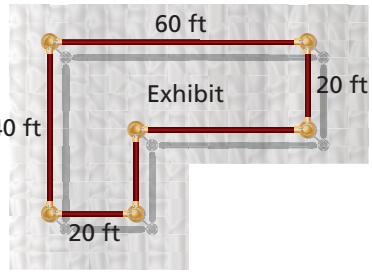
10.



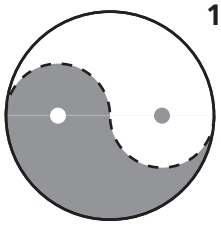
11.



12. MUSEUM A museum plans to rope off the perimeter of the L-shaped exhibit. How much rope does it need?



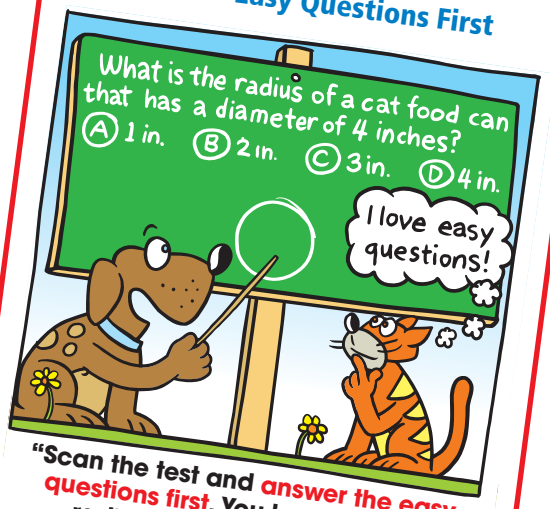
13. ANIMAL PEN You unfold chicken wire to make a circular pen with a diameter of 2.9 meters. How many meters of chicken wire do you need?



14. YIN AND YANG In the Chinese symbol for yin and yang, the dashed curve shows two semicircles formed by the curve separating the yin (dark) and the yang (light). Is the circumference of the entire yin and yang symbol *less than*, *greater than*, or *equal to* the perimeter of the yin?

13 Standards Assessment

Test-Taking Strategy
Answer Easy Questions First



"Scan the test and answer the easy questions first. You know that the radius is half the diameter."

1. To make 6 servings of soup, you need 5 cups of chicken broth. You want to know how many servings you can make with 2 quarts of chicken broth. Which proportion should you use? *(7.RP.2c)*

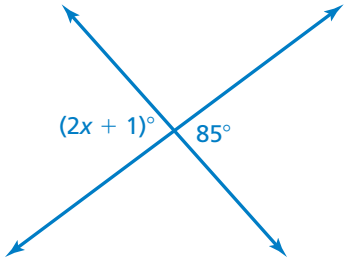
A. $\frac{6}{5} = \frac{2}{x}$

B. $\frac{6}{5} = \frac{x}{2}$

C. $\frac{6}{5} = \frac{x}{8}$

D. $\frac{5}{6} = \frac{x}{8}$

2. What is the value of x? *(7.G.5)*



3. Your mathematics teacher described an equation in words. Her description is in the box below.

> "5 less than the product of 7 and an unknown number is equal to 42."

Which equation matches your mathematics teacher's description? *(7.EE.4a)*

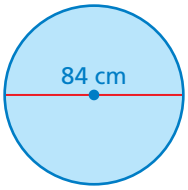
F. $(5 - 7)n = 42$

G. $(7 - 5)n = 42$

H. $5 - 7n = 42$

I. $7n - 5 = 42$

4. What is the area of the circle below? $\left(\text{Use } \frac{22}{7} \text{ for } \pi.\right)$ *(7.G.4)*



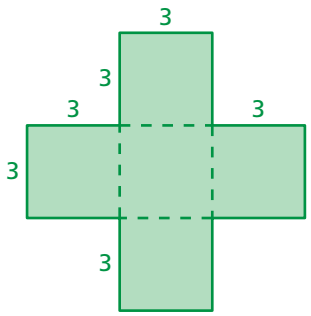
A. 132 cm^2

B. 264 cm^2

C. 5544 cm^2

D. 22,176 cm^2

5. John was finding the area of the figure below.



John's work is in the box below.

area of horizontal rectangle

$A = 3 \times (3 + 3 + 3)$

$= 3 \times 9$

$= 27$ square units

area of vertical rectangle

$A = (3 + 3 + 3) \times 3$

$= 9 \times 3$

$= 27$ square units

total area of figure

$A = 27 + 27$

$= 54$ square units

What should John do to correct the error that he made? *(7.G.6)*

F. Add the area of the center square to the 54 square units.

G. Find the area of one square and multiply this number by 4.

H. Subtract the area of the center square from the 54 square units.

I. Subtract 54 from the area of a large square that is 9 units on each side.

6. Which value of x makes the equation below true? *(7.EE.4a)*

$$5x - 3 = 11$$

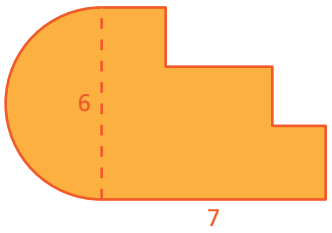
A. 1.6

B. 2.8

C. 40

D. 70

7. What is the perimeter of the figure below? (Use 3.14 for π.) *(7.G.4)*



8. Which inequality has 5 in its solution set? *(7.EE.4b)*

F. $5 - 2x \geq 3$

G. $3x - 4 \geq 8$

H. $8 - 3x > -7$

I. $4 - 2x < -6$

9. Four jewelry stores are selling an identical pair of earrings.

- Store A: original price of $75; 20% off during sale
- Store B: original price of $100; 35% off during sale
- Store C: original price of $70; 10% off during sale
- Store D: original price of $95; 30% off during sale

Which store has the least sale price for the pair of earrings? *(7.RP.3)*

A. Store A

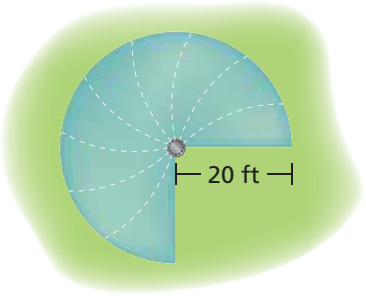
B. Store B

C. Store C

D. Store D

10. A lawn sprinkler sprays water onto part of a circular region, as shown below. *(7.G.4)*

Think Solve Explain



Part A What is the area, in square feet, of the region that the sprinkler sprays with water? Show your work and explain your reasoning. (Use 3.14 for π.)

Part B What is the perimeter, in feet, of the region that the sprinkler sprays with water? Show your work and explain your reasoning. (Use 3.14 for π.)