

11 Inequalities

11.1 Writing and Graphing Inequalities

11.2 Solving Inequalities Using Addition or Subtraction

11.3 Solving Inequalities Using Multiplication or Division

11.4 Solving Two-Step Inequalities



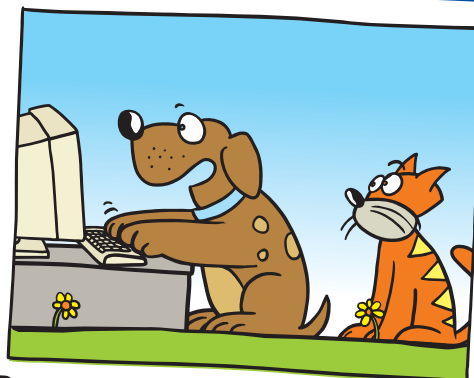
"If you reached into your water bowl and found more than \$20..."



"And then reached into your cat food bowl and found more than \$40..."



"What would you have?"

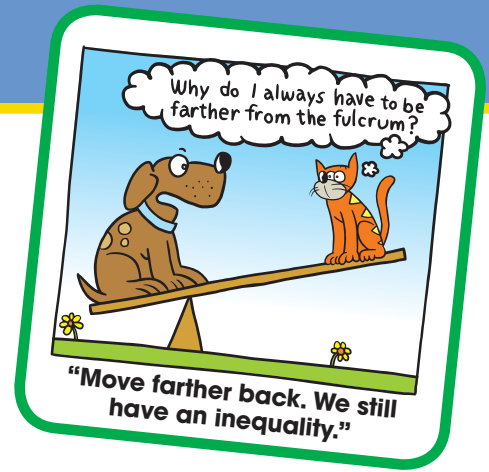


"Dear Precious Pet World: Your ad says 'Up to 75% off on selected items.'"



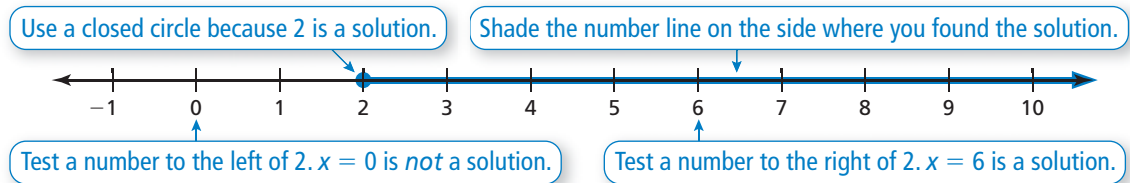
"I select Yummy Tummy Bacon-Flavored Dog Biscuits."

What You Learned Before



Graphing Inequalities (6.EE.8)

Example 1 Graph $x \geq 2$.



Try It Yourself

Graph the inequality.

1. $x \geq 1$

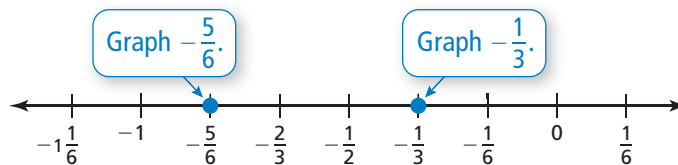
2. $x < 5$

3. $x \leq 20$

4. $x > 13$

Comparing Numbers (6.NS.7a)

Example 2 Compare $-\frac{1}{3}$ and $-\frac{5}{6}$.



$-\frac{5}{6}$ is to the left of $-\frac{1}{3}$.

∴ So, $-\frac{5}{6} < -\frac{1}{3}$.

Try It Yourself

Copy and complete the statement using $<$ or $>$.

5. $-\frac{2}{3}$ $\frac{3}{8}$

6. $-\frac{1}{2}$ $-\frac{7}{8}$

7. $-\frac{1}{5}$ $\frac{1}{10}$

8. -1.4 1.2

9. -2.2 -4.6

10. -1.9 -1.1

11.1 Writing and Graphing Inequalities

Essential Question How can you use a number line to represent solutions of an inequality?

1 ACTIVITY: Understanding Inequality Statements

Work with a partner. Read the statement. Circle each number that makes the statement true, and then answer the questions.

a. “You are in **at least** 5 of the photos.”

–3 –2 –1 0 1 2 3 4 5 6

- What do you notice about the numbers that you circled?
- Is the number 5 included? Why or why not?
- Write four other numbers that make the statement true.



b. “The temperature is **less than** -4 degrees Fahrenheit.”

–7 –6 –5 –4 –3 –2 –1 0 1 2

- What do you notice about the numbers that you circled?
- Can the temperature be exactly -4 degrees Fahrenheit? Explain.
- Write four other numbers that make the statement true.



c. “**More than** 3 students from our school are in the chess tournament.”

–3 –2 –1 0 1 2 3 4 5 6

- What do you notice about the numbers that you circled?
- Is the number 3 included? Why or why not?
- Write four other numbers that make the statement true.



d. “The balance in a yearbook fund is **no more than** $-\$5$.”

–7 –6 –5 –4 –3 –2 –1 0 1 2

- What do you notice about the numbers that you circled?
- Is the number -5 included? Why or why not?
- Write four other numbers that make the statement true.



COMMON
CORE

Inequalities

In this lesson, you will

- write and graph inequalities.
- use substitution to check whether a number is a solution of an inequality.

Preparing for Standard
7.EE.4b

2 ACTIVITY: Understanding Inequality Symbols

Work with a partner.

- a. Consider the statement “ x is a number such that $x > -1.5$.”
- Can the number be exactly -1.5 ? Explain.
 - Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
 - Write four other numbers that are not integers that make the statement true.
- b. Consider the statement “ x is a number such that $x \leq \frac{5}{2}$.”
- Can the number be exactly $\frac{5}{2}$? Explain.
 - Make a number line. Shade the part of the number line that shows the numbers that make the statement true.
 - Write four other numbers that are not integers that make the statement true.

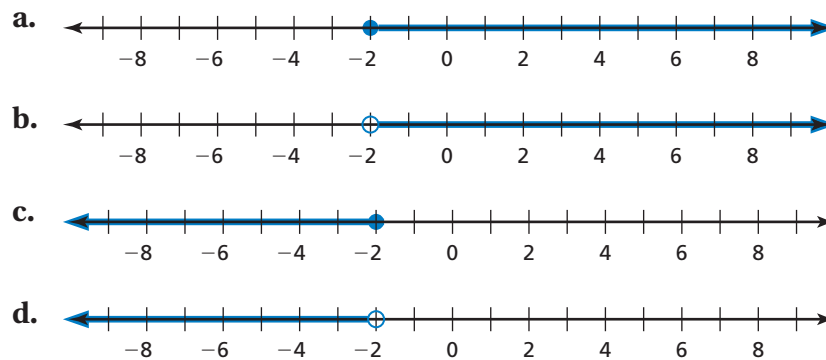
3 ACTIVITY: Writing and Graphing Inequalities

Math Practice 1

Check Progress

All the graphs are similar. So, what can you do to make sure that you have correctly written each inequality?

Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of x that make the inequality true.



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a number line to represent solutions of an inequality?
5. **STRUCTURE** Is $x \geq -1.4$ the same as $-1.4 \leq x$? Explain.

Practice

Use what you learned about writing and graphing inequalities to complete Exercises 4 and 5 on page 468.

11.1 Lesson

Key Vocabulary

inequality, p. 466
solution of an inequality, p. 466
solution set, p. 466
graph of an inequality, p. 467

An **inequality** is a mathematical sentence that compares expressions. It contains the symbols $<$, $>$, \leq , or \geq . To write an inequality, look for the following phrases to determine where to place the inequality symbol.

Inequality Symbols				
Symbol	$<$	$>$	\leq	\geq
Key Phrases	<ul style="list-style-type: none"> is less than 	<ul style="list-style-type: none"> is greater than 	<ul style="list-style-type: none"> is less than or equal to 	<ul style="list-style-type: none"> is greater than or equal to
	<ul style="list-style-type: none"> is fewer than 	<ul style="list-style-type: none"> is more than 	<ul style="list-style-type: none"> is at most is no more than 	<ul style="list-style-type: none"> is at least is no less than

EXAMPLE 1 Writing an Inequality

A number q plus 5 is greater than or equal to -7.9 . Write this word sentence as an inequality.

$$\underbrace{\text{A number } q \text{ plus } 5}_{q + 5} \underbrace{\text{is greater than or equal to}}_{\geq} \underbrace{-7.9}_{-7.9}$$

 An inequality is $q + 5 \geq -7.9$.




On Your Own

Write the word sentence as an inequality.

- A number x is at most -10 .
- Twice a number y is more than $-\frac{5}{2}$.

 **Now You're Ready**
Exercises 6–9

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 2 \leq -1$	Is the inequality true?
-2	$-2 + 2 \stackrel{?}{\leq} -1$ $0 \not\leq -1$ 	no
-3	$-3 + 2 \stackrel{?}{\leq} -1$ $-1 \leq -1$ 	yes
-4	$-4 + 2 \stackrel{?}{\leq} -1$ $-2 \leq -1$ 	yes

Reading

The symbol $\not\leq$ means is not less than or equal to.

EXAMPLE 2 Checking Solutions

Tell whether -2 is a solution of each inequality.

a. $y - 5 \geq -6$

$$y - 5 \geq -6$$

$$-2 - 5 \stackrel{?}{\geq} -6$$

$$-7 \not\geq -6 \quad \times$$

-7 is *not* greater than or equal to -6 .

∴ So, -2 is *not* a solution of the inequality.

Write the inequality.

Substitute -2 for y .

Simplify.

b. $-5.5y < 14$

$$-5.5y < 14$$

$$-5.5(-2) \stackrel{?}{<} 14$$

$$11 < 14 \quad \checkmark$$

11 is less than 14 .

∴ So, -2 is a solution of the inequality.

On Your Own

Now You're Ready
Exercises 11–16

Tell whether -5 is a solution of the inequality.

3. $x + 12 > 7$

4. $1 - 2p \leq -9$

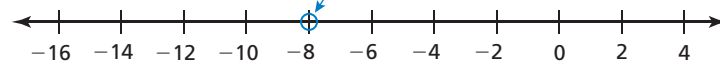
5. $n \div 2.5 \geq -3$

The **graph of an inequality** shows all the solutions of the inequality on a number line. An open circle \circ is used when a number is *not* a solution. A closed circle \bullet is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 Graphing an Inequality

Graph $y > -8$.

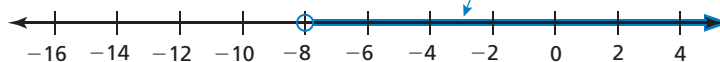
Use an open circle because -8 is *not* a solution.



Test a number to the left of -8 .
 $y = -12$ is *not* a solution.

Test a number to the right of -8 .
 $y = 0$ is a solution.

Shade the number line on the side where you found the solution.



Study Tip

The graph in Example 3 shows that the inequality has *infinitely many* solutions.

On Your Own

Now You're Ready
Exercises 17–20

Graph the inequality on a number line.

6. $x < -1$

7. $z \geq 4$

8. $s \leq 1.4$

9. $-\frac{1}{2} < t$

11.1 Exercises

Vocabulary and Concept Check

- PRECISION** Should you use an open circle or a closed circle in the graph of the inequality $b \geq -42$? Explain.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Write “both” inequalities.

k is less than or equal to -3 .

k is no more than -3 .

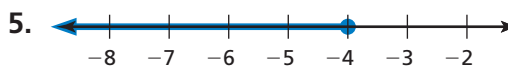
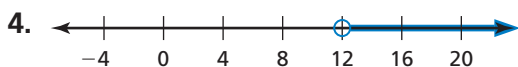
k is at most -3 .

k is at least -3 .

- REASONING** Do $x < 5$ and $5 < x$ represent the same inequality? Explain.

Practice and Problem Solving

Write an inequality for the graph. Then, in words, describe all the values of x that make the inequality true.



Write the word sentence as an inequality.

6. A number y is no more than -8 .
7. A number w added to 2.3 is more than 18 .
8. A number t multiplied by -4 is at least $-\frac{2}{5}$.
9. A number b minus 4.2 is less than -7.5 .
10. **ERROR ANALYSIS** Describe and correct the error in writing the word sentence as an inequality.



Twice a number x is at most -24 .

$$2x \geq -24$$

Tell whether the given value is a solution of the inequality.

11. $n + 8 \leq 13$; $n = 4$
12. $5h > -15$; $h = -5$
13. $p + 1.4 \leq 0.5$; $p = 0.1$
14. $\frac{a}{6} > -4$; $a = -18$
15. $-\frac{2}{3}s \geq 6$; $s = -9$
16. $\frac{7}{8} - 3k < -\frac{1}{2}$; $k = \frac{1}{4}$

Graph the inequality on a number line.

17. $r \leq -9$
18. $g > 2.75$
19. $x \geq -3\frac{1}{2}$
20. $z < 1\frac{1}{4}$
21. **FOOD TRUCK** Each day at lunchtime, at least 53 people buy food from a food truck. Write an inequality that represents this situation.

Tell whether the given value is a solution of the inequality.

22. $4k < k + 8; k = 3$

23. $\frac{w}{3} \geq w - 12; w = 15$

24. $7 - 2y > 3y + 13; y = -1$

25. $\frac{3}{4}b - 2 \leq 2b + 8; b = -4$



26. **MODELING** A subway ride for a student costs \$1.25. A monthly pass costs \$35.

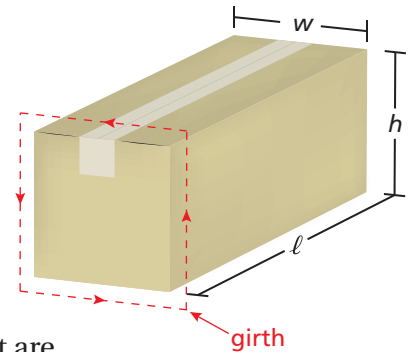
- Write an inequality that represents the number of times you must ride the subway for the monthly pass to be a better deal.
- You ride the subway about 45 times per month. Should you buy the monthly pass? Explain.

27. **LOGIC** Consider the inequality $b > -2$.

- Describe the values of b that are solutions of the inequality.
- Describe the values of b that are *not* solutions of the inequality. Write an inequality for these values.
- What do all the values in parts (a) and (b) represent? Is this true for any inequality?

28. **Critical Thinking** A postal service says that a rectangular package can have a maximum combined length and *girth* of 108 inches. The *girth* of a package is the distance around the perimeter of a face that does not include the length.

- Write an inequality that represents the allowable dimensions for the package.
- Find three different sets of allowable dimensions that are reasonable for the package. Find the volume of each package.



Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. (Section 1.1)

29. $p - 8 = 3$

30. $8.7 + w = 5.1$

31. $x - 2 = -9$

32. **MULTIPLE CHOICE** Which expression has a value less than -5 ? (Skills Review Handbook)

(A) $5 + 8$

(B) $-9 + 5$

(C) $1 + (-8)$

(D) $7 + (-2)$

11.2 Solving Inequalities Using Addition or Subtraction

Essential Question How can you use addition or subtraction to solve an inequality?

1 ACTIVITY: Writing an Inequality

Work with a partner. Members of the Boy Scouts must be less than 18 years old. In 4 years, your friend will still be eligible to be a scout.

- a. Which of the following represents your friend's situation? What does x represent? Explain your reasoning.



$$x + 4 > 18$$

$$x + 4 < 18$$

$$x + 4 \geq 18$$

$$x + 4 \leq 18$$

- b. Graph the possible ages of your friend on a number line. Explain how you decided what to graph.

2 ACTIVITY: Writing an Inequality

Work with a partner. Supercooling is the process of lowering the temperature of a liquid or a gas below its freezing point without it becoming a solid. Water can be supercooled to 86°F below its normal freezing point (32°F) and still not freeze.

- a. Let x represent the temperature of water. Which inequality represents the temperature at which water can be a liquid or a gas? Explain your reasoning.

$$x - 32 > -86$$

$$x - 32 < -86$$

$$x - 32 \geq -86$$

$$x - 32 \leq -86$$



- b. On a number line, graph the possible temperatures at which water can be a liquid or a gas. Explain how you decided what to graph.



COMMON CORE

Inequalities

In this lesson, you will

- solve inequalities using addition or subtraction.
- solve real-life problems.

Learning Standard
7.EE.4b

3 ACTIVITY: Solving Inequalities

Math Practice 4

Interpret Results

What does the solution of the inequality represent?

Work with a partner. Complete the following steps for Activity 1. Then repeat the steps for Activity 2.

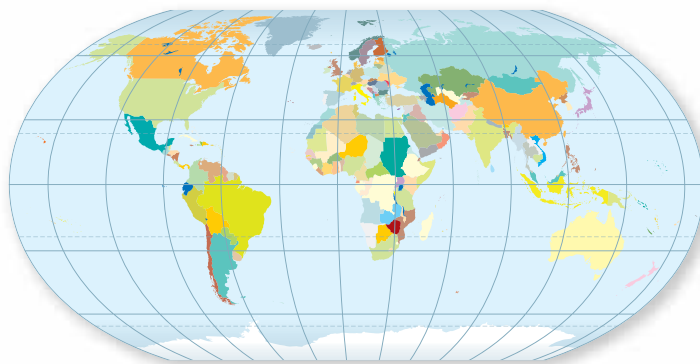
- Use your inequality from part (a). Replace the inequality symbol with an equal sign.
- Solve the equation.
- Replace the equal sign with the original inequality symbol.
- Graph this new inequality.
- Compare the graph with your graph in part (b). What do you notice?

4 ACTIVITY: Temperatures of Continents

Work with a partner. The table shows the lowest recorded temperature on each continent. Write an inequality that represents each statement. Then solve and graph the inequality.

- The temperature at a weather station in Asia is more than 150°F greater than the record low in Asia.
- The temperature at a research station in Antarctica is at least 80°F greater than the record low in Antarctica.

Continent	Lowest Temperature
Africa	-11°F
Antarctica	-129°F
Asia	-90°F
Australia	-9.4°F
Europe	-67°F
North America	-81.4°F
South America	-27°F



What Is Your Answer?

- IN YOUR OWN WORDS** How can you use addition or subtraction to solve an inequality?
- Describe a real-life situation that you can represent with an inequality. Write the inequality. Graph the solution on a number line.

Practice

Use what you learned about solving inequalities to complete Exercises 3–5 on page 474.

Key Ideas

Study Tip

You can solve inequalities in the same way you solve equations. Use inverse operations to get the variable by itself.

Addition Property of Inequality

Words When you add the same number to each side of an inequality, the inequality remains true.

Numbers	$-4 < 3$	Algebra	If $a < b$, then $a + c < b + c$.
	$\begin{array}{r} +2 \\ -4 < 3 \\ +2 \\ \hline -2 < 5 \end{array}$		If $a > b$, then $a + c > b + c$.

Subtraction Property of Inequality

Words When you subtract the same number from each side of an inequality, the inequality remains true.

Numbers	$-2 < 2$	Algebra	If $a < b$, then $a - c < b - c$.
	$\begin{array}{r} -3 \\ -2 < 2 \\ -3 \\ \hline -5 < -1 \end{array}$		If $a > b$, then $a - c > b - c$.

These properties are also true for \leq and \geq .

EXAMPLE 1 Solving an Inequality Using Addition

Solve $x - 5 < -3$. Graph the solution.

$$x - 5 < -3$$

Write the inequality.

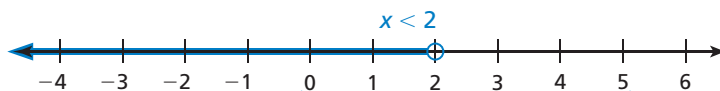
Undo the subtraction.

$$\begin{array}{r} +5 \\ x - 5 < -3 \\ +5 \\ \hline x < 2 \end{array}$$

Addition Property of Inequality

Simplify.

The solution is $x < 2$.



$x = 0$ is a solution.

$x = 5$ is not a solution.

Check:

$$x = 0: 0 - 5 \stackrel{?}{<} -3$$

$$-5 < -3 \quad \checkmark$$

$$x = 5: 5 - 5 \stackrel{?}{<} -3$$

$$0 \not< -3 \quad \times$$

On Your Own

Solve the inequality. Graph the solution.

1. $y - 6 > -7$

2. $b - 3.8 \leq 1.7$

3. $-\frac{1}{2} > z - \frac{1}{4}$

EXAMPLE 2 Solving an Inequality Using Subtraction

Solve $13 \leq x + 14$. Graph the solution.

$$13 \leq x + 14 \quad \text{Write the inequality.}$$

Undo the addition.

$$\xrightarrow{-14} \quad \underline{-14} \quad \underline{-14} \quad \text{Subtraction Property of Inequality}$$

$$-1 \leq x \quad \text{Simplify.}$$

∴ The solution is $x \geq -1$.



Reading

The inequality $-1 \leq x$ is the same as $x \geq -1$.

On Your Own

Solve the inequality. Graph the solution.

4. $w - 7 \leq -10$

5. $-7.5 \geq d - 10$

6. $x + \frac{3}{4} > 1\frac{1}{2}$

Now You're Ready
Exercises 3–17

EXAMPLE 3 Real-Life Application

A person can be no taller than 6.25 feet to become an astronaut pilot for NASA. Your friend is 5 feet 9 inches tall. Write and solve an inequality that represents how much your friend can grow and still meet the requirement.

Words Current height plus amount your friend can grow is no more than the height limit.

Variable Let h be the possible amounts your friend can grow.

Inequality 5.75 + h ≤ 6.25

$$5 \text{ ft } 9 \text{ in.} = 60 + 9 = 69 \text{ in.}$$

$$69 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 5.75 \text{ ft}$$

$$5.75 + h \leq 6.25 \quad \text{Write the inequality.}$$

$$\underline{-5.75} \quad \underline{-5.75} \quad \text{Subtraction Property of Inequality}$$

$$h \leq 0.5 \quad \text{Simplify.}$$

∴ So, your friend can grow no more than 0.5 foot, or 6 inches.

On Your Own

7. Your cousin is 5 feet 3 inches tall. Write and solve an inequality that represents how much your cousin can grow and still meet the requirement.



Vocabulary and Concept Check

- REASONING** Is the inequality $c + 3 > 5$ the same as $c > 5 - 3$? Explain.
- WHICH ONE DOESN'T BELONG?** Which inequality does *not* belong with the other three? Explain your reasoning.

$$w + \frac{7}{4} > \frac{3}{4}$$

$$w - \frac{3}{4} > -\frac{7}{4}$$

$$w + \frac{7}{4} < \frac{3}{4}$$


$$\frac{3}{4} < w + \frac{7}{4}$$

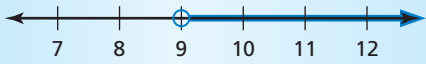
Practice and Problem Solving


Solve the inequality. Graph the solution.


- | | | | | |
|---|---|------------------------------------------|------------------------------|--------------------------------------|
| 1 | 2 | 3. $x + 7 \geq 18$ | 4. $a - 2 > 4$ | 5. $3 \leq 7 + g$ |
| | | 6. $8 + k \leq -3$ | 7. $-12 < y - 6$ | 8. $n - 4 < 5$ |
| | | 9. $t - 5 \leq -7$ | 10. $p + \frac{1}{4} \geq 2$ | 11. $\frac{2}{7} > b + \frac{5}{7}$ |
| | | 12. $z - 4.7 \geq -1.6$ | 13. $-9.1 < d - 6.3$ | 14. $\frac{8}{5} > s + \frac{12}{5}$ |
| | | 15. $-\frac{7}{8} \geq m - \frac{13}{8}$ | 16. $r + 0.2 < -0.7$ | 17. $h - 6 \leq -8.4$ |

ERROR ANALYSIS Describe and correct the error in solving the inequality or graphing the solution of the inequality.

18. 
$$\begin{array}{r} x - 7 > -2 \\ +7 \quad +7 \\ \hline x > 9 \end{array}$$



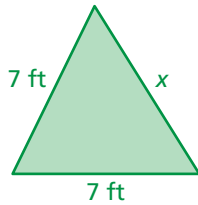
19. 
$$\begin{array}{r} 8 \leq x + 3 \\ -3 \quad -3 \\ \hline 5 \leq x \end{array}$$



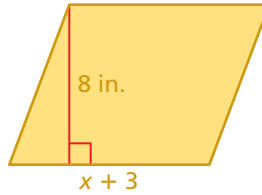

20. **AIRPLANE** A small airplane can hold 44 passengers. Fifteen passengers board the plane.
- Write and solve an inequality that represents the additional number of passengers that can board the plane.
 - Can 30 more passengers board the plane? Explain.

Write and solve an inequality that represents x .

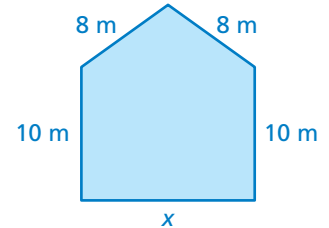
21. The perimeter is less than 28 feet.



22. The base is greater than the height.



23. The perimeter is less than or equal to 51 meters.



24. **REASONING** The solution of $d + s > -3$ is $d > -7$. What is the value of s ?

25. **BIRDFEEDER** The hole for a birdfeeder post is 3 feet deep. The top of the post needs to be at least 5 feet above the ground. Write and solve an inequality that represents the required length of the post.



26. **SHOPPING** You can spend up to \$35 on a shopping trip.

- You want to buy a shirt that costs \$14. Write and solve an inequality that represents the amount of money you will have left if you buy the shirt.
- You notice that the shirt is on sale for 30% off. How does this change the inequality?
- Do you have enough money to buy the shirt that is on sale and a pair of pants that costs \$23? Explain.

27. **POWER** A circuit overloads at 2400 watts of electricity. A portable heater that uses 1050 watts of electricity is plugged into the circuit.

- Write and solve an inequality that represents the additional number of watts you can plug in without overloading the circuit.
- In addition to the portable heater, what two other items in the table can you plug in at the same time without overloading the circuit? Is there more than one possibility? Explain.

Item	Watts
Aquarium	200
Hair dryer	1200
Television	150
Vacuum cleaner	1100

28. **Number Sense** The possible values of x are given by $x + 8 \leq 6$. What is the greatest possible value of $7x$?



Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. (Section 1.1)

29. $4x = 36$

30. $\frac{w}{3} = -9$

31. $-2b = 44$

32. $60 = \frac{3}{4}h$

33. **MULTIPLE CHOICE** Which fraction is equivalent to -2.4 ? (Skills Review Handbook)

(A) $-\frac{12}{5}$

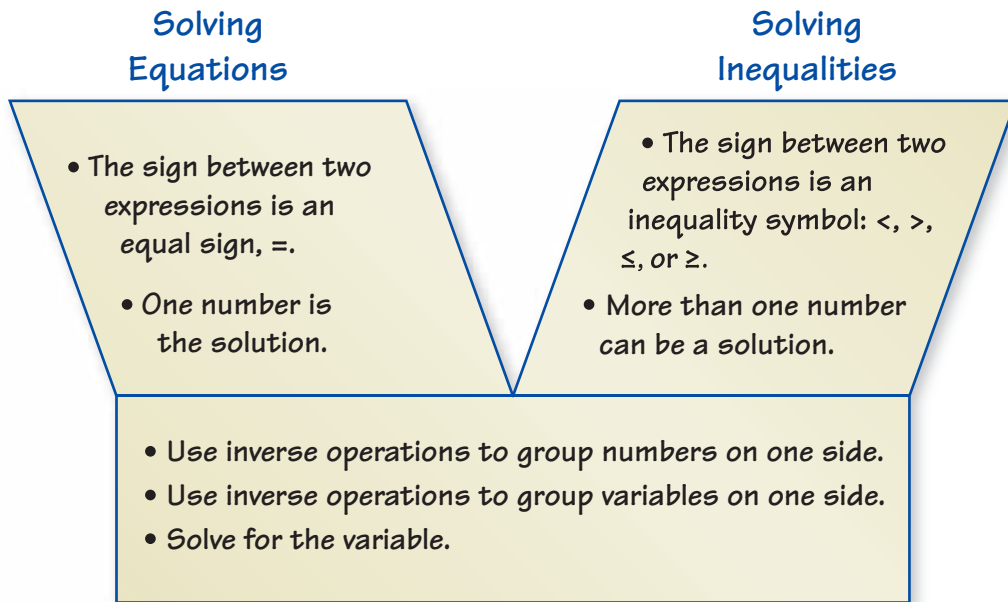
(B) $-\frac{51}{25}$

(C) $-\frac{8}{5}$

(D) $-\frac{6}{25}$

11 Study Help

You can use a **Y chart** to compare two topics. List differences in the branches and similarities in the base of the Y. Here is an example of a Y chart that compares solving equations and solving inequalities.



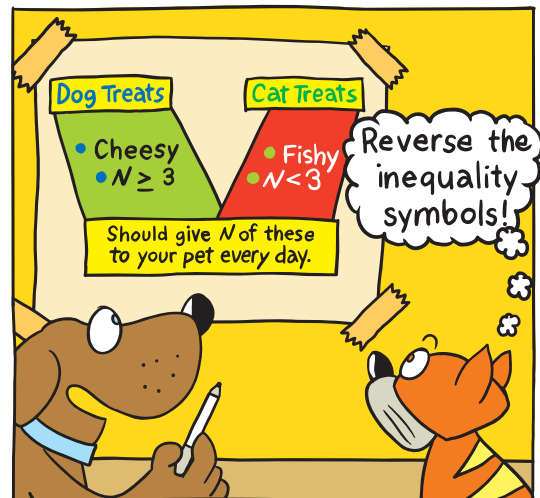
On Your Own

Make Y charts to help you study and compare these topics.

1. writing equations and writing inequalities
2. graphing the solution of an equation and graphing the solution of an inequality
3. graphing inequalities that use $>$ and graphing inequalities that use $<$
4. graphing inequalities that use $>$ or $<$ and graphing inequalities that use \geq or \leq
5. solving inequalities using addition and solving inequalities using subtraction

After you complete this chapter, make Y charts for the following topics.

6. solving inequalities using multiplication and solving inequalities using division
7. solving two-step equations and solving two-step inequalities
8. Pick two other topics that you studied earlier in this course and make a Y chart to compare them.



"Hey Descartes, do you have any suggestions for the **Y chart** I am making?"

11.1–11.2 Quiz

Write the word sentence as an inequality. (Section 11.1)

1. A number y plus 2 is greater than -5 . 2. A number s minus 2.4 is at least 8.

Tell whether the given value is a solution of the inequality. (Section 11.1)

3. $8p < -3$; $p = -2$ 4. $z + 2 > -4$; $z = -8$

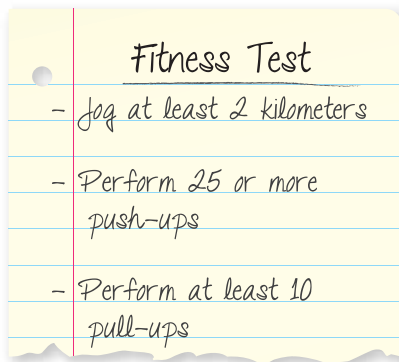
Graph the inequality on a number line. (Section 11.1)

5. $x < -12$ 6. $v > \frac{5}{4}$
7. $b \geq -\frac{1}{3}$ 8. $q \leq 4.2$

Solve the inequality. Graph the solution. (Section 11.2)

9. $n + 2 \leq -6$ 10. $t - \frac{3}{7} > \frac{6}{7}$
11. $-\frac{3}{4} \geq w + 1$ 12. $y - 2.6 < -3.4$

13. **STUDYING** You plan to study at least 1.5 hours for a geography test. Write an inequality that represents this situation. (Section 11.1)



14. **FITNESS TEST** The three requirements to pass a fitness test are shown. (Section 11.1)

- a. Write and graph three inequalities that represent the requirements.
b. You can jog 2500 meters, perform 30 push-ups, and perform 20 pull-ups. Do you satisfy the requirements of the test? Explain.

15. **NUMBER LINE** Use tape on the floor to make the number line shown. All units are in feet. You are standing at $-\frac{7}{2}$. You want to move to a number greater than $-\frac{3}{2}$. Write and solve an inequality that represents the distance you must move. (Section 11.2)



Essential Question How can you use multiplication or division to solve an inequality?

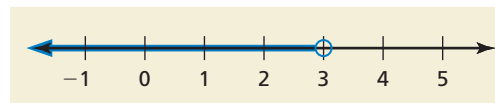
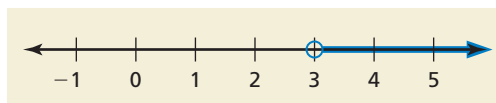
1 ACTIVITY: Using a Table to Solve an Inequality

Work with a partner.

- Copy and complete the table.
- Decide which graph represents the solution of the inequality.
- Write the solution of the inequality.

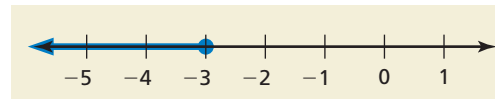
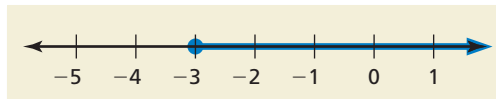
a. $4x > 12$

x	-1	0	1	2	3	4	5
$4x$							
$4x > 12$							



b. $-3x \leq 9$

x	-5	-4	-3	-2	-1	0	1
$-3x$							
$-3x \leq 9$							



COMMON
CORE

Inequalities

In this lesson, you will

- solve inequalities using multiplication or division.
- solve real-life problems.

Learning Standard
7.EE.4b

2 ACTIVITY: Solving an Inequality

Work with a partner.

- Solve $-3x \leq 9$ by adding $3x$ to each side of the inequality first. Then solve the resulting inequality.
- Compare the solution in part (a) with the solution in Activity 1(b).

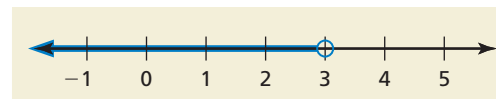
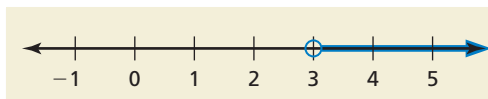
3 ACTIVITY: Using a Table to Solve an Inequality

Work with a partner.

- Copy and complete the table.
- Decide which graph represents the solution of the inequality.
- Write the solution of the inequality.

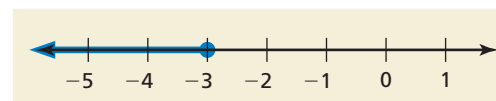
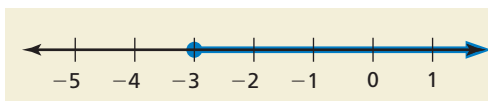
a. $\frac{x}{3} < 1$

x	-1	0	1	2	3	4	5
$\frac{x}{3}$							
$\frac{x}{3} ? < 1$							



b. $\frac{x}{-4} \geq \frac{3}{4}$

x	-5	-4	-3	-2	-1	0	1
$\frac{x}{-4}$							
$\frac{x}{-4} ? \geq \frac{3}{4}$							



4 ACTIVITY: Writing Rules

Work with a partner. Use a table to solve each inequality.

a. $-2x \leq 10$ b. $-6x > 0$ c. $\frac{x}{-4} < 1$ d. $\frac{x}{-8} \geq \frac{1}{8}$

Write a set of rules that describes how to solve inequalities like those in Activities 1 and 3. Then use your rules to solve each of the four inequalities above.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use multiplication or division to solve an inequality?

Math Practice 3

Analyze Conjectures

When you apply your rules to parts (a)–(d), do you get the same solutions? Explain.

Practice

Use what you learned about solving inequalities using multiplication or division to complete Exercises 4–9 on page 483.

Key Idea

Remember

Multiplication and division are inverse operations.

Multiplication and Division Properties of Inequality (Case 1)

Words When you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Numbers $-4 < 6$ $4 > -6$

$$2 \cdot (-4) < 2 \cdot 6 \qquad \frac{4}{2} > \frac{-6}{2}$$

$$-8 < 12 \qquad 2 > -3$$

Algebra If $a < b$ and c is positive, then

$$a \cdot c < b \cdot c \qquad \text{and} \qquad \frac{a}{c} < \frac{b}{c}$$

If $a > b$ and c is positive, then

$$a \cdot c > b \cdot c \qquad \text{and} \qquad \frac{a}{c} > \frac{b}{c}$$

These properties are also true for \leq and \geq .

EXAMPLE 1 Solving an Inequality Using Multiplication

Solve $\frac{x}{5} \leq -3$. Graph the solution.

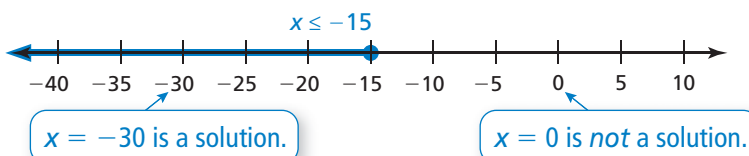
$$\frac{x}{5} \leq -3 \qquad \text{Write the inequality.}$$

Undo the division.

$$\rightarrow 5 \cdot \frac{x}{5} \leq 5 \cdot (-3) \qquad \text{Multiplication Property of Inequality}$$

$$x \leq -15 \qquad \text{Simplify.}$$

∴ The solution is $x \leq -15$.



On Your Own

Solve the inequality. Graph the solution.

1. $n \div 3 < 1$

2. $-0.5 \leq \frac{m}{10}$

3. $-3 > \frac{2}{3}p$

EXAMPLE 2 Solving an Inequality Using Division

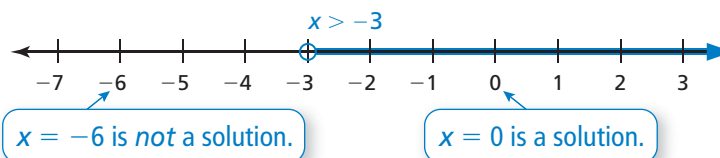
Solve $6x > -18$. Graph the solution.

$$6x > -18 \quad \text{Write the inequality.}$$

Undo the multiplication. $\rightarrow \frac{6x}{6} > \frac{-18}{6}$ Division Property of Inequality

$$x > -3 \quad \text{Simplify.}$$

The solution is $x > -3$.



On Your Own

Solve the inequality. Graph the solution.

4. $4b \geq 2$

5. $12k \leq -24$

6. $-15 < 2.5q$

Now You're Ready
Exercises 10–18

Key Idea

Multiplication and Division Properties of Inequality (Case 2)

Words When you multiply or divide each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed for the inequality to remain true.

Numbers $-4 < 6$ $4 > -6$

$$-2 \cdot (-4) > -2 \cdot 6 \quad \frac{4}{-2} < \frac{-6}{-2}$$

$$8 > -12 \quad -2 < 3$$

Algebra If $a < b$ and c is negative, then

$$a \cdot c > b \cdot c \quad \text{and} \quad \frac{a}{c} > \frac{b}{c}$$

If $a > b$ and c is negative, then

$$a \cdot c < b \cdot c \quad \text{and} \quad \frac{a}{c} < \frac{b}{c}$$

These properties are also true for \leq and \geq .

Common Error

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol.

Only reverse the inequality symbol when you multiply or divide both sides by a negative number.

EXAMPLE 3 Solving an Inequality Using Multiplication

Solve $-\frac{3}{2}n \leq 6$. Graph the solution.

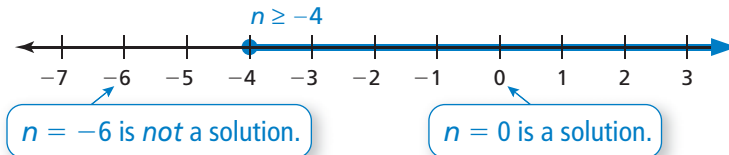
$$\begin{aligned} -\frac{3}{2}n &\leq 6 \\ -\frac{2}{3} \cdot \left(-\frac{3}{2}n\right) &\geq -\frac{2}{3} \cdot 6 \\ n &\geq -4 \end{aligned}$$

Write the inequality.

Use the Multiplication Property of Inequality.
Reverse the inequality symbol.

Simplify.

∴ The solution is $n \geq -4$.



On Your Own

Solve the inequality. Graph the solution.

7. $\frac{x}{-3} > -4$

8. $0.5 \leq -\frac{y}{2}$

9. $-12 \geq \frac{6}{5}m$

10. $-\frac{2}{5}h \leq -8$

EXAMPLE 4 Solving an Inequality Using Division

Solve $-3z > -4.5$. Graph the solution.

$$\begin{aligned} -3z &> -4.5 \\ \frac{-3z}{-3} &< \frac{-4.5}{-3} \\ z &< 1.5 \end{aligned}$$

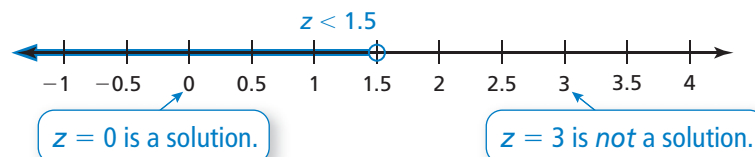
Write the inequality.

Use the Division Property of Inequality.
Reverse the inequality symbol.

Simplify.

Undo the multiplication.

∴ The solution is $z < 1.5$.



On Your Own

Solve the inequality. Graph the solution.

11. $-5z < 35$

12. $-2a > -9$

13. $-1.5 < 3n$

14. $-4.2 \geq -0.7w$

Now You're Ready
Exercises 27–35

11.3 Exercises

Vocabulary and Concept Check

- 1. WRITING** Explain how to solve $\frac{x}{3} < -2$.
- 2. PRECISION** Explain how solving $4x < -16$ is different from solving $-4x < 16$.
- 3. OPEN-ENDED** Write an inequality that you can solve using the Division Property of Inequality where the direction of the inequality symbol must be reversed.

Practice and Problem Solving

Use a table to solve the inequality.

- | | | |
|--------------------------|---------------------------------|-----------------------------------|
| 4. $2x < 2$ | 5. $-3x \leq 3$ | 6. $-6x > 18$ |
| 7. $\frac{x}{-5} \geq 7$ | 8. $\frac{x}{-1} > \frac{2}{5}$ | 9. $\frac{x}{3} \leq \frac{1}{2}$ |

Solve the inequality. Graph the solution.

- | | | |
|----------------------------|-----------------------------|----------------------------|
| 10. $2n > 20$ | 11. $\frac{c}{9} \leq -4$ | 12. $2.2m < 11$ |
| 13. $-16 > x \div 2$ | 14. $\frac{1}{6}w \geq 2.5$ | 15. $7 < 3.5k$ |
| 16. $3x \leq -\frac{5}{4}$ | 17. $4.2y \leq -12.6$ | 18. $11.3 > \frac{b}{4.3}$ |

19. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

X

$$\frac{x}{3} < -9$$
$$3 \cdot \frac{x}{3} > 3 \cdot (-9)$$
$$x > -27$$

Write the word sentence as an inequality. Then solve the inequality.

20. The quotient of a number and 4 is at most 5.
21. A number divided by 7 is less than -3 .
22. Six times a number is at least -24 .
23. The product of -2 and a number is greater than 30.
24. **SMART PHONE** You earn \$9.20 per hour at your summer job. Write and solve an inequality that represents the number of hours you need to work in order to buy a smart phone that costs \$299.



25. **AVOCADOS** You have \$9.60 to buy avocados for a guacamole recipe. Avocados cost \$2.40 each.

- Write and solve an inequality that represents the number of avocados you can buy.
- Are there infinitely many solutions in this context? Explain.



26. **SCIENCE PROJECT** Students in a science class are divided into 6 equal groups with at least 4 students in each group for a project. Write and solve an inequality that represents the number of students in the class.

Solve the inequality. Graph the solution.

3 4 27. $-5n \leq 15$

28. $-7w > 49$

29. $-\frac{1}{3}h \geq 8$

30. $-9 < -\frac{1}{5}x$

31. $-3y < -14$

32. $-2d \geq 26$

33. $4.5 > -\frac{m}{6}$

34. $\frac{k}{-0.25} \leq 36$

35. $-2.4 > \frac{b}{-2.5}$

36. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

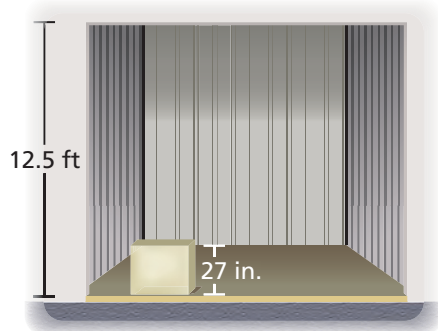
37. **TEMPERATURE** It is currently 0°C outside. The temperature is dropping 2.5°C every hour. Write and solve an inequality that represents the number of hours that must pass for the temperature to drop below -20°C .

X

$$-3m \geq 9$$

$$\frac{-3m}{-3} \geq \frac{9}{-3}$$

$$m \geq -3$$

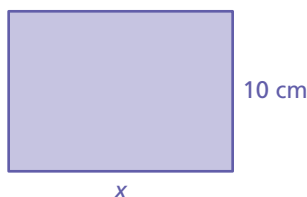


38. **STORAGE** You are moving some of your belongings into a storage facility.

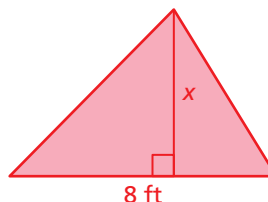
- Write and solve an inequality that represents the number of boxes that you can stack vertically in the storage unit.
- Can you stack 6 boxes vertically in the storage unit? Explain.

Write and solve an inequality that represents x .

39. Area $\geq 120 \text{ cm}^2$



40. Area $< 20 \text{ ft}^2$



41. **AMUSEMENT PARK** You and four friends are planning a visit to an amusement park. You want to keep the cost below \$100 per person. Write and solve an inequality that represents the total cost of visiting the amusement park.
42. **LOGIC** When you multiply or divide each side of an inequality by the same negative number, you must reverse the direction of the inequality symbol. Explain why.
43. **PROJECT** Choose two novels to research.
- Use the Internet or a magazine to complete the table.
 - Use the table to find and compare the average number of copies sold per month for each novel. Which novel do you consider to be the most successful? Explain.
 - Assume each novel continues to sell at the average rate. Write and solve an inequality that represents the number of months it will take for the total number of copies sold to exceed twice the current number sold.



Author	Name of Novel	Release Date	Current Number of Copies Sold
1.			
2.			

Number Sense Describe all numbers that satisfy *both* inequalities. Include a graph with your description.

44. $4m > -4$ and $3m < 15$

45. $\frac{n}{3} \geq -4$ and $\frac{n}{-5} \geq 1$

46. $2x \geq -6$ and $2x \geq 6$

47. $-\frac{1}{2}s > -7$ and $\frac{1}{3}s < 12$



Fair Game Review

What you learned in previous grades & lessons

Solve the equation. Check your solution. (Section 1.2)

48. $-2w + 4 = -12$

49. $\frac{v}{5} - 6 = 3$

50. $3(x - 1) = 18$

51. $\frac{m + 200}{4} = 51$

52. **MULTIPLE CHOICE** What is the value of $\frac{2}{3} + \left(-\frac{5}{7}\right)$? (Skills Review Handbook)

(A) $-\frac{3}{4}$

(B) $-\frac{1}{21}$

(C) $\frac{7}{10}$

(D) $1\frac{8}{21}$

11.4 Solving Two-Step Inequalities

Essential Question How can you use an inequality to describe the dimensions of a figure?

1 ACTIVITY: Areas and Perimeters of Figures

Work with a partner.

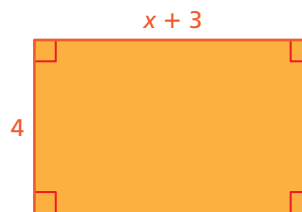
- Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.
 - Write four values of the variable that satisfy the inequality you chose.
- a. You want to find the values of x so that the area of the rectangle is more than 22 square units.

$$4x + 12 > 22$$

$$4x + 3 > 22$$

$$4x + 12 \geq 22$$

$$2x + 14 > 22$$



- b. You want to find the values of x so that the perimeter of the rectangle is greater than or equal to 28 units.

$$x + 7 \geq 28$$

$$4x + 12 \geq 28$$

$$2x + 14 \geq 28$$

$$2x + 14 \leq 28$$

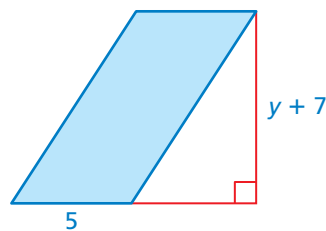
- c. You want to find the values of y so that the area of the parallelogram is fewer than 41 square units.

$$5y + 7 < 41$$

$$5y + 35 < 41$$

$$5y + 7 \leq 41$$

$$5y + 35 \leq 41$$



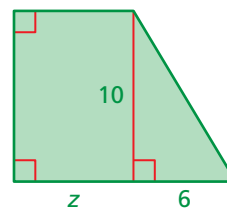
- d. You want to find the values of z so that the area of the trapezoid is at most 100 square units.

$$5z + 30 \leq 100$$

$$10z + 30 \leq 100$$

$$5z + 30 < 100$$

$$10z + 30 < 100$$



Inequalities

In this lesson, you will

- solve multi-step inequalities.
- solve real-life problems.

Learning Standard
7.EE.4b

2 ACTIVITY: Volumes of Rectangular Prisms

Work with a partner.

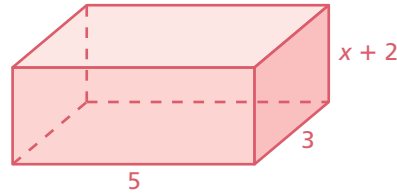
- Use the given condition to choose the inequality that you can use to find the possible values of the variable. Justify your answer.
- Write four values of the variable that satisfy the inequality you chose.

Math Practice 6

State the Meaning of Symbols

What inequality symbols do the phrases *at least* and *no more than* represent? Explain.

- a. You want to find the values of x so that the volume of the rectangular prism is at least 50 cubic units.



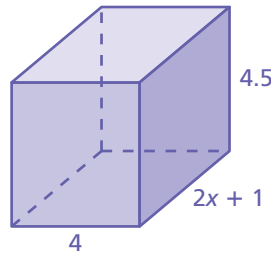
$15x + 30 > 50$

$x + 10 \geq 50$

$15x + 30 \geq 50$

$15x + 2 \geq 50$

- b. You want to find the values of x so that the volume of the rectangular prism is no more than 36 cubic units.



$8x + 4 < 36$

$36x + 18 < 36$

$2x + 9.5 \leq 36$

$36x + 18 \leq 36$

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use an inequality to describe the dimensions of a figure?
4. Use what you know about solving equations and inequalities to describe how you can solve a two-step inequality. Give an example to support your explanation.

Practice

Use what you learned about solving two-step inequalities to complete Exercises 3 and 4 on page 490.

You can solve two-step inequalities in the same way you solve two-step equations.

EXAMPLE 1 Solving Two-Step Inequalities

a. Solve $5x - 4 \geq 11$. Graph the solution.

Step 1: Undo the subtraction.

$$5x - 4 \geq 11$$

$$\xrightarrow{+4 \quad +4}$$

$$5x \geq 15$$

Write the inequality.

Addition Property of Inequality

Simplify.

Step 2: Undo the multiplication.

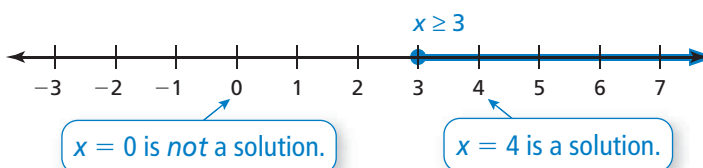
$$\xrightarrow{\frac{5x}{5} \geq \frac{15}{5}}$$

$$x \geq 3$$

Division Property of Inequality

Simplify.

∴ The solution is $x \geq 3$.



b. Solve $\frac{b}{-3} + 4 < 13$. Graph the solution.

Step 1: Undo the addition.

$$\frac{b}{-3} + 4 < 13$$

$$\xrightarrow{-4 \quad -4}$$

$$\frac{b}{-3} < 9$$

Write the inequality.

Subtraction Property of Inequality

Simplify.

Step 2: Undo the division.

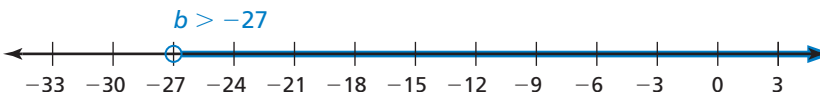
$$\xrightarrow{-3 \cdot \frac{b}{-3} > -3 \cdot 9}$$

$$b > -27$$

Use the Multiplication Property of Inequality.
Reverse the inequality symbol.

Simplify.

∴ The solution is $b > -27$.



On Your Own

Solve the inequality. Graph the solution.

1. $6y - 7 > 5$

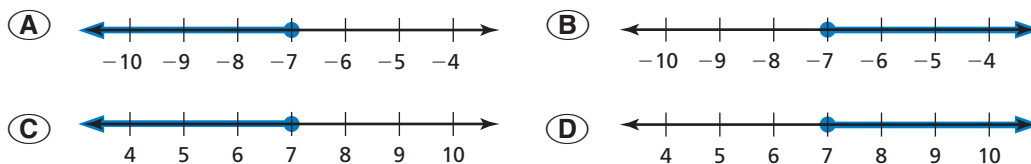
2. $4 - 3d \geq 19$

3. $\frac{w}{-4} + 8 > 9$

Now You're Ready
Exercises 5–10

EXAMPLE 2 Graphing an Inequality

Which graph represents the solution of $-7(x + 3) \leq 28$?



$$-7(x + 3) \leq 28$$

$$-7x - 21 \leq 28$$

Step 1: Undo the subtraction.

$$\xrightarrow{+21 \quad +21}$$

$$-7x \leq 49$$

Step 2: Undo the multiplication.

$$\xrightarrow{\frac{-7x}{-7} \geq \frac{49}{-7}}$$

$$x \geq -7$$

Write the inequality.

Distributive Property

Addition Property of Inequality

Simplify.

Use the Division Property of Inequality.
Reverse the inequality symbol.

Simplify.

∴ The correct answer is (B).

EXAMPLE 3 Real-Life Application

Progress Report	
Month	Pounds Lost
1	12
2	9
3	5
4	8

A contestant in a weight-loss competition wants to lose an average of at least 8 pounds per month during a 5-month period. How many pounds must the contestant lose in the fifth month to meet the goal?

Write and solve an inequality. Let x be the number of pounds lost in the fifth month.

$$\frac{12 + 9 + 5 + 8 + x}{5} \geq 8$$

The phrase *at least* means *greater than or equal to*.

$$\frac{34 + x}{5} \geq 8$$

Simplify.

$$5 \cdot \frac{34 + x}{5} \geq 5 \cdot 8$$

Multiplication Property of Inequality

$$34 + x \geq 40$$

Simplify.

$$x \geq 6$$

Subtract 34 from each side.

∴ So, the contestant must lose at least 6 pounds to meet the goal.

Remember

In Example 3, the average is equal to the sum of the pounds lost divided by the number of months.

On Your Own

Solve the inequality. Graph the solution.

4. $2(k - 5) < 6$

5. $-4(n - 10) < 32$

6. $-3 \leq 0.5(8 + y)$

7. **WHAT IF?** In Example 3, the contestant wants to lose an average of at least 9 pounds per month. How many pounds must the contestant lose in the fifth month to meet the goal?

Now You're Ready
Exercises 12–17

11.4 Exercises

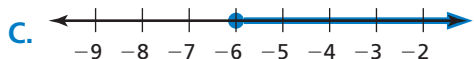
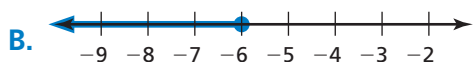
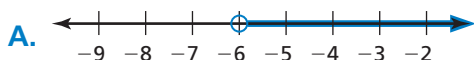
Vocabulary and Concept Check

- WRITING** Compare and contrast solving two-step inequalities and solving two-step equations.
- OPEN-ENDED** Describe how to solve the inequality $3(a + 5) < 9$.

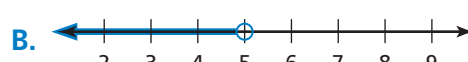
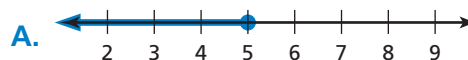
Practice and Problem Solving

Match the inequality with its graph.

3. $\frac{t}{3} - 1 \geq -3$



4. $5x + 7 \leq 32$



Solve the inequality. Graph the solution.

1 5. $8y - 5 < 3$

6. $3p + 2 \geq -10$

7. $2 > 8 - \frac{4}{3}h$

8. $-2 > \frac{m}{6} - 7$

9. $-1.2b - 5.3 \geq 1.9$

10. $-1.3 \geq 2.9 - 0.6r$

11. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

X $\frac{x}{3} + 4 < 6$
 $x + 4 < 18$
 $x < 14$

Solve the inequality. Graph the solution.

2 12. $5(g + 4) > 15$

13. $4(w - 6) \leq -12$

14. $-8 \leq \frac{2}{5}(k - 2)$

15. $-\frac{1}{4}(d + 1) < 2$

16. $7.2 > 0.9(n + 8.6)$

17. $20 \geq -3.2(c - 4.3)$



18. **UNICYCLE** The first jump in a unicycle high-jump contest is shown. The bar is raised 2 centimeters after each jump. Solve the inequality $2n + 10 \geq 26$ to find the number of additional jumps needed to meet or exceed the goal of clearing a height of 26 centimeters.

Solve the inequality. Graph the solution.

19. $9x - 4x + 4 \geq 36 - 12$

20. $3d - 7d + 2.8 < 5.8 - 27$

21. **SCUBA DIVER** A scuba diver is at an elevation of -38 feet. The diver starts moving at a rate of -12 feet per minute. Write and solve an inequality that represents how long it will take the diver to reach an elevation deeper than -200 feet.

22. **KILLER WHALES** A killer whale has eaten 75 pounds of fish today. It needs to eat at least 140 pounds of fish each day.

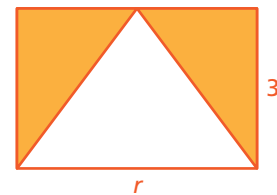
- a. A bucket holds 15 pounds of fish. Write and solve an inequality that represents how many more buckets of fish the whale needs to eat.
- b. Should the whale eat *four* or *five* more buckets of fish? Explain.



23. **REASONING** A student theater charges \$9.50 per ticket.

- a. The theater has already sold 70 tickets. Write and solve an inequality that represents how many more tickets the theater needs to sell to earn at least \$1000.
- b. The theater increases the ticket price by \$1. Without solving an inequality, describe how this affects the total number of tickets needed to earn at least \$1000.

24. **Problem Solving** For what values of r will the area of the shaded region be greater than or equal to 12 square units?



Fair Game Review what you learned in previous grades & lessons

Find the missing values in the ratio table. Then write the equivalent ratios.

(Skills Review Handbook)

25.

Flutes	7		28
Clarinets	4	12	

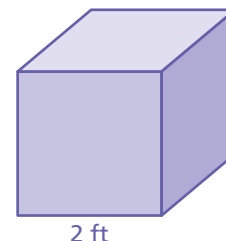
26.

Boys	6	3	
Girls	10		50

27. **MULTIPLE CHOICE** What is the volume of the cube?

(Skills Review Handbook)

- (A) 8 ft^3
- (B) 16 ft^3
- (C) 24 ft^3
- (D) 32 ft^3



11.3–11.4 Quiz

Solve the inequality. Graph the solution. (Section 11.3 and Section 11.4)

1. $3p \leq 18$

2. $2x > -\frac{3}{5}$

3. $\frac{r}{3} \geq -5$

4. $-\frac{z}{8} < 1.5$

5. $3n + 2 \leq 11$

6. $-2 < 5 - \frac{k}{2}$

7. $1.3m - 3.8 < -1.2$

8. $4.8 \geq 0.3(12 - y)$

Write the word sentence as an inequality. Then solve the inequality. (Section 11.3)

9. The quotient of a number and 5 is less than 4.

10. Six times a number is at least -14 .

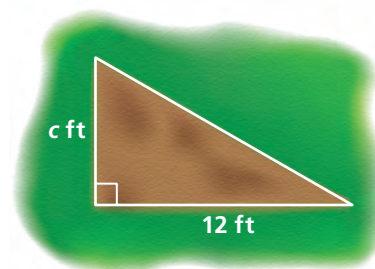


11. **PEPPERS** You have \$18 to buy peppers. Peppers cost \$1.50 each. Write and solve an inequality that represents the number of peppers you can buy. (Section 11.3)

12. **MOVIES** You have a gift card worth \$90. You want to buy several movies that cost \$12 each. Write and solve an inequality that represents the number of movies you can buy and still have at least \$30 on the gift card. (Section 11.4)

13. **ORANGES** Your class sells boxes of oranges to raise \$500 for a field trip. You earn \$6.25 for each box of oranges sold. Write and solve an inequality that represents the number of boxes your class must sell to meet or exceed the fundraising goal. (Section 11.3)

14. **FENCE** You want to put up a fence that encloses a triangular region with an area greater than or equal to 60 square feet. What is the least possible value of c ? Explain. (Section 11.3)



11 Chapter Review

Review Key Vocabulary

inequality, p. 466
solution of an inequality, p. 466

solution set, p. 466
graph of an inequality, p. 467

Review Examples and Exercises

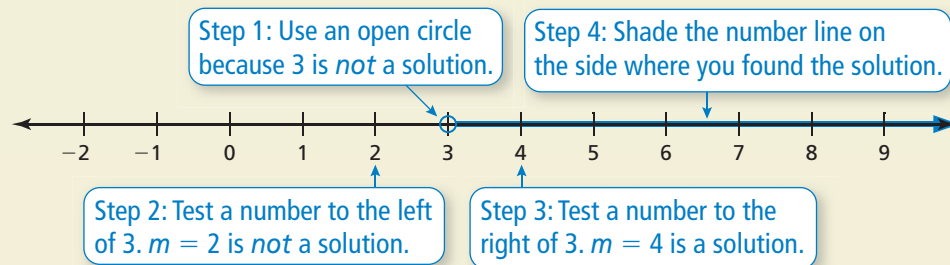
11.1 Writing and Graphing Inequalities (pp. 464–469)

- a. Six plus a number x is at most $-\frac{1}{4}$. Write this word sentence as an inequality.

$$\underbrace{\text{Six plus a number } x}_{6 + x} \text{ is at most } \underbrace{-\frac{1}{4}}_{\leq -\frac{1}{4}}.$$

••• An inequality is $6 + x \leq -\frac{1}{4}$.

- b. Graph $m > 3$.



Exercises

Write the word sentence as an inequality.

1. A number w is greater than -3 .
2. A number y minus $\frac{1}{2}$ is no more than $-\frac{3}{2}$.

Tell whether the given value is a solution of the inequality.

3. $5 + j > 8; j = 7$
4. $6 \div n \leq -5; n = -3$

Graph the inequality on a number line.

5. $q > -1.3$
6. $s < 1\frac{3}{4}$
7. **BUMPER CARS** You must be at least 42 inches tall to ride the bumper cars at an amusement park. Write an inequality that represents this situation.

11.2 Solving Inequalities Using Addition or Subtraction (pp. 470–475)

Solve $-5 < m - 3$. Graph the solution.

Undo the subtraction.

$$-5 < m - 3$$

$$\xrightarrow{+3} \quad \quad \quad +3$$

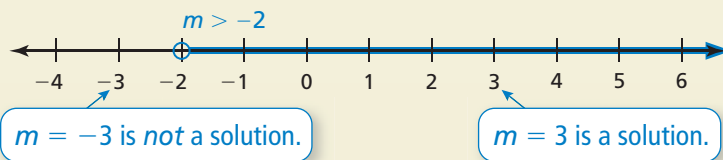
$$-2 < m$$

Write the inequality.

Addition Property of Inequality

Simplify.

••• The solution is $m > -2$.



Exercises

Solve the inequality. Graph the solution.

8. $d + 12 < 19$

9. $t - 4 \leq -14$

10. $-8 \leq z + 6.4$

11.3 Solving Inequalities Using Multiplication or Division (pp. 478–485)

Solve $\frac{c}{-3} \geq -2$. Graph the solution.

Undo the division.

$$\frac{c}{-3} \geq -2$$

$$\xrightarrow{-3 \cdot} \quad \frac{c}{-3} \leq -3 \cdot (-2)$$

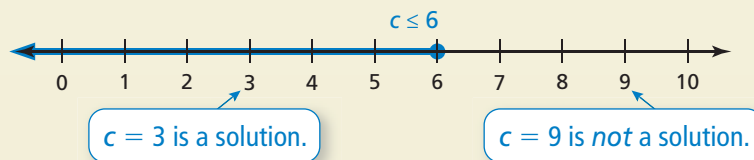
$$c \leq 6$$

Write the inequality.

Use the Multiplication Property of Inequality.
Reverse the inequality symbol.

Simplify.

••• The solution is $c \leq 6$.



Exercises

Solve the inequality. Graph the solution.

11. $6q < -18$

12. $-\frac{r}{3} \leq 6$

13. $-4 > -\frac{4}{3}s$

11.4 Solving Two-Step Inequalities (pp. 486–491)

a. Solve $6x - 8 \leq 10$. Graph the solution.

Step 1: Undo the subtraction.

$$6x - 8 \leq 10$$

$$\xrightarrow{+8 \quad +8} 6x \leq 18$$

$$6x \leq 18$$

Write the inequality.

Addition Property of Inequality

Simplify.

Step 2: Undo the multiplication.

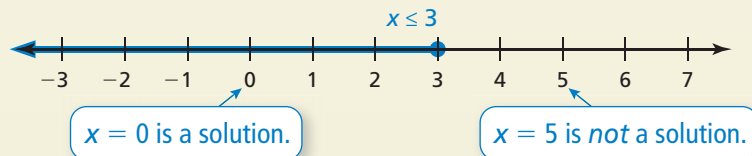
$$\xrightarrow{\frac{6x}{6} \leq \frac{18}{6}} \frac{6x}{6} \leq \frac{18}{6}$$

$$x \leq 3$$

Division Property of Inequality

Simplify.

••• The solution is $x \leq 3$.



b. Solve $\frac{q}{-4} + 7 < 11$. Graph the solution.

Step 1: Undo the addition.

$$\frac{q}{-4} + 7 < 11$$

$$\xrightarrow{-7 \quad -7} \frac{q}{-4} < 4$$

$$\frac{q}{-4} < 4$$

Write the inequality.

Subtraction Property of Inequality

Simplify.

Step 2: Undo the division.

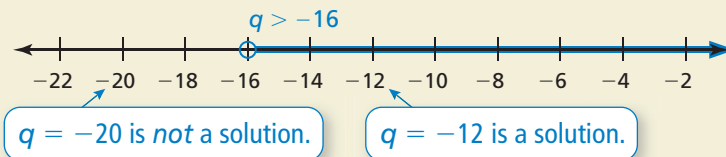
$$\xrightarrow{-4 \cdot \frac{q}{-4} > -4 \cdot 4} -4 \cdot \frac{q}{-4} > -4 \cdot 4$$

$$q > -16$$

Use the Multiplication Property of Inequality.
Reverse the inequality symbol.

Simplify.

••• The solution is $q > -16$.



Exercises

Solve the inequality. Graph the solution.

14. $3x + 4 > 16$

15. $\frac{z}{-2} - 6 \leq -2$

16. $-2t - 5 < 9$

17. $7(q + 2) < -77$

18. $-\frac{1}{3}(p + 9) \leq 4$

19. $1.2(j + 3.5) \geq 4.8$

11 Chapter Test

Write the word sentence as an inequality.

1. A number k plus 19.5 is less than or equal to 40.
2. A number q multiplied by $\frac{1}{4}$ is greater than -16 .

Tell whether the given value is a solution of the inequality.

3. $n - 3 \leq 4$; $n = 7$
4. $-\frac{3}{7}m < 1$; $m = -7$
5. $-4c \geq 7$; $c = -2$
6. $-2.4m > -6.8$; $m = -3$

Solve the inequality. Graph the solution.

7. $w + 4 \leq 3$
8. $x - 4 > -6$
9. $-\frac{2}{9} + y \leq \frac{5}{9}$
10. $-6z \geq 36$
11. $-5.2 \geq \frac{p}{4}$
12. $4k - 8 \geq 20$
13. $\frac{4}{7} - b \geq -\frac{1}{7}$
14. $-0.6 > -0.3(d + 6)$

15. **SUGAR-FREE GUMBALLS** You have \$2.50. Each sugar-free gumball in a gumball machine costs \$0.25. Write and solve an inequality that represents the number of gumballs you can buy.

16. **PARTY** You can spend no more than \$100 on a party you are hosting. The cost per guest is \$8.
 - a. Write and solve an inequality that represents the number of guests you can invite to the party.
 - b. What is the greatest number of guests that you can invite to the party? Explain your reasoning.



17. **BASEBALL CARDS** You have \$30 to buy baseball cards. Each pack of cards costs \$5. Write and solve an inequality that represents the number of packs of baseball cards you can buy and still have at least \$10 left.

11 Standards Assessment

1. What is the value of the expression below when $x = -5$, $y = 3$, and $z = -1$? (7.NS.3)

$$\frac{x^2 - 3y}{z}$$

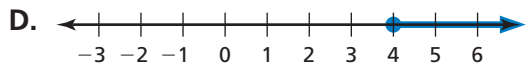
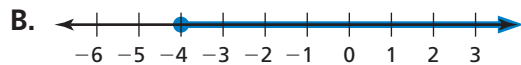
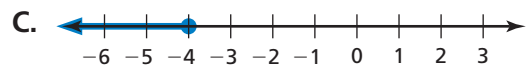
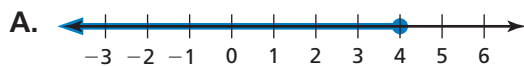
- A. -34
B. -16
C. 16
D. 34
2. What is the value of the expression below? (7.NS.2a)

$$-\frac{3}{8} \cdot \frac{2}{5}$$

- F. $-\frac{20}{3}$
G. $-\frac{16}{15}$
H. $-\frac{15}{16}$
I. $-\frac{3}{20}$

3. Which graph represents the inequality below? (7.EE.4b)

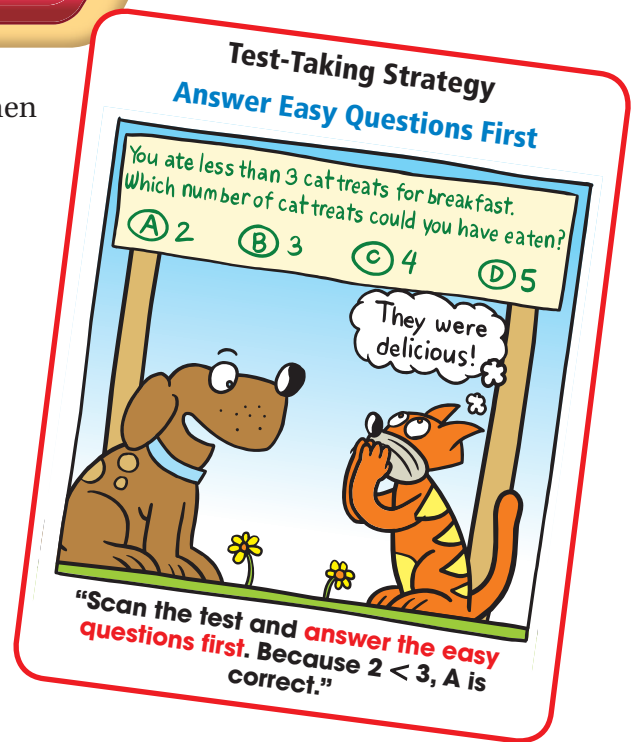
$$\frac{x}{-4} - 8 \geq -9$$



4. Which value of p makes the equation below true? (7.EE.4a)

$$5(p + 6) = 25$$

- F. -1
G. $3\frac{4}{5}$
H. 11
I. 14



5. You set up the lemonade stand. Your profit is equal to your revenue from lemonade sales minus your cost to operate the stand. Your cost is \$8. How many cups of lemonade must you sell to earn a profit of \$30?

(7.EE.4a)



- A. 4
B. 44
C. 60
D. 76
6. Which value is a solution of the inequality below? (7.EE.4b)

$$3 - 2y < 7$$

- F. -6
G. -3
H. -2
I. -1

7. What value of y makes the equation below true? (7.EE.4a)



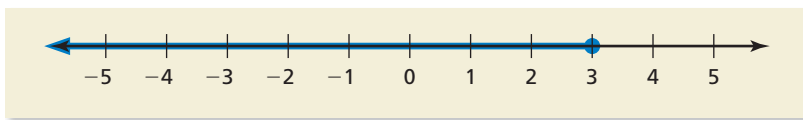
$$12 - 3y = -6$$

8. What is the mean distance of the four points from -3 ? (7.NS.3)



- A. $-\frac{1}{2}$
B. $2\frac{1}{2}$
C. 3
D. $7\frac{1}{8}$

9. Martin graphed the solution of the inequality $-4x + 18 > 6$ in the box below.



What should Martin do to correct the error that he made? (7.EE.4b)

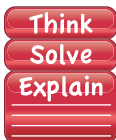
- F. Use an open circle at 3 and shade to the left of 3.
- G. Use an open circle at 3 and shade to the right of 3.
- H. Use a closed circle and shade to the right of 3.
- I. Use an open circle and shade to the left of -3 .

10. What is the value of the expression below? (7.NS.1c)



$$\frac{5}{12} - \frac{7}{8}$$

11. You are selling T-shirts to raise money for a charity. You sell the T-shirts for \$10 each. (7.EE.4b)



Part A You have already sold 2 T-shirts. How many more T-shirts must you sell to raise at least \$500? Explain.

Part B Your friend is raising money for the same charity. He sells the T-shirts for \$8 each. What is the total number of T-shirts he must sell to raise at least \$500? Explain.

Part C Who has to sell more T-shirts in total? How many more? Explain.

12. Which expression is equivalent to the expression below? (7.NS.3)

$$-\frac{2}{3} - \left(-\frac{4}{9}\right)$$

A. $-\frac{1}{3} + \frac{1}{9}$

C. $-\frac{1}{3} - \frac{7}{9}$

B. $-\frac{2}{3} \times \left(-\frac{1}{3}\right)$

D. $\frac{3}{2} \div \left(-\frac{1}{3}\right)$