6 Functions

"Here's a math anagram."

"Here's another one."

"It is my treat-converter function machine. However many cat treats I input, the machine outputs TWICE that many dog biscuits. Isn't that cool?"

What You Learned Before

"Do you think the stripes in this shirt make me look too linear?"

Identifying Patterns

Example 1 Find the missing value in the table.

x	y
30	0
40	10
50	20
60	

Each y-value is 30 less than the x-value.

So, the missing value is $60 - 30 = 30$.

Try It Yourself

Find the missing value in the table.

1.

x	y
5	10
7	14
10	20
40	

2.

x	y
0.5	1
1.5	2
3	3.5
9.5	

3.

x	y
15	5
30	10
45	15
60	

Evaluating Algebraic Expressions

Example 2 Evaluate $2x - 12$ when $x = 5$.

$$2x - 12 = 2(5) - 12 \qquad \text{Substitute 5 for } x.$$
$$= 10 - 12 \qquad \text{Using order of operations, multiply 2 and 5.}$$
$$= 10 + (-12) \qquad \text{Add the opposite of 12.}$$
$$= -2 \qquad \text{Add.}$$

Try It Yourself

Evaluate the expression when $y = 4$.

4. $-4y + 2$
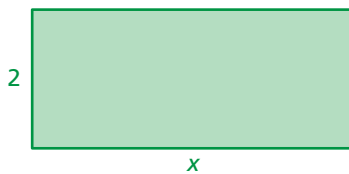
5. $\dfrac{y}{2} - 8$

6. $-10 - 6y$

6.1 Relations and Functions

Essential Question How can you use a mapping diagram to show the relationship between two data sets?
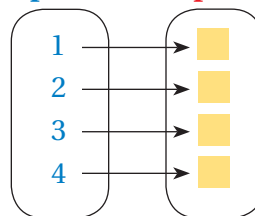
1 ACTIVITY: Constructing Mapping Diagrams

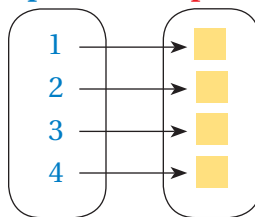**Work with a partner. Copy and complete the mapping diagram.**
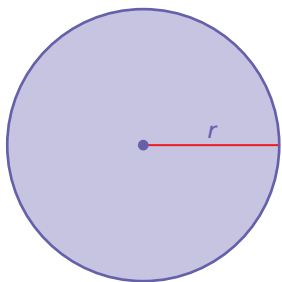
a. Area A
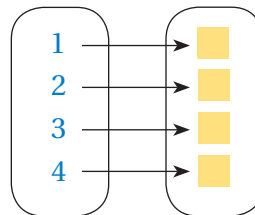

Input, x	Output, A
1	☐
2	☐
3	☐
4	☐

b. Perimeter P

Input, x	Output, P
1	☐
2	☐
3	☐
4	☐

c. Circumference C

Input, r	Output, C
1	☐
2	☐
3	☐
4	☐

Functions

In this lesson, you will
- define relations and functions.
- determine whether relations are functions.
- describe patterns in mapping diagrams.

d. Volume V

| Input, h	Output, V
1	☐
2	☐
3	☐
4	☐

2 ACTIVITY: Describing Situations

Math Practice

View as Components

What are the input values? Do any of the input values point to more than one output value? How does this help you describe a possible situation?

Work with a partner. How many outputs are assigned to each input? Describe a possible situation for each mapping diagram.
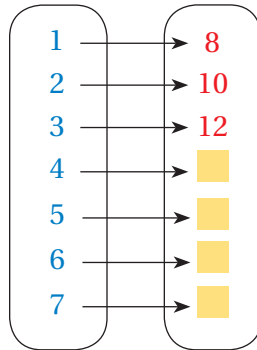
a. **Input, x Output, y**

Input, x	Output, y
12	6
23	13
30	15
48	20

b. **Input, x Output, y**
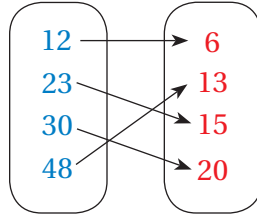
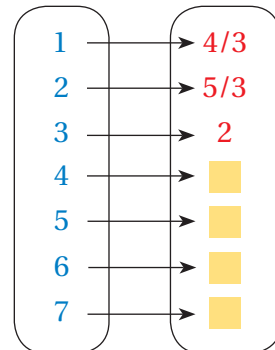| Input, x	Output, y
10	0
11	1
12	2
13	3
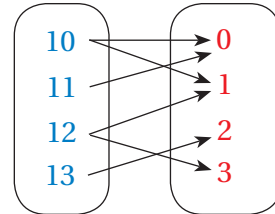
3 ACTIVITY: Interpreting Mapping Diagrams

Work with a partner. Describe the pattern in the mapping diagram. Copy and complete the diagram.
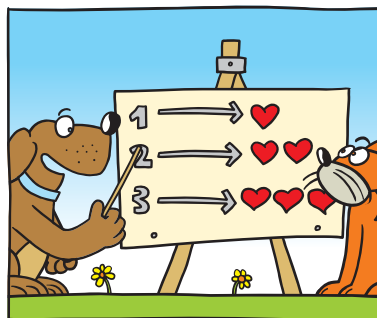
a. **Input, t Output, M**

Input, t	Output, M
1	8
2	10
3	12
4	☐
5	☐
6	☐
7	☐

b. **Input, x Output, A**

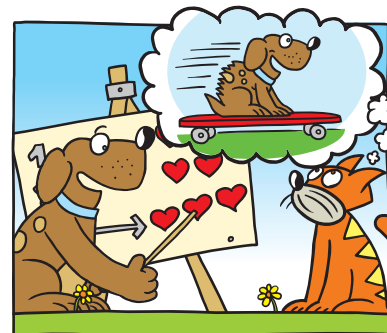| Input, x	Output, A
1	4/3
2	5/3
3	2
4	☐
5	☐
6	☐
7	☐

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use a mapping diagram to show the relationship between two data sets?

"I made a mapping diagram."

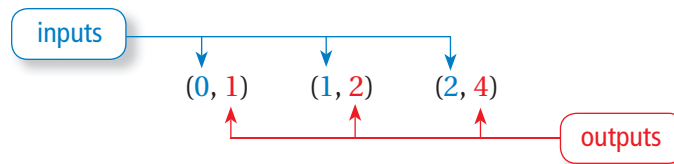"It shows how I feel about my skateboard with each passing day."

Practice

Use what you learned about mapping diagrams to complete Exercises 3–5 on page 246.

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Key Vocabulary
input, *p. 244*
output, *p. 244*
relation, *p. 244*
mapping diagram, *p. 244*
function, *p. 245*

Ordered pairs can be used to show **inputs** and **outputs**.

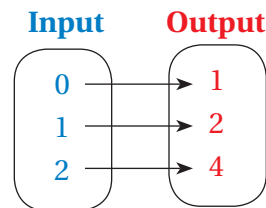inputs

$(0, 1)$ $(1, 2)$ $(2, 4)$

outputs

Key Idea

Relations and Mapping Diagrams

A **relation** pairs inputs with outputs. A relation can be represented by ordered pairs or a **mapping diagram**.
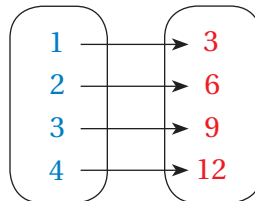
Ordered Pairs
$(0, 1)$
$(1, 2)$
$(2, 4)$

Mapping Diagram

Input	Output
0	1
1	2
2	4

EXAMPLE 1   Listing Ordered Pairs of a Relation

List the ordered pairs shown in the mapping diagram.

a.
Input	Output
1	3
2	6
3	9
4	12

b.
Input	Output
0	1
2	0
4	−2
	−3

∴ The ordered pairs are $(1, 3)$, $(2, 6)$, $(3, 9)$, and $(4, 12)$.
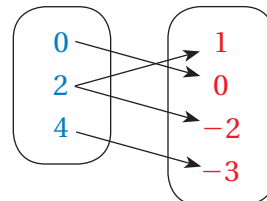
∴ The ordered pairs are $(0, 0)$, $(2, 1)$, $(2, -2)$, and $(4, -3)$.

On Your Own

Now You're Ready
Exercises 6–8

List the ordered pairs shown in the mapping diagram.

1.
Input	Output
0	12
2	10
4	8
6	6
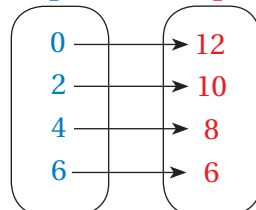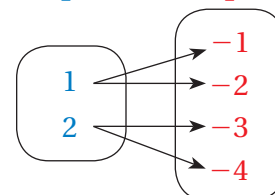
2.
Input	Output
	−1
1	−2
2	−3
	−4

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A relation that pairs each input with *exactly one* output is a **function**.

EXAMPLE **2** **Determining Whether Relations Are Functions**

Determine whether each relation is a function.

a. **Input**   **Output**

−9 → 0
−2 → 5
5 → 10
12
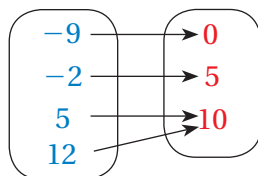
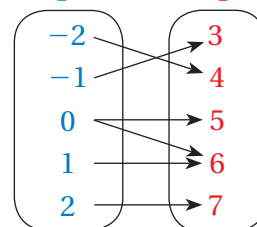∵ Each input has exactly one output. So, the relation is a function.

b. **Input**   **Output**

−2 → 3
−1 → 4
0 → 5
1 → 6
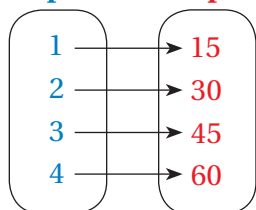2 → 7

∵ The input 0 has two outputs, 5 and 6. So, the relation is *not* a function.

EXAMPLE **3** **Describing a Mapping Diagram**

Input   **Output**

1 → 15
2 → 30
3 → 45
4 → 60

Consider the mapping diagram at the left.

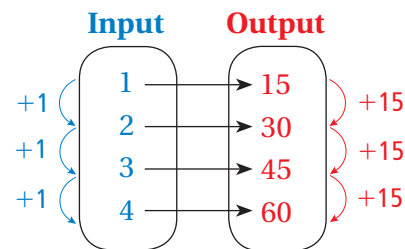a. **Determine whether the relation is a function.**

Each input has exactly one output.

∵ So, the relation is a function.

b. **Describe the pattern of inputs and outputs in the mapping diagram.**

Look at the relationship between the inputs and the outputs.

∵ As each input increases by 1, the output increases by 15.

Input   **Output**

+1 (1 → 15) +15
+1 (2 → 30) +15
+1 (3 → 45) +15
+1 (4 → 60) +15

⬤ **On Your Own**

Now You're Ready
Exercises 9–11 and 13–15

Determine whether the relation is a function.

3. **Input**   **Output**

$-\dfrac{1}{2}$ → 2
→ 4
$\dfrac{1}{3}$ → 7
→ 9

4. **Input**   **Output**

2 → 6
4 → 3
6 → 0
8 → −3
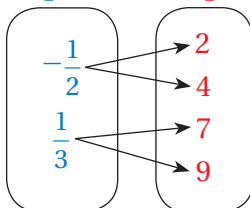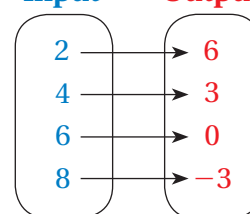
5. Describe the pattern of inputs and outputs in the mapping diagram in On Your Own 4.

✔ Vocabulary and Concept Check

1. **VOCABULARY** In an ordered pair, which number represents the input? the output?

2. **PRECISION** Describe how relations and functions are different.

Practice and Problem Solving

Describe the pattern in the mapping diagram. Copy and complete the diagram.

3.
Input	Output
1	4
2	8
3	12
4	☐
5	☐
6	☐

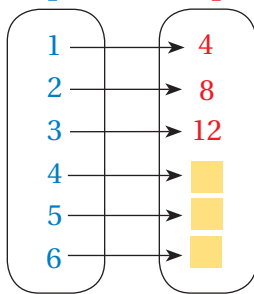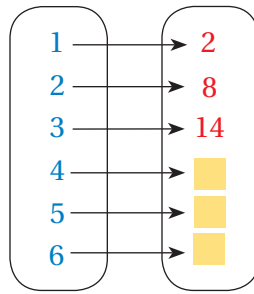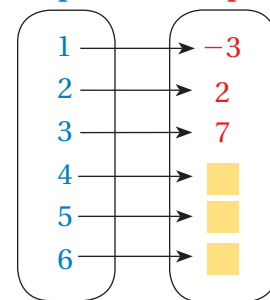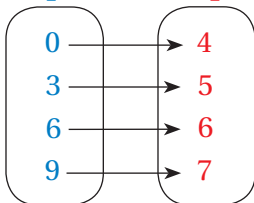4.
Input	Output
1	2
2	8
3	14
4	☐
5	☐
6	☐

5.
Input	Output
1	−3
2	2
3	7
4	☐
5	☐
6	☐

List the ordered pairs shown in the mapping diagram.

6.
Input	Output
0	4
3	5
6	6
9	7

7.
Input	Output
1	8
3	6
5	4
7	2

8.
Input	Output
6	
7	−5
8	−10
9	

Determine whether the relation is a function.

9.
Input	Output
−2	5
0	10
2	15
4	20

10.
Input	Output
0	−18
4	−9
8	0
12	9

11.
Input	Output
−3	
−2	7
−1	14
0	
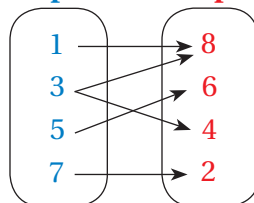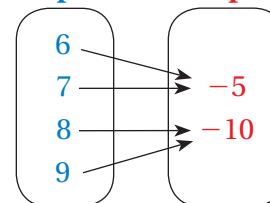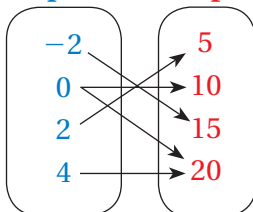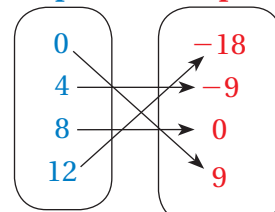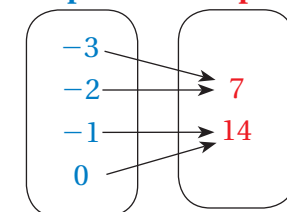
12. **ERROR ANALYSIS** Describe and correct the error in determining whether the relation is a function.

✗
Input	Output
4	5
	6
	7
	8

Each output is paired with exactly one input. So, the relation is a function.

Draw a mapping diagram for the graph. Then describe the pattern of inputs and outputs.

3 **13.**

14.

15.

16. SCUBA DIVING The normal pressure at sea level is one atmosphere of pressure (1 ATM). As you dive below sea level, the pressure increases by 1 ATM for each 10 meters of depth.

  a. Complete the mapping diagram.

  b. Is the relation a function? Explain.

  c. List the ordered pairs. Then plot the ordered pairs in a coordinate plane.

  d. Compare the mapping diagram and graph. Which do you prefer? Why?

  e. **RESEARCH** What are common depths for people who are just learning to scuba dive? What are common depths for experienced scuba divers?

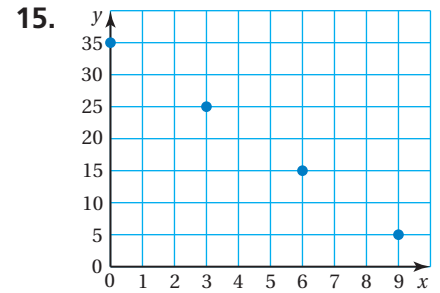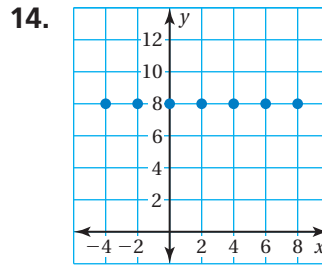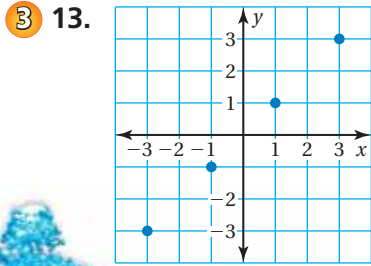| Input, Depth	Output, Pressure
0 m	1 ATM
10 m	2 ATM
20 m	▨
30 m	
40 m	▨
50 m	▨
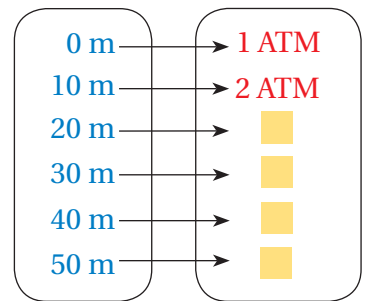
17. MOVIES A store sells previously viewed movies. The table shows the cost of buying 1, 2, 3, or 4 movies.

  a. Use the table to draw a mapping diagram.

  b. Is the relation a function? Explain.

  c. Describe the pattern. How does the cost per movie change as you buy more movies?

Movies	Cost
1	$10
2	$18
3	$24
4	$28

18. **Repeated Reasoning** The table shows the outputs for several inputs. Use two methods to find the output for an input of 200.

Input, x	0	1	2	3	4
Output, y	25	30	35	40	45

Fair Game Review What you learned in previous grades & lessons

The coordinates of a point and its image are given. Is the reflection in the x-axis or y-axis? *(Section 2.3)*

19. $(3, -3) \longrightarrow (-3, -3)$ **20.** $(-5, 1) \longrightarrow (-5, -1)$ **21.** $(-2, -4) \longrightarrow (-2, 4)$

22. MULTIPLE CHOICE Which word best describes two figures that have the same size and the same shape? *(Section 2.1)*

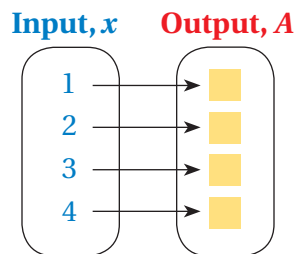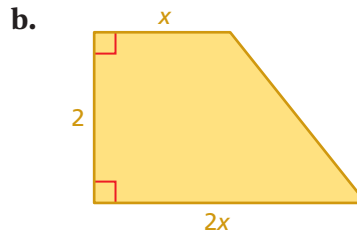 (A) congruent   (B) dilation   (C) parallel   (D) similar

Essential Question How can you represent a function in different ways?

1 ACTIVITY: Describing a Function

Work with a partner. Copy and complete the mapping diagram for the area of the figure. Then write an equation that describes the function.

a.

2 ———— 2

x

b.

x

2

$2x$

Input, x	Output, A
1	□
2	□
3	□
4	□

Input, x	Output, A
1	□
2	□
3	□
4	□
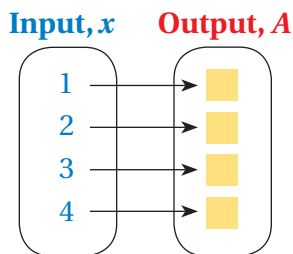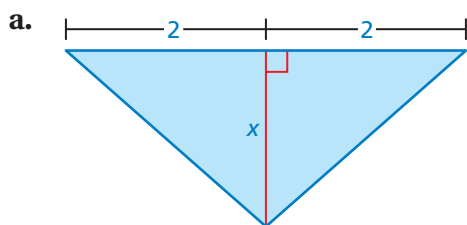
2 ACTIVITY: Using a Table

Work with a partner. Make a table that shows the pattern for the area, where the input is the figure number x and the output is the area A. Write an equation that describes the function. Then use your equation to find which figure has an area of 81 when the pattern continues.

□ 1 square unit
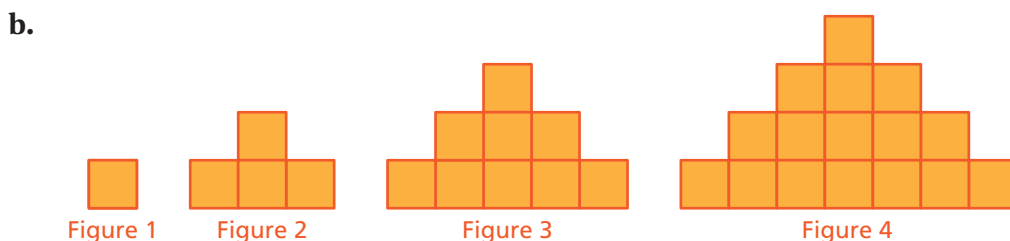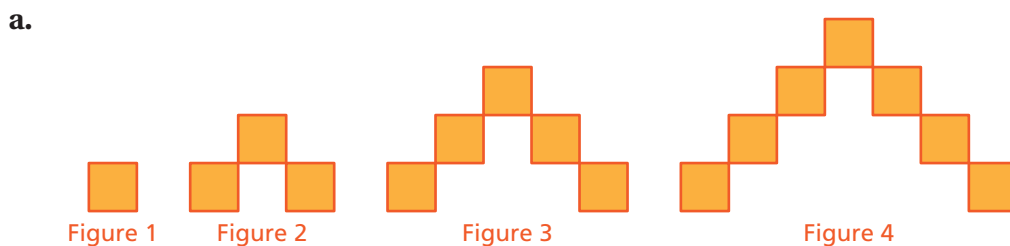
Functions

In this lesson, you will
- write function rules.
- use input-output tables to represent functions.
- use graphs to represent functions.

a.

Figure 1   Figure 2   Figure 3   Figure 4

b.

Figure 1   Figure 2   Figure 3   Figure 4

3 ACTIVITY: Using a Graph

Work with a partner. Graph the data. Use the graph to test the truth of each statement. If the statement is true, write an equation that shows how to obtain one measurement from the other measurement.

a. "You can find the horsepower of a race car engine if you know its volume in cubic inches."

Volume (cubic inches), x	200	350	350	500
Horsepower, y	375	650	250	600

b. "You can find the volume of a race car engine in cubic centimeters if you know its volume in cubic inches."

Volume (cubic inches), x	100	200	300
Volume (cubic centimeters), y	1640	3280	4920

4 ACTIVITY: Interpreting a Graph

Work with a partner. The table shows the average speeds of the winners of the Daytona 500. Graph the data. Can you use the graph to predict future winning speeds? Explain why or why not.

Year, x	2004	2005	2006	2007	2008	2009	2010	2011	2012
Speed (mi/h), y	156	135	143	149	153	133	137	130	140

What Is Your Answer?

5. IN YOUR OWN WORDS How can you represent a function in different ways?
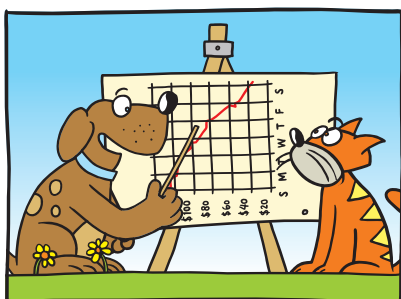
"I graphed our profits."
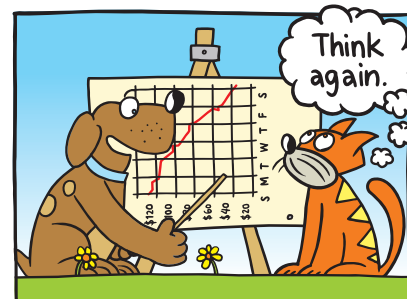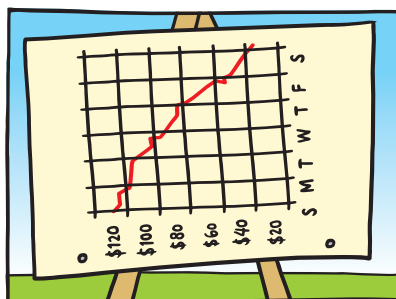
"And I am happy to say that they are going up every day!"

Think again.

Practice Use what you learned about representing functions to complete Exercises 4–6 on page 253.

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Key Vocabulary 🔊
function rule, *p. 250*

Remember

An independent variable represents a quantity that can change freely. A dependent variable *depends* on the independent variable.

🔑 Key Idea

Functions as Equations

A **function rule** is an equation that describes the relationship between inputs (independent variable) and outputs (dependent variable).

Input $\xrightarrow{-2}$ Function Rule: $y = 3x$ $\xrightarrow{-6}$ Output

EXAMPLE 1 — Writing Function Rules

a. **Write a function rule for "The output is five less than the input."**

Words The output is five less than the input.

Equation $y \qquad = \qquad x - 5$
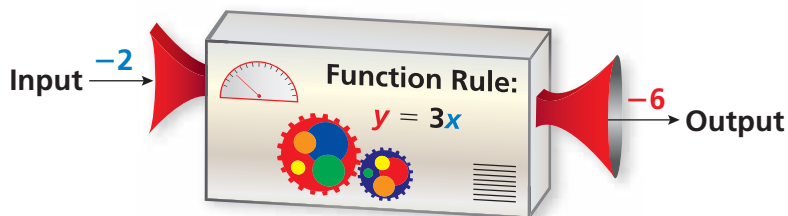
∴ A function rule is $y = x - 5$.

b. **Write a function rule for "The output is the square of the input."**

Words The output is the square of the input.

Equation $y \qquad = \qquad x^2$

∴ A function rule is $y = x^2$.

EXAMPLE 2 — Evaluating a Function

What is the value of $y = 2x + 5$ when $x = 3$?

$y = 2x + 5$ Write the equation.

$\quad = 2(3) + 5$ Substitute 3 for x.

$\quad = 11$ Simplify.

∴ When $x = 3$, $y = 11$.

🔴 On Your Own

Now You're Ready
Exercises 7–18

1. Write a function rule for "The output is one-fourth of the input."

Find the value of y when $x = 5$.
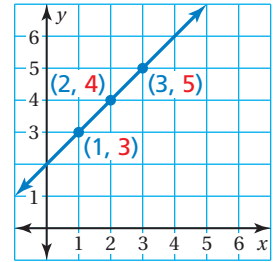
2. $y = 4x - 1$ **3.** $y = 10x$ **4.** $y = 7 - 3x$

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Key Idea

Functions as Tables and Graphs

A function can be represented by an input-output table and by a graph. The table and graph below represent the function $y = x + 2$.

Input, x	Output, y	Ordered Pair, (x, y)
1	3	$(1, 3)$
2	4	$(2, 4)$
3	5	$(3, 5)$

By drawing a line through the points, you graph *all* of the solutions of the function $y = x + 2$.

EXAMPLE **3** **Graphing a Function**

Graph the function $y = -2x + 1$ using inputs of $-1, 0, 1,$ and 2.

Make an input-output table.

Input, x	$-2x + 1$	Output, y	Ordered Pair, (x, y)
-1	$-2(-1) + 1$	3	$(-1, 3)$
0	$-2(0) + 1$	1	$(0, 1)$
1	$-2(1) + 1$	-1	$(1, -1)$
2	$-2(2) + 1$	-3	$(2, -3)$

Plot the ordered pairs and draw a line through the points.
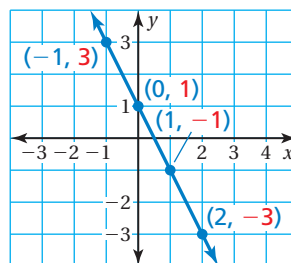
On Your Own

Graph the function.

5. $y = x + 1$ 　　　　 **6.** $y = -3x$ 　　　　 **7.** $y = 3x + 2$

EXAMPLE 4 — Real-Life Application

The number of pounds p of carbon dioxide produced by a car is 20 times the number of gallons g of gasoline used by the car. Write and graph a function that describes the relationship between g and p.

Write a function rule using the variables g and p.

| Words	The number of pounds of carbon dioxide	is	20	times	the number of gallons of gasoline used.
Equation	p	$=$ 20		$\bullet$	g

Make an input-output table that represents the function $p = 20g$.

Input, g	$20g$	Output, p	Ordered Pair, (g, p)
1	20(1)	20	(1, 20)
2	20(2)	40	(2, 40)
3	20(3)	60	(3, 60)

Plot the ordered pairs and draw a line through the points.

Because you cannot have a negative number of gallons, use only positive values of g.
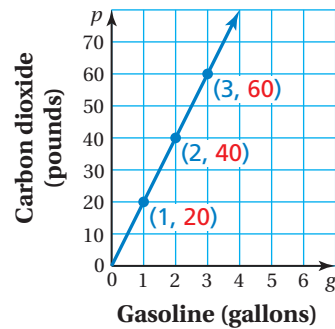
On Your Own

8. **WHAT IF?** For a truck, p is 25 times g. Write and graph a function that describes the relationship between g and p.

Summary

Representations of Functions

Words   An output is 2 more than the input.

Equation $y = x + 2$

Input-Output Table
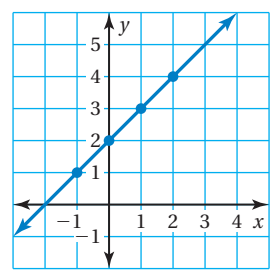
Input, x	Output, y
-1	1
0	2
1	3
2	4

Mapping Diagram

Input, x	Output, y
-1	1
0	2
1	3
2	4
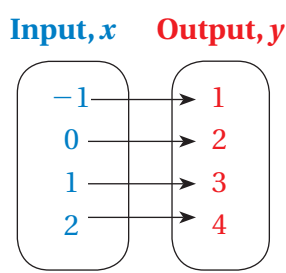
Graph

✓ Vocabulary and Concept Check

1. **VOCABULARY** Identify the input variable and the output variable for the function rule $y = 2x + 5$.

2. **WRITING** Describe five ways to represent a function.

3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

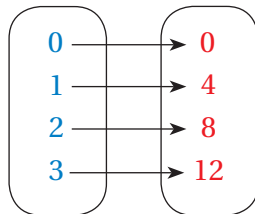|	
What output is 4 more than twice the input 3?	What output is twice the sum of the input 3 and 4?
What output is the sum of 2 times the input 3 and 4?	What output is 4 increased by twice the input 3?

Practice and Problem Solving

Write an equation that describes the function.

4.
Input, x	Output, y
0	0
1	4
2	8
3	12

5.
Input, x	Output, y
1	8
2	9
3	10
4	11

6.
Input, x	Output, y
1	0
3	−2
5	−4
7	−6

Write a function rule for the statement.

① 7. The output is half of the input.

8. The output is eleven more than the input.

9. The output is three less than the input.

10. The output is the cube of the input.

11. The output is six times the input.

12. The output is one more than twice the input.

Find the value of y for the given value of x.

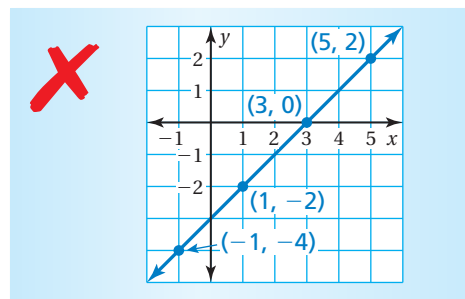② 13. $y = x + 5;\ x = 3$

14. $y = 7x;\ x = -5$

15. $y = 1 - 2x;\ x = 9$

16. $y = 3x + 2;\ x = 0.5$

17. $y = 2x^3;\ x = 3$

18. $y = \dfrac{x}{2} + 9;\ x = -12$

Graph the function.

③ 19. $y = x + 4$

20. $y = 2x$

21. $y = -5x + 3$

22. $y = \dfrac{x}{4}$

23. $y = \dfrac{3}{2}x + 1$

24. $y = 1 + 0.5x$

25. ERROR ANALYSIS Describe and correct the error in graphing the function represented by the input-output table.

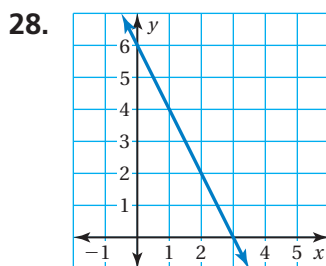| Input, x	-4	-2	0	2
Output, y	-1	1	3	5

4 26. DOLPHIN A dolphin eats 30 pounds of fish per day.

   a. Write and graph a function that relates the number of pounds p of fish that a dolphin eats in d days.

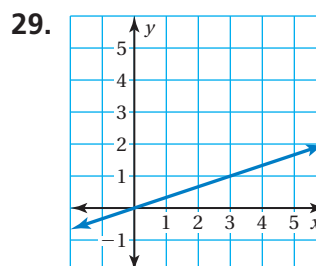   **b.** How many pounds of fish does a dolphin eat in 30 days?

Match the graph with the function it represents.

27.

28.

29.
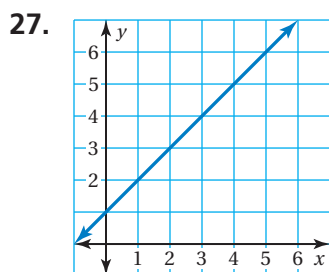
A. $y = \dfrac{x}{3}$ **B.** $y = x + 1$ **C.** $y = -2x + 6$

Find the value of x for the given value of y.

30. $y = 5x - 7$; $y = -22$ **31.** $y = 9 - 7x$; $y = 37$ **32.** $y = \dfrac{x}{4} - 7$; $y = 2$

33. BRACELETS You decide to make and sell bracelets. The cost of your materials is $84. You charge $3.50 for each bracelet.

   a. Write a function that represents the profit P for selling b bracelets.

   b. Which variable is independent? dependent? Explain.

   c. You will *break even* when the cost of your materials equals your income. How many bracelets must you sell to break even?

34. SALE A furniture store is having a sale where everything is 40% off.

   a. Write a function that represents the amount of discount d on an item with a regular price p.

   b. Graph the function using the inputs 100, 200, 300, 400, and 500 for p.

   c. You buy a bookshelf that has a regular price of $85. What is the sale price of the bookshelf?

35. **AIRBOAT TOURS** You want to take a two-hour airboat tour.

   a. Write a function that represents the cost *G* of a tour at Gator Tours.

   b. Write a function that represents the cost *S* of a tour at Snake Tours.

   c. Which is a better deal? Explain.

Gator Tours
$35 boarding fee plus $5 each 1/2 hour
All rates are per person.

Snake Tours
$25 per hour
All rates are per person.

36. **REASONING** The graph of a function is a line that goes through the points (3, 2), (5, 8), and (8, *y*). What is the value of *y*?

37. **CRITICAL THINKING** Make a table where the independent variable is the side length of a square and the dependent variable is the *perimeter*. Make a second table where the independent variable is the side length of a square and the dependent variable is the *area*. Graph both functions in the same coordinate plane. Compare the functions and graphs.

38. **Puzzle** The blocks that form the diagonals of each square are shaded. Each block is one square unit. Find the "green area" of Square 20. Find the "green area" of Square 21. Explain your reasoning.
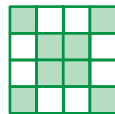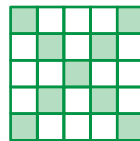
Square 1   Square 2   Square 3   Square 4   Square 5
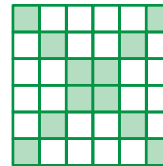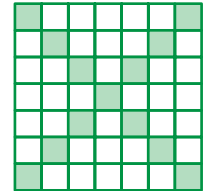
Fair Game Review *What you learned in previous grades & lessons*

Find the slope of the line. *(Section 4.2)*

39.

40.

41.
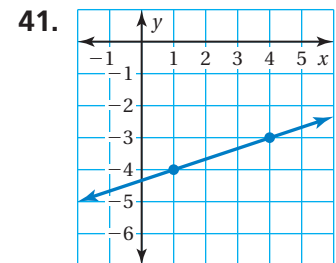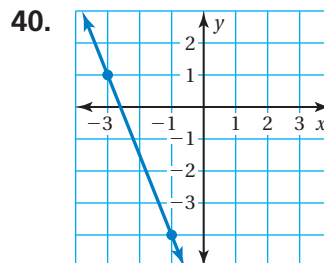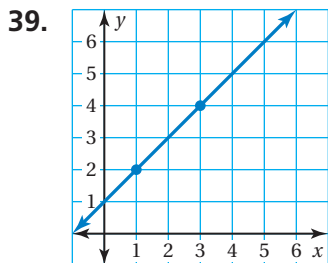
42. **MULTIPLE CHOICE** You want to volunteer for at most 20 hours each month. So far, you have volunteered for 7 hours this month. Which inequality represents the number of hours *h* you can volunteer for the rest of this month? *(Skills Review Handbook)*

   Ⓐ $h \geq 13$ Ⓑ $h \geq 27$ Ⓒ $h \leq 13$ Ⓓ $h < 27$

Essential Question How can you use a function to describe a linear pattern?

1 ACTIVITY: Finding Linear Patterns

Work with a partner.

- Plot the points from the table in a coordinate plane.
- Write a linear equation for the function represented by the graph.

a.

x	0	2	4	6	8
y	150	125	100	75	50
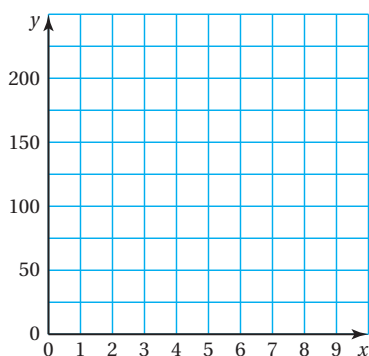
b.

x	4	6	8	10	12
y	15	20	25	30	35
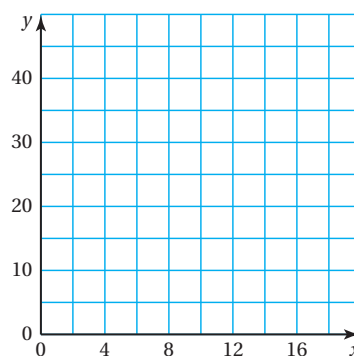
c.

x	−4	−2	0	2	4
y	4	6	8	10	12
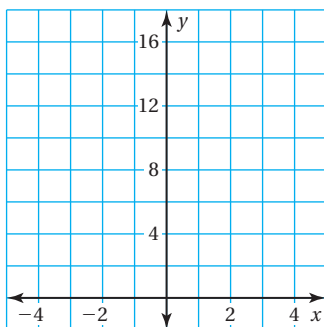
d.

x	−4	−2	0	2	4
y	1	0	−1	−2	−3
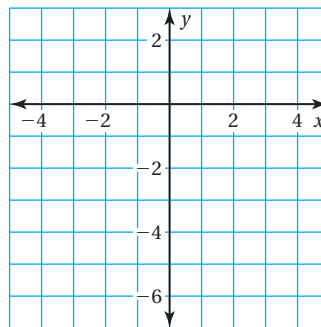
Functions

In this lesson, you will
- understand that the equation $y = mx + b$ defines a linear function.
- write linear functions using graphs or tables.
- compare linear functions.

Math Practice

Label Axes
How do you know what to label the axes? How does this help you accurately graph the data?

Work with a partner. The table shows a familiar linear pattern from geometry.

- **Write a function that relates y to x.**
- **What do the variables x and y represent?**
- **Graph the function.**

a.

x	1	2	3	4	5
y	2π	4π	6π	8π	10π
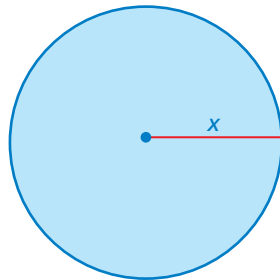
b.

x	1	2	3	4	5
y	10	12	14	16	18
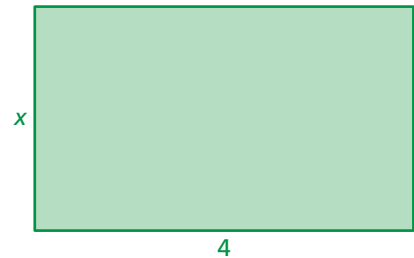
c.

x	1	2	3	4	5
y	5	6	7	8	9
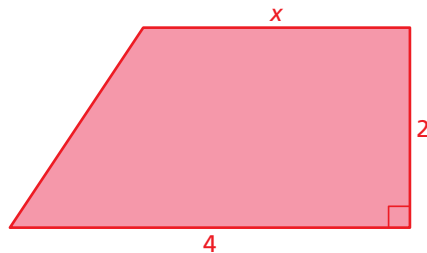
d.

x	1	2	3	4	5
y	28	40	52	64	76
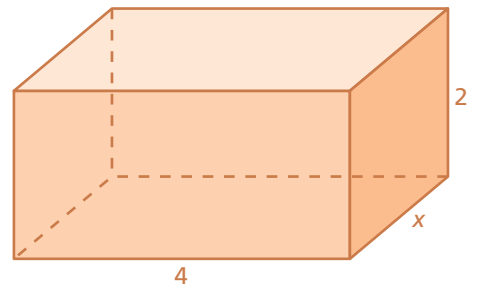
What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use a function to describe a linear pattern?

4. Describe the strategy you used to find the functions in Activities 1 and 2.

Practice

Use what you learned about linear patterns to complete Exercises 3 and 4 on page 261.

6.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
linear function, p. 258

A **linear function** is a function whose graph is a nonvertical line. A linear function can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept.

EXAMPLE 1 — Writing a Linear Function Using a Graph

Use the graph to write a linear function that relates y to x.

The points lie on a line. Find the slope by using the points $(2, 0)$ and $(4, 3)$.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{3-0}{4-2} = \frac{3}{2}$$

Because the line crosses the y-axis at $(0, -3)$, the y-intercept is -3.

So, the linear function is $y = \frac{3}{2}x - 3$.
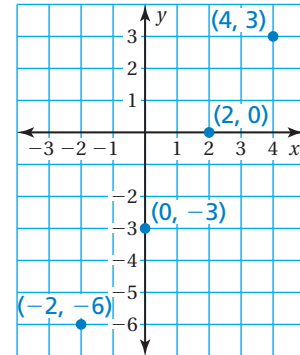
EXAMPLE 2 — Writing a Linear Function Using a Table

Use the table to write a linear function that relates y to x.

x	−3	−2	−1	0
y	9	7	5	3

Plot the points in the table.

The points lie on a line. Find the slope by using the points $(-2, 7)$ and $(-3, 9)$.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{9-7}{-3-(-2)} = \frac{2}{-1} = -2$$

Because the line crosses the y-axis at $(0, 3)$, the y-intercept is 3.

So, the linear function is $y = -2x + 3$.
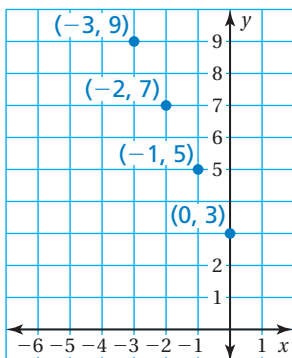
On Your Own

Now You're Ready
Exercises 5–10

Use the graph or table to write a linear function that relates y to x.

1.

2.

x	−2	−1	0	1
y	2	2	2	2

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 3 — **Real-Life Application**

Minutes, x	Height (thousands of feet), y
0	65
10	60
20	55
30	50

You are controlling an unmanned aerial vehicle (UAV) for surveillance. The table shows the height y (in thousands of feet) of the UAV x minutes after you start its descent from cruising altitude.

a. Write a linear function that relates y to x. Interpret the slope and the y-intercept.

You can write a linear function that relates the dependent variable y to the independent variable x because the table shows a constant rate of change. Find the slope by using the points $(0, 65)$ and $(10, 60)$.
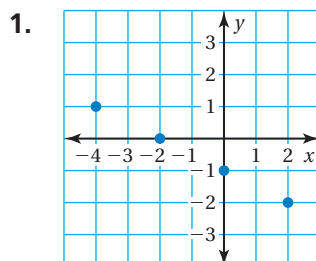
$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{60 - 65}{10 - 0} = \frac{-5}{10} = -0.5$$

Because the line crosses the y-axis at $(0, 65)$, the y-intercept is 65.

Common Error

Make sure you consider the units when interpreting the slope and the y-intercept.

So, the linear function is $y = -0.5x + 65$. The slope indicates that the height decreases 500 feet per minute. The y-intercept indicates that the descent begins at a cruising altitude of 65,000 feet.

b. Graph the linear function.

Plot the points in the table and draw a line through the points.

Because time cannot be negative in this context, use only positive values of x.

UAV Flight
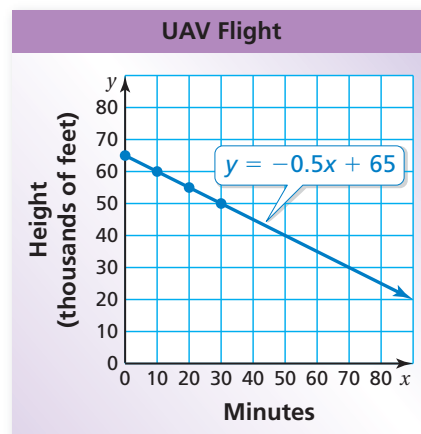
$y = -0.5x + 65$

c. Find the height of the UAV when you stop the descent after 1 hour.

Because 1 hour = 60 minutes, find the value of y when $x = 60$.

$y = -0.5x + 65$ Write the equation.

$\quad = -0.5(60) + 65$ Substitute 60 for x.

$\quad = 35$ Simplify.

So, the descent of the UAV stops at a height of 35,000 feet.

On Your Own

Now You're Ready
Exercises 11–13

3. WHAT IF? You double the rate of descent. Repeat parts (a)–(c).

EXAMPLE 4   Comparing Linear Functions

The earnings y (in dollars) of a nighttime employee working x hours are represented by the linear function $y = 7.5x + 30$. The table shows the earnings of a daytime employee.

	+1	+1	+1	
Time (hours), x	1	2	3	4
Earnings (dollars), y	12.50	25.00	37.50	50.00

+12.50 +12.50 +12.50

a. Which employee has a higher hourly wage?

Nighttime Employee

$y = 7.5x + 30$

The slope is 7.5.

The nighttime employee earns $7.50 per hour.

Daytime Employee

$$\frac{\text{change in earnings}}{\text{change in time}} = \frac{\$12.50}{1 \text{ hour}}$$

The daytime employee earns $12.50 per hour.

So, the daytime employee has a higher hourly wage.

b. Write a linear function that relates the daytime employee's earnings to the number of hours worked. In the same coordinate plane, graph the linear functions that represent the earnings of the two employees. Interpret the graphs.
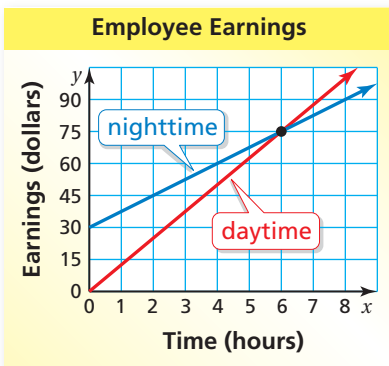
Use a verbal model to write a linear function that represents the earnings of the daytime employee.

$$\text{Earnings} = \frac{\text{Hourly}}{\text{wage}} \cdot \frac{\text{Hours}}{\text{worked}}$$

$$y = 12.5x$$

Employee Earnings

The graph shows that the daytime employee has a higher hourly wage but does not earn more money than the nighttime employee until each person has worked more than 6 hours.

On Your Own

Now You're Ready
Exercise 14

4. Manager A earns $15 per hour and receives a $50 bonus. The graph shows the earnings of Manager B.

  a. Which manager has a higher hourly wage?

  b. After how many hours does Manager B earn more money than Manager A?

Earnings of Manager B

(3, 75)
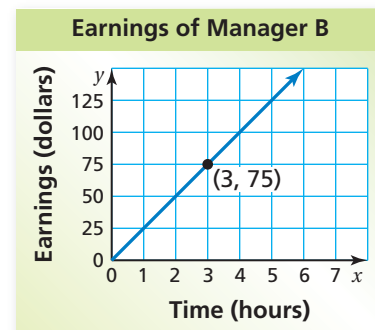
Vocabulary and Concept Check

1. **STRUCTURE** Is $y = mx + b$ a linear function when $b = 0$? Explain.

2. **WRITING** Explain why the vertical line does not represent a linear function.

Practice and Problem Solving

The table shows a familiar linear pattern from geometry. Write a function that relates y to x. What do the variables x and y represent? Graph the function.

3.

x	1	2	3	4	5
y	π	2π	3π	4π	5π

4.

x	1	2	3	4	5
y	2	4	6	8	10

Use the graph or table to write a linear function that relates y to x.

5.

6.

7.
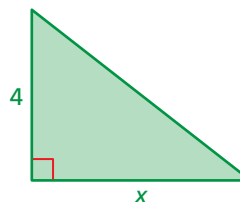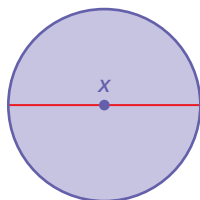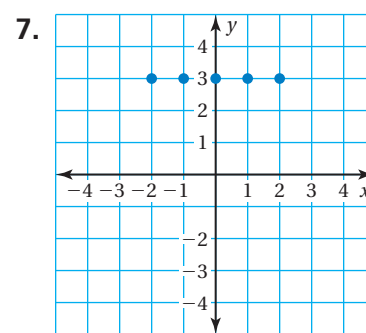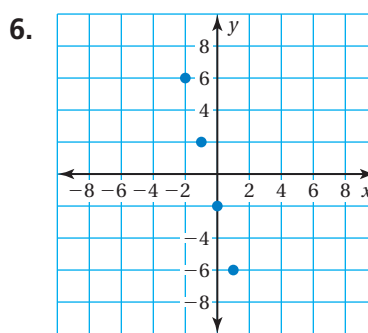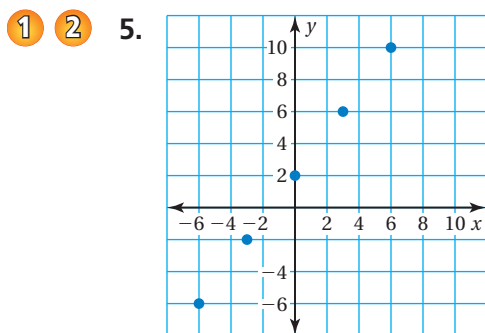
8.

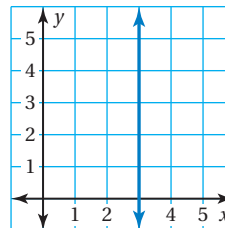| x	-2	-1	0	1
y	-4	-2	0	2

9.

x	-8	-4	0	4
y	2	1	0	-1

10.

x	-3	0	3	6
y	3	5	7	9

11. **MOVIES** The table shows the cost y (in dollars) of renting x movies.

   a. Which variable is independent? dependent?
   b. Write a linear function that relates y to x. Interpret the slope.
   c. Graph the linear function.
   d. How much does it cost to rent three movies?

Number of Movies, x	0	1	2	4
Cost, y	0	3	6	12

12. **BIKE JUMPS** A *bunny hop* is a bike trick in which the rider brings both tires off the ground without using a ramp. The table shows the height y (in inches) of a bunny hop on a bike that weighs x pounds.

Weight (pounds), x	19	21	23
Height (inches), y	10.2	9.8	9.4

a. Write a linear function that relates the height of a bunny hop to the weight of the bike.

b. Graph the linear function.

c. What is the height of a bunny hop on a bike that weighs 21.5 pounds?

13. **BATTERY** The graph shows the percent y (in decimal form) of battery power remaining x hours after you turn on a laptop computer.

a. Write a linear function that relates y to x.

b. Interpret the slope, the x-intercept, and the y-intercept.

c. After how many hours is the battery power at 75%?

Laptop Battery

Power remaining (in decimal form) vs. Hours
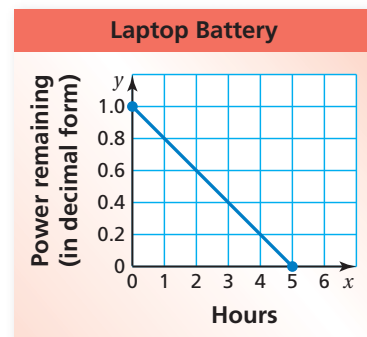
④ 14. **RACE** You and a friend race each other. You give your friend a 50-foot head start. The distance y (in feet) your friend runs after x seconds is represented by the linear function $y = 14x + 50$. The table shows the distances you run.

Time (seconds), x	2	4	6	8
Distance (feet), y	38	76	114	152

a. Who runs at a faster rate? What is that rate?

b. Write a linear function that relates your distance to the number of seconds. In the same coordinate plane, graph the linear functions that represent the distances of you and your friend.

c. For what distances will you win the race? Explain.

15. **CALORIES** The number of calories burned y after x minutes of kayaking is represented by the linear function $y = 4.5x$. The graph shows the calories burned by hiking.

a. Which activity burns more calories per minute?

b. How many more calories are burned by doing the activity in part (a) than the other activity for 45 minutes?

Hiking

Calories burned vs. Minutes
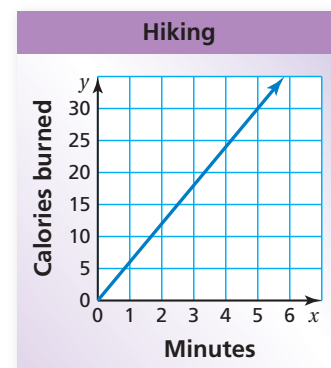
16. SAVINGS You and your friend are saving money to buy bicycles that cost $175 each. The amount y (in dollars) you save after x weeks is represented by the equation $y = 5x + 45$. The graph shows your friend's savings.

Friend's Savings

a. Who has more money to start? Who saves more per week?

b. Who can buy a bicycle first? Explain.
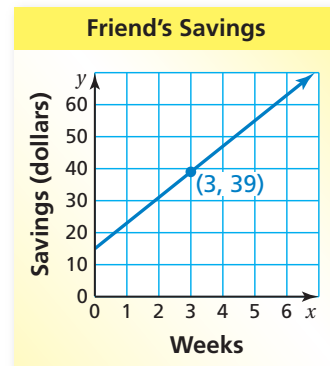
17. REASONING Can the graph of a linear function be a horizontal line? Explain your reasoning.

Years of Education, x	Annual Salary, y
0	28
2	40
4	52
6	64
10	88

18. SALARY The table shows a person's annual salary y (in thousands of dollars) after x years of education beyond high school.

a. Graph the data. Then describe the pattern.

b. What is the annual salary of the person after 8 years of education beyond high school?

c. Find the annual salary of a person with 30 years of education beyond high school. Do you think this situation makes sense? Explain.

19. *Problem Solving* The Heat Index is calculated using the relative humidity and the temperature. For every 1 degree increase in the temperature from $94°F$ to $98°F$ at 75% relative humidity, the Heat Index rises $4°F$.

a. On a summer day, the relative humidity is 75%, the temperature is $94°F$, and the Heat Index is $122°F$. Construct a table that relates the temperature t to the Heat Index H. Start the table at $94°F$ and end it at $98°F$.

b. Identify the independent and dependent variables.

c. Write a linear function that represents this situation.

d. Estimate the Heat Index when the temperature is $100°F$.
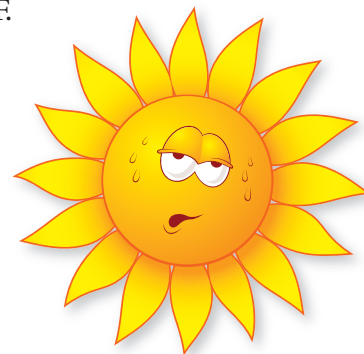
Fair Game Review What you learned in previous grades & lessons

Solve the equation. *(Section 1.1)*

20. $b - 1.6 \div 4 = -3$

21. $w + |-2.8| = 4.3$

22. $\frac{3}{4} = y - \frac{1}{5}(8)$

23. MULTIPLE CHOICE Which of the following describes the translation from the red figure to the blue figure? *(Section 2.2)*
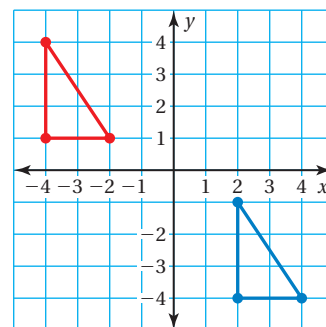
(A) $(x - 6, y + 5)$

(B) $(x - 5, y + 6)$

(C) $(x + 6, y - 5)$

(D) $(x + 5, y - 6)$

You can use a **comparison chart** to compare two topics. Here is an example of a comparison chart for relations and functions.

	Relations	Functions
Definition	A relation pairs inputs with outputs.	A relation that pairs each input with *exactly one* output is a function.
Ordered pairs	(1, 0) (3, –1) (3, 6) (7, 14)	(1, 0) (2, –1) (5, 7) (6, 20)
Mapping diagram		

On Your Own

Make comparison charts to help you study and compare these topics.

1. functions as tables and functions as graphs

2. linear functions with positive slopes and linear functions with negative slopes

After you complete this chapter, make comparison charts for the following topics.

3. linear functions and nonlinear functions

4. graphs with numerical values on the axes and graphs without numerical values on the axes

Comparison of Beagles & Calicos		
	Beagles	Calicos
Example	Newton	Descartes
Treats per day	Lots	1 or 2

Either that... or you just wanted more doggy treats.
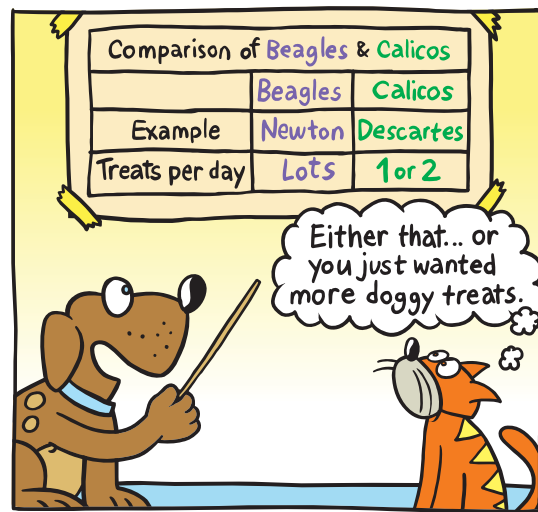
"Creating a comparison chart causes canines to crystalize concepts."

List the ordered pairs shown in the mapping diagram. Then determine whether the relation is a function. *(Section 6.1)*
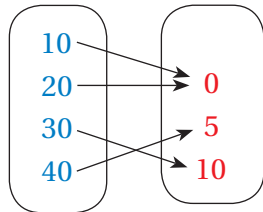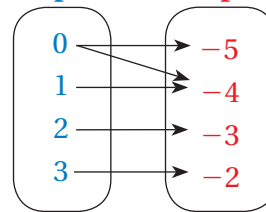
1.

Input	Output
10	0
20	5
30	10
40	

2.

Input	Output
0	−5
1	−4
2	−3
3	−2

Find the value of y for the given value of x. *(Section 6.2)*

3. $y = 10x;\ x = -3$

4. $y = 6 - 2x;\ x = 11$

5. $y = 4x + 5;\ x = \dfrac{1}{2}$
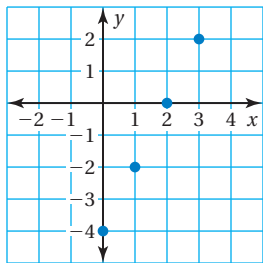
Graph the function. *(Section 6.2)*

6. $y = x - 10$

7. $y = 2x + 3$

8. $y = \dfrac{x}{2}$

Use the graph or table to write a linear function that relates y to x. *(Section 6.3)*

9.

10.

x	y
−3	−3
0	−1
3	1
6	3

11. PUPPIES The table shows the ages of four puppies and their weights. Use the table to draw a mapping diagram. *(Section 6.1)*

Age (weeks)	Weight (oz)
3	11
4	85
6	85
10	480

12. MUSIC An online music store sells songs for $0.90 each. *(Section 6.2)*

  a. Write a function that you can use to find the cost C of buying s songs.

  b. What is the cost of buying 5 songs?

13. ADVERTISING The table shows the revenue R (in millions of dollars) of a company when it spends A (in millions of dollars) on advertising. *(Section 6.3)*

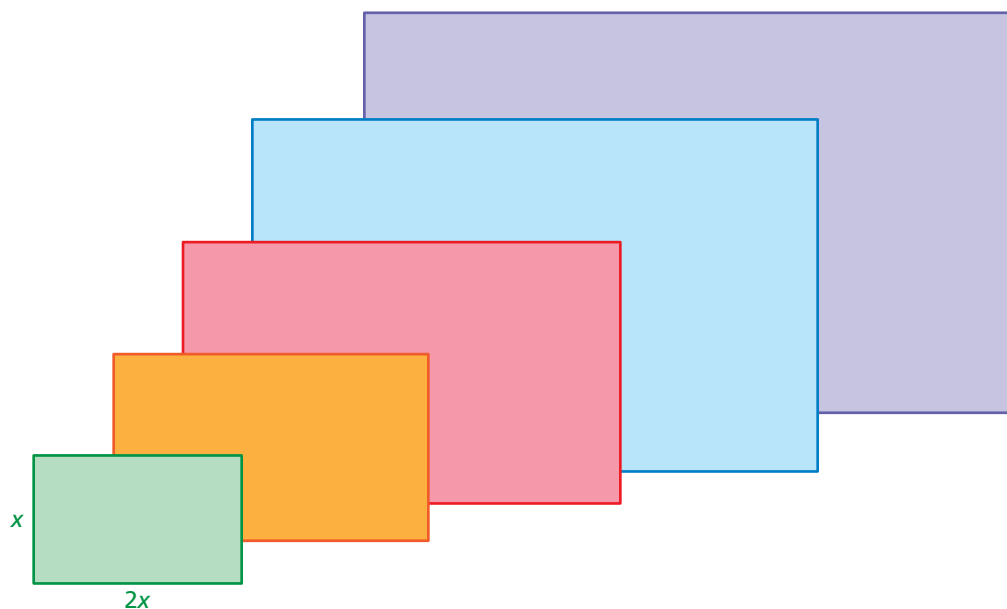| Advertising, A	Revenue, R
0	2
2	6
4	10
6	14
8	18

  a. Write and graph a linear function that relates the revenue to the advertising cost.

  b. What is the revenue of the company when it spends $15 million on advertising?

Comparing Linear and Nonlinear Functions

Essential Question How can you recognize when a pattern in real life is linear or nonlinear?
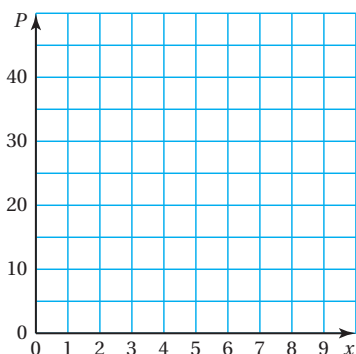
1 ACTIVITY: Finding Patterns for Similar Figures

Work with a partner. Copy and complete each table for the sequence of similar rectangles. Graph the data in each table. Decide whether each pattern is linear or nonlinear.

x

2x

Functions

In this lesson, you will
- identify linear and nonlinear functions from tables or graphs.
- compare linear and nonlinear functions.

a. Perimeters of similar rectangles

x	1	2	3	4	5
P					

b. Areas of similar rectangles

x	1	2	3	4	5
A					

ACTIVITY: Comparing Linear and Nonlinear Functions

Work with a partner. Each table shows the height *h* (in feet) of a falling object at *t* seconds.

- **Graph the data in each table.**
- **Decide whether each graph is linear or nonlinear.**
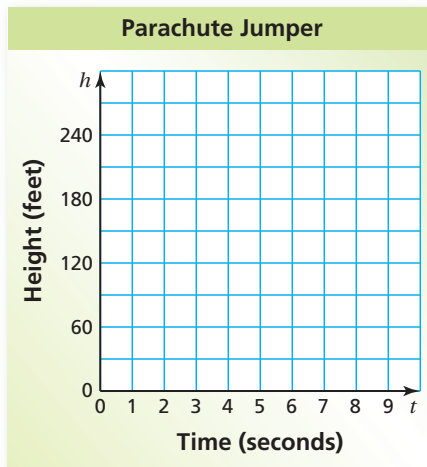- **Compare the two falling objects. Which one has an increasing speed?**

a. Falling parachute jumper

t	0	1	2	3	4
h	300	285	270	255	240

b. Falling bowling ball

t	0	1	2	3	4
h	300	284	236	156	44
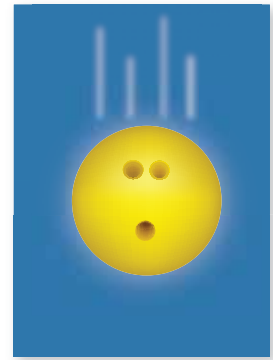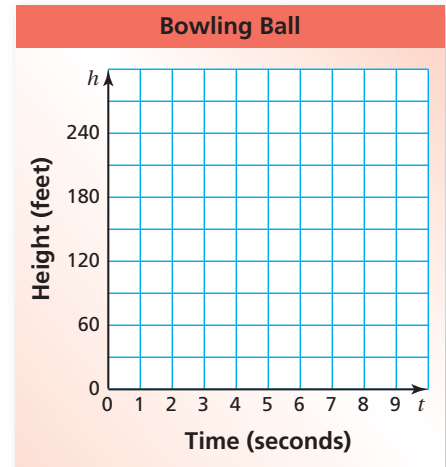
Math Practice

Apply Mathematics
What will the graph look like for an object that has a constant speed? an increasing speed? Explain.

Parachute Jumper

Bowling Ball

What Is Your Answer?

3. IN YOUR OWN WORDS How can you recognize when a pattern in real life is linear or nonlinear? Describe two real-life patterns: one that is linear and one that is nonlinear. Use patterns that are different from those described in Activities 1 and 2.

Practice ➤ Use what you learned about comparing linear and nonlinear functions to complete Exercises 3–6 on page 270.

Key Vocabulary
nonlinear function, *p. 268*

The graph of a linear function shows a constant rate of change. A **nonlinear function** does not have a constant rate of change. So, its graph is *not* a line.
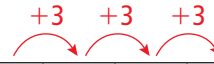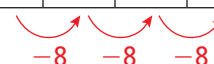
EXAMPLE 1 — Identifying Functions from Tables

Does the table represent a *linear* or *nonlinear* function? Explain.

Study Tip

A constant rate of change describes a quantity that changes by equal amounts over equal intervals.
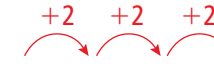
a.

+3 +3 +3

x	3	6	9	12
y	40	32	24	16

−8 −8 −8

⋰ As *x* increases by 3, *y* decreases by 8. The rate of change is constant. So, the function is linear.

b.

+2 +2 +2

x	1	3	5	7
y	2	11	33	88

+9 +22 +55

⋰ As *x* increases by 2, *y* increases by different amounts. The rate of change is *not* constant. So, the function is nonlinear.

EXAMPLE 2 — Identifying Functions from Graphs

Does the graph represent a *linear* or *nonlinear* function? Explain.

a.
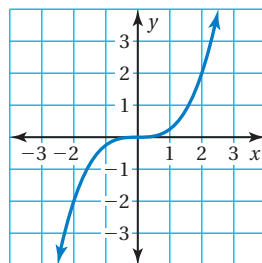
⋰ The graph is *not* a line. So, the function is nonlinear.

b.
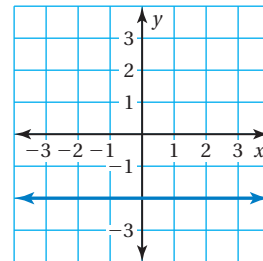
⋰ The graph is a line. So, the function is linear.

On Your Own

Does the table or graph represent a *linear* or *nonlinear* function? Explain.

1.

x	y
0	25
7	20
14	15
21	10

2.

x	y
2	8
4	4
6	0
8	−4

3.

🔊 Multi-Language Glossary at BigIdeasMath.com
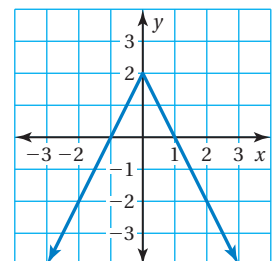
EXAMPLE 3

Identifying a Nonlinear Function

Which equation represents a *nonlinear* function?

(A) $y = 4.7$

(B) $y = \pi x$

(C) $y = \dfrac{4}{x}$

(D) $y = 4(x - 1)$

You can rewrite the equations $y = 4.7$, $y = \pi x$, and $y = 4(x - 1)$ in slope-intercept form. So, they are linear functions.

You cannot rewrite the equation $y = \dfrac{4}{x}$ in slope-intercept form.

So, it is a nonlinear function.

∴ The correct answer is (C).

EXAMPLE 4

Real-Life Application

Account A earns simple interest. Account B earns compound interest. The table shows the balances for 5 years. Graph the data and compare the graphs.

Year, t	Account A Balance	Account B Balance
0	$100	$100
1	$110	$110
2	$120	$121
3	$130	$133.10
4	$140	$146.41
5	$150	$161.05

Savings Account

Both graphs show that the balances are positive and increasing.

The balance of Account A has a constant rate of change of $10. So, the function representing the balance of Account A is linear.

The balance of Account B increases by different amounts each year. Because the rate of change is not constant, the function representing the balance of Account B is nonlinear.
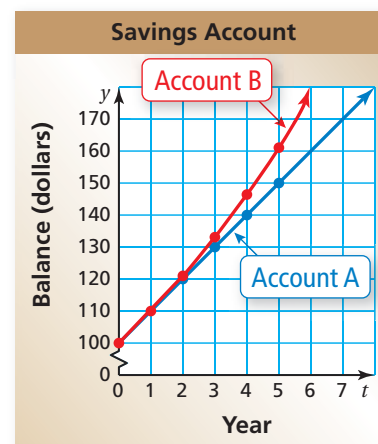
On Your Own

Now You're Ready
Exercises 12–14

Does the equation represent a *linear* or *nonlinear* function? Explain.

4. $y = x + 5$

5. $y = \dfrac{4x}{3}$

6. $y = 1 - x^2$

✓ Vocabulary and Concept Check

1. **VOCABULARY** Describe how linear functions and nonlinear functions are different.

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$$5y = 2x \qquad y = \frac{2}{5}x \qquad 10y = 4x \qquad 5xy = 2$$

Practice and Problem Solving

Graph the data in the table. Decide whether the graph is *linear* or *nonlinear*.

3.

x	0	1	2	3
y	4	8	12	16

4.

x	1	2	3	4
y	1	2	6	24

5.

x	6	5	4	3
y	21	15	10	6

6.

x	−1	0	1	2
y	−7	−3	1	5

Does the table or graph represent a *linear* or *nonlinear* function? Explain.

① ② 7.

8.
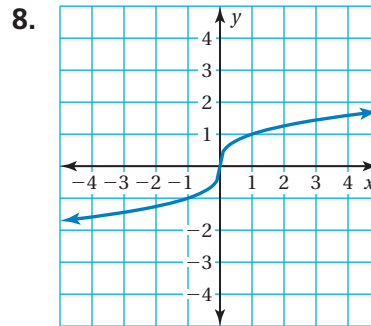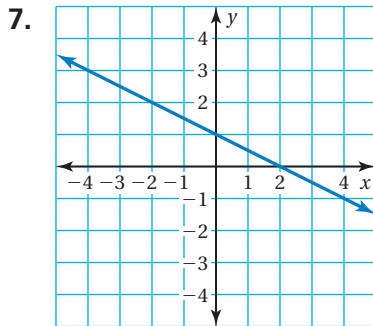
9.

x	5	11	17	23
y	7	11	15	19

10.

x	−3	−1	1	3
y	9	1	1	9

11. **VOLUME** The table shows the volume V (in cubic feet) of a cube with an edge length of x feet. Does the table represent a linear or nonlinear function? Explain.

Edge Length, x	1	2	3	4	5	6	7	8
Volume, V	1	8	27	64	125	216	343	512

Does the equation represent a *linear* or *nonlinear* function? Explain.

③ 12. $2x + 3y = 7$

13. $y + x = 4x + 5$

14. $y = \dfrac{8}{x^2}$

15. LIGHT The frequency y (in terahertz) of a light wave is a function of its wavelength x (in nanometers). Does the table represent a linear or nonlinear function? Explain.

Color	Red	Yellow	Green	Blue	Violet
Wavelength, x	660	595	530	465	400
Frequency, y	454	504	566	645	749

16. MODELING The table shows the cost y (in dollars) of x pounds of sunflower seeds.

Pounds, x	Cost, y
2	2.80
3	?
4	5.60

  a. What is the missing y-value that makes the table represent a linear function?

  b. Write a linear function that represents the cost y of x pounds of seeds. Interpret the slope.

  c. Does the function have a maximum value? Explain your reasoning.

17. TREES Tree A is 5 feet tall and grows at a rate of 1.5 feet per year. The table shows the height h (in feet) of Tree B after x years.

Years, x	Height, h
0	5
1	11
4	17
9	23

  a. Does the table represent a linear or nonlinear function? Explain.

  b. Which tree is taller after 10 years? Explain.

18. **Number Sense** The ordered pairs represent a function.

$$(0, -1), (1, 0), (2, 3), (3, 8), \text{ and } (4, 15)$$

  a. Graph the ordered pairs and describe the pattern. Is the function linear or nonlinear?

  b. Write an equation that represents the function.

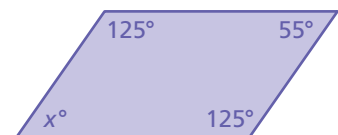## Fair Game Review What you learned in previous grades & lessons

The vertices of a figure are given. Draw the figure and its image after a dilation with the given scale factor k. Identify the type of dilation. *(Section 2.7)*

19. $A(-3, 1), B(-1, 3), C(-1, 1); k = 3$

20. $J(-8, -4), K(2, -4), L(6, -10), M(-8, -10); k = \dfrac{1}{4}$

21. MULTIPLE CHOICE What is the value of x? *(Section 3.3)*

  Ⓐ 25

  Ⓑ 35

  Ⓒ 55

  Ⓓ 125

6.5 Analyzing and Sketching Graphs

Essential Question How can you use a graph to represent relationships between quantities without using numbers?

1 ACTIVITY: Interpreting a Graph

Work with a partner. Use the graph shown.

a. How is this graph different from the other graphs you have studied?

b. Write a short paragraph that describes how the water level changes over time.

c. What situation can this graph represent?
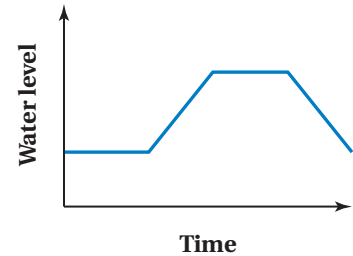
2 ACTIVITY: Matching Situations to Graphs

Work with a partner. You are riding your bike. Match each situation with the appropriate graph. Explain your reasoning.

A.

B.

C.

D.
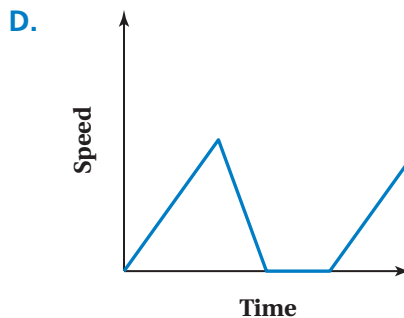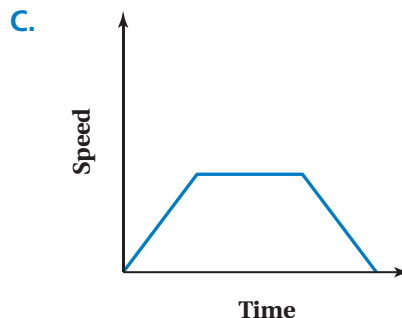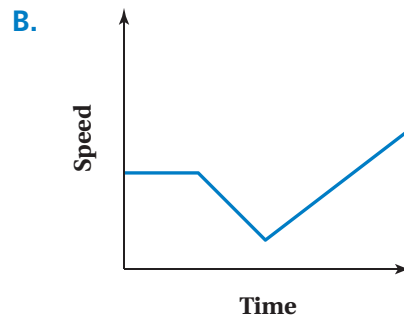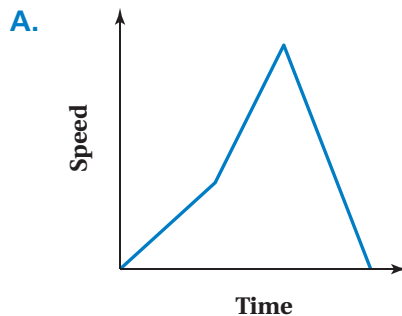
Functions

In this lesson, you will
- analyze the relationship between two quantities using graphs.
- sketch graphs to represent the relationship between two quantities.

a. You gradually increase your speed, then ride at a constant speed along a bike path. You then slow down until you reach your friend's house.

b. You gradually increase your speed, then go down a hill. You then quickly come to a stop at an intersection.

c. You gradually increase your speed, then stop at a store for a couple of minutes. You then continue to ride, gradually increasing your speed.

d. You ride at a constant speed, then go up a hill. Once on top of the hill, you gradually increase your speed.

3 ACTIVITY: Comparing Graphs

Work with a partner. The graphs represent the heights of a rocket and a weather balloon after they are launched.

a. How are the graphs similar? How are they different? Explain.
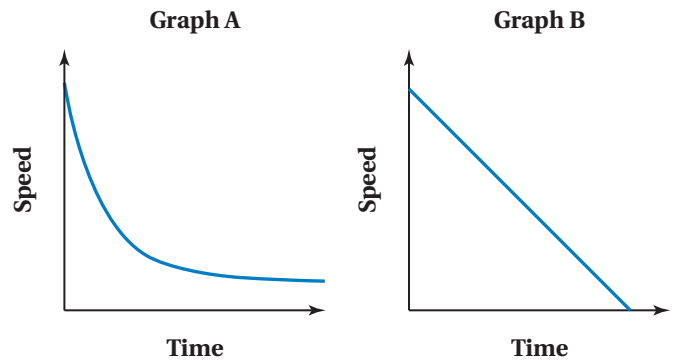
b. Compare the steepness of each graph.

c. Which graph do you think represents the height of the rocket? Explain.

Graph A

Height above ground

Time

Graph B

Height above ground

Time

4 ACTIVITY: Comparing Graphs

Math Practice

Consider Similar Problems

How is this activity similar to the previous activity?

Work with a partner. The graphs represent the speeds of two cars. One car is approaching a stop sign. The other car is approaching a yield sign.

a. How are the graphs similar? How are they different? Explain.

b. Compare the steepness of each graph.

**c.** Which graph do you think represents the car approaching a stop sign? Explain.

Graph A

Speed

Time

Graph B

Speed

Time

What Is Your Answer?

5. IN YOUR OWN WORDS How can you use a graph to represent relationships between quantities without using numbers?

6. Describe a possible situation represented by the graph shown.

7. Sketch a graph similar to the graphs in Activities 1 and 2. Exchange graphs with a classmate and describe a possible situation represented by the graph. Discuss the results.

Height above ground

Time
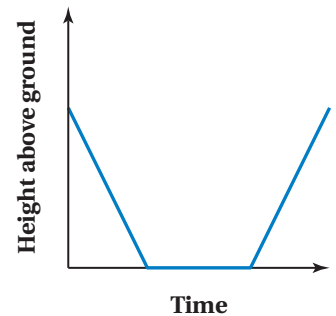
Practice

Use what you learned about analyzing and sketching graphs to complete Exercises 7–9 on page 276.

Graphs can show the relationship between quantities without using specific numbers on the axes.

increasing

no change (constant)

decreasing

Distance

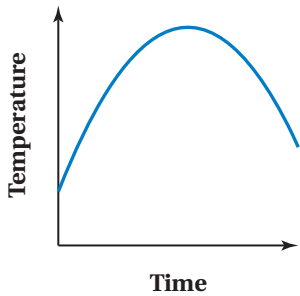**Time**
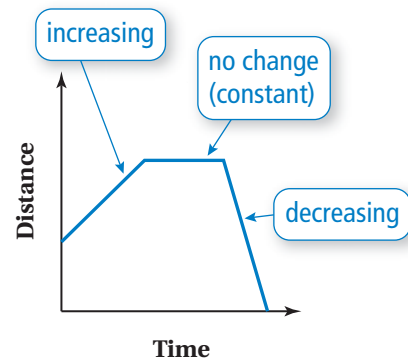
EXAMPLE 1 Analyzing Graphs

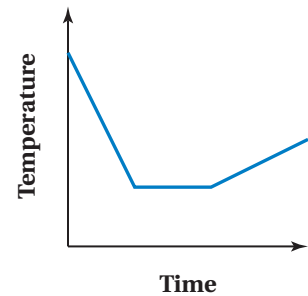**Belfast, Maine**

Temperature

Time

The graphs show the temperatures throughout the day in two cities.

a. **Describe the change in temperature in each city.**

   Belfast: The temperature increases at the beginning of the day. Then the temperature begins to decrease at a faster and faster rate for the rest of the day.

   Newport: The temperature decreases at a constant rate at the beginning of the day. Then the temperature stays the same for a while before increasing at a constant rate for the rest of the day.

b. **Make three comparisons from the graphs.**

   Three possible comparisons follow:

   • Both graphs show increasing and decreasing temperatures.

   • Both graphs are nonlinear, but the graph of the temperatures in Newport consists of three linear sections.

   • In Belfast, it was warmer at the end of the day than at the beginning. In Newport, it was colder at the end of the day than at the beginning.

Newport, Oregon

Temperature

Time

Study Tip

The comparisons given in Example 1(b) are sample answers. You can make many other correct comparisons.
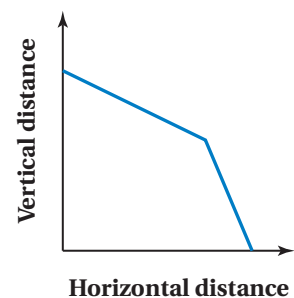
On Your Own

Now You're Ready
Exercises 7–12

1. The graphs show the paths of two birds diving to catch fish.

   a. Describe the path of each bird.

   b. Make three comparisons from the graphs.

Pelican

Vertical distance

Horizontal distance

Osprey

Vertical distance

Horizontal distance
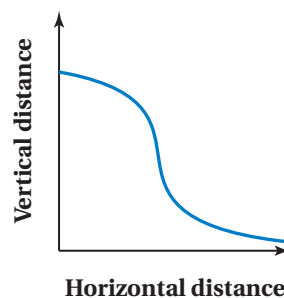
You can sketch graphs showing relationships between quantities that are described verbally.

EXAMPLE 2 Sketching Graphs

Sketch a graph that represents each situation.

a. **A stopped subway train gains speed at a constant rate until it reaches its maximum speed. It travels at this speed for a while, and then slows down at a constant rate until coming to a stop at the next station.**

Step 1: Draw the axes. Label the vertical axis "Speed" and the horizontal axis "Time."

Step 2: Sketch the graph.

| Words	Graph
A stopped subway train gains speed at a constant rate . . .	increasing line segment starting at the origin
until it reaches its maximum speed. It travels at this speed for a while, . . .	horizontal line segment
and then slows down at a constant rate until coming to a stop at the next station.	decreasing line segment ending at the horizontal axis
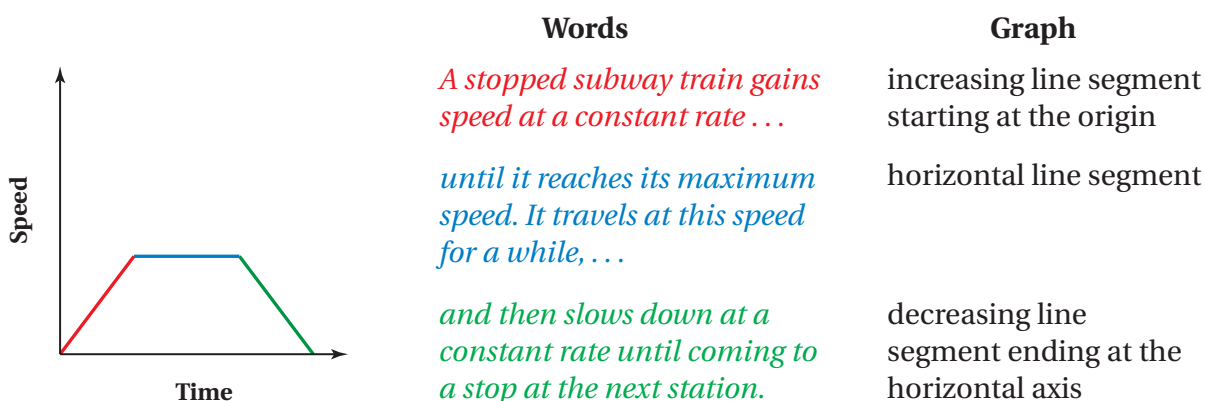
b. **As television size increases, the price increases at an increasing rate.**

Step 1: Draw the axes. Label the vertical axis "Price" and the horizontal axis "TV size."

Step 2: Sketch the graph.

The price *increases at an increasing rate.* So, the graph is nonlinear and becomes steeper and steeper as the TV size increases.
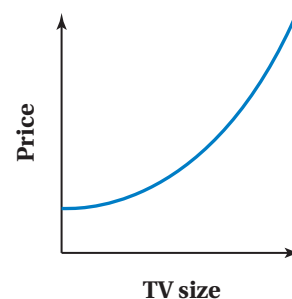

🔴 On Your Own

Sketch a graph that represents the situation.

2. A fully charged battery loses its charge at a constant rate until it has no charge left. You plug it in and recharge it fully. Then it loses its charge at a constant rate until it has no charge left.

3. As the available quantity of a product increases, the price decreases at a decreasing rate.

Vocabulary and Concept Check

MATCHING **Match the verbal description with the part of the graph it describes.**

1. stays the same

2. slowly decreases at a constant rate

3. slowly increases at a constant rate

4. increases at an increasing rate

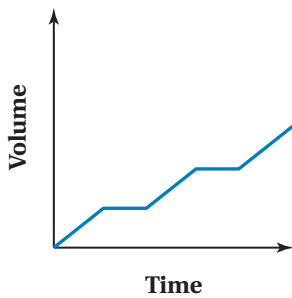**5.** quickly decreases at a constant rate

6. quickly increases at a constant rate
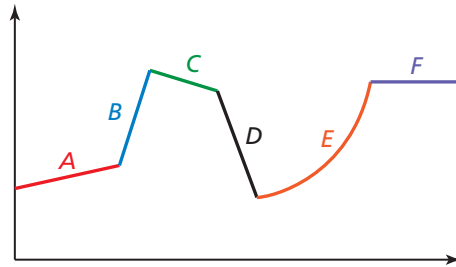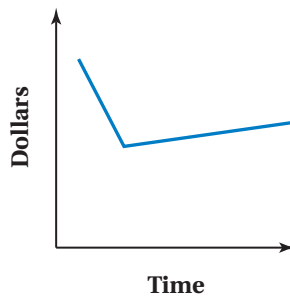
Practice and Problem Solving

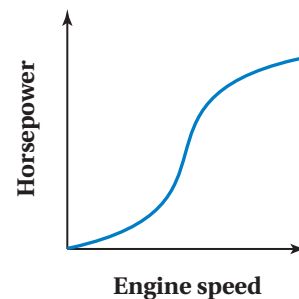**Describe the relationship between the two quantities.**

① **7.**   **Balloon**

8.   **Sales**

9.   **Engine Power**

10.   **Decay**
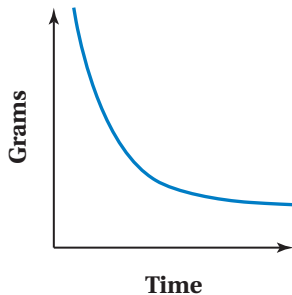
11.   **Hair**

12.   **Loan**

13. **NATURAL GAS** The graph shows the natural gas usage for a house.

   a. Describe the change in usage from January to March.

   b. Describe the change in usage from March to May.

14. **REASONING** The graph shows two bowlers' averages during a bowling season.

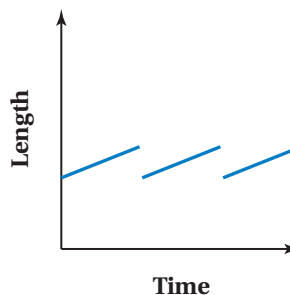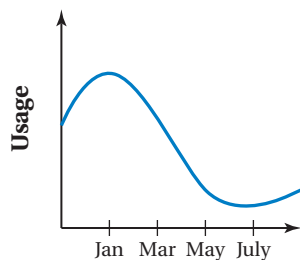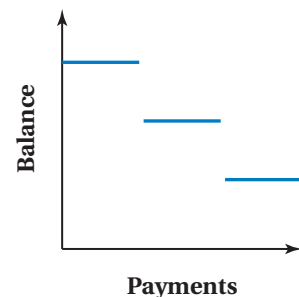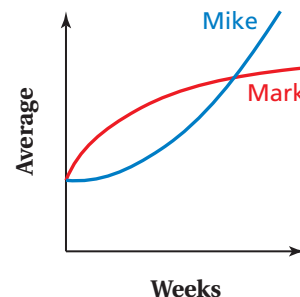   a. Describe each bowler's performance.

   b. Who had a greater average most of the season? Who had a greater average at the end of the season?

Sketch a graph that represents the situation.

15. The value of a car depreciates. The value decreases quickly at first and then more slowly.

16. The distance from the ground changes as your friend swings on a swing.

17. The value of a rare coin increases at an increasing rate.

18. You are typing at a constant rate. You pause to think about your next paragraph, and then you resume typing at the same constant rate.

19. **Economics** You can use a *supply and demand model* to understand how the price of a product changes in a market. The *supply curve* of a particular product represents the quantity suppliers will produce at various prices. The *demand curve* for the product represents the quantity consumers are willing to buy at various prices.
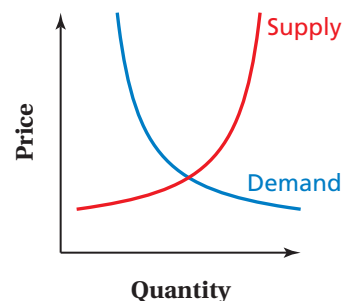
   a. Describe and interpret each curve.

   b. Which part of the graph represents a surplus? a shortage? Explain your reasoning.

   c. The curves intersect at the *equilibrium point*, which is where the quantity produced equals the quantity demanded. Suppose that demand for a product suddenly increases, causing the entire demand curve to shift to the right. What happens to the equilibrium point?

Fair Game Review What you learned in previous grades & lessons

Solve the system of linear equations by graphing. *(Section 5.1)*

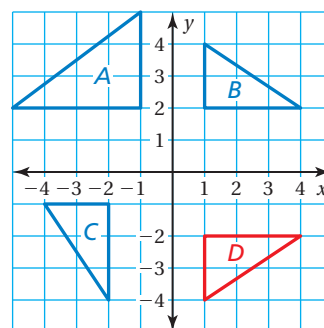20. $y = x + 2$
    $y = -x - 4$

21. $x - y = 3$
    $-2x + y = -5$

22. $3x + 2y = 2$
    $5x - 3y = -22$

23. **MULTIPLE CHOICE** Which triangle is a rotation of Triangle D? *(Section 2.4)*
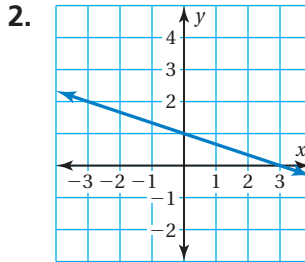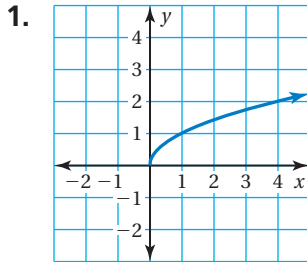
   Ⓐ Triangle A

   Ⓑ Triangle B

   Ⓒ Triangle C

   Ⓓ none

Does the table or graph represent a *linear* or *nonlinear* function? Explain.
(Section 6.4)

1.

2.

3.

| x	y
0	3
3	0
6	3
9	6

4.

x	y
−1	3
1	7
3	11
5	15

5. CHICKEN SALAD The equation $y = 7.9x$ represents the cost y (in dollars) of buying x pounds of chicken salad. Does this equation represent a linear or nonlinear function? Explain.   *(Section 6.4)*

6. HEIGHTS The graphs show the heights of two people over time.   *(Section 6.5)*

   a. Describe the change in height of each person.

   b. Make three comparisons from the graphs.
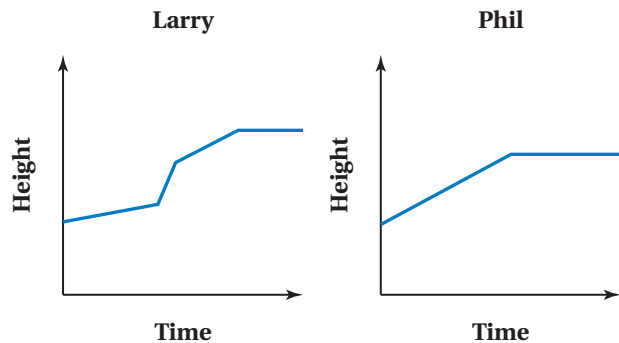
You are snowboarding down a hill. Sketch a graph that represents the situation.   *(Section 6.5)*

   7. You gradually increase your speed at a constant rate over time but fall about halfway down the hill. You take a short break, then get up, and gradually increase your speed again.

   8. You gradually increase your speed at a constant rate over time. You come to a steep section of the hill and rapidly increase your speed at a constant rate. You then decrease your speed at a constant rate until you come to a stop.

Review Key Vocabulary

input, *p. 244*   function, *p. 245*
output, *p. 244*   function rule, *p. 250*
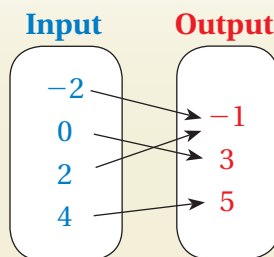relation, *p. 244*   linear function, *p. 258*
mapping diagram, *p. 244*   nonlinear function, *p. 268*

Review Examples and Exercises

6.1 Relations and Functions *(pp. 242–247)*

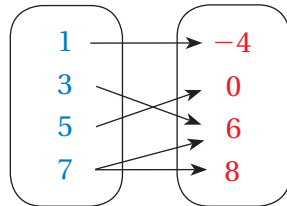**Determine whether the relation is a function.**

Input	Output
−2	−1
0	3
2	5
4	

Each input has exactly one output.

∴ So, the relation is a function.

Exercises

Determine whether the relation is a function.

1.
Input	Output
1	−4
3	0
5	6
7	8

2.
Input	Output
0	0
1	5
2	10
3	15
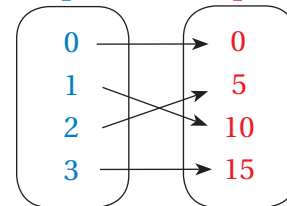
6.2 Representations of Functions *(pp. 248–255)*

Graph the function $y = x - 1$ using inputs of −1, 0, 1, and 2.

Make an input-output table.

Input, x	$x - 1$	Output, y	Ordered Pair, (x, y)
−1	−1 − 1	−2	(−1, −2)
0	0 − 1	−1	(0, −1)
1	1 − 1	0	(1, 0)
2	2 − 1	1	(2, 1)

Plot the ordered pairs and draw a line through the points.
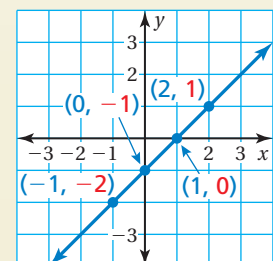
Exercises

Find the value of y for the given value of x.

3. $y = 2x - 3$; $x = -4$

4. $y = 2 - 9x$; $x = \dfrac{2}{3}$

5. $y = \dfrac{x}{3} + 5$; $x = 6$

Graph the function.

6. $y = x + 3$

7. $y = -5x$

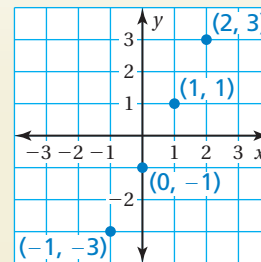**8.** $y = 3 - 3x$

6.3   Linear Functions   *(pp. 256–263)*

Use the graph to write a linear function that relates y to x.

The points lie on a line. Find the slope by using the points (1, 1) and (2, 3).

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{3 - 1}{2 - 1} = \frac{2}{1} = 2$$

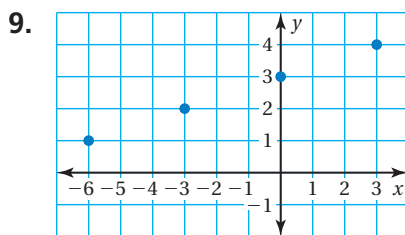Because the line crosses the y-axis at $(0, -1)$, the y-intercept is -1.

⋮ So, the linear function is $y = 2x - 1$.

Exercises

Use the graph or table to write a linear function that relates y to x.

9.

10.

x	−2	0	2	4
y	−7	−7	−7	−7

6.4   Comparing Linear and Nonlinear Functions   *(pp. 266–271)*

Does the table represent a *linear* or *nonlinear* function? Explain.

a.

+2   +2   +2

x	0	2	4	6
y	0	1	4	9

+1   +3   +5

⋮ As x increases by 2, y increases by different amounts. The rate of change is *not* constant. So, the function is nonlinear.

b.

+5   +5   +5

x	0	5	10	15
y	50	40	30	20

−10   −10   −10

⋮ As x increases by 5, y decreases by 10. The rate of change is constant. So, the function is linear.

Exercises

Does the table represent a *linear* or *nonlinear* function? Explain.

11.

x	3	6	9	12
y	1	10	19	28

12.

| x	1	3	5	7
y	3	1	1	3

6.5 **Analyzing and Sketching Graphs** *(pp. 272–277)*

The graphs show the populations of two cities over several years.

a. Describe the change in population in each city.

Lake City: The population gradually decreases at a constant rate, then gradually increases at a constant rate. Then the population rapidly increases at a constant rate.

Gold Point: The population rapidly increases at a constant rate. Then the population stays the same for a short period of time before gradually decreasing at a constant rate.
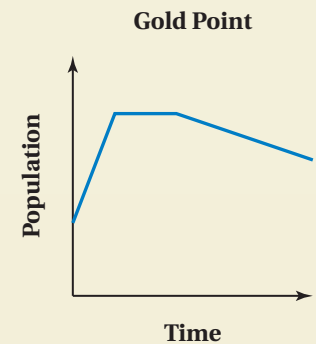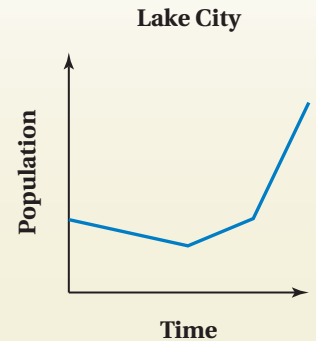
b. Make three comparisons from the graphs.

- Both graphs show increasing and decreasing populations.

- Both graphs are nonlinear, but both graphs consist of three linear sections.

- Both populations at the end of the time period are greater than the populations at the beginning of the time period.

Lake City

Gold Point

Exercises

Company A

Company B

13. SALES The graphs show the sales of two companies.

 a. Describe the sales of each company.

 **b.** Make three comparisons from the graphs.
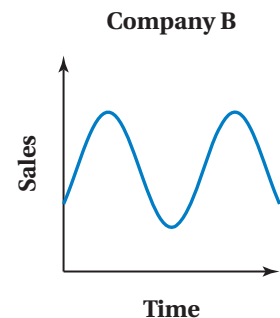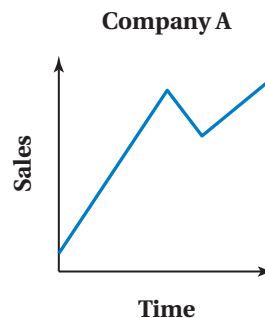
**Sketch a graph that represents the situation.**

14. You climb up a climbing wall. You gradually climb halfway up the wall at a constant rate, then stop and take a break. You then climb to the top of the wall at a constant rate.

15. The price of a stock steadily increases at a constant rate for several months before the stock market crashes. The price then quickly decreases at a constant rate.

Determine whether the relation is a function.

1.

Input	Output
2	9
4	10
6	11
8	

2.
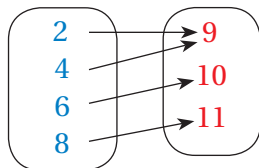
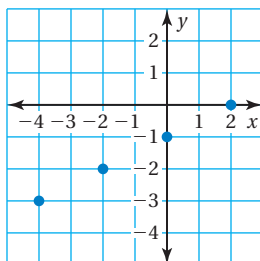| Input	Output
−3	1
−1	3
1	5
3	7
	9
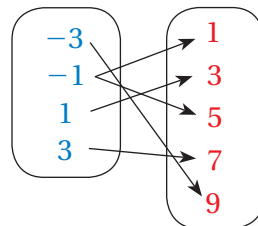
Graph the function.

3. $y = x + 8$ **4.** $y = 1 - 3x$ **5.** $y = x - 4$

**6.** Use the graph to write a linear function that relates y to x.

7. Does the table represent a *linear* or *nonlinear* function? Explain.

x	0	2	4	6
y	8	0	−8	−16

8. WATER SKI The table shows the number of meters a water skier travels in x minutes.

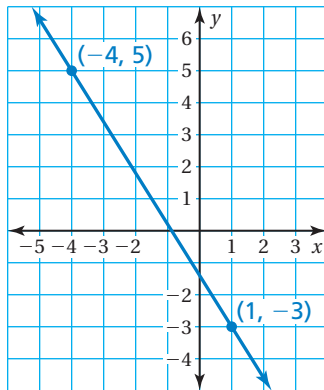| Minutes, x	1	2	3	4	5
Meters, y	600	1200	1800	2400	3000

   a. Write a function that relates x to y.

   b. Graph the linear function.

   c. At this rate, how many *kilometers* would the water skier travel in 12 minutes?

9. STOCKS The graphs show the prices of two stocks during one day.
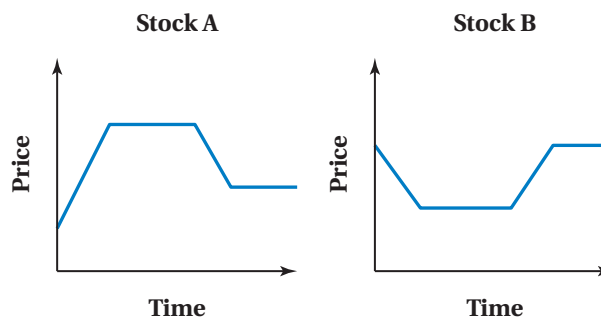
   a. Describe the prices of each stock.

   b. Make three comparisons from the graphs.

Stock A

Stock B

10. RACE You are competing in a race. You begin the race by increasing your speed at a constant rate. You then run at a constant speed until you get a cramp and have to stop. You wait until your cramp goes away before you start gradually increasing your speed again at a constant rate. Sketch a graph that represents the situation.

1. What is the slope of the line shown in the graph below?

A. $-\dfrac{8}{3}$

B. $-\dfrac{8}{5}$

C. $-\dfrac{2}{3}$

D. $-\dfrac{2}{5}$

Test-Taking Strategy
Work Backwards

For x cats, a litter box is changed $y = 3x$ times per month. How many cats are there when $y = 12$?

Ⓐ 1　Ⓑ 2　Ⓒ 3　Ⓓ 4

Share a litter box? Please!

KEEP OFF!

"**Work backwards** by trying 1, 2, 3, and 4. You will see that 3(4) = 12. So, D is correct."

2. Which value of a makes the equation below true?
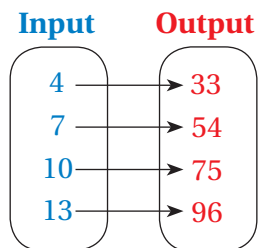
$$24 = \frac{a}{3} - 9$$

F. 5

G. 11

H. 45

I. 99

3. A mapping diagram is shown.

What number belongs in the box below so that the equation will correctly describe the function represented by the mapping diagram?

$$y = \boxed{}\,x + 5$$

Input	Output
4	33
7	54
10	75
13	96

4. What is the solution of the system of linear equations shown below?

$$y = 2x - 1$$
$$y = 3x + 5$$

A. $(-13, -6)$

B. $(-6, -13)$

C. $(-13, 6)$

D. $(-6, 13)$

5. A system of two linear equations has no solution. What can you conclude about the graphs of the two equations?

   F. The lines have the same slope and the same y-intercept.

   G. The lines have the same slope and different y-intercepts.

   H. The lines have different slopes and the same y-intercept.

   I. The lines have different slopes and different y-intercepts.

6. Which graph shows a nonlinear function?

   A.

   C.

   B.

   D.
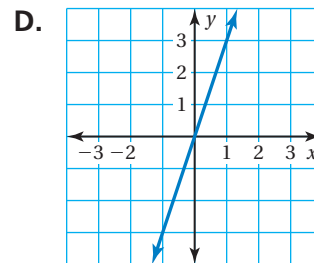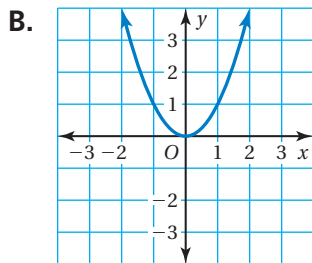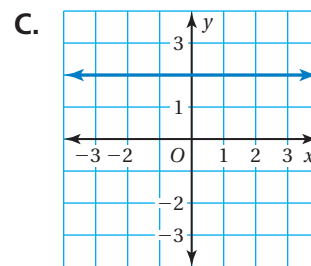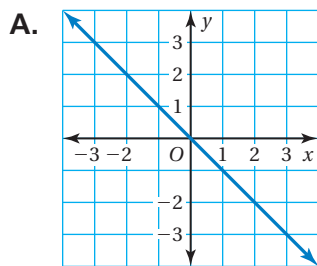
7. What is the value of x?

   F. 40   **H.** 140

   G. 50   **I.** 220

8. The tables show the sales (in millions of dollars) for two companies over a 5-year period. Examine the data in the tables.

Year	1	2	3	4	5
Sales	2	4	6	8	10

Part A   Does the first table show a linear function? Explain your reasoning.

Year	1	2	3	4	5
Sales	1	1	2	3	5

Part B   Does the second table show a linear function? Explain your reasoning.

9. The equations $y = -x + 4$ and $y = \frac{1}{2}x - 8$ form a system of linear equations. The table below shows the y-value for each equation at six different values of x.

x	0	2	4	6	8	10
$y = -x + 4$	4	2	0	-2	-4	-6
$y = \frac{1}{2}x - 8$	-8	-7	-6	-5	-4	-3

What can you conclude from the table?

A. The system has one solution, when $x = 0$.

B. The system has one solution, when $x = 4$.

C. The system has one solution, when $x = 8$.

D. The system has no solution.

10. In the diagram below, Triangle *ABC* is a dilation of Triangle *DEF*. What is the value of x?