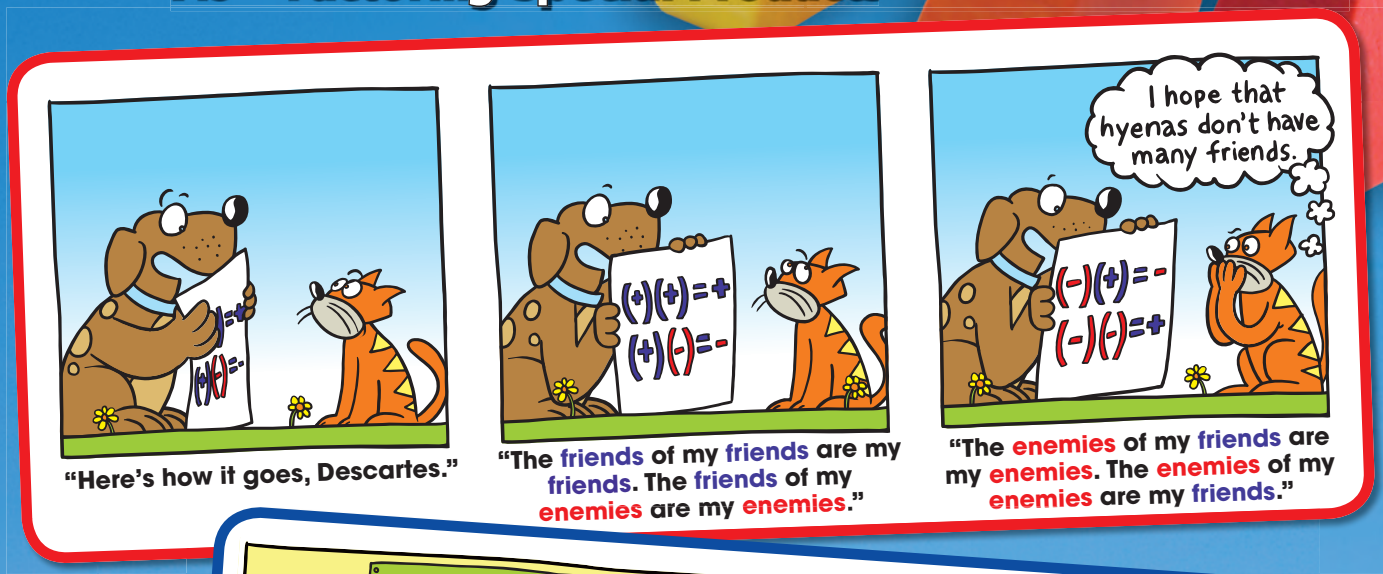
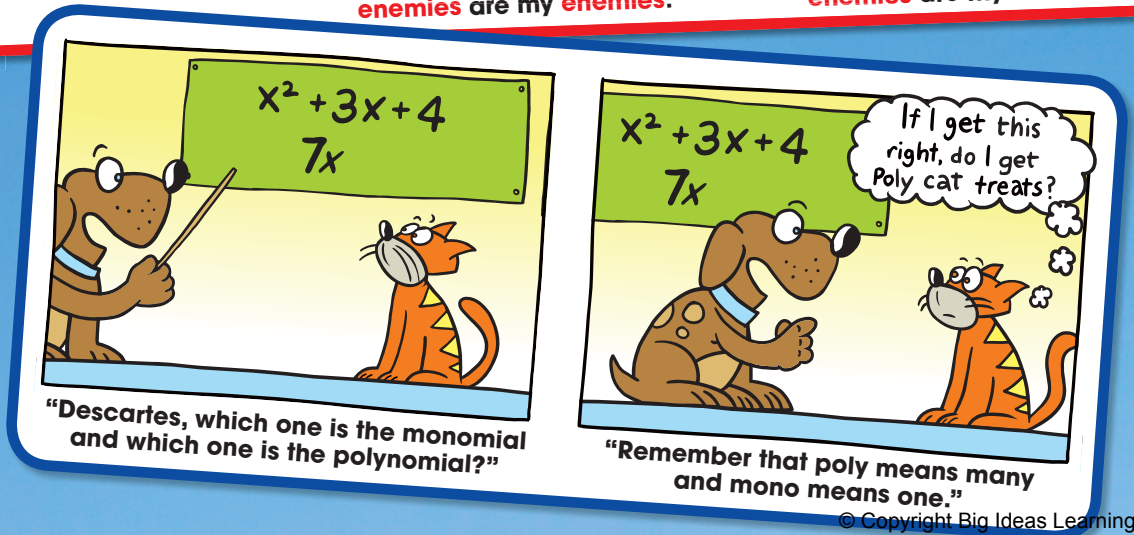
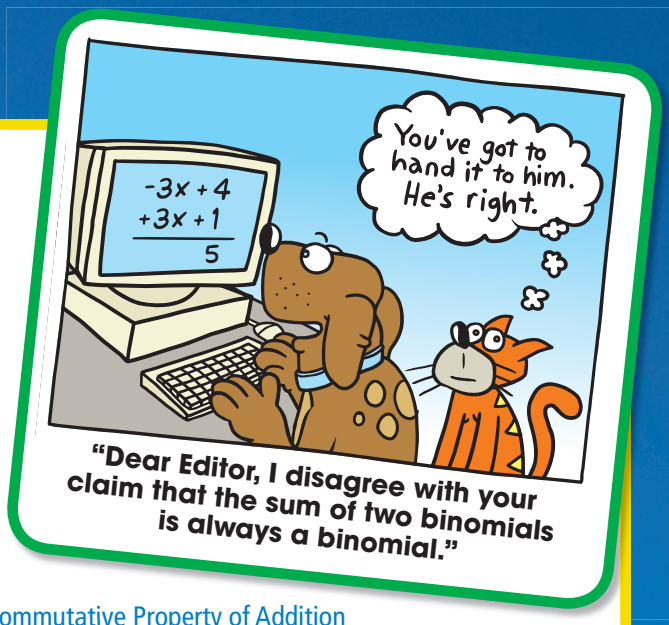
7 Polynomial Equations and Factoring





What You Learned Before



Simplifying Algebraic Expressions (7.EE.1)

Example 1 **Simplify $5x + 7 - 2x - 3$.**

$5x + 7 - 2x - 3 = 5x - 2x + 7 - 3$ Commutative Property of Addition

$= (5 - 2)x + 7 - 3$ Distributive Property

$= 3x + 4$ Simplify.

Example 2 **Simplify $-7(y - 2) + 3y$.**

$-7(y - 2) + 3y = -7(y) - (-7)(2) + 3y$ Distributive Property

$= -7y + 14 + 3y$ Multiply.

$= -7y + 3y + 14$ Commutative Property of Addition

$= (-7 + 3)y + 14$ Distributive Property

$= -4y + 14$ Add coefficients.

Try It Yourself

Simplify the expression.

1. $3x - 8 + 4x$
2. $3t - 4 - 6t + 7$
3. $-7z + 3 + 2z + 4z + 5$
4. $3(w + 2) - 5$
5. $4g - 2(g + 6)$
6. $3(n + 1) - 4(n - 3)$

Finding the Greatest Common Factor (6.NS.4)

Example 3 **Find the greatest common factor of 50 and 75.**

$50 = 2 \cdot 5 \cdot 5$

$75 = 3 \cdot 5 \cdot 5$

∴ The GCF is $5 \cdot 5 = 25$.

Example 4 **Find the greatest common factor of 30 and 42.**

$30 = 2 \cdot 3 \cdot 5$

$42 = 2 \cdot 3 \cdot 7$

∴ The GCF is $2 \cdot 3 = 6$.

Try It Yourself

Find the greatest common factor.

7. 28, 64
8. 60, 72
9. 24, 27

7.1 Polynomials

Essential Question How can you use algebra tiles to model and classify polynomials?

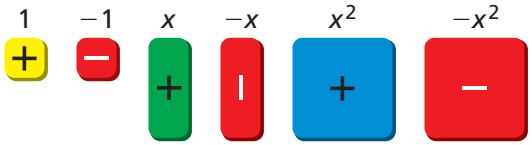
1 ACTIVITY: Meaning of Prefixes

Work with a partner. Think of a word that uses one of the prefixes with one of the base words. Then define the word and write a sentence that uses the word.

Prefix	Base Word
Mono	Dactyl
Bi	Cycle
Tri	Ped
Poly	Syllabic

2 ACTIVITY: Classifying Polynomials Using Algebra Tiles

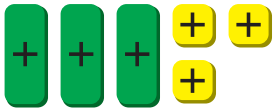
Work with a partner. Six different algebra tiles are shown at the right.



Write the polynomial that is modeled by the algebra tiles. Then classify the polynomial as a monomial, binomial, or trinomial. Explain your reasoning.

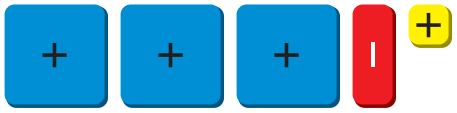
a.

b.



c.

d.



e.

f.

Common Core

Polynomials

In this lesson, you will
- find the degrees of monomials.
- classify polynomials.

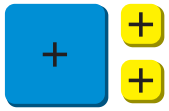
Applying Standard A.SSE.1a

3 ACTIVITY: Solving an Algebra Tile Puzzle

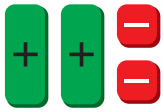
Math Practice 2
Use Expressions
What do the shapes and colors of the tiles represent? How does this help you write a polynomial?

Work with a partner. Write the polynomial modeled by the algebra tiles, evaluate the polynomial at the given value, and write the result in the corresponding square of the Sudoku puzzle. Then solve the puzzle.

A3, H7
Value when $x = 2$

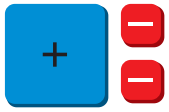


A4, B3, E5, G6, I7
Value when $x = 2$



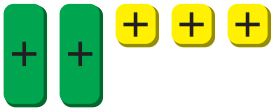
	1	2	3	4	5	6	7	8	9
A									
B									
C									
D									1
E									
F									
G									
H									
I									

A6, D7, E2, H5
Value when $x = -3$



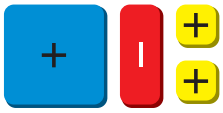
B5, F1, H3
Value when $x = -1$

A7, F9, I4
Value when $x = 3$

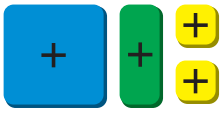


E8, F3, I6
Value when $x = -1$

C4, I3
Value when $x = 3$



B7, D1
Value when $x = -2$



What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use algebra tiles to model and classify polynomials? Explain why algebra tiles have the dimensions, shapes, and colors that they have.

Practice Use what you learned about modeling polynomials to complete Exercises 5 and 6 on page 332.

7.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
monomial, *p. 330*
degree of a monomial, *p. 330*
polynomial, *p. 331*
binomial, *p. 331*
trinomial, *p. 331*
degree of a polynomial, *p. 331*

A **monomial** is a number, a variable, or a product of a number and one or more variables with whole number exponents.

Monomials
-4
$\frac{1}{2}y^2$
$2.5x^2y$

Not monomials	Reason
$x^{1.5}$	Monomials must have whole number exponents.
$-\frac{2}{z}$	Monomials cannot have variables in the denominator.
7^y	Monomials cannot have variable exponents.

The **degree of a monomial** is the sum of the exponents of the variables in the monomial.

EXAMPLE 1 Finding the Degrees of Monomials

Find the degree of each monomial.

a. $5x^2$

The exponent of x is 2.

∴ So, the degree of the monomial is 2.

b. $-\frac{1}{2}xy^3$

The exponent of x is 1 and the exponent of y is 3.

The sum of the exponents is $1 + 3 = 4$.

∴ So, the degree of the monomial is 4.

c. -3

You can rewrite -3 as $-3x^0$.

The exponent of x is 0.

∴ So, the degree of the monomial is 0.

Remember
For any nonzero number a, $a^0 = 1$.

On Your Own

Now You're Ready
Exercises 7–14

Find the degree of the monomial.

1. $-3x^4$
2. $7c^3d^2$
3. $\frac{5}{3}y$
4. -20.5

Multi-Language Glossary at BigIdeasMath.com

A **polynomial** is a monomial or a sum of monomials. Each monomial is called a *term* of the polynomial.

A polynomial with two terms is a **binomial**. $5x + 2$

A polynomial with three terms is a **trinomial**. $x^2 + 5x + 2$

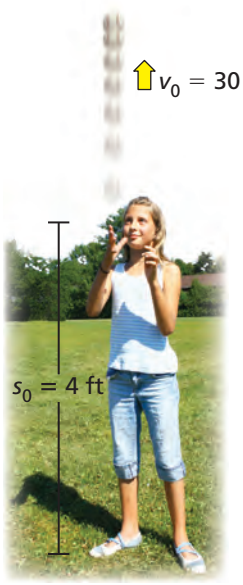
The **degree of a polynomial** is the greatest degree of its terms. A polynomial in one variable is in *standard form* when the exponents of the terms decrease from left to right.

EXAMPLE 2 Classifying Polynomials

Write each polynomial in standard form. Identify the degree and classify each polynomial by the number of terms.

	Polynomial	*Standard Form*	*Degree*	*Type of Polynomial*
a.	$-3z^4$	$-3z^4$	4	monomial
b.	$4 + 5x^2 - x$	$5x^2 - x + 4$	2	trinomial
c.	$8q + q^5$	$q^5 + 8q$	5	binomial

EXAMPLE 3 Real-Life Application



The polynomial $-16t^2 + v_0t + s_0$ represents the height (in feet) of an object, where v_0 is the initial vertical velocity (in feet per second), s_0 is the initial height of the object (in feet), and t is the time (in seconds).

a. Write a polynomial that represents the height of the baseball.

$-16t^2 + v_0t + s_0 = -16t^2 + 30t + 4$ Substitute 30 for v_0 and 4 for s_0.

b. What is the height of the baseball after 1 second?

$-16t^2 + 30t + 4 = -16(1)^2 + 30(1) + 4$ Substitute 1 for t.

$= -16 + 30 + 4$ Simplify.

$= 18$ Add.

∴ The height of the baseball after 1 second is 18 feet.

On Your Own

Now You're Ready
Exercises 15–23

Write the polynomial in standard form. Identify the degree and classify the polynomial by the number of terms.

5. $4 - 9z$ **6.** $t^2 - t^3 - 10t$ **7.** $2.8x + x^3$

8. In Example 3, the initial height is 5 feet. What is the height of the baseball after 2 seconds?

7.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Is $-\frac{\pi}{3}$ a monomial? Explain your reasoning.
2. **VOCABULARY** When is a polynomial in one variable in standard form?
3. **OPEN-ENDED** Write a trinomial of degree 5 in standard form.
4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$a^3 + 4a$ $\quad$ 8^x $\quad$ $b - 2^{-1}$ $\quad$ $-6y^8z$

Practice and Problem Solving

Use algebra tiles to represent the polynomial.

5. $x^2 + 2x - 4$
6. $2x^2 - x + 3$

Find the degree of the monomial.

1 7. $4g$ 8. $23x^4$ 9. s^8t 10. $-\frac{4}{9}$

11. $1.75k^2$ 12. $\frac{1}{8}m^2n^4$ 13. 2π 14. $-3q^4rs^6$

Write the polynomial in standard form. Identify the degree and classify the polynomial by the number of terms.

2 15. $7 + 3p^2$ 16. $2w^6$ 17. $8d - 2 - 4d^3$

18. $6.5c^2 + 1.2c^4 - c$ 19. $4v^{11} - v^{12}$ 20. $-\frac{1}{4}y - \frac{3}{8}y^2$

21. $7.4z^5$ 22. $\sqrt{3}n^7 - 19 + \sqrt{2}n^3$ 23. $\pi r^2 - \frac{5}{7}r^8 + 2r^5$

24. **ERROR ANALYSIS** Describe and correct the error in writing the polynomial in standard form.

✗ polynomial: $3m^2 - 5m^5 + m^4$
standard form: $-5m^5 + 3m^2 + m^4$

25. **SPHERE** The expression $\frac{4}{3}\pi r^3$ represents the volume of a sphere with radius r. Why is this expression a monomial? What is its degree?

Tell whether the expression is a polynomial. If so, identify the degree and classify the polynomial by the number of terms.

26. $-g^3$

27. $7^x - 2x^2$

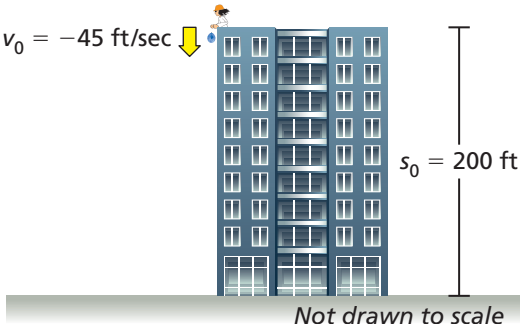
28. $y^{-3} + 1.5$

29. $8k^5 + 4k^3 - k$

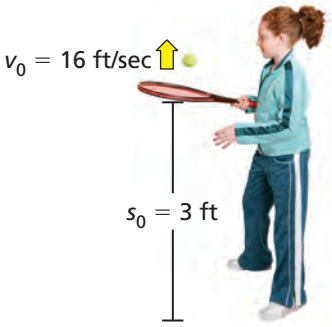
30. LOGIC The polynomial $d^2 - \pi r^2$ represents the area of a region, where d is the diameter of a circle and r is the radius of the circle. How can this happen? Justify your answer with a diagram.

Use the polynomial $-16t^2 + v_0t + s_0$ to write a polynomial that represents the height of the object. Then find the height of the object after 1 second.

31. WATER BALLOON You throw a water balloon from a building.



32. TENNIS You bounce a tennis ball on a racket.



33. Number Sense The polynomial $-w^2 + 28w$ represents the area of a rectangular garden with a width of w feet.

a. Use guess, check, and revise to find the width of the garden with the maximum area. (*Hint:* The width is between 10 feet and 18 feet.)

b. What is the perimeter of the garden?

c. How many seed packets do you need for the garden?

Fair Game Review What you learned in previous grades & lessons

Simplify the expression. *(Skills Review Handbook)*

34. $2x + 4y + 3x + 13$

35. $4x - x + 5y - 7y$

36. $-11 + 5x - 3x + x$

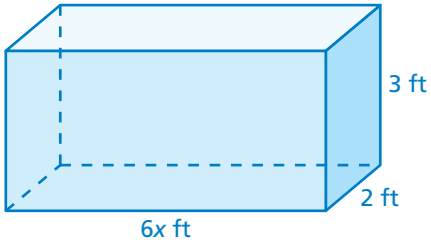
37. MULTIPLE CHOICE What is the surface area of the prism? *(Skills Review Handbook)*

A $11x$ ft

B $(24x + 36)$ ft

C $(36x + 24)$ ft

D $(60x + 12)$ ft

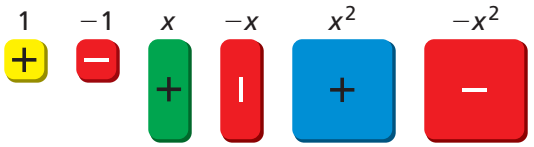


7.2 Adding and Subtracting Polynomials

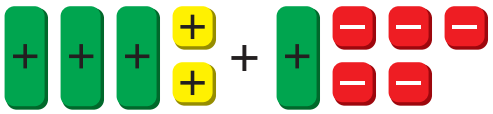
Essential Question How can you add polynomials? How can you subtract polynomials?

1 EXAMPLE: Adding Polynomials Using Algebra Tiles

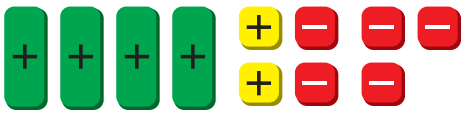
Work with a partner. Six different algebra tiles are shown at the right.



Write the polynomial addition steps shown by the algebra tiles.

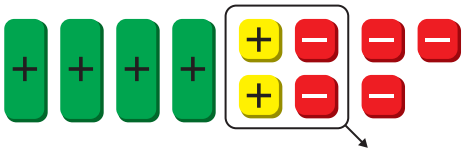


Step 1:



Group like tiles.

Step 2:



Remove zero pairs.

Step 3:

Simplify.

Common Core

Polynomials

In this lesson, you will

- add and subtract polynomials.

Learning Standard A.APR.1

2 ACTIVITY: Adding Polynomials Using Algebra Tiles

Use algebra tiles to find the sum of the polynomials.

a. $(x^2 + 2x - 1) + (2x^2 - 2x + 1)$

b. $(4x + 3) + (x - 2)$

c. $(x^2 + 2) + (3x^2 + 2x + 5)$

d. $(2x^2 - 3x) + (x^2 - 2x + 4)$

e. $(x^2 - 3x + 2) + (x^2 + 4x - 1)$

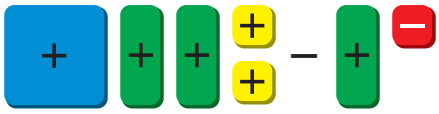
f. $(4x - 3) + (2x + 1) + (-3x + 2)$

g. $(-x^2 + 3x) + (2x^2 - 2x)$

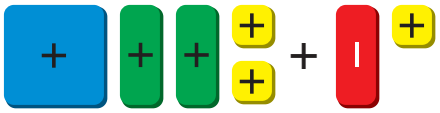
h. $(x^2 + 2x - 5) + (-x^2 - 2x + 5)$

3 EXAMPLE: Subtracting Polynomials Using Algebra Tiles

Write the polynomial subtraction steps shown by the algebra tiles.



Step 1:

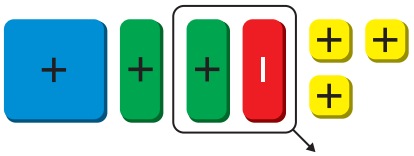


To subtract, add the opposite.

Step 2:

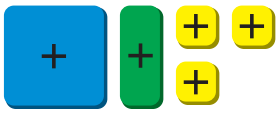
Group like tiles.

Step 3:



Remove zero pairs.

Step 4:



Simplify.

Math Practice 7

View as Components

How can you use algebra tiles to represent the sums and differences of polynomials?

4 ACTIVITY: Subtracting Polynomials Using Algebra Tiles

Use algebra tiles to find the difference of the polynomials.

a. $(x^2 + 2x - 1) - (2x^2 - 2x + 1)$

b. $(4x + 3) - (x - 2)$

c. $(x^2 + 2) - (3x^2 + 2x + 5)$

d. $(2x^2 - 3x) - (x^2 - 2x + 4)$

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you add polynomials? Use the results of Activity 2 to summarize a procedure for adding polynomials without using algebra tiles.

6. **IN YOUR OWN WORDS** How can you subtract polynomials? Use the results of Activity 4 to summarize a procedure for subtracting polynomials without using algebra tiles.

Practice Use what you learned about adding and subtracting polynomials to complete Exercises 3 and 4 on page 338.

7.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

You can add polynomials using a vertical or horizontal method to combine like terms.

EXAMPLE 1 Adding Polynomials

Find each sum.

a. $(3a^2 + 8) + (5a - 1)$ **b.** $(-x^2 + 5x + 4) + (3x^2 - 8x + 9)$

a. Vertical method: Align like terms vertically and add.

$$\begin{array}{rr} & 3a^2 \quad + 8 \\ + & \quad 5a - 1 \\ \hline & 3a^2 + 5a + 7 \end{array}$$

Leave a space for the missing term.

b. Horizontal method: Group like terms and simplify.

$$\begin{aligned}(-x^2 + 5x + 4) + (3x^2 - 8x + 9) &= (-x^2 + 3x^2) + [5x + (-8x)] + (4 + 9) \\ &= 2x^2 - 3x + 13\end{aligned}$$

To subtract one polynomial from another polynomial, add the opposite.

EXAMPLE 2 Subtracting Polynomials

Find each difference.

a. $(y^2 + 4y + 2) - (2y^2 - 5y - 3)$ **b.** $(5x^2 + 4x - 1) - (2x^2 - 6)$

Study Tip

You can think of finding the opposite of a polynomial as finding the opposite of each term's coefficient.

a. Use the vertical method.

$$\begin{array}{rr} & (y^2 + 4y + 2) \\ - & (2y^2 - 5y - 3) \\ \hline & \end{array} \quad \text{Add the opposite.} \quad \begin{array}{rr} & y^2 + 4y + 2 \\ + & -2y^2 + 5y + 3 \\ \hline & -y^2 + 9y + 5 \end{array}$$

b. Use the horizontal method.

$$\begin{aligned}(5x^2 + 4x - 1) - (2x^2 - 6) &= (5x^2 + 4x - 1) + (-2x^2 + 6) \\ &= [5x^2 + (-2x^2)] + 4x + (-1 + 6) \\ &= 3x^2 + 4x + 5\end{aligned}$$

On Your Own

Now You're Ready
Exercises 5–10 and 12–17

Find the sum or difference.

1. $(b - 10) + (4b - 3)$
2. $(x^2 - x - 2) + (7x^2 - x)$
3. $(p^2 + p + 3) - (-4p^2 - p + 3)$
4. $(-k + 5) - (3k^2 - 6)$

EXAMPLE 3 Adding Polynomials

Which polynomial represents the sum of $x^2 - 2xy - y^2$ and $x^2 + xy + y^2$?

(A) $-3xy$ (B) $-3xy - 2y^2$ (C) $2x^2 - xy$ (D) $2x^2 + 3xy + 2y^2$

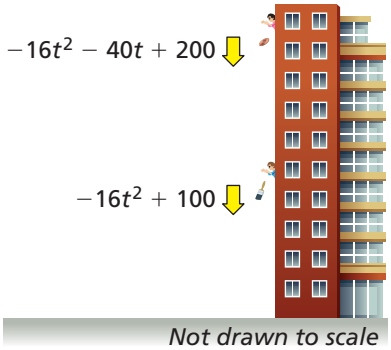
Use the horizontal method to find the sum.

$$(x^2 - 2xy - y^2) + (x^2 + xy + y^2) = (x^2 + x^2) + (-2xy + xy) + (-y^2 + y^2)$$
$$= 2x^2 - xy$$

∴ The correct answer is (C).

EXAMPLE 4 Real-Life Application

A penny is thrown straight downward from a height of 200 feet. At the same time, a paintbrush falls from a height of 100 feet. The polynomials represent the heights (in feet) of the objects after t seconds.



a. Write a polynomial that represents the distance between the penny and the paintbrush after t seconds.

To find the distance between the objects after t seconds, subtract the polynomials.

Penny	*Paintbrush*

$$(-16t^2 - 40t + 200) - (-16t^2 + 100)$$
$$= (-16t^2 - 40t + 200) + (16t^2 - 100)$$
$$= (-16t^2 + 16t^2) - 40t + [200 + (-100)]$$
$$= -40t + 100$$

∴ The polynomial $-40t + 100$ represents the distance between the objects after t seconds.

b. What is the distance between the objects after 2 seconds?

Find the value of $-40t + 100$ when $t = 2$.

$$-40t + 100 = -40(2) + 100 \quad \text{Substitute 2 for } t.$$
$$= 20 \quad \text{Simplify.}$$

∴ After 2 seconds, the distance between the objects is 20 feet.

Study Tip

To check your answer, substitute 2 into the original polynomials and verify that the difference of the heights is 20.

On Your Own

Now You're Ready
Exercises 20–23

5. In Example 3, which polynomial represents the difference of the two polynomials?
6. In Example 4, the polynomial $-16t^2 - 25t + 200$ represents the height of the penny after t seconds. What is the distance between the objects after 1 second?

7.2 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **WRITING** How do you add $(4x^2 - 3 + 2y^3)$ and $(-6x^2 - 15)$ using a vertical method? a horizontal method?
2. **REASONING** Describe how subtracting polynomials is similar to subtracting integers.

Practice and Problem Solving

Use algebra tiles to find the sum or difference of the polynomials.

3. $(x^2 - 3x + 2) + (x^2 + 4x - 1)$
4. $(x^2 + 2x - 5) - (-x^2 - 2x + 5)$

Find the sum.

1 5. $(5y + 4) + (-2y + 6)$
6. $(3g^2 - g) + (3g^2 - 8g + 4)$
7. $(2n^2 - 5n - 6) + (-n^2 - 3n + 11)$
8. $(-3p^2 + 5p - 2) + (-p^2 - 8p - 15)$
9. $(-a^3 + 4a - 3) + (5a^3 - a)$
10. $\left(-s^2 - \frac{2}{9}s + 1\right) + \left(-\frac{5}{9}s - 4\right)$
11. **ERROR ANALYSIS** Describe and correct the error in finding the sum of the polynomials.

✗
$$\begin{array}{r} -5x^2 + 1 \\ +\ 2x - 8 \\ \hline -3x - 7 \end{array}$$

Find the difference.

2 12. $(d^2 - 9) - (3d - 1)$
13. $(k^2 - 7k + 2) - (k^2 - 12)$
14. $(x^2 - 4x + 9) - (3x^2 - 6x - 7)$
15. $(-r - 10) - (-4r^2 + r + 7)$
16. $(t^4 - t^2 + t) - (-9t^2 + 7t - 12)$
17. $\left(\frac{1}{6}q^2 + \frac{2}{3}\right) - \left(\frac{1}{12}q^2 - \frac{1}{3}\right)$
18. **ERROR ANALYSIS** Describe and correct the error in finding the difference of the polynomials.

✗
$$\begin{aligned} (x^2 - 5x) - (-3x^2 + 2x) &= (x^2 - 5x) + (3x^2 + 2x) \\ &= (x^2 + 3x^2) + (-5x + 2x) \\ &= 4x^2 - 3x \end{aligned}$$

19. **COST** The cost (in dollars) of making b bracelets is represented by $4 + 5b$. The cost (in dollars) of making b necklaces is $8b + 6$. Write a polynomial that represents how much more it costs to make b necklaces than b bracelets.

Find the sum or difference.

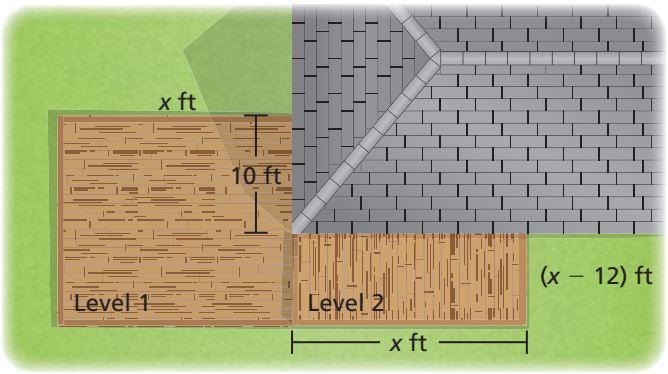
20. $(c^2 - 6d^2) + (c^2 - 2cd + 2d^2)$

21. $(-x^2 + 9xy) - (x^2 + 6xy - 8y^2)$

22. $(2s^2 - 5st - t^2) - (s^2 + 7st - t^2)$

23. $(a^2 - 3ab + 2b^2) + (-4a^2 + 5ab - b^2)$

24. MODELING You are building a multi-level deck.



a. Write a polynomial that represents the area of each level.

b. Write a polynomial that represents the total area of the deck.

c. What is the total area of the deck when $x = 20$?

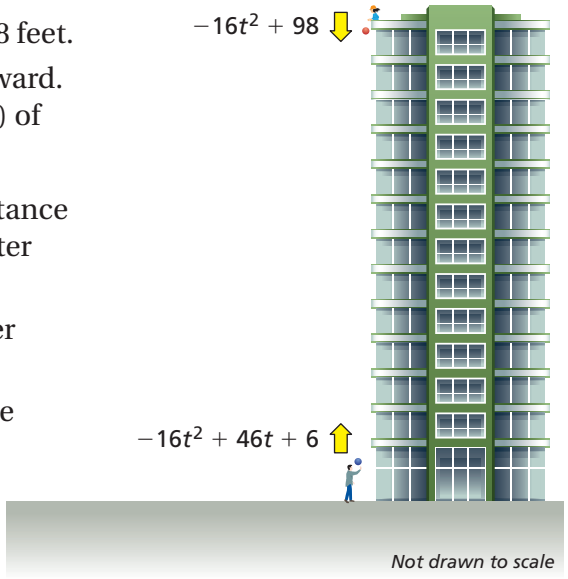
d. A gallon of deck sealant covers 400 square feet. How many gallons of sealant do you need to cover the deck once? Explain.

25. Problem Solving You drop a ball from a height of 98 feet. At the same time, your friend throws a ball upward. The polynomials represent the heights (in feet) of the balls after t seconds.

a. Write a polynomial that represents the distance between your ball and your friend's ball after t seconds.

b. What is the distance between the balls after 1.5 seconds?

c. After how many seconds are the balls at the same height? How far are they from the ground? Explain your reasoning.



Fair Game Review What you learned in previous grades & lessons

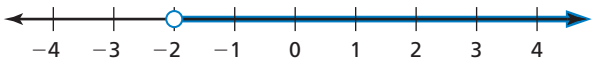
Simplify the expression. *(Skills Review Handbook)*

26. $2(x - 1) + 3(x + 2)$

27. $(4y - 3) - 2(y - 5)$

28. $-5(2w + 1) - 3(-4w + 2)$

29. MULTIPLE CHOICE Which inequality is represented by the graph? *(Section 3.1)*



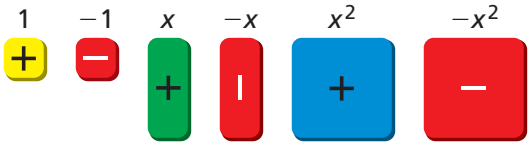
(A) $x < -2$ (B) $x > -2$ (C) $x \leq -2$ (D) $x \geq -2$

7.3 Multiplying Polynomials

Essential Question How can you multiply two binomials?

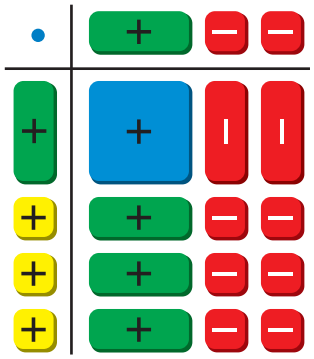
1 ACTIVITY: Multiplying Binomials Using Algebra Tiles

Work with a partner. Six different algebra tiles are shown below.

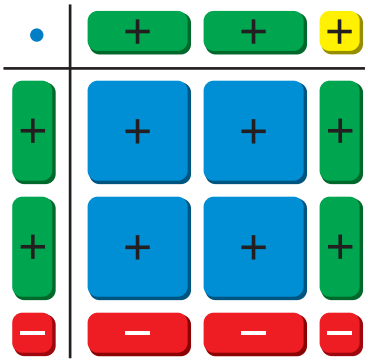


Write the product of the two binomials shown by the algebra tiles.

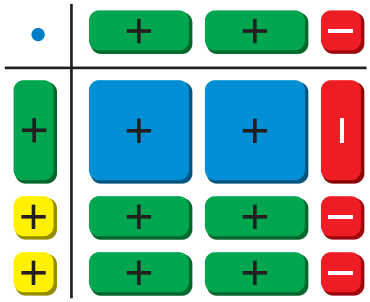
a. $(x + 3)(x - 2) =$ _____



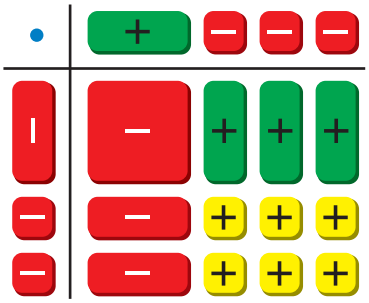
b. $(2x - 1)(2x + 1) =$ _____



c. $(x + 2)(2x - 1) =$ _____



d. $(-x - 2)(x - 3) =$ _____



Common Core

Polynomials

In this lesson, you will

- multiply binomials using the Distributive Property, a table, or the FOIL method.
- multiply binomials and trinomials.

Learning Standard
A.APR.1

2 ACTIVITY: Multiplying Monomials Using Algebra Tiles

Math Practice 4

Use a Diagram

How can you represent the product of polynomials using diagrams?

Work with a partner. Write each product. Explain your reasoning.

a. $+ \cdot + =$ ☐

b. $+ \cdot - =$ ☐

c. $- \cdot - =$ ☐

d. $+ \cdot + =$ ☐

e. $+ \cdot - =$ ☐

f. $- \cdot + =$ ☐

g. $- \cdot - =$ ☐

h. $+ \cdot + =$ ☐

i. $+ \cdot - =$ ☐

j. $- \cdot - =$ ☐

3 ACTIVITY: Multiplying Binomials Using Algebra Tiles

Use algebra tiles to find each product.

a. $(2x - 2)(2x + 1)$

b. $(4x + 3)(x - 2)$

c. $(-x + 2)(2x + 2)$

d. $(2x - 3)(x + 4)$

e. $(3x + 2)(-x - 1)$

f. $(2x + 1)(-3x + 2)$

g. $(x - 2)^2$

h. $(2x - 3)^2$

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you multiply two binomials? Use the results of Activity 3 to summarize a procedure for multiplying binomials without using algebra tiles.

5. Find two binomials with the given product.

   a. $x^2 - 3x + 2$

   b. $x^2 - 4x + 4$

Practice Use what you learned about multiplying binomials to complete Exercises 3 and 4 on page 345.

7.3 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
FOIL Method, *p. 343*

In Section 1.2, you used the Distributive Property to multiply a binomial by a monomial. You can also use the Distributive Property to multiply two binomials.

EXAMPLE 1 Multiplying Binomials Using the Distributive Property

Find each product.

a. $(x + 2)(x + 5)$

Use the horizontal method.

$(x + 2)(x + 5) = x(x + 5) + 2(x + 5)$ — Distribute $(x + 5)$ to each term of $(x + 2)$.

$= x(x) + x(5) + 2(x) + 2(5)$ — Distributive Property

$= x^2 + 5x + 2x + 10$ — Multiply.

$= x^2 + 7x + 10$ — Combine like terms.

b. $(x + 3)(x - 4)$

Use the vertical method.

$$\begin{array}{rl} & x + 3 \\ \times & x - 4 \\ \hline & -4x - 12 \\ & x^2 + 3x \\ \hline & x^2 - x - 12 \end{array}$$

Align like terms vertically.

Multiply $-4(x + 3)$. → $-4x - 12$ — Distributive Property

Multiply $x(x + 3)$. → $x^2 + 3x$ — Distributive Property

$x^2 - x - 12$ — Combine like terms.

∴ The product is $x^2 - x - 12$.

EXAMPLE 2 Multiplying Binomials Using a Table

Find $(2x - 3)(x + 5)$.

Step 1: Write each binomial as a sum of terms.

$(2x - 3)(x + 5) = [2x + (-3)](x + 5)$

Step 2: Make a table of products.

	$2x$	-3
x	$2x^2$	$-3x$
5	$10x$	-15

∴ The product is $2x^2 - 3x + 10x - 15$, or $2x^2 + 7x - 15$.

On Your Own

Now You're Ready
Exercises 5–13 and 16–21

Use the Distributive Property to find the product.

1. $(y + 4)(y + 1)$
2. $(z - 2)(z + 6)$

Use a table to find the product.

3. $(p + 3)(p - 8)$
4. $(r - 5)(2r - 1)$

The **FOIL Method** is a shortcut for multiplying two binomials.

Key Idea

FOIL Method

To multiply two binomials using the FOIL Method, find the sum of the products of the

First terms, $(x + 1)(x + 2)$ → $x(x) = x^2$

Outer terms, $(x + 1)(x + 2)$ → $x(2) = 2x$

Inner terms, and $(x + 1)(x + 2)$ → $1(x) = x$

Last terms. $(x + 1)(x + 2)$ → $1(2) = 2$

$$(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

EXAMPLE 3 Multiplying Binomials Using the FOIL Method

Find each product.

a. $(x - 3)(x - 6)$

	First Outer Inner Last	
$(x - 3)(x - 6)$	$= x(x) + x(-6) + (-3)(x) + (-3)(-6)$	Use the FOIL Method.
	$= x^2 + (-6x) + (-3x) + 18$	Multiply.
	$= x^2 - 9x + 18$	Combine like terms.

b. $(2x + 1)(3x - 5)$

	First Outer Inner Last	
$(2x + 1)(3x - 5)$	$= 2x(3x) + 2x(-5) + 1(3x) + 1(-5)$	Use the FOIL Method.
	$= 6x^2 + (-10x) + 3x + (-5)$	Multiply.
	$= 6x^2 - 7x - 5$	Combine like terms.

On Your Own

Now You're Ready
Exercises 22–30

Use the FOIL Method to find the product.

5. $(m + 5)(m - 6)$

6. $(x - 4)(x + 2)$

7. $(k + 5)(6k + 3)$

8. $\left(2u + \frac{1}{2}\right)\left(u - \frac{3}{2}\right)$

EXAMPLE 4 Multiplying a Binomial and a Trinomial

Find $(x + 5)(x^2 - 3x - 2)$.

$$\begin{array}{rl} & x^2 - 3x - 2 \\ \times & \quad x + 5 \\ \hline & 5x^2 - 15x - 10 \\ & x^3 - 3x^2 - 2x \\ \hline & x^3 + 2x^2 - 17x - 10 \end{array}$$

Align like terms vertically.

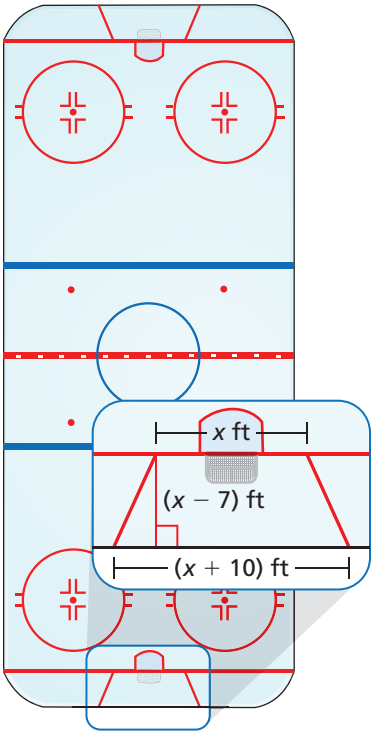
Multiply $5(x^2 - 3x - 2)$. → Distributive Property

Multiply $x(x^2 - 3x - 2)$. → Distributive Property

Combine like terms.

∴ The product is $x^3 + 2x^2 - 17x - 10$.

EXAMPLE 5 Real-Life Application



In hockey, a goalie behind the goal line can only play a puck in a trapezoidal region.

a. Write a polynomial that represents the area of the trapezoidal region.

$$\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(x - 7)[x + (x + 10)]$$ Substitute.

$$= \frac{1}{2}(x - 7)(2x + 10)$$ Combine like terms.

F O I L

$$= \frac{1}{2}[2x^2 + 10x + (-14x) + (-70)]$$ Use the FOIL Method.

$$= \frac{1}{2}(2x^2 - 4x - 70)$$ Combine like terms.

$$= x^2 - 2x - 35$$ Distributive Property

b. Find the area of the trapezoidal region when the shorter base is 18 feet.

Find the value of $x^2 - 2x - 35$ when $x = 18$.

$$x^2 - 2x - 35 = 18^2 - 2(18) - 35$$ Substitute 18 for x.

$$= 324 - 36 - 35$$ Simplify.

$$= 253$$ Subtract.

∴ The area of the trapezoidal region is 253 square feet.

On Your Own

Now You're Ready
Exercises 40–45

Find the product.

9. $(x + 1)(x^2 + 5x + 8)$ **10.** $(n - 3)(n^2 - 2n + 4)$

11. WHAT IF? How does the polynomial in Example 5 change if the longer base is extended by 1 foot? Explain.

7.3 Exercises

Check It Out
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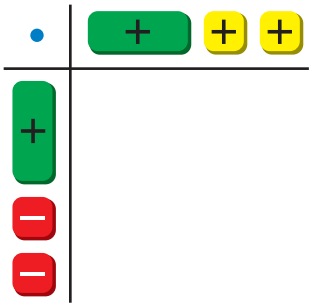
Vocabulary and Concept Check

1. **VOCABULARY** Describe two ways to find the product of two binomials.
2. **WRITING** Explain how the letters of the word FOIL can help you remember how to multiply two binomials.

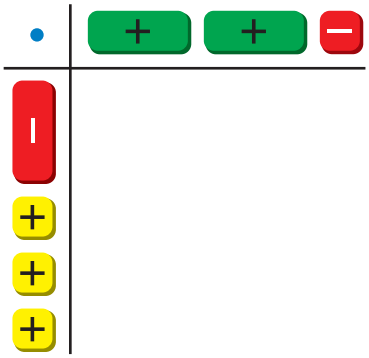
Practice and Problem Solving

Write the product of the two binomials shown by the algebra tiles.

3. $(x - 2)(x + 2) =$



4. $(-x + 3)(2x - 1) =$



Use the Distributive Property to find the product.

5. $(x + 1)(x + 3)$
6. $(y + 6)(y + 4)$
7. $(z - 5)(z + 3)$
8. $(a + 8)(a - 3)$
9. $(g - 7)(g - 2)$
10. $(n - 6)(n - 4)$
11. $(3m + 1)(m + 9)$
12. $(2p - 4)(3p + 2)$
13. $(6 - 5s)(2 - s)$

14. **ERROR ANALYSIS** Describe and correct the error in finding the product.

$$(t - 2)(t + 5) = t - 2(t + 5)$$
$$= t - 2t - 10$$
$$= -t - 10$$

15. **CALCULATOR** The width of a calculator can be represented by $(3x + 1)$ inches. The length of the calculator is twice the width. Write a polynomial that represents the area of the calculator.

Use a table to find the product.

16. $(x + 3)(x + 1)$ **17.** $(y + 10)(y - 5)$ **18.** $(h - 8)(h - 9)$

19. $(-3 + 2j)(4j - 7)$ **20.** $(5c + 6)(6c + 5)$ **21.** $(5d - 12)(-7 + 3d)$

Use the FOIL Method to find the product.

22. $(b + 3)(b + 7)$ **23.** $(w + 9)(w + 6)$ **24.** $(k + 5)(k - 1)$

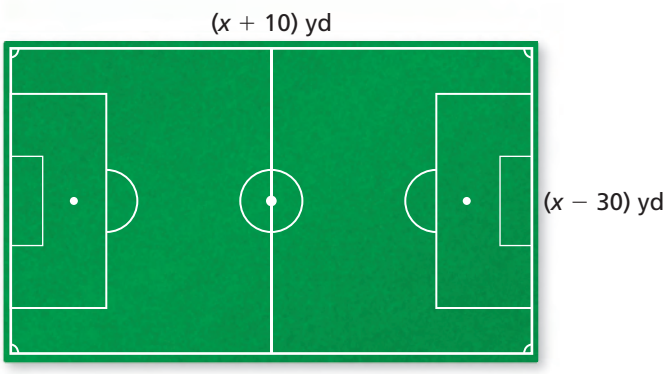
25. $(x - 4)(x + 8)$ **26.** $(q - 3)(q - 4)$ **27.** $(z - 5)(z - 9)$

28. $(t + 2)(2t + 1)$ **29.** $(5v - 3)(2v + 4)$ **30.** $(9 - r)(2 - 3r)$

31. ERROR ANALYSIS Describe and correct the error in finding the product.

$$(r + 6)(r - 7) = r(r) + r(7) + 6(r) + 6(7)$$
$$= r^2 + 7r + 6r + 42$$
$$= r^2 + 13r + 42$$

32. OPEN-ENDED Write two binomials whose product includes the term 12.



33. SOCCER The soccer field is rectangular.

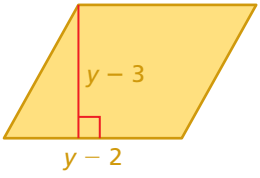
a. Write a polynomial that represents the area of the soccer field.

b. Use the polynomial in part (a) to find the area of the field when $x = 90$.

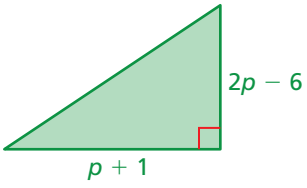
c. A groundskeeper mows 200 square yards in 3 minutes. How long does it take the groundskeeper to mow the field?

Write a polynomial that represents the area of the shaded region.

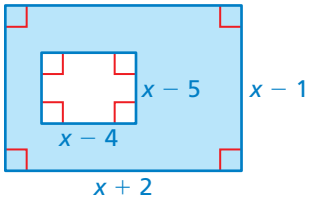
34.



35.



36.



Find the product.

37. $(n + 3)(2n^2 + 1)$ **38.** $(x + y)(2x - y)$ **39.** $(2r + s)(r - 3s)$

40. $(x - 4)(x^2 - 3x + 2)$ **41.** $(f^2 + 4f - 8)(f - 1)$ **42.** $(3 + i)(i^2 + 8i - 2)$

43. $(t^2 - 5t + 1)(-3 + t)$ **44.** $(b - 4)(5b^2 - 5b + 4)$ **45.** $(3e^2 - 5e + 7)(6e + 1)$

46. REASONING Can you use the FOIL method to multiply a binomial by a trinomial? a trinomial by a trinomial? Explain your reasoning.

47. AMUSEMENT PARK You go to an amusement park $(x + 1)$ times each year and pay $(x + 40)$ dollars each time, where x is the number of years after 2011.

a. Write a polynomial that represents your yearly admission cost.

b. What is your yearly admission cost in 2013?

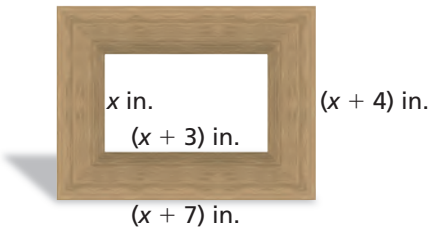
48. PRECISION You use the Distributive Property to multiply $(x + 3)(x - 5)$. Your friend uses the FOIL Method to multiply $(x - 5)(x + 3)$. Should your answers be equivalent? Justify your answer.

49. REASONING The product of $(x + m)(x + n)$ is $x^2 + bx + c$.

a. What do you know about m and n when $c < 0$?

b. What do you know about m and n when $c > 0$?

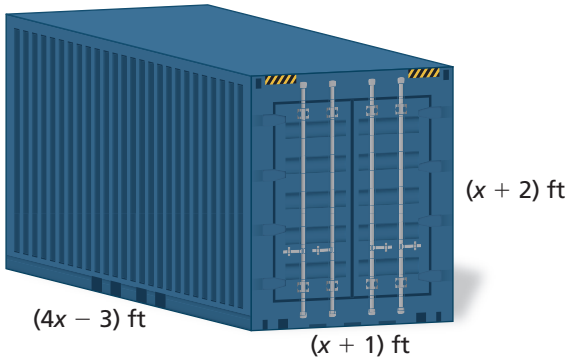
50. PICTURE You design the wooden picture frame and paint the front surface.



a. Write a polynomial that represents the area of wood you paint.

b. You design the picture frame to display a 5-inch by 8-inch photograph. How much wood do you paint?

51. Number Sense The shipping container is a rectangular prism. Write a polynomial that represents the volume of the container.



Fair Game Review What you learned in previous grades & lessons

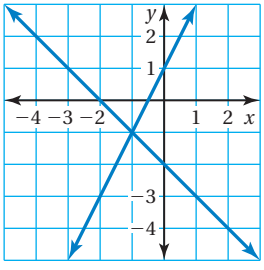
Write the polynomial in standard form. Identify the degree and classify the polynomial by the number of terms. *(Section 7.1)*

52. $2x - 5x^2 - x^3$

53. $z^2 - \frac{5}{7}z$

54. $-15y^7$

55. MULTIPLE CHOICE Which system of linear equations does the graph represent? *(Section 4.1)*



(A) $y = 3x + 4$, $y = -2x - 6$

(B) $y = 2x + 1$, $y = -x - 2$

(C) $y = -x + 7$, $y = 4x - 8$

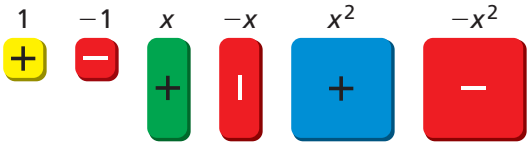
(D) $y = x + 10$, $y = -3x + 2$

7.4 Special Products of Polynomials

Essential Question What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$?

1 ACTIVITY: Finding a Sum and Difference Pattern

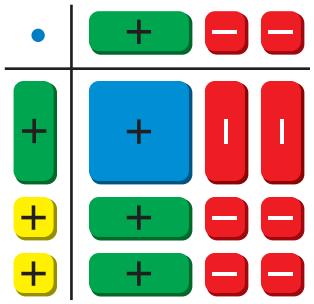
Work with a partner. Six different algebra tiles are shown below.

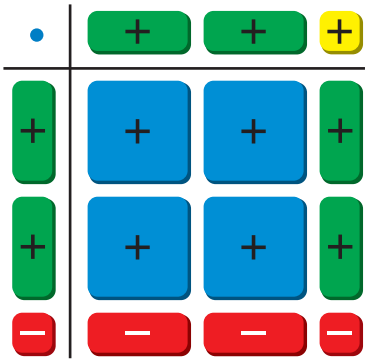


Write the product of the two binomials shown by the algebra tiles.

a. $(x + 2)(x - 2) =$ _____

b. $(2x - 1)(2x + 1) =$ _____





2 ACTIVITY: Describing a Sum and Difference Pattern

Work with a partner.

a. Describe the pattern for the special product: $(a + b)(a - b)$.

b. Use the pattern you described to find each product. Check your answers using algebra tiles.

i. $(x + 3)(x - 3)$ **ii.** $(x - 4)(x + 4)$ **iii.** $(3x + 1)(3x - 1)$

iv. $(3y + 4)(3y - 4)$ **v.** $(2x - 5)(2x + 5)$ **vi.** $(z + 1)(z - 1)$

COMMON CORE

Polynomials

In this lesson, you will

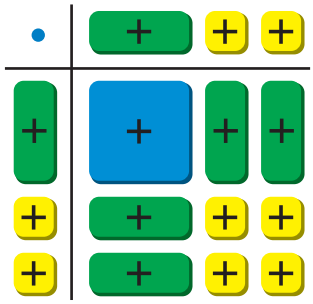
- use patterns to multiply polynomials.

Learning Standard A.APR.1

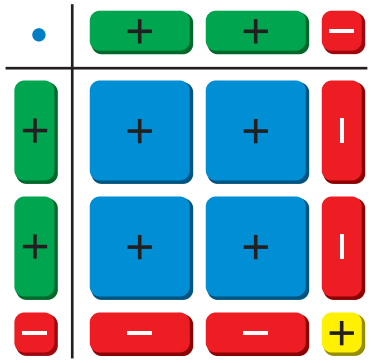
3 ACTIVITY: Finding the Square of a Binomial Pattern

Write the product of the two binomials shown by the algebra tiles.

a. $(x + 2)^2 =$ _____



b. $(2x - 1)^2 =$ _____



4 ACTIVITY: Describing the Square of a Binomial Pattern

Work with a partner.

a. Describe the pattern for the special product: $(a + b)^2$.

b. Describe the pattern for the special product: $(a - b)^2$.

c. Use the patterns you described to find each product. Check your answers using algebra tiles.

i. $(x + 3)^2$ **ii.** $(x - 2)^2$ **iii.** $(3x + 1)^2$

iv. $(3y + 4)^2$ **v.** $(2x - 5)^2$ **vi.** $(z + 1)^2$

Math Practice 8

Find General Methods

What did the products of the binomials in the previous activity have in common? How does this help in describing the pattern?

What Is Your Answer?

5. IN YOUR OWN WORDS What are the patterns in the special products $(a + b)(a - b)$, $(a + b)^2$, and $(a - b)^2$? Use the results of Activities 2 and 4 to write formulas for these special products.

Practice Use what you learned about the patterns in special products to complete Exercises 3–5 on page 352.

7.4 Lesson

Check It Out
Lesson Tutorials
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Some pairs of binomials show patterns when multiplied. You can use these patterns to multiply other similar pairs of binomials.

Study Tip

Because multiplication is commutative, the pattern also applies to $(a - b)(a + b)$.

Key Idea

Sum and Difference Pattern

Algebra

$(a + b)(a - b) = a^2 - b^2$

Example

$(x + 3)(x - 3) = x^2 - 3^2$
$= x^2 - 9$

EXAMPLE 1 Using the Sum and Difference Pattern

Find each product.

a. $(x + 7)(x - 7)$

$(a + b)(a - b) = a^2 - b^2$ Sum and Difference Pattern

$(x + 7)(x - 7) = x^2 - 7^2$ Use pattern.

$= x^2 - 49$ Simplify.

b. $(3x - 1)(3x + 1)$

$(a - b)(a + b) = a^2 - b^2$ Sum and Difference Pattern

$(3x - 1)(3x + 1) = (3x)^2 - 1^2$ Use pattern.

$= 9x^2 - 1$ Simplify.

Check

Use the FOIL Method.

$(3x - 1)(3x + 1)$

$= 9x^2 + 3x - 3x - 1$

$= 9x^2 - 1$ ✓

On Your Own

Now You're Ready
Exercises 6–14

Find the product.

1. $(x - 4)(x + 4)$ **2.** $(b + 10)(b - 10)$ **3.** $(2g + 5)(2g - 5)$

Key Idea

Square of a Binomial Pattern

Algebra

$(a + b)^2 = a^2 + 2ab + b^2$

Example

$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$
$= x^2 + 6x + 9$

$(a - b)^2 = a^2 - 2ab + b^2$

$(x - 3)^2 = x^2 - 2(x)(3) + 3^2$
$= x^2 - 6x + 9$

EXAMPLE 2 Using the Square of a Binomial Pattern

Find each product.

a. $(y + 1)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ Square of a Binomial Pattern

$(y + 1)^2 = y^2 + 2(y)(1) + 1^2$ Use pattern.

$= y^2 + 2y + 1$ Simplify.

Check

Use the FOIL Method.

$(2z - 3)^2 = (2z - 3)(2z - 3)$

$= 4z^2 - 6z - 6z + 9$

$= 4z^2 - 12z + 9$ ✓

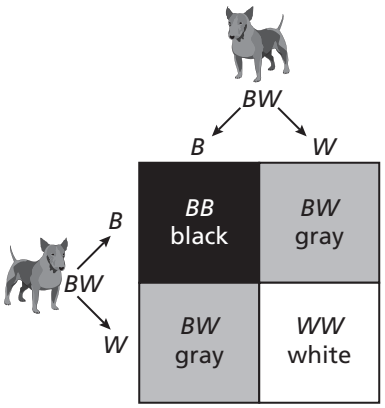
b. $(2z - 3)^2$

$(a - b)^2 = a^2 - 2ab + b^2$ Square of a Binomial Pattern

$(2z - 3)^2 = (2z)^2 - 2(2z)(3) + 3^2$ Use pattern.

$= 4z^2 - 12z + 9$ Simplify.

EXAMPLE 3 Real-Life Application



A diagram that models possible gene combinations in offspring is called a Punnett square.

Each of two dogs has one black gene (B) and one white gene (W). The diagram shows the possible gene combinations of an offspring and the resulting colors.

a. What percent of the possible gene combinations result in black?

Use the diagram. One of the four possible gene combinations results in black.

∴ So, $\frac{1}{4}$ or 25% of the possible gene combinations result in black.

b. The genetic makeup of an offspring can be modeled by $(0.5B + 0.5W)^2$. Use the square of a binomial pattern to model the possible gene combinations of an offspring.

$(a + b)^2 = a^2 + 2ab + b^2$ Square of a Binomial Pattern

$(0.5B + 0.5W)^2 = (0.5B)^2 + 2(0.5B)(0.5W) + (0.5W)^2$ Use pattern.

$= 0.25B^2 + 0.5BW + 0.25W^2$ Simplify.

- $0.25B^2$: 25% *BB* (black)
- $0.5BW$: 50% *BW* (gray)
- $0.25W^2$: 25% *WW* (white)

On Your Own

Now You're Ready
Exercises 16–24

Find the product.

4. $(w + 2)^2$ **5.** $(x - 7)^2$ **6.** $(3y - 1)^2$ **7.** $(5z + 4)^2$

7.4 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **OPEN-ENDED** Write two binomials whose product can be found using the sum and difference pattern.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$(x + 1)(x - 1)$ $(3x + 2)(3x - 2)$ $(x + 2)(x - 3)$ $(2x + 5)(2x - 5)$

Practice and Problem Solving

Use algebra tiles to find the product.

3. $(x + 6)(x - 6)$
4. $(3y - 2)(3y + 2)$
5. $(2z + 2)^2$

Find the product.

1 6. $(x + 2)(x - 2)$
7. $(g - 5)(g + 5)$
8. $(z - 8)(z + 8)$
9. $(b + 12)(b - 12)$
10. $(2x + 1)(2x - 1)$
11. $(3x - 4)(3x + 4)$
12. $(6x + 7)(6x - 7)$
13. $(9 - c)(9 + c)$
14. $(8 - 3m)(8 + 3m)$

15. **REASONING** Write two binomials whose product is $x^2 - 16$. Explain how you found your answer.

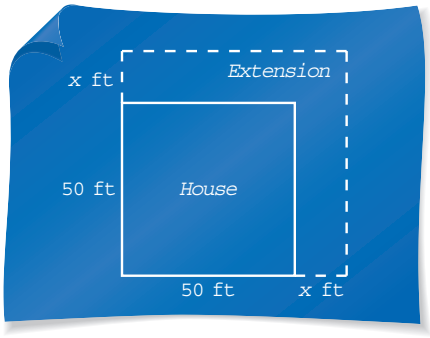
Find the product.

2 16. $(b - 2)^2$
17. $(y + 8)^2$
18. $(n + 6)^2$
19. $(d - 10)^2$
20. $(2f - 1)^2$
21. $(5p + 2)^2$
22. $(4b - 5)^2$
23. $(12 - x)^2$
24. $(4 + 7t)^2$

ERROR ANALYSIS **Describe and correct the error in finding the product.**

25. ✗ $(k + 4)^2 = k^2 + 4^2$
$= k^2 + 16$

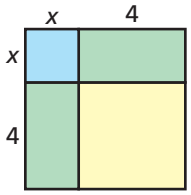
26. ✗ $(s + 5)(s - 5)$
$= s^2 + 2(s)(5) - 5^2$
$= s^2 + 10s - 25$



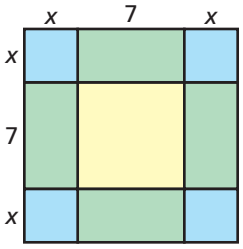
27. **CONSTRUCTION** A contractor extends a house on two sides.
   a. The area of the first level of the house after the renovation is represented by $(x + 50)^2$. Find this product.
   b. Use the polynomial in part (a) to find the area of the first level when $x = 15$. What is the area of the extension?

Write a polynomial that represents the area of the figure.

28.



29.

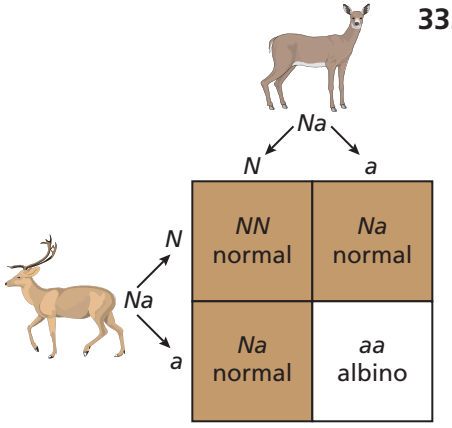


Find the product.

30. $(x^2 + 1)(x^2 - 1)$

31. $(x + y)(x - y)$

32. $(2x - y)^2$

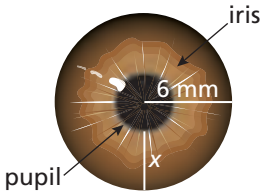


33. GENETICS In deer, the gene N is for normal coloring and the gene a is for no coloring, or albino. Any gene combination with an N results in normal coloring. The diagram shows the possible gene combinations of an offspring and the resulting colors from parents that both have the gene combination Na.

a. What percent of the possible gene combinations result in normal coloring?

b. The genetic makeup of an offspring can be modeled by $(0.5N + 0.5a)^2$. Use the square of a binomial pattern to model the possible gene combinations of an offspring.

34. VISION Your iris controls the amount of light that enters your eye by changing the size of your pupil.



a. Write a polynomial that represents the area of your pupil. Write your answer in terms of π.

b. The width x of your iris decreases from 4 millimeters to 2 millimeters when you enter a dark room. How many times greater is the area of your pupil after entering the room than before entering the room? Explain.

35. Repeated Reasoning Find $(x + 1)^3$ and $(x + 2)^3$. Find a pattern in the terms and use it to write a pattern for the cube of a binomial $(a + b)^3$.

Fair Game Review What you learned in previous grades & lessons

Find the product. *(Section 7.3)*

36. $(x + 4)(x + 9)$

37. $(y - 7)(y + 3)$

38. $(z - 10)(z - 1)$

39. MULTIPLE CHOICE What is the solution of the linear system? *(Section 4.2)*

$$y = 2x - 5$$
$$3x - 8y = 1$$

Ⓐ $(-3, -1)$ Ⓑ $(-3, 1)$

Ⓒ $(3, -1)$ Ⓓ $(3, 1)$

7 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **idea and examples chart** to organize information about a concept. Here is an example of an idea and examples chart for using the FOIL Method to multiply binomials.

FOIL Method: To multiply two binomials using the FOIL Method, find the sum of the products of the First terms, Outer terms, Inner terms, and Last terms.

Example

$$\begin{aligned}(x-2)(x+3) &= \underset{\text{First}}{x(x)} + \underset{\text{Outer}}{x(3)} + \underset{\text{Inner}}{(-2)(x)} + \underset{\text{Last}}{(-2)(3)} && \text{Use the FOIL Method.}\\ &= x^2 + (3x) + (-2x) + (-6) && \text{Multiply.}\\ &= x^2 + x - 6 && \text{Combine like terms.}\end{aligned}$$

Example

$$\begin{aligned}(3x-1)(2x-2) &= \underset{\text{First}}{3x(2x)} + \underset{\text{Outer}}{3x(-2)} + \underset{\text{Inner}}{(-1)(2x)} + \underset{\text{Last}}{(-1)(-2)} && \text{Use the FOIL Method.}\\ &= 6x^2 + (-6x) + (-2x) + 2 && \text{Multiply.}\\ &= 6x^2 - 8x + 2 && \text{Combine like terms.}\end{aligned}$$

On Your Own

Make idea and examples charts to help you study these topics.

1. degree of a polynomial
2. adding and subtracting polynomials
3. special products of polynomials

After you complete this chapter, make idea and examples charts for the following topics.

4. factored form of a polynomial
5. factoring polynomials using the GCF
6. factoring polynomials of the form $x^2 + bx + c$
7. factoring polynomials of the form $ax^2 + bx + c$

"I made an idea and examples chart to give my owner ideas for my birthday next week."

7.1–7.4 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Write the polynomial in standard form. Identify the degree and classify the polynomial by the number of terms. *(Section 7.1)*

1. $-8q^3$

2. $-9 + d^2 - 3d$

3. $\frac{2}{3}m^4 - \frac{5}{6}m^6$

4. $-1.3z + 2z^4 + 7.4z^2$

Find the sum or difference. *(Section 7.2)*

5. $(2x^2 + 5) + (-x^2 + 4)$

6. $(-3n^2 + n) - (2n^2 + 7)$

7. $(-p^2 + 4p) - (p^2 - 3p + 15)$

8. $(a^2 - 3ab + b^2) + (-a^2 + ab + b^2)$

Find the product. *(Section 7.3 and Section 7.4)*

9. $(w + 6)(w + 7)$

10. $(y + 9)(y - 3)$

11. $(d - 2)(d - 5)$

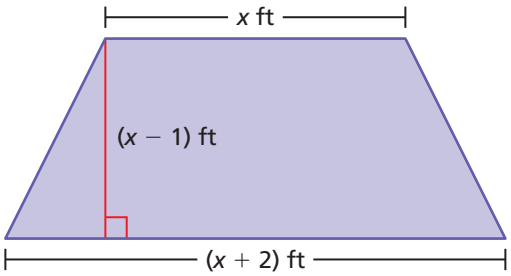
12. $(2z - 3)(3z + 5)$

13. $(h - 1)(h + 1)$

14. $(p + 9)(p - 9)$

15. $(t + 5)^2$

16. $(q - 2)^2$



17. WINDOW SEAT A window seat is in the shape of a trapezoid. *(Section 7.3)*

a. Write a polynomial that represents the area of the window seat.

b. What is the area of the window seat when $x = 3$?

18. COMPOUND INTEREST You are saving for a guitar. You deposit \$100 in an account that earns interest compounded annually. The expression $100(1 + r)^2$ represents the balance after 2 years, where r is the annual interest rate in decimal form. *(Section 7.4)*

a. Write a polynomial that represents the balance of your account.

b. What is the balance of your account when the interest rate is 12%?

c. How much more money do you need to save to buy the guitar?

7.5 Solving Polynomial Equations in Factored Form

Essential Question How can you solve a polynomial equation that is written in factored form?

Two polynomial equations are equivalent when they have the same solutions. For instance, the following equations are equivalent because the only solutions of each equation are $x = 1$ and $x = 2$.

Factored Form	Standard Form	Nonstandard Form
$(x - 1)(x - 2) = 0$	$x^2 - 3x + 2 = 0$	$x^2 - 3x = -2$

✓ Check this by substituting 1 and 2 for x in each equation.

1 ACTIVITY: Matching Equivalent Forms of an Equation

Work with a partner. Match each factored form of the equation with two other forms of equivalent equations. Notice that an equation is considered to be in factored form only when the product of the factors is equal to 0.

	Factored Form		Standard Form		Nonstandard Form
a.	$(x - 1)(x - 3) = 0$	**A.**	$x^2 - x - 2 = 0$	**1.**	$x^2 - 5x = -6$
b.	$(x - 2)(x - 3) = 0$	**B.**	$x^2 + x - 2 = 0$	**2.**	$(x - 1)^2 = 4$
c.	$(x + 1)(x - 2) = 0$	**C.**	$x^2 - 4x + 3 = 0$	**3.**	$x^2 - x = 2$
d.	$(x - 1)(x + 2) = 0$	**D.**	$x^2 - 5x + 6 = 0$	**4.**	$x(x + 1) = 2$
e.	$(x + 1)(x - 3) = 0$	**E.**	$x^2 - 2x - 3 = 0$	**5.**	$x^2 - 4x = -3$

2 ACTIVITY: Writing a Conjecture

Work with a partner. Substitute

1, 2, 3, 4, 5, and 6 for x

in each equation. Write a conjecture describing what you discovered.

a. $(x - 1)(x - 2) = 0$ **b.** $(x - 2)(x - 3) = 0$ **c.** $(x - 3)(x - 4) = 0$

d. $(x - 4)(x - 5) = 0$ **e.** $(x - 5)(x - 6) = 0$ **f.** $(x - 6)(x - 1) = 0$

COMMON CORE

Polynomial Equations

In this lesson, you will

- solve polynomial equations in factored form.

Learning Standard A.REI.4b

3 ACTIVITY: Special Properties of 0 and 1

Work with a partner. The numbers 0 and 1 have special properties that are shared by no other numbers. For each of the following, decide whether the property is true for 0, 1, both, or neither. Explain your reasoning.

a. If you add ____ to a number n, you get n.

b. If the product of two numbers is ____, then one or both numbers are 0.

c. The square of ____ is equal to itself.

d. If you multiply a number n by ____, you get n.

e. If you multiply a number n by ____, you get 0.

f. The opposite of ____ is equal to itself.

4 ACTIVITY: Writing About Solving Equations

Math Practice 3

Use Definitions

What previous examples, information, and definitions can you use to reply to the student's comment?

Work with a partner. Imagine that you are part of a study group in your algebra class. One of the students in the group makes the following comment.

"I don't see why we spend so much time solving equations that are equal to zero. Why don't we spend more time solving equations that are equal to other numbers?"

Write an answer for this student.

What Is Your Answer?

5. One of the properties in Activity 3 is called the Zero-Product Property. It is one of the most important properties in all of algebra. Which property is it? Explain how it is used in algebra and why it is so important.

6. **IN YOUR OWN WORDS** How can you solve a polynomial equation that is written in factored form?

Practice Use what you learned about solving polynomial equations to complete Exercises 4–6 on page 360.

7.5 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
factored form, *p. 358*
Zero-Product Property, *p. 358*
root, *p. 358*

A polynomial is in **factored form** when it is written as a product of factors.

Standard form	*Factored form*
$x^2 + 2x$	$x(x + 2)$
$x^2 + 5x - 24$	$(x - 3)(x + 8)$

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of a polynomial equation are also called **roots**.

Key Idea

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 Solving Polynomial Equations

Solve each equation.

a. $x(x + 8) = 0$

$x(x + 8) = 0$ — Write equation.

$x = 0 \quad or \quad x + 8 = 0$ — Use Zero-Product Property.

$x = -8$ — Solve for x.

∴ The roots are $x = 0$ and $x = -8$.

Check
Substitute each solution in the original equation.

$0(0 + 8) \stackrel{?}{=} 0$

$0(8) \stackrel{?}{=} 0$

$0 = 0$ ✓

$-8(-8 + 8) \stackrel{?}{=} 0$

$-8(0) \stackrel{?}{=} 0$

$0 = 0$ ✓

b. $(x + 6)(x - 5) = 0$

$(x + 6)(x - 5) = 0$ — Write equation.

$x + 6 = 0 \quad or \quad x - 5 = 0$ — Use Zero-Product Property.

$x = -6 \quad or \quad x = 5$ — Solve for x.

∴ The roots are $x = -6$ and $x = 5$.

On Your Own

Now You're Ready
Exercises 4–9

Solve the equation.

1. $x(x - 1) = 0$
2. $3t(t + 2) = 0$
3. $(z - 4)(z - 6) = 0$
4. $(b + 7)^2 = 0$

EXAMPLE 2 Solving a Polynomial Equation

What are the solutions of $(2a + 7)(2a - 7) = 0$?

Ⓐ -7 and 7

Ⓑ $-\frac{7}{2}$ and $\frac{7}{2}$

Ⓒ -2 and 2

Ⓓ $-\frac{2}{7}$ and $\frac{2}{7}$

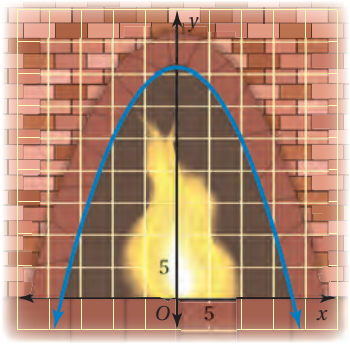
$(2a + 7)(2a - 7) = 0$ Write equation.

$2a + 7 = 0$ *or* $2a - 7 = 0$ Use Zero-Product Property.

$a = -\frac{7}{2}$ *or* $a = \frac{7}{2}$ Solve for a.

∴ The correct answer is Ⓑ.

EXAMPLE 3 Real-Life Application



The arch of a fireplace can be modeled by $y = -\frac{1}{9}(x + 18)(x - 18)$, where x and y are measured in inches. The x-axis represents the floor. Find the width of the arch at floor level.

Use the x-coordinates at floor level to find the width. At floor level, $y = 0$. So, substitute 0 for y and solve for x.

$y = -\frac{1}{9}(x + 18)(x - 18)$ Write equation.

$0 = -\frac{1}{9}(x + 18)(x - 18)$ Substitute 0 for y.

$0 = (x + 18)(x - 18)$ Multiply each side by -9.

$x + 18 = 0$ *or* $x - 18 = 0$ Use Zero-Product Property.

$x = -18$ *or* $x = 18$ Solve for x.

The width is the distance between the x-coordinates, -18 and 18.

∴ So, the width of the arch at floor level is $18 - (-18) = 36$ inches.

On Your Own

Now You're Ready
Exercises 10–15

Solve the equation.

5. $(3p + 5)(3p - 5) = 0$

6. $(12 - 6x)^2 = 0$

7. The entrance to a mine shaft can be modeled by $y = -\frac{1}{2}(x + 4)(x - 4)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the entrance at ground level.

7.5 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** Is $x = 3$ a solution of $(x - 3)(x + 6) = 0$? Explain.
2. **WRITING** Describe how to solve $(x - 2)(x + 1) = 0$ using the Zero-Product Property.
3. **WHICH ONE DOESN'T BELONG?** Which statement does *not* belong with the other three? Explain your reasoning.

$(n - 9)(n + 3)$ $\quad$ $(2k + 5)(k - 3)$

$(g + 2)^2$ $\quad$ $x^2 + 4x$

Practice and Problem Solving

Solve the equation.

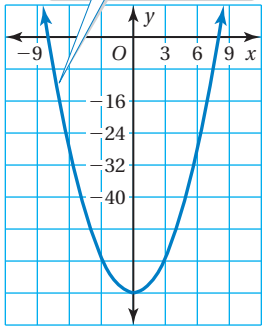
4. $x(x + 7) = 0$
5. $12t(t - 5) = 0$
6. $(s - 9)(s - 1) = 0$
7. $(q + 3)(q - 2) = 0$
8. $(h - 8)^2 = 0$
9. $(m + 4)^2 = 0$
10. $(5 - k)(5 + k) = 0$
11. $(3 - g)(7 - g) = 0$
12. $(3p + 6)^2 = 0$
13. $(4z - 12)^2 = 0$
14. $\left(\frac{1}{2}y + 4\right)(y - 8) = 0$
15. $\left(\frac{1}{3}d - 2\right)\left(\frac{1}{3}d + 2\right) = 0$

16. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

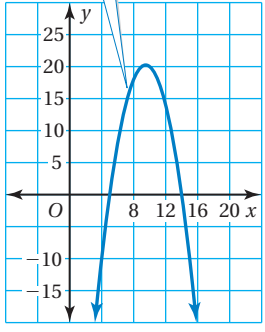
✗
$6x(x + 5) = 0$
$x + 5 = 0$
$x = -5$
The root is $x = -5$.

Find the x-coordinates of the points where the graph crosses the x-axis.

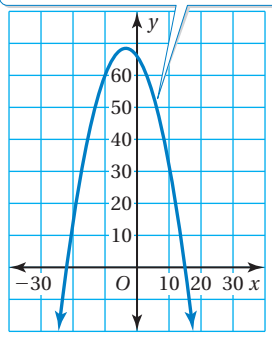
17. $y = (x - 8)(x + 8)$



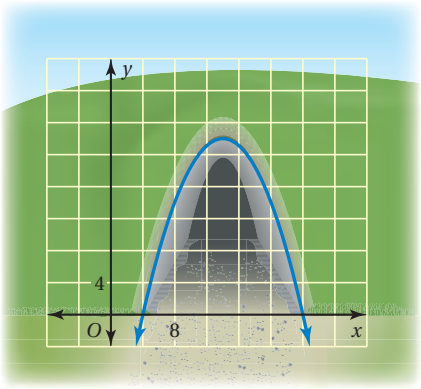
18. $y = -(x - 14)(x - 5)$



19. $y = -0.2(x + 22)(x - 15)$



20. CHOOSE TOOLS The entrance of a tunnel can be modeled by $y = -\frac{11}{50}(x - 4)(x - 24)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the tunnel at ground level.



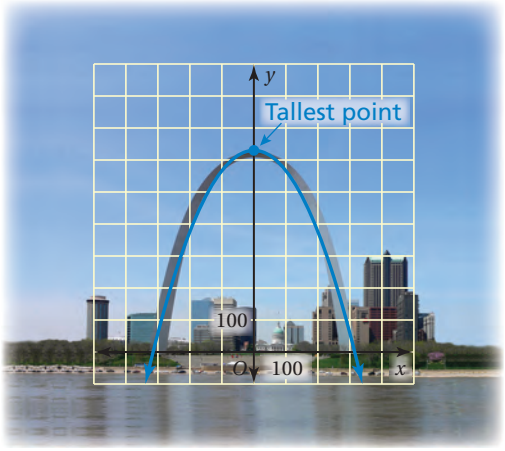
Solve the equation.

21. $5z(z + 2)(z - 1) = 0$

22. $w(w - 6)^2 = 0$

23. $(r - 4)(r + 4)(r + 8) = 0$

24. $(2p + 3)(2p - 3)(p + 7) = 0$



25. GATEWAY ARCH The Gateway Arch in St. Louis can be modeled by $y = -\frac{2}{315}(x + 315)(x - 315)$, where x and y are measured in feet. The x-axis represents the ground.

a. Find the width of the arch at ground level.

b. How tall is the arch?

26. Algebra Find the values of x in terms of y that are solutions of the equation.

a. $(x + y)(2x - y) = 0$

b. $(x^2 - y^2)(4x + 16y) = 0$

Fair Game Review What you learned in previous grades & lessons

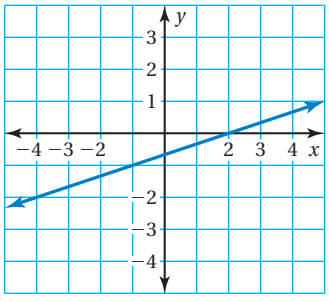
Find the greatest common factor of the numbers. *(Skills Review Handbook)*

27. 21 and 63

28. 12 and 27

29. 30, 75, and 90

30. MULTIPLE CHOICE What is the slope of the line? *(Section 2.2)*



Ⓐ -3

Ⓑ $-\frac{1}{3}$

Ⓒ $\frac{1}{3}$

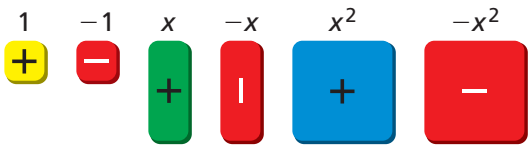
Ⓓ 3

7.6 Factoring Polynomials Using the GCF

Essential Question How can you use common factors to write a polynomial in factored form?

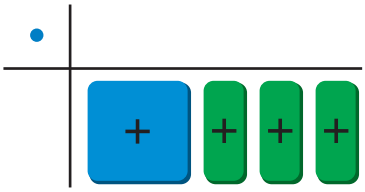
1 ACTIVITY: Finding Monomial Factors

Work with a partner. Six different algebra tiles are shown below.

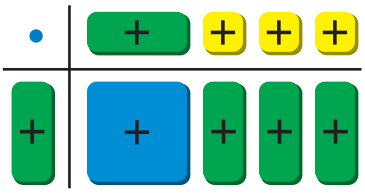


Sample:

Step 1: Look at the rectangular array for $x^2 + 3x$.



Step 2: Use algebra tiles to label the dimensions of the rectangle.



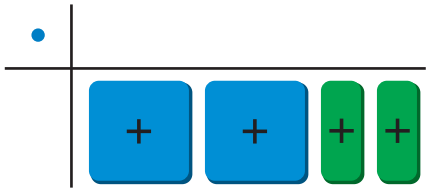
Step 3: Write the polynomial in factored form by finding the dimensions of the rectangle.

width, length

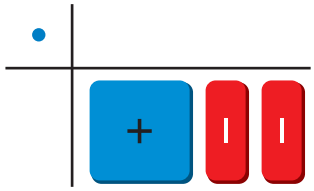
$$\text{Area} = x^2 + 3x = x(x + 3)$$

Use algebra tiles to write each polynomial in factored form.

a.

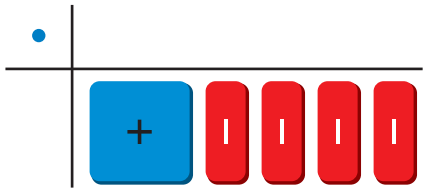


b.



c.

d.



COMMON CORE

Polynomial Equations

In this lesson, you will

- factor polynomials using the greatest common factor.
- solve polynomial equations by factoring.

Learning Standards
A.REI.4b
A.SSE.3a

2 ACTIVITY: Finding Monomial Factors

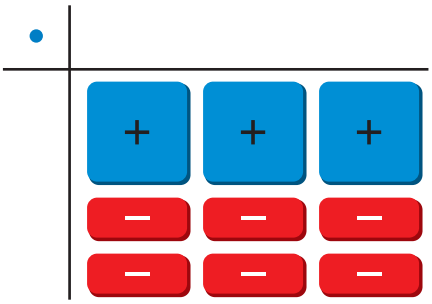
Math Practice 4

Interpret Results

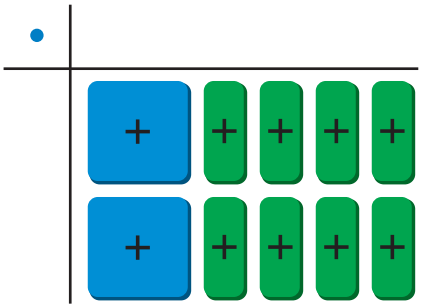
What does your answer represent? How can you make sure your answer makes sense?

Work with a partner. Use algebra tiles to write each polynomial in factored form.

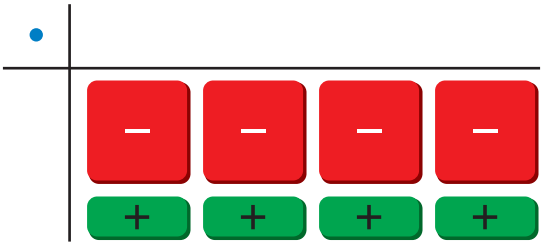
a.



b.



c.



3 ACTIVITY: Finding Monomial Factors

Work with a partner. Use algebra tiles to model each polynomial as a rectangular array. Then write the polynomial in factored form by finding the dimensions of the rectangle.

a. $3x^2 - 9x$ **b.** $7x + 14x^2$ **c.** $-2x^2 + 6x$

What Is Your Answer?

4. Consider the polynomial $4x^2 + 8x$.

a. What are the terms of the polynomial?

b. List all the factors that are common to both terms.

c. Of the common factors, which is the greatest? Explain your reasoning.

5. **IN YOUR OWN WORDS** How can you use common factors to write a polynomial in factored form?

Practice Use what you learned about factoring polynomials to complete Exercises 3–5 on page 366.

7.6 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Writing a polynomial as a product of factors is called *factoring*. When the terms of a polynomial have a common factor, you can factor the polynomial as shown below.

Key Idea

Factoring Polynomials Using the GCF

Step 1: Find the greatest common factor (GCF) of the terms.

Step 2: Use the Distributive Property to write the polynomial as a product of the GCF and its remaining factors.

EXAMPLE 1 Factoring Polynomials

Factor each polynomial.

a. $2x^2 + 18$

Step 1: Find the GCF of the terms.

$$2x^2 = 2 \cdot x \cdot x$$
$$18 = 2 \cdot 3 \cdot 3$$

The GCF is 2.

Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$2x^2 + 18 = 2(x^2) + 2(9) \quad \text{Factor out GCF.}$$
$$= 2(x^2 + 9) \quad \text{Distributive Property}$$

Study Tip

When you factor a polynomial, you *undo* the multiplication of its factors.

b. $15y^3 + 10y^2$

Step 1: Find the GCF of the terms.

$$15y^3 = 3 \cdot 5 \cdot y \cdot y \cdot y$$
$$10y^2 = 2 \cdot 5 \cdot y \cdot y$$

The GCF is $5 \cdot y \cdot y = 5y^2$.

Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$15y^3 + 10y^2 = 5y^2(3y) + 5y^2(2) \quad \text{Factor out GCF.}$$
$$= 5y^2(3y + 2) \quad \text{Distributive Property}$$

On Your Own

Now You're Ready
Exercises 6–11

Factor the polynomial.

1. $5z^2 + 30$ **2.** $3x^2 + 14x$ **3.** $8y^2 - 24y$

To solve an equation using the Zero-Product Property, you may need to first collect the terms on one side of the equation and then factor.

EXAMPLE 2 Solving an Equation by Factoring

Solve $4g^2 = -6g$.

$$4g^2 = -6g$$ Write equation.

$$4g^2 + 6g = 0$$ Add $6g$ to each side.

$$2g(2g + 3) = 0$$ Factor the polynomial.

$$2g = 0 \quad or \quad 2g + 3 = 0$$ Use Zero-Product Property.

$$g = 0 \quad or \quad g = -\frac{3}{2}$$ Solve for g.

∴ The solutions are $g = 0$ and $g = -\frac{3}{2}$.

On Your Own

Now You're Ready
Exercises 14–22

Solve the equation.

4. $3x^2 + 21x = 0$ **5.** $5z^2 = 5z$ **6.** $18y = 6y^2$

EXAMPLE 3 Real-Life Application

A female athlete tests her vertical jump by jumping straight into the air. Her height y (in feet) after t seconds can be modeled by $y = -16t^2 + 12t$. How many seconds is she in the air?

She is on the ground when $y = 0$. So, substitute 0 for y and solve for t.

$$y = -16t^2 + 12t$$ Write equation.

$$0 = -16t^2 + 12t$$ Substitute 0 for y.

$$0 = 4t(-4t + 3)$$ Factor the polynomial.

$$4t = 0 \quad or \quad -4t + 3 = 0$$ Use Zero-Product Property.

$$t = 0 \quad or \quad t = 0.75$$ Solve for t.

She starts the jump at $t = 0$ and lands when $t = 0.75$.

∴ So, she is in the air for 0.75 second.

On Your Own

7. WHAT IF? The height of a male athlete testing his vertical jump can be modeled by $y = -16t^2 + 14t$. How many seconds is he in the air?

7.6 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** What is the greatest common factor of $12y$ and $30y^2$?
2. **WRITING** Describe how to factor a polynomial using the greatest common factor.

Practice and Problem Solving

Use algebra tiles to factor the polynomial.

3. $4x + 8$
4. $2x^2 + 4x$
5. $x^2 - 4x$

Factor the polynomial.

6. $5z^2 + 45z$
7. $8m^2 + 4m$
8. $3y^3 - 9y^2$
9. $20x^3 + 30x^2$
10. $4w^3 - 8w + 12$
11. $5t^2 + 20t + 50$

12. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$$2x^2 + 2x = 2(x^2) + 2(x)$$
$$= 2(x^2 + x)$$

13. **INTEREST** You deposit \$100 in a savings account that earns simple interest. The balance of the account can be represented by $100 + 100rt$, where r is the annual interest rate and t is the time in years. Factor the polynomial.

Solve the equation.

14. $2q + 10 = 0$
15. $10x + 15 = 0$
16. $4p^2 - p = 0$
17. $6m^2 + 12m = 0$
18. $3n^2 = 9n$
19. $4r^2 = -28r$
20. $4a^3 = 44a^2$
21. $6k^3 + 39k^2 = 0$
22. $2y^2 = 2\pi y$

23. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

$$3x^2 = 15x$$
$$3x^2 - 15x = 0$$
$$3x(x - 15) = 0$$
$$3x = 0 \text{ or } x - 15 = 0$$
$$x = 0 \text{ or } x = 15$$

The roots are $x = 0$ and $x = 15$.

24. **AGES** Your brother is y years old. Your older cousins are $2y^2$ and $6y$ years old. The difference between your cousins' ages is zero. Your brother is older than 1 year old. How old is he?

Solve the equation.

25. $5b^2 - 20b = b^2$

26. $5n^2 + 40n = 5n$

27. $2s^3 + 15s^2 = 3s^2$

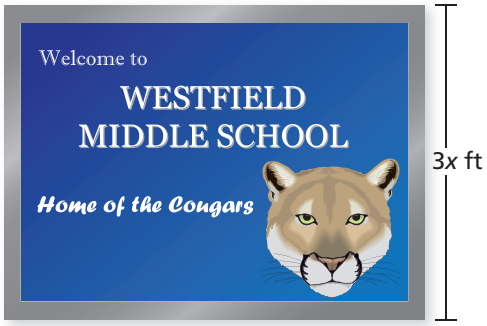
28. $8g^3 - 2g^2 = 2g^3 - 5g^2$

29. OPEN-ENDED Write a binomial whose terms have a GCF of $3x$.

30. SCHOOL SIGN The area (in square feet) of the school sign can be represented by $15x^2 - 6x$.

a. Write an expression that represents the length of the sign.

b. Describe two ways to find the area of the sign when $x = 2$.



31. DOLPHIN A dolphin jumps straight into the air during a performance. The dolphin's height y (in feet) after t seconds can be modeled by $y = -16t^2 + 24t$.

a. How many seconds is the dolphin in the air?

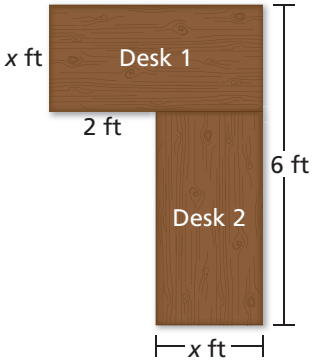
b. The dolphin reaches its maximum height after 0.75 second. What is the maximum height of the jump?

32. Modeling Your teacher's work station is made up of two identical desks arranged as shown.

a. Write an equation in terms of x that relates the area of Desk 1 to the area of Desk 2.

b. What is the value of x?

c. Find the area of the top of your teacher's work station.



Fair Game Review What you learned in previous grades & lessons

Find the product. *(Section 7.3)*

33. $(y + 4)(y + 6)$

34. $(m - 2)(m - 9)$

35. $(2k + 1)(2k - 3)$

36. MULTIPLE CHOICE An African elephant weighs 5,200,000 grams. Write this number in scientific notation. *(Skills Review Handbook)*

A 0.52×10^{-7} g

B 5.2×10^{-6} g

C 52×10^{5} g

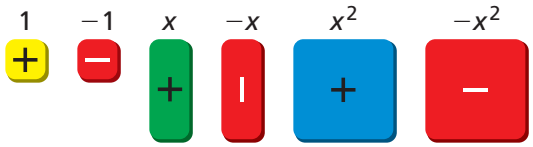
D 5.2×10^{6} g

7.7 Factoring $x^2 + bx + c$

Essential Question How can you factor the trinomial $x^2 + bx + c$ into the product of two binomials?

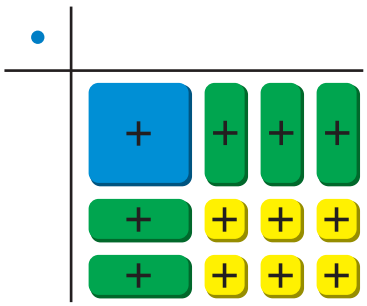
1 ACTIVITY: Finding Binomial Factors

Work with a partner. Six different algebra tiles are shown below.

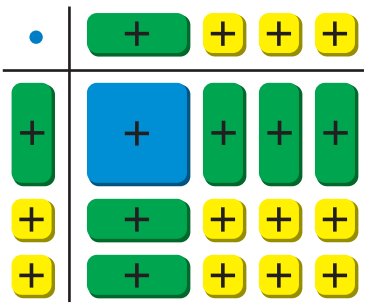


Sample:

Step 1: Arrange the algebra tiles into a rectangular array to model $x^2 + 5x + 6$.



Step 2: Use algebra tiles to label the dimensions of the rectangle.



Step 3: Write the polynomial in factored form by finding the dimensions of the rectangle.

width, length

$$\text{Area} = x^2 + 5x + 6 = (x + 2)(x + 3)$$

Common Core

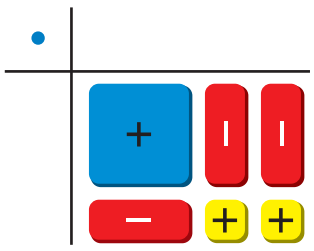
Polynomial Equations

In this lesson, you will
- factor trinomials of the form $x^2 + bx + c$.

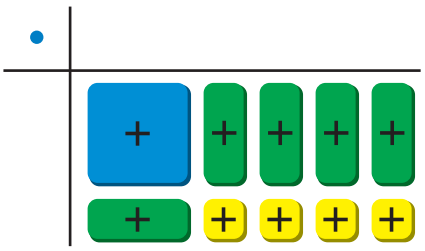
Learning Standards
A.REI.4b
A.SSE.3a

Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

a.



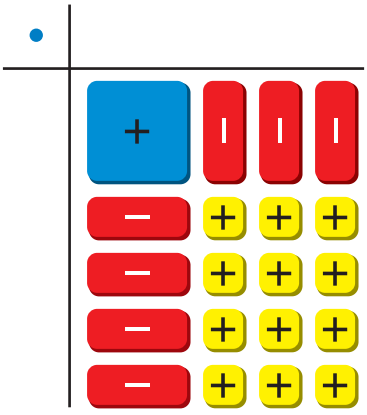
b.



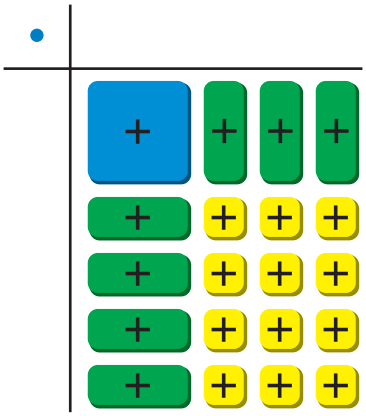
2 ACTIVITY: Finding Binomial Factors

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

a.



b.



3 ACTIVITY: Finding Binomial Factors

Work with a partner. Write each polynomial as the product of two binomials. Check your answer by multiplying.

a. $x^2 + 6x + 9$ **b.** $x^2 - 6x + 9$ **c.** $x^2 + 6x + 8$

d. $x^2 - 6x + 8$ **e.** $x^2 + 6x + 5$ **f.** $x^2 - 6x + 5$

Math Practice 2

Make Sense of Quantities

What is the relationship between a polynomial and its binomial factors?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you factor the trinomial $x^2 + bx + c$ into the product of two binomials?

a. Describe a strategy that uses algebra tiles.

b. Describe a strategy that does not use algebra tiles.

5. Use one of your strategies to factor each trinomial.

a. $x^2 + 6x - 16$ **b.** $x^2 - 6x - 16$ **c.** $x^2 + 6x - 27$

Practice Use what you learned about factoring trinomials to complete Exercises 3–5 on page 373.

7.7 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Consider the polynomial $x^2 + bx + c$, where b and c are integers. To factor this polynomial as $(x + p)(x + q)$, you need to find integers p and q such that $p + q = b$ and $pq = c$.

$$(x + p)(x + q) = x^2 + px + qx + pq$$
$$= x^2 + (p + q)x + pq$$

Key Idea

Factoring $x^2 + bx + c$ When c Is Positive

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is positive, p and q have the same sign as b.

Examples $x^2 + 6x + 5 = (x + 1)(x + 5)$

$x^2 - 6x + 5 = (x - 1)(x - 5)$

EXAMPLE 1 Factoring $x^2 + bx + c$ When b and c Are Positive

Factor $x^2 + 10x + 16$.

Notice that $b = 10$ and $c = 16$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is also positive, p and q must each be positive so that $p + q$ is positive.

Find two positive integer factors of 16 whose sum is 10.

Factors of 16	Sum of Factors
1, 16	17
2, 8	10
4, 4	8

The values of p and q are 2 and 8.

∴ So, $x^2 + 10x + 16 = (x + 2)(x + 8)$.

Check

Use the FOIL Method.

$(x + 2)(x + 8)$

$= x^2 + 8x + 2x + 16$

$= x^2 + 10x + 16$ ✓

On Your Own

Now You're Ready
Exercises 3–8

Factor the polynomial.

1. $x^2 + 2x + 1$
2. $x^2 + 9x + 8$
3. $y^2 + 6y + 8$
4. $z^2 + 11z + 24$

EXAMPLE 2 Factoring $x^2 + bx + c$ When b Is Negative and c Is Positive

Factor $x^2 - 8x + 12$.

Notice that $b = -8$ and $c = 12$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is negative, p and q must each be negative so that $p + q$ is negative.

Find two negative integer factors of 12 whose sum is -8.

Factors of 12	$-1, -12$	$-2, -6$	$-3, -4$
Sum of Factors	-13	-8	-7

The values of p and q are -2 and -6.

∴ So, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Check

Use the FOIL Method.

$(x - 2)(x - 6)$

$= x^2 - 6x - 2x + 12$

$= x^2 - 8x + 12$ ✓

On Your Own

Now You're Ready
Exercises 10–15

Factor the polynomial.

5. $w^2 - 4w + 3$ **6.** $n^2 - 12n + 35$ **7.** $x^2 - 14x + 24$

Key Idea

Factoring $x^2 + bx + c$ When c Is Negative

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is negative, p and q have different signs.

Example $x^2 - 4x - 5 = (x + 1)(x - 5)$

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor $x^2 + 4x - 21$.

Notice that $b = 4$ and $c = -21$. Because c is negative, the factors p and q must have different signs so that pq is negative.

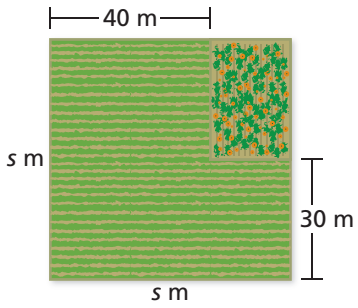
Find two integer factors of -21 whose sum is 4.

Factors of -21	$-21, 1$	$-1, 21$	$-7, 3$	$-3, 7$
Sum of Factors	-20	20	-4	4

The values of p and q are -3 and 7.

∴ So, $x^2 + 4x - 21 = (x - 3)(x + 7)$.

EXAMPLE 4 Real-Life Application



A farmer plants a rectangular pumpkin patch in the northeast corner of the square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?

The length of the pumpkin patch is $(s - 30)$ meters and the width is $(s - 40)$ meters. Write and solve an equation for its area.

$$600 = (s - 30)(s - 40) \qquad \text{Write an equation.}$$

$$600 = s^2 - 70s + 1200 \qquad \text{Multiply.}$$

$$0 = s^2 - 70s + 600 \qquad \text{Subtract 600 from each side.}$$

$$0 = (s - 10)(s - 60) \qquad \text{Factor the polynomial.}$$

$$s - 10 = 0 \quad \text{or} \quad s - 60 = 0 \qquad \text{Use Zero-Product Property.}$$

$$s = 10 \quad \text{or} \quad s = 60 \qquad \text{Solve for } s.$$

The diagram shows that the side length is at least 30 meters, so 10 meters does not make sense in this situation. The width is 60 meters.

∴ So, the area of the square plot of land is $60(60) = 3600$ square meters.

On Your Own

Now You're Ready
Exercises 21–29

Factor the polynomial.

8. $x^2 + 2x - 15$ **9.** $y^2 + 13y - 30$ **10.** $v^2 + v - 20$

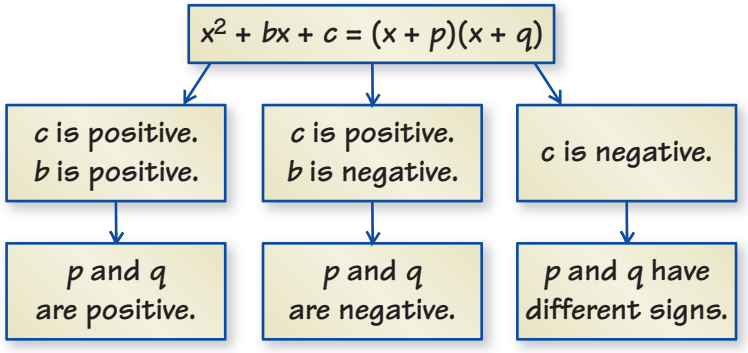
11. $z^2 - z - 12$ **12.** $m^2 - 11m - 26$ **13.** $x^2 - 3x - 40$

14. WHAT IF? In Example 4, the area of the pumpkin patch is 200 square meters. What is the area of the square plot of land?

Summary

Factoring $x^2 + bx + c$ as $(x + p)(x + q)$

The diagram shows the relationships between the signs of b and c and the signs of p and q.



7.7 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **WRITING** You are factoring $x^2 + 11x - 26$. What do the signs of the terms tell you about the factors? Explain.
2. **OPEN-ENDED** Write a trinomial that can be factored as $(x + p)(x + q)$ where p and q are positive.

Practice and Problem Solving

Factor the polynomial.

3. $x^2 + 8x + 7$
4. $z^2 + 7z + 12$
5. $n^2 + 8n + 12$
6. $s^2 + 11s + 30$
7. $h^2 + 11h + 18$
8. $y^2 + 13y + 40$

9. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$t^2 + 14t + 48 = (t + 4)(t + 12)$

Factor the polynomial.

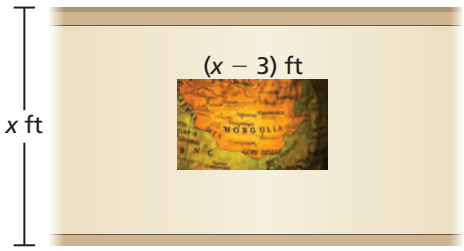
10. $v^2 - 5v + 4$
11. $x^2 - 9x + 20$
12. $d^2 - 5d + 6$
13. $k^2 - 10k + 24$
14. $w^2 - 17w + 72$
15. $j^2 - 13j + 42$

Solve the equation.

16. $m^2 + 3m + 2 = 0$
17. $x^2 + 11x + 28 = 0$
18. $n^2 - 9n + 18 = 0$

19. **PROFIT** A company's profit (in millions of dollars) can be represented by $x^2 - 6x + 8$, where x is the number of years since the company started. When did the company have a profit of \$3 million?

20. **PROJECTION** A projector displays an image on a wall. The area (in square feet) of the rectangular projection can be represented by $x^2 - 8x + 15$.
    a. Write a binomial that represents the height of the projection.
    b. Find the perimeter of the projection when the height of the wall is 8 feet.



Factor the polynomial.

21. $x^2 + 3x - 4$ **22.** $z^2 + 7z - 18$ **23.** $n^2 + 4n - 12$

24. $s^2 + 3s - 40$ **25.** $h^2 + 6h - 27$ **26.** $y^2 + 2y - 48$

27. $m^2 - 6m - 7$ **28.** $x^2 - x - 20$ **29.** $t^2 - 6t - 16$

Solve the equation.

30. $v^2 + 3v - 4 = 0$ **31.** $x^2 + 5x - 14 = 0$ **32.** $n^2 - 5n = 24$

33. ERROR ANALYSIS Describe and correct the error in solving the equation.

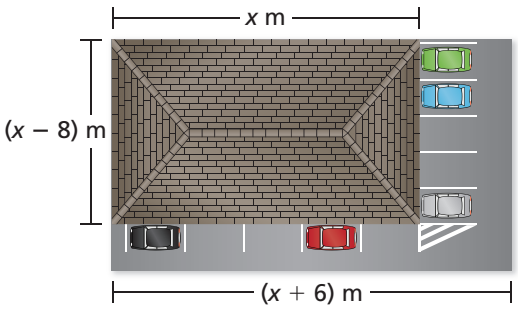
$$x^2 - 2x - 15 = 20$$
$$(x - 5)(x + 3) = 20$$
$$x - 5 = 20 \quad or \quad x + 3 = 20$$
$$x = 25 \quad or \quad x = 17$$

34. DENTIST A dentist's office and parking lot are on a rectangular piece of land. The area (in square meters) of the land can be represented by $x^2 + x - 30$.

a. Write a binomial that represents the width of the land.

b. Write an expression that represents the area of the parking lot.

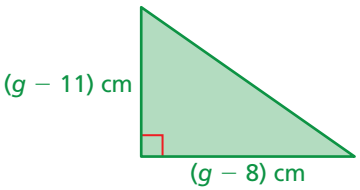
c. Evaluate the expressions in parts (a) and (b) when $x = 20$.



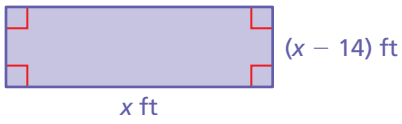
Find the dimensions of the polygon with the given area.

35. Area = 44 square feet

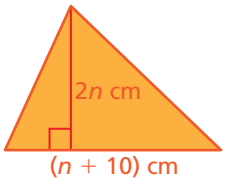
36. Area = 35 square centimeters



37. Area = 120 square feet



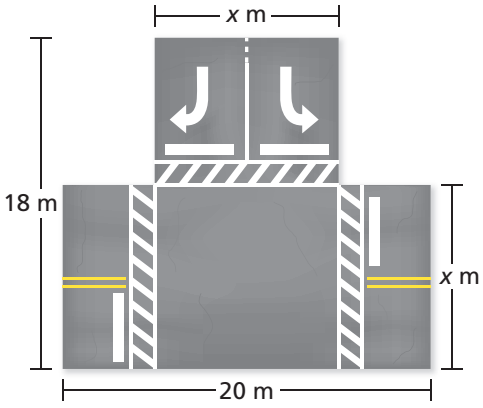
38. Area = 75 square centimeters



39. COMPUTER A web browser is open on your computer screen.

a. The area of the browser is 24 square inches. Find the value of x.

b. The browser covers $\frac{3}{13}$ of the screen. What are the dimensions of the screen?



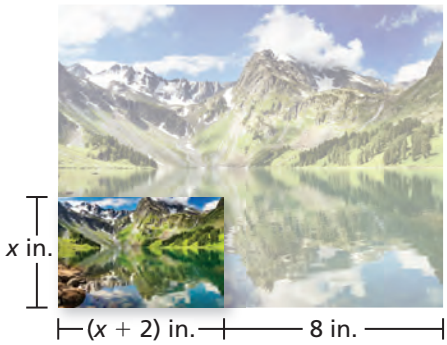
40. LOGIC Road construction workers are paving the area shown.

a. Write an expression that represents the area being paved.

b. The area being paved is 280 square meters. Write and solve an equation to find x.

c. The equation in part (b) has two solutions. Explain why one of the solutions is not reasonable.

41. PHOTOGRAPHY You enlarge a photograph on a computer. The area (in square inches) of the enlarged photograph can be represented by $x^2 + 17x + 70$.



a. Write binomials that represent the length and width of the enlarged photograph.

b. How many inches greater is the length of the enlarged photograph than the width? Explain.

c. The area of the enlarged photograph is 154 square inches. Find the dimensions of each photograph.

42. Number Sense Find all of the integer values of b for which the trinomial $x^2 + bx - 12$ has two binomial factors of the form $(x + p)$ and $(x + q)$.

Fair Game Review What you learned in previous grades & lessons

Factor the polynomial. *(Section 7.6)*

43. $2y - 18$

44. $7n^2 + 23n$

45. $8z^3 + 28z^2$

46. MULTIPLE CHOICE Which expression is *not* equivalent to $\sqrt{\frac{9}{4}}$? *(Section 6.1)*

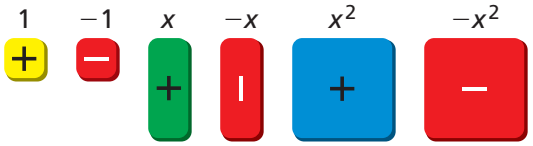
Ⓐ $\frac{3}{2}$ Ⓑ $\sqrt{2.25}$ Ⓒ $2\sqrt{3}$ Ⓓ $3\sqrt{\frac{1}{4}}$

7.8 Factoring $ax^2 + bx + c$

Essential Question How can you factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

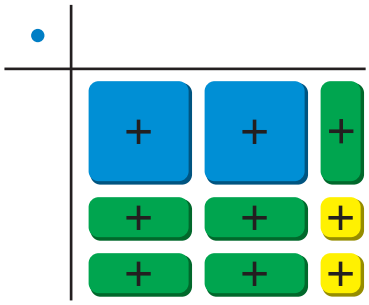
1 ACTIVITY: Finding Binomial Factors

Work with a partner. Six different algebra tiles are shown below.

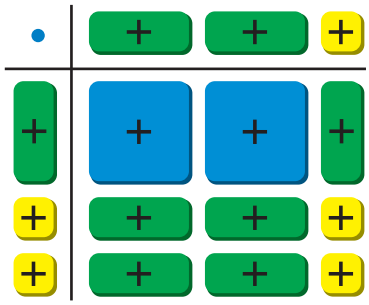


Sample:

Step 1: Arrange the algebra tiles into a rectangular array to model $2x^2 + 5x + 2$.



Step 2: Use algebra tiles to label the dimensions of the rectangle.

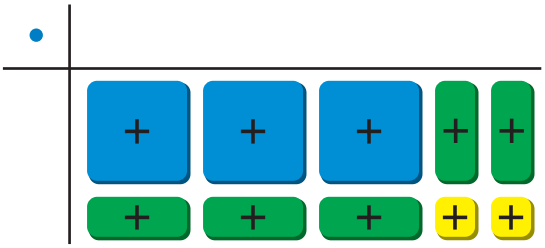


Step 3: Write the polynomial in factored form by finding the dimensions of the rectangle.

length width

$$\text{Area} = 2x^2 + 5x + 2 = (2x + 1)(x + 2)$$

Use algebra tiles to write the polynomial as the product of two binomials. Check your answer by multiplying.



Common Core

Polynomial Equations

In this lesson, you will

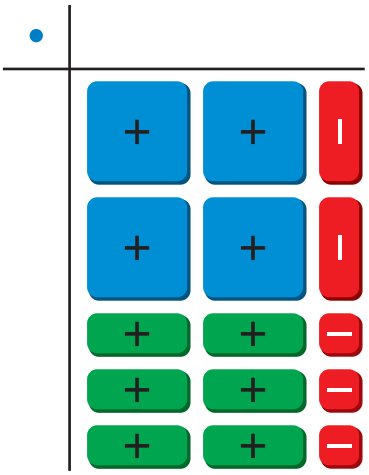
- factor trinomials of the form $ax^2 + bx + c$.

Learning Standards
A.REI.4b
A.SSE.3a

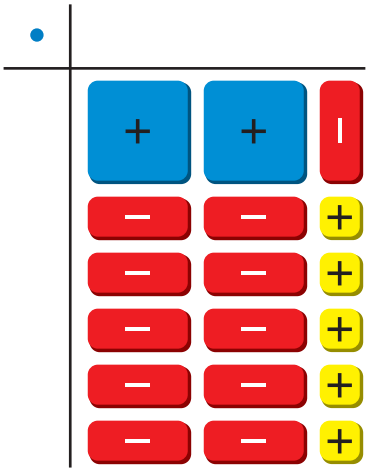
2 ACTIVITY: Finding Binomial Factors

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

a.



b.



3 ACTIVITY: Finding Binomial Factors

Work with a partner. Write each polynomial as the product of two binomials. Check your answer by multiplying.

a. $2x^2 + 5x - 3$ **b.** $3x^2 + 10x - 8$ **c.** $4x^2 + 4x - 3$

d. $2x^2 + 11x + 15$ **e.** $9x^2 - 6x + 1$ **f.** $4x^2 + 11x - 3$

Math Practice 1

Find Entry Points

What should you do first when factoring a polynomial using algebra tiles?

What Is Your Answer?

4. IN YOUR OWN WORDS How can you factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

5. Use your strategy to factor each trinomial.

a. $4x^2 + 4x + 1$ **b.** $3x^2 + 5x - 2$ **c.** $2x^2 - 13x + 15$

Practice

Use what you learned about factoring trinomials to complete Exercises 3–5 on page 380.

7.8 Lesson

Check It Out
Lesson Tutorials
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In Section 7.7, you factored polynomials of the form $ax^2 + bx + c$, where $a = 1$. To factor polynomials of the form $ax^2 + bx + c$, where $a \neq 1$, first look for the GCF of the terms of the polynomial.

EXAMPLE 1 Factoring Out the GCF

Factor $5x^2 + 15x + 10$.

Notice that the GCF of the terms $5x^2$, $15x$, and 10 is 5.

$$\begin{aligned} 5x^2 + 15x + 10 &= 5(x^2 + 3x + 2) && \text{Factor out GCF.} \\ &= 5(x + 1)(x + 2) && \text{Factor } x^2 + 3x + 2. \end{aligned}$$

∴ So, $5x^2 + 15x + 10 = 5(x + 1)(x + 2)$.

When there is no GCF, consider the possible factors of a and c.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When ac Is Positive

a. Factor $4x^2 + 13x + 3$.

Consider the possible factors of $a = 4$ and $c = 3$.

Factors are 1, 2, and 4. → $4x^2 + 13x + 3$ ← Factors are 1 and 3.

These factors lead to the following possible products.

$$(1x + 1)(4x + 3) \qquad (1x + 3)(4x + 1) \qquad (2x + 1)(2x + 3)$$

Multiply to find the product that is equal to the original polynomial.

$(x + 1)(4x + 3) = 4x^2 + 7x + 3$ ✗ $(2x + 1)(2x + 3) = 4x^2 + 8x + 3$ ✗

$(x + 3)(4x + 1) = 4x^2 + 13x + 3$ ✓

∴ So, $4x^2 + 13x + 3 = (x + 3)(4x + 1)$.

b. Factor $3x^2 - 7x + 2$.

Consider the possible factors of $a = 3$ and $c = 2$. Because b is negative and c is positive, both factors of c must be negative.

Factors are 1 and 3. → $3x^2 - 7x + 2$ ← Factors are -2 and -1.

These factors lead to the following possible products.

$$(1x - 1)(3x - 2) \qquad (1x - 2)(3x - 1)$$

Multiply to find the product that is equal to the original polynomial.

$(x - 1)(3x - 2) = 3x^2 - 5x + 2$ ✗ $(x - 2)(3x - 1) = 3x^2 - 7x + 2$ ✓

∴ So, $3x^2 - 7x + 2 = (x - 2)(3x - 1)$.

Study Tip

When ac is positive, the sign of b determines whether the factors of c are positive or negative.

Now You're Ready
Exercises 6–11
and 13–15

On Your Own

Factor the polynomial.

1. $8x^2 - 56x + 48$
2. $2x^2 + 11x + 5$
3. $2x^2 - 7x + 5$
4. $3x^2 - 14x + 8$

EXAMPLE 3 Factoring $ax^2 + bx + c$ When ac Is Negative

Study Tip

For polynomials of the form $ax^2 + bx + c$, where a is negative, factor out -1 first to make factoring easier. Just be sure to put -1 back in your final answer.

Factor $2x^2 - 5x - 7$.

Consider the possible factors of $a = 2$ and $c = -7$. Because b and c are both negative, the factors of c must have different signs.

Factors are 1 and 2. → $2x^2 - 5x - 7$ ← Factors are ± 1 and ± 7.

These factors lead to the following possible products.

$(x + 1)(2x - 7)$ $(x + 7)(2x - 1)$ $(x - 1)(2x + 7)$ $(x - 7)(2x + 1)$

Multiply to find the product that is equal to the original polynomial.

$(x + 1)(2x - 7) = 2x^2 - 5x - 7$ ✓ $(x - 1)(2x + 7) = 2x^2 + 5x - 7$ ✗

$(x + 7)(2x - 1) = 2x^2 + 13x - 7$ ✗ $(x - 7)(2x + 1) = 2x^2 - 13x - 7$ ✗

∴ So, $2x^2 - 5x - 7 = (x + 1)(2x - 7)$.

EXAMPLE 4 Real-Life Application

The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. How wide is the reserve?

(A) 2 mi (B) 2.5 mi (C) 5 mi (D) 5.5 mi

Write an equation that represents the area of the reserve. Then solve by factoring. Let w represent the width. Then $2w + 1$ represents the length.

$w(2w + 1) = 55$	Area of the reserve
$2w^2 + w - 55 = 0$	Multiply. Then subtract 55 from each side.
$(w - 5)(2w + 11) = 0$	Factor left side of the equation.
$w - 5 = 0$ *or* $2w + 11 = 0$	Use Zero-Product Property.
$w = 5$ *or* $w = -\frac{11}{2}$	Solve for w. Use the positive solution.

∴ The correct answer is (C).

On Your Own

Now You're Ready
Exercises 16–21

Factor the polynomial.

5. $6x^2 + x - 12$
6. $4x^2 - 19x - 5$
7. **WHAT IF?** In Example 4, the area of the reserve is 136 square miles. How wide is the reserve?

7.8 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Describe how to factor polynomials of the form $ax^2 + bx + c$.
2. **WHICH ONE DOESN'T BELONG?** Which factored polynomial does *not* belong with the other three? Explain your reasoning.

$(2x - 3)(x + 2)$ | $x(2x - 3) + 2(2x - 3)$ | $(2x + 3)(x - 2)$ | $2x(x + 2) - 3(x + 2)$

Practice and Problem Solving

Use algebra tiles to write the polynomial as the product of two binomials.

3. $2x^2 - 3x + 1$
4. $3x^2 + x - 2$
5. $4x^2 + 11x + 6$

Factor the polynomial.

1 6. $3x^2 + 3x - 6$
7. $8v^2 + 8v - 48$
8. $4k^2 + 28k + 48$
9. $6y^2 - 24y + 18$
10. $9r^2 - 36r - 45$
11. $7d^2 - 63d + 140$

12. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$$2x^2 + 2x - 4 = 2x(x + 1 - 2)$$
$$= 2x(x - 1)$$

Factor the polynomial.

2 3 13. $3h^2 + 11h + 6$
14. $6x^2 - 5x + 1$
15. $8m^2 + 30m + 7$
16. $18v^2 - 15v - 18$
17. $2n^2 - 5n - 3$
18. $4z^2 - 4z - 3$
19. $8g^2 - 10g - 12$
20. $10w^2 + 19w - 15$
21. $14d^2 + 3d - 2$

22. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$$6x^2 - 7x - 3 = (3x - 3)(2x + 1)$$

23. **DANCE FLOOR** The area (in square feet) of a rectangular lighted dance floor can be represented by $8x^2 + 22x + 5$. Write the expressions that represent the dimensions of the dance floor.

Solve the equation.

24. $5x^2 - 5x - 30 = 0$ **25.** $2k^2 - 5k - 18 = 0$ **26.** $12m^2 + 11m = 15$

Factor the polynomial.

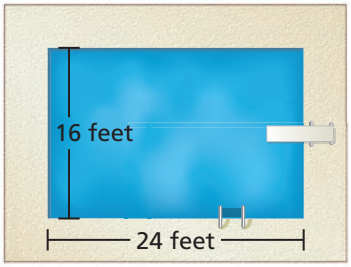
27. $-3w^2 - 2w + 8$ **28.** $-12x^2 + 48x + 27$ **29.** $-40n^2 + 70n - 15$

30. CLIFF DIVING The height h (in feet) above the water of a cliff diver is modeled by $h = -16t^2 + 8t + 80$, where t is the time (in seconds). How long is the diver in the air?

31. REASONING For what values of t can $2x^2 + tx + 10$ be written as the product of two binomials?

32. INVITATION The length of a rectangular birthday party invitation is 1 inch less than twice its width. The area of the invitation is 15 square inches. Will the invitation fit in a $3\frac{5}{8}$-inch by $5\frac{1}{8}$-inch envelope without being folded? Explain your reasoning.

33. SWIMMING POOL A rectangular swimming pool is bordered by a concrete patio. The width of the patio is the same on every side. The surface area of the pool is equal to the area of the patio border. What is the width of the patio border?



34. REASONING When is it *not* possible to factor $ax^2 + bx + c$, where $a \neq 1$? Give an example.

35. CHOOSE TOOLS A vendor can sell 50 bobbleheads per day when the price is \$40 each. For every \$2 decrease in price, 5 more bobbleheads are sold each day.

a. The revenue from yesterday was \$2160. What was the price per bobblehead? (*Note:* revenue = units sold × unit price)

b. How much should the vendor charge per bobblehead to maximize the daily revenue? Explain how you found your answer.

Structure **Factor the polynomial.**

36. $40k^3 + 6k^2 - 4k$ **37.** $6x^2 + 5xy - 4y^2$ **38.** $18m^3 + 39m^2n - 15mn^2$

Fair Game Review What you learned in previous grades & lessons

Find the product. *(Section 7.4)*

39. $(2x - 7)(2x + 7)$ **40.** $(k + 5)^2$ **41.** $(3b - 4)^2$

42. MULTIPLE CHOICE Two angles are supplementary. The measure of one of the angles is 58°. What is the measure of the other angle? *(Skills Review Handbook)*

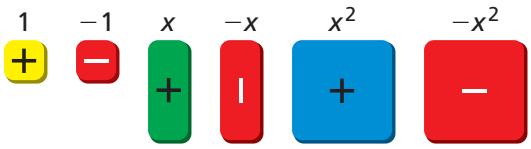
Ⓐ 22° Ⓑ 32° Ⓒ 58° Ⓓ 122°

7.9 Factoring Special Products

Essential Question How can you recognize and factor special products?

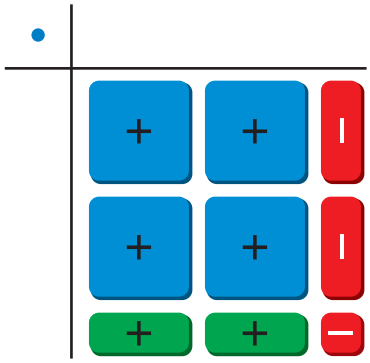
1 ACTIVITY: Factoring Special Products

Work with a partner. Six different algebra tiles are shown below.

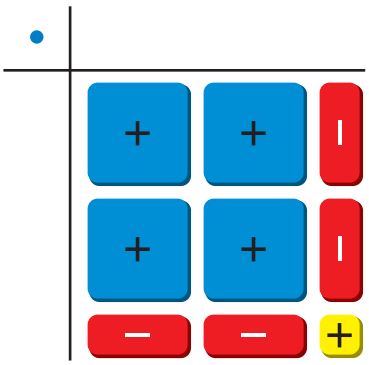


Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a "special product" that you studied in Lesson 7.4.

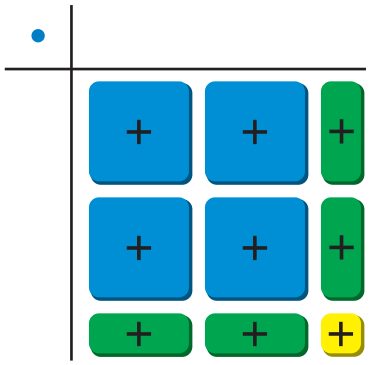
a.



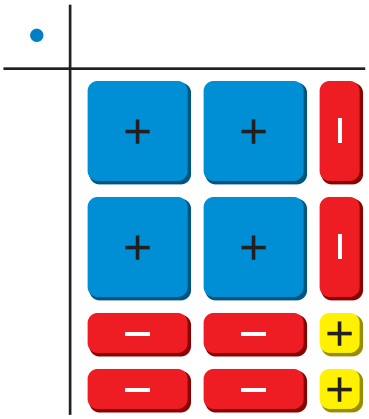
b.



c.



d.



Common Core

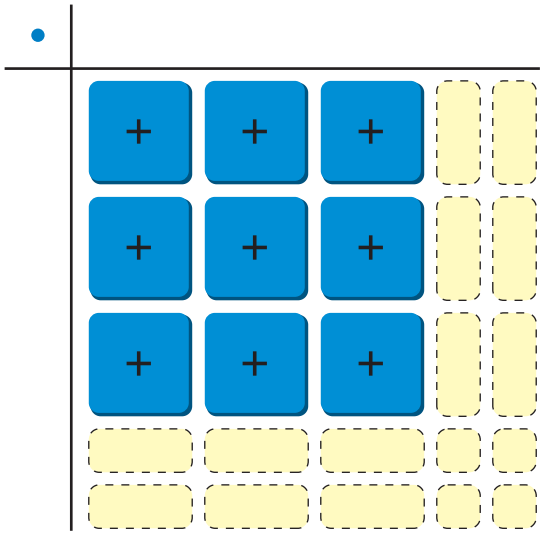
Polynomial Equations

In this lesson, you will
- factor differences of two squares.
- factor perfect square trinomials.

Learning Standards
A.REI.4b
A.SSE.2
A.SSE.3a

2 ACTIVITY: Factoring Special Products

Work with a partner. Use algebra tiles to complete the rectangular array in three different ways, so that each way represents a different special product. Write each special product in polynomial form and also in factored form.



3 ACTIVITY: Finding Binomial Factors

Work with a partner. Write each polynomial as the product of two binomials. Check your answer by multiplying.

a. $4x^2 - 12x + 9$ **b.** $4x^2 - 9$ **c.** $4x^2 + 12x + 9$

Math Practice 8

Maintain Oversight

How is factoring a special product similar to factoring a polynomial?

What Is Your Answer?

4. IN YOUR OWN WORDS How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.

5. Use your strategy to factor each polynomial.

a. $25x^2 + 10x + 1$ **b.** $25x^2 - 10x + 1$ **c.** $25x^2 - 1$

Practice Use what you learned about factoring polynomials as special products to complete Exercises 4–6 on page 386.

7.9 Lesson

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You can use special product patterns to factor polynomials.

Key Idea

Difference of Two Squares Pattern

Algebra

$a^2 - b^2 = (a + b)(a - b)$

Example

$x^2 - 9 = x^2 - 3^2$
$= (x + 3)(x - 3)$

EXAMPLE 1 Factoring the Difference of Two Squares

Factor each polynomial.

a. $x^2 - 25$

$x^2 - 25 = x^2 - 5^2$ Write as $a^2 - b^2$.

$= (x + 5)(x - 5)$ Difference of Two Squares Pattern

b. $64 - y^2$

$64 - y^2 = 8^2 - y^2$ Write as $a^2 - b^2$.

$= (8 + y)(8 - y)$ Difference of Two Squares Pattern

c. $4z^2 - 1$

$4z^2 - 1 = (2z)^2 - 1^2$ Write as $a^2 - b^2$.

$= (2z + 1)(2z - 1)$ Difference of Two Squares Pattern

Remember

You can check your answers using the FOIL Method.

On Your Own

Now You're Ready
Exercises 4–8

Factor the polynomial.

1. $x^2 - 36$ **2.** $100 - m^2$ **3.** $9n^2 - 16$ **4.** $16h^2 - 49$

Key Idea

Perfect Square Trinomial Pattern

Algebra

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Example

$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$
$= (x + 3)^2$

$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$
$= (x - 3)^2$

EXAMPLE 2 Factoring Perfect Square Trinomials

Factor each polynomial.

a. $n^2 + 8n + 16$

$$n^2 + 8n + 16 = n^2 + 2(n)(4) + 4^2 \quad \text{Write as } a^2 + 2ab + b^2.$$
$$= (n + 4)^2 \quad \text{Perfect Square Trinomial Pattern}$$

b. $x^2 - 18x + 81$

$$x^2 - 18x + 81 = x^2 - 2(x)(9) + 9^2 \quad \text{Write as } a^2 - 2ab + b^2.$$
$$= (x - 9)^2 \quad \text{Perfect Square Trinomial Pattern}$$

On Your Own

Now You're Ready
Exercises 9–12

Factor the polynomial.

5. $m^2 - 2m + 1$ **6.** $d^2 - 10d + 25$ **7.** $z^2 + 20z + 100$

EXAMPLE 3 Real-Life Application

A bird picks up a golf ball and drops it while flying. The function represents the height y (in feet) of the golf ball t seconds after it is dropped. The ball hits the top of a 32-foot tall pine tree. After how many seconds does the ball hit the tree?

Substitute 32 for y and solve for t.

$$y = 81 - 16t^2 \quad \text{Write equation.}$$
$$32 = 81 - 16t^2 \quad \text{Substitute 32 for } y.$$
$$0 = 49 - 16t^2 \quad \text{Subtract 32 from each side.}$$
$$0 = 7^2 - (4t)^2 \quad \text{Write as } a^2 - b^2.$$
$$0 = (7 + 4t)(7 - 4t) \quad \text{Difference of Two Squares Pattern}$$
$$7 + 4t = 0 \quad \text{or} \quad 7 - 4t = 0 \quad \text{Use Zero-Product Property.}$$
$$t = -\frac{7}{4} \quad \text{or} \quad t = \frac{7}{4} \quad \text{Solve for } t.$$

A negative time does not make sense in this situation.

∴ So, the golf ball hits the tree after $\frac{7}{4}$, or 1.75 seconds.

On Your Own

8. WHAT IF? The golf ball does not hit the pine tree. After how many seconds does the ball hit the ground?

7.9 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **WRITING** Describe two ways to show that $x^2 - 16$ is equal to $(x + 4)(x - 4)$.
2. **REASONING** Can you use the perfect square trinomial pattern to factor $y^2 + 16y + 64$? Explain.
3. **WHICH ONE DOESN'T BELONG?** Which polynomial does *not* belong with the other three? Explain your reasoning.

$n^2 - 4$ | $g^2 - 6g + 9$ | $r^2 + 12r + 36$ | $k^2 + 25$

Practice and Problem Solving

Factor the polynomial.

4. $m^2 - 49$
5. $9 - r^2$
6. $4x^2 - 25$
7. $81d^2 - 64$
8. $121 - 16t^2$
9. $h^2 + 12h + 36$
10. $x^2 - 4x + 4$
11. $w^2 - 14w + 49$
12. $g^2 + 24g + 144$

13. **ERROR ANALYSIS** Describe and correct the error in factoring the polynomial.

$$n^2 - 16n + 64 = n^2 - 2(n)(8) + 8^2$$
$$= (n + 8)^2$$

Solve the equation.

14. $z^2 - 4 = 0$
15. $s^2 + 20s + 100 = 0$
16. $k^2 - 16k + 64 = 0$
17. $4x^2 = 49$
18. $n^2 + 9 = -6n$
19. $y^2 = 12y - 36$

20. **REASONING** Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial can be factored using the perfect square trinomial pattern.
    a. $w^2 + 18w + 84$
    b. $y^2 - 10y + 23$
    c. $x^2 - 14x + 50$

21. **COASTER** The area (in square centimeters) of a square coaster can be represented by $d^2 + 8d + 16$. Write an expression that represents the side length of the coaster.

Factor the polynomial.

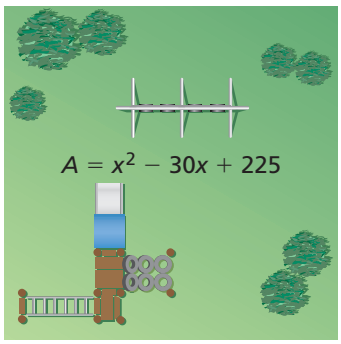
22. $3z^2 - 27$

23. $2m^3 - 50m$

24. $x^4 + 8x^3 + 16x^2$

25. $5f^3 - 20f^2 + 20f$

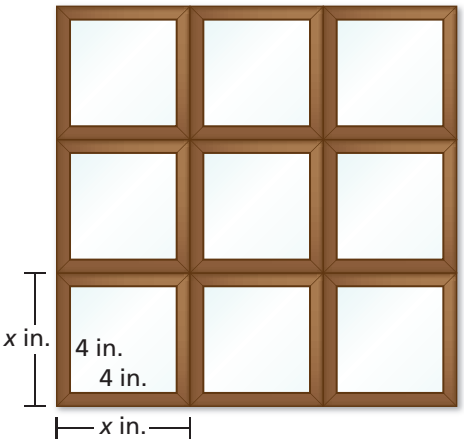
26. **PROBLEM SOLVING** The polynomial represents the area (in square feet) of the square playground.



a. Write a polynomial that represents the side length of the playground.

b. Write an expression for the perimeter of the playground.

27. **NUMBER SENSE** Solve $28 = 64 - 9x^2$ in two ways.



28. **INTERIOR DESIGN** You hang 9 identical square picture frames on a wall.

a. Write a polynomial that represents the area of the picture frames, not including the pictures.

b. The area in part (a) is 81 square inches. What is the side length of one of the picture frames? Explain your reasoning.

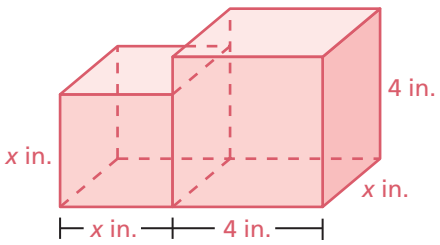
Factor the polynomial.

29. $4y^2 + 4y + 1$

30. $16v^2 - 24v + 9$

31. $9m^2 + 36m + 36$

32. Geometry A composite solid is made up of a cube and a rectangular prism.



a. Write a polynomial that represents the volume of the composite solid.

b. The volume of the composite solid is equal to $25x$. What is the value of x? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Factor the polynomial. *(Section 7.7)*

33. $w^2 + w - 12$

34. $x^2 - 5x - 36$

35. $d^2 - 4d - 60$

36. **MULTIPLE CHOICE** You deposit \$3000 in a savings account. The account earns 4% simple interest per year. What is the balance after 2 years? *(Skills Review Handbook)*

(A) \$240 (B) \$3000 (C) \$3240 (D) \$5400

Extension 7.9 Factoring Polynomials Completely

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Key Vocabulary

factoring by grouping, *p. 388*

prime polynomial, *p. 389*

factored completely, *p. 389*

To factor polynomials with four terms, group the terms into pairs, factor the GCF out of each pair of terms, and look for a common binomial factor. This process is called **factoring by grouping.**

EXAMPLE 1 Factoring by Grouping

Factor each polynomial.

a. $x^3 + 3x^2 + 2x + 6$

$x^3 + 3x^2 + 2x + 6 = (x^3 + 3x^2) + (2x + 6)$ — Group terms with common factors.

Common binomial factor is $x + 3$. → $= x^2(x + 3) + 2(x + 3)$ — Factor out GCF of each pair of terms.

$= (x + 3)(x^2 + 2)$ — Factor out $(x + 3)$.

b. $x^3 - 7 - x^2 + 7x$

The terms x^3 and -7 do not have a common factor. Rearrange the terms of the polynomial so you can group terms with common factors.

$x^3 - 7 - x^2 + 7x = x^3 - x^2 + 7x - 7$ — Rewrite polynomial.

$= (x^3 - x^2) + (7x - 7)$ — Group terms with common factors.

Common binomial factor is $x - 1$. → $= x^2(x - 1) + 7(x - 1)$ — Factor out GCF of each pair of terms.

$= (x - 1)(x^2 + 7)$ — Factor out $(x - 1)$.

c. $x^2 + y + x + xy$

$x^2 + y + x + xy = x^2 + x + xy + y$ — Rewrite polynomial.

$= (x^2 + x) + (xy + y)$ — Group terms with common factors.

$= x(x + 1) + y(x + 1)$ — Factor out GCF of each pair of terms.

$= (x + 1)(x + y)$ — Factor out $(x + 1)$.

Practice

Factor the polynomial by grouping.

1. $n^3 + 2n^2 + 5n + 10$
2. $p^3 - 7p^2 + 3p - 21$
3. $2y^3 + 8y^2 + 3y + 12$
4. $6s^3 - 16s^2 + 21s - 56$
5. $8v^3 + 48v - 5v^2 - 30$
6. $2w^3 - w^2 - 18w + 9$
7. $x^2 + xy + 3x + 3y$
8. $a - ab + a^2 - b$
9. $4xy + 20y + 3x + 15$

Multi-Language Glossary at BigIdeasMath.com

A **prime polynomial** is a polynomial that cannot be factored as a product of polynomials with integer coefficients. A factorable polynomial with integer coefficients is said to be **factored completely** when no more factors can be found and it is written as the product of prime factors.

EXAMPLE 2 Factoring Completely

COMMON CORE

Polynomial Equations

In this extension, you will
- factor polynomials by grouping.
- factor polynomials completely.

Learning Standards
A.REI.4b
A.SSE.2
A.SSE.3a

Factor each polynomial completely.

a. $3x^3 - 18x^2 + 24x$

$3x^3 - 18x^2 + 24x = 3x(x^2 - 6x + 8)$ Factor out $3x$.

$= 3x(x - 2)(x - 4)$ Factor $x^2 - 6x + 8$.

b. $7x^4 - 28x^2$

$7x^4 - 28x^2 = 7x^2(x^2 - 4)$ Factor out $7x^2$.

$= 7x^2(x^2 - 2^2)$ Write as $a^2 - b^2$.

$= 7x^2(x + 2)(x - 2)$ Difference of Two Squares Pattern

c. $p^2 + 4p - 2$

The terms of $p^2 + 4p - 2$ have no common factors. There are no integer factors of -2 whose sum is 4. So, this polynomial is already factored completely.

EXAMPLE 3 Solving an Equation by Factoring Completely

$2x^3 + 8x^2 = 10x$ Original equation

$2x^3 + 8x^2 - 10x = 0$ Subtract $10x$ from each side.

$2x(x^2 + 4x - 5) = 0$ Factor out $2x$.

$2x(x + 5)(x - 1) = 0$ Factor $x^2 + 4x - 5$.

$2x = 0$ *or* $x + 5 = 0$ *or* $x - 1 = 0$ Use Zero-Product Property.

$x = 0$ *or* $x = -5$ *or* $x = 1$ Solve for x.

∴ The solutions are $x = -5$, $x = 0$, and $x = 1$.

Practice

Factor the polynomial completely, if possible.

10. $2x^3 + 10x^2 - 48x$

11. $5z^4 - 5z^2$

12. $20c + 4c^3 - 24c^2$

13. $y^2 + 6y - 5$

14. $q^2 - q + 7$

15. $3n^4 - 48n^2$

Solve the equation.

16. $k^3 - 6k^2 + 9k = 0$

17. $3x^3 + 6x^2 = 72x$

18. $4y^3 - 12y^2 - 40y = 0$

7.5–7.9 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Factor the polynomial. *(Sections 7.6–7.9)*

1. $3d^2 + 11d$

2. $9z^2 - 18z$

3. $x^2 + 9x + 20$

4. $r^2 - 3r - 18$

5. $2x^2 - 3x + 1$

6. $3b^2 - 13b + 4$

7. $x^2 - 9$

8. $z^2 + 22z + 121$

Solve the equation. *(Sections 7.5–7.9)*

9. $m^2 - 11m + 18 = 0$

10. $w^3 - 9w^2 = 0$

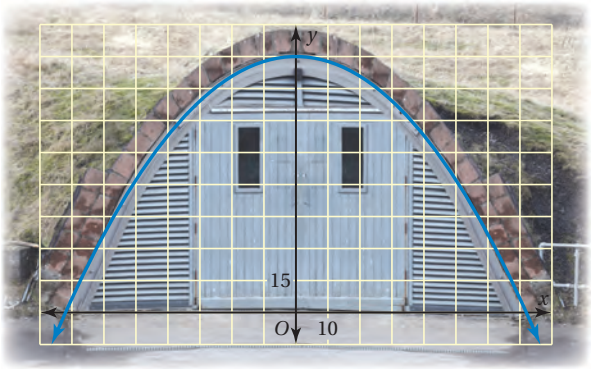
11. $6m^2 - 5m + 1 = 0$

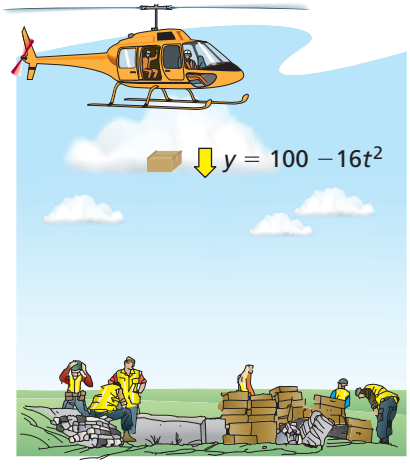
12. $h^2 - 8 = -3h + 10$

13. $4s^2 = 144$

14. $k^2 + 100 = 20k$

15. STORAGE The front of a storage bunker can be modeled by $y = -\frac{5}{216}(x - 72)(x + 72)$, where x and y are measured in inches. The x-axis represents the ground. Find the width of the bunker at ground level. *(Section 7.5)*





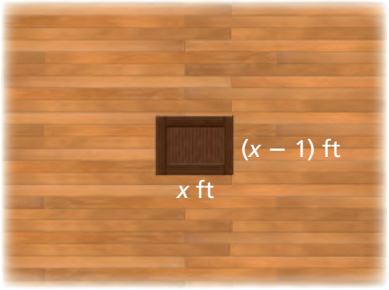
16. DISASTER RELIEF A helicopter drops a box of supplies after a disaster. The function represents the height y (in feet) of the box t seconds after it is dropped. After how many seconds does the box hit the ground? *(Section 7.9)*

17. MAGIC SHOW A magician's stage has a trap door. *(Section 7.7)*

a. The total area of the stage can be represented by $x^2 + 27x + 176$. Write an expression for the width of the stage.

b. The area of the trap door is 12 square feet. Find the value of x.

c. What fraction of the area of the stage is the area of the trap door?



7 Chapter Review

Check It Out
Vocabulary Help
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Review Key Vocabulary

monomial, *p. 330*
degree of a monomial, *p. 330*
polynomial, *p. 331*
binomial, *p. 331*
trinomial, *p. 331*
degree of a polynomial, *p. 331*
FOIL Method, *p. 343*
factored form, *p. 358*
Zero-Product Property, *p. 358*
root, *p. 358*
factoring by grouping, *p. 388*
prime polynomial, *p. 389*
factored completely, *p. 389*

Review Examples and Exercises

7.1 Polynomials *(pp. 328–333)*

a. Find the degree of $4x^2y$.

The exponent of x is 2 and the exponent of y is 1.
The sum of the exponents is $2 + 1 = 3$.

∴ So, the degree of the monomial is 3.

b. Write $x + 1 + 2x^3$ in standard form. Identify the degree and classify the polynomial by the number of terms.

Polynomial	*Standard Form*	*Degree*	*Type of Polynomial*
$x + 1 + 2x^3$	$2x^3 + x + 1$	3	trinomial

Exercises

Write the polynomial in standard form. Identify the degree and classify the polynomial by the number of terms.

1. $2w^3 + 3 - 4w$ **2.** $-6y^2$ **3.** $-6.2 + 3t^5$

7.2 Adding and Subtracting Polynomials *(pp. 334–339)*

a. $(2d^2 - 3) + (4d^2 + 2)$

$$(2d^2 - 3) + (4d^2 + 2) = (2d^2 + 4d^2) + (-3 + 2)$$
$$= 6d^2 - 1$$

b. $(c^2 + 5c + 1) - (c^2 - 2)$

$$(c^2 + 5c + 1) - (c^2 - 2) = (c^2 + 5c + 1) + (-c^2 + 2)$$
$$= [c^2 + (-c^2)] + 5c + (1 + 2) = 5c + 3$$

Exercises

Find the sum or difference.

4. $(3a + 7) + (a - 1)$ **5.** $(x^2 + 4x - 2) + (6x^2 + 6)$

6. $(-y^2 + y + 2) - (y^2 - 5y - 2)$ **7.** $(p - 9) - (-8p^2 + 7)$

7.3 Multiplying Polynomials *(pp. 340–347)*

Find $(x + 1)(x - 4)$.

First Outer Inner Last

$(x + 1)(x - 4) = x(x) + x(-4) + (1)(x) + (1)(-4)$ Use the FOIL Method.

$= x^2 + (-4x) + (x) + (-4)$ Multiply.

$= x^2 - 3x - 4$ Combine like terms.

Exercises

Find the product.

8. $(y + 4)(y - 2)$ **9.** $(q - 3)(2q + 7)$ **10.** $(-3v + 1)(v^2 - v - 2)$

7.4 Special Products of Polynomials *(pp. 348–353)*

Find each product.

a. $(x + 3)(x - 3)$

$(a + b)(a - b) = a^2 - b^2$ Sum and Difference Pattern

$(x + 3)(x - 3) = x^2 - 3^2$ Use pattern.

$= x^2 - 9$ Simplify.

b. $(y + 2)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ Square of a Binomial Pattern

$(y + 2)^2 = y^2 + 2(y)(2) + 2^2$ Use pattern.

$= y^2 + 4y + 4$ Simplify.

Exercises

Find the product.

11. $(y + 9)(y - 9)$ **12.** $(2x + 4)(2x - 4)$

13. $(h + 4)^2$ **14.** $(-1 + 2d)^2$

7.5 Solving Polynomial Equations in Factored Form *(pp. 356–361)*

Solve $(x + 4)(x - 3) = 0$.

$(x + 4)(x - 3) = 0$ Write equation.

$x + 4 = 0$ *or* $x - 3 = 0$ Use Zero-Product Property.

$x = -4$ *or* $x = 3$ Solve for x.

∴ The roots are $x = -4$ and $x = 3$.

Exercises

Solve the equation.

15. $x(x + 2) = 0$

16. $(t - 3)(t - 8) = 0$

17. $(a + 10)^2 = 0$

18. $2s(s + 1)(s - 4) = 0$

7.6 Factoring Polynomials Using the GCF *(pp. 362–367)*

Factor $4z^2 + 32$.

Step 1: Find the GCF of the terms.

$$4z^2 = 2 \cdot 2 \cdot z \cdot z$$
$$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

The GCF is $2 \cdot 2 = 4$.

Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$4z^2 + 32 = 4(z^2) + 4(8) \qquad \text{Factor out GCF.}$$
$$= 4(z^2 + 8) \qquad \text{Distributive Property}$$

Exercises

Factor the polynomial.

19. $6t^2 + 36$

20. $2x^2 - 20x$

21. $15y^3 + 3y^2$

7.7 Factoring $x^2 + bx + c$ *(pp. 368–375)*

Factor $x^2 + 12x + 27$.

Notice that $b = 12$ and $c = 27$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is also positive, p and q must each be positive so that $p + q$ is positive.

Find two positive integer factors of 27 whose sum is 12.

Factors of 27	Sum of Factors
1, 27	28
3, 9	12

The values of p and q are 3 and 9.

∴ So, $x^2 + 12x + 27 = (x + 3)(x + 9)$.

Exercises

Factor the polynomial.

22. $p^2 + 2p - 35$ **23.** $b^2 + 9b + 20$ **24.** $z^2 - 4z - 21$

7.8 Factoring $ax^2 + bx + c$ *(pp. 376–381)*

a. Factor $2x^2 + 13x + 15$.

Consider the possible factors of $a = 2$ and $c = 15$.

Factors are 1 and 2. → $2x^2 + 13x + 15$ ← Factors are 1, 3, 5, and 15.

These factors lead to the following possible products.

$(1x + 1)(2x + 15)$ $(1x + 3)(2x + 5)$

$(1x + 15)(2x + 1)$ $(1x + 5)(2x + 3)$

Multiply to find the product that is equal to the original polynomial.

$(x + 1)(2x + 15) = 2x^2 + 17x + 15$ ✗

$(x + 15)(2x + 1) = 2x^2 + 31x + 15$ ✗

$(x + 3)(2x + 5) = 2x^2 + 11x + 15$ ✗

$(x + 5)(2x + 3) = 2x^2 + 13x + 15$ ✓

∴ So, $2x^2 + 13x + 15 = (x + 5)(2x + 3)$.

b. Factor $5x^2 + 4x - 9$.

Consider the possible factors of $a = 5$ and $c = -9$. Because b is positive and c is negative, the factors of c must have different signs.

Factors are 1 and 5. → $5x^2 + 4x - 9$ ← Factors are ± 1, ± 3, and ± 9.

These factors lead to the following possible products.

$(1x + 1)(5x - 9)$ $(1x - 1)(5x + 9)$ $(1x - 3)(5x + 3)$

$(1x + 9)(5x - 1)$ $(1x - 9)(5x + 1)$ $(1x + 3)(5x - 3)$

Multiply to find the product that is equal to the original polynomial.

$(x + 1)(5x - 9) = 5x^2 - 4x - 9$ ✗

$(x + 9)(5x - 1) = 5x^2 + 44x - 9$ ✗

$(x - 1)(5x + 9) = 5x^2 + 4x - 9$ ✓

$(x - 9)(5x + 1) = 5x^2 - 44x - 9$ ✗

$(x - 3)(5x + 3) = 5x^2 - 12x - 9$ ✗

$(x + 3)(5x - 3) = 5x^2 + 12x - 9$ ✗

∴ So, $5x^2 + 4x - 9 = (x - 1)(5x + 9)$.

Exercises

Factor the polynomial.

25. $10a^2 + 11a + 3$

26. $4z^2 + 11z + 6$

27. $2x^2 - 27x - 14$

28. $-2p^2 + 2p + 4$

29. OUTSIDE PATIO You are installing new tile on an outside patio. The area (in square feet) of the rectangular patio can be represented by $8x^2 + 33x + 4$. Write the expressions that represent the dimensions of the patio.

7.9 Factoring Special Products *(pp. 382–389)*

Factor each polynomial.

a. $x^2 - 16$

$x^2 - 16 = x^2 - 4^2$ Write as $a^2 - b^2$.

$= (x + 4)(x - 4)$ Difference of Two Squares Pattern

b. $x^2 - 2x + 1$

$x^2 - 2x + 1 = x^2 - 2(x)(1) + 1^2$ Write as $a^2 - 2ab + b^2$.

$= (x - 1)^2$ Perfect Square Trinomial Pattern

c. $x^3 + 4x^2 + 3x + 12$

$x^3 + 4x^2 + 3x + 12 = (x^3 + 4x^2) + (3x + 12)$ Group terms with common factors.

Common binomial factor is $x + 4$. → $= x^2(x + 4) + 3(x + 4)$ Factor out GCF of each pair of terms.

$= (x + 4)(x^2 + 3)$ Factor out $(x + 4)$.

d. $2x^4 - 8x^2$

$2x^4 - 8x^2 = 2x^2(x^2 - 4)$ Factor out $2x^2$.

$= 2x^2(x^2 - 2^2)$ Write as $a^2 - b^2$.

$= 2x^2(x + 2)(x - 2)$ Difference of Two Squares Pattern

Exercises

Factor the polynomial.

30. $x^2 - 9$

31. $y^2 - 100$

32. $z^2 + 6z + 9$

33. $m^2 + 16m + 64$

34. $x^2 - 3x + 4ax - 12a$

35. $n^3 - 9n$

7 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Write the polynomial in standard form. Identify the degree and classify the polynomial by the number of terms.

1. $-2.1w^3$
2. $7k + 4 - 3k^2$
3. $-c^8 + 9c^{12}$

Find the sum or difference.

4. $(-2p + 4) - (p^2 - 6p + 8)$
5. $(4s^2 + 2st + t) + (-3s^2 + 5st - 4t)$

Find the product.

6. $(h - 5)(h - 8)$
7. $(2w - 3)(2w + 5)$
8. $(z + 11)(z - 11)$

Factor the polynomial.

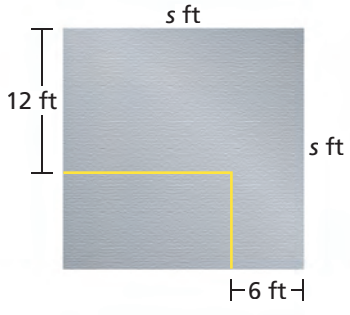
9. $7x^2 - 21x$
10. $n^2 + 7n + 10$
11. $m^2 - 2m - 24$
12. $6g^2 + 23g + 7$
13. $y^2 - 100$
14. $b^3 - 2b^2 + 3b - 6$

Solve the equation.

15. $(n - 1)(n + 6) = 0$
16. $3h^2 = -12h$
17. $s^2 - 15s + 50 = 0$
18. $5k^2 + 22k - 15 = 0$
19. $d^2 + 14d + 49 = 0$
20. $6x^4 + 8x^2 = 26x^3$

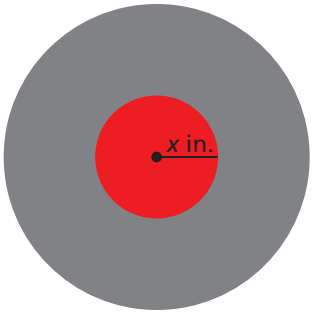
21. **TIME** The expression $\pi(r - 3)^2$ represents the area covered by the hour hand on a clock in one rotation, where r is the radius of the entire clock. Write a polynomial that represents the area covered by the hour hand in one rotation.

22. **TRAMPOLINE** You are jumping on a trampoline. Your height y (in feet) above the trampoline after t seconds can be represented by $y = -16t^2 + 24t$. How many seconds are you in the air?

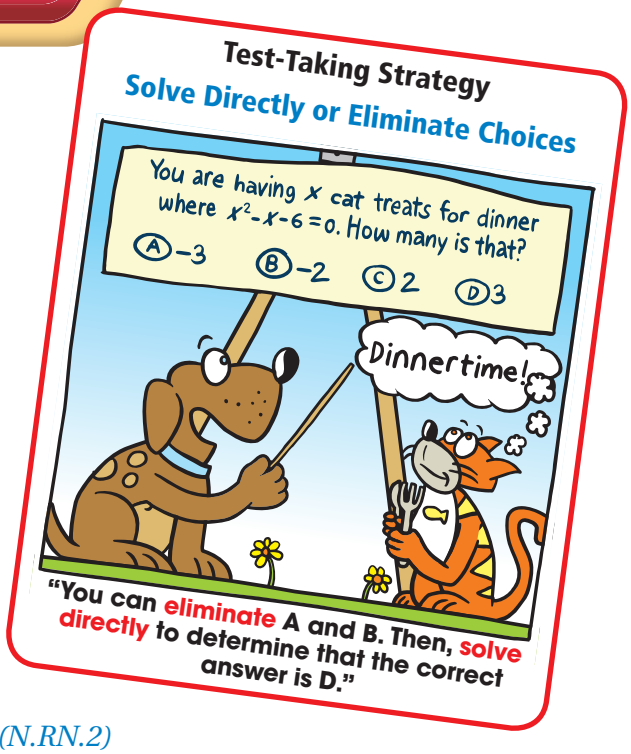


23. **CEMENT** You pour cement in a rectangular region of a square garage. The area of the rectangular region is 112 square feet.
    a. What is the area of the garage floor?
    b. You place caution tape along the two sides of the newly cemented region that are not on the wall. How many feet of caution tape do you use?

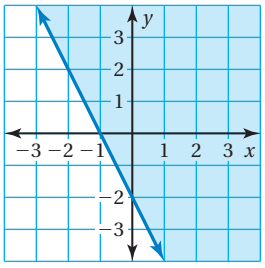
24. **ARCHERY** The area (in square inches) of the target can be represented by $\pi(x^2 + 6x + 9)$.
    a. Find the areas of the red bull's eye and the gray ring when the area of the target is 25π square inches. Write your answer in terms of π.
    b. Write a binomial that represents the radius of the target.
    c. What is the width of the gray ring? Does it change as x changes? Does its area change as x changes? Explain.



7 Standards Assessment



1. Which inequality is shown in the coordinate plane? *(A.REI.12)*



A. $y > -2x - 2$

B. $y < -2x - 2$

C. $y \geq -2x - 2$

D. $y \leq -2x - 2$

2. Which expression is equivalent to $\left(\frac{a^3}{a^{-2}}\right)^{-3}$? *(N.RN.2)*

F. a^2

G. $\frac{1}{a^3}$

H. $\frac{1}{a^2}$

I. $\frac{1}{a^{15}}$

3. What is the degree of the polynomial shown below? *(A.SSE.1a)*

$$p^3 + 2p - 5p^4$$

4. Which of the following is the equation of the line that passes through the points $(-1, -6)$ and $(2, 6)$? *(A.CED.2)*

A. $y = -\frac{1}{4}x - \frac{25}{4}$

B. $y = -4x - 10$

C. $y = \frac{1}{4}x - \frac{23}{4}$

D. $y = 4x - 2$

5. What are the roots of $(5b + 3)(5b - 3) = 0$? *(A.REI.4b)*

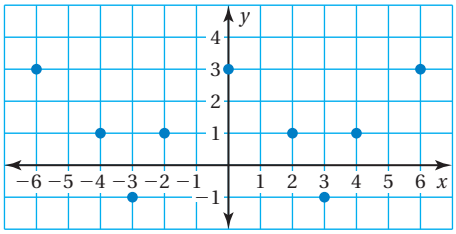
F. $-\frac{3}{5}$ and $\frac{3}{5}$

G. -3 and 3

H. -5 and 5

I. $-\frac{5}{3}$ and $\frac{5}{3}$

6. What is the range of the function graphed in the coordinate plane below? *(F.IF.1)*



A. 1, 3

B. −1, 1, 3

C. 0, 2, 3, 4, 6

D. −6, −4, −3, −2, 0, 2, 3, 4, 6

7. Which polynomial represents the product of $2x - 4$ and $x^2 + 6x - 2$? *(A.APR.1)*

F. $2x^3 + 8x^2 - 4x + 8$

G. $2x^3 + 8$

H. $2x^3 + 8x^2 - 28x + 8$

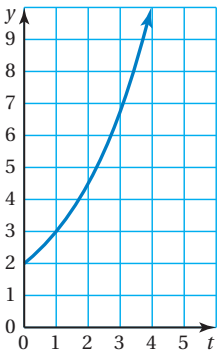
I. $2x^3 - 24x - 2$

8. For what value of b does the system of linear equations shown below have no solution? *(8.EE.8a)*

$$y = 6x + 3$$

$$bx - 2y = -10$$

9. The graph of which equation is shown in the coordinate plane? *(F.IF.7e)*



A. $y = 2(1.5)^t$

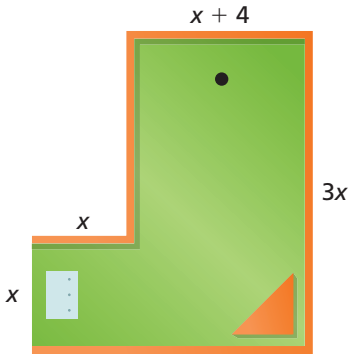
B. $y = 2^t$

C. $y = 2(0.5)^t$

D. $y = (0.5)^t$

10. The playing area of a hole on a miniature golf course is 216 square feet. What is the perimeter of the playing area? Explain. *(A.REI.4b)*

Think
Solve
Explain

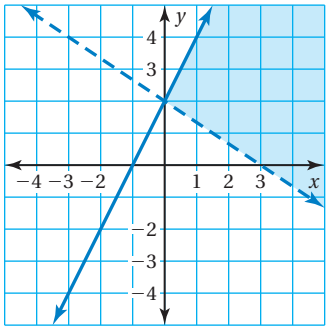


11. Which graph shows the solution of the system of linear inequalities shown below? *(A.REI.12)*

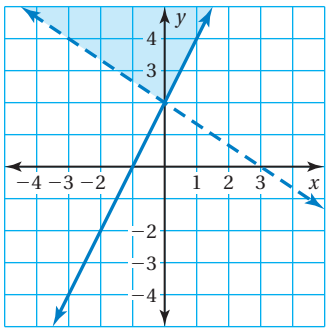
$$2x + 3y > 6$$

$$2x - y \leq -2$$

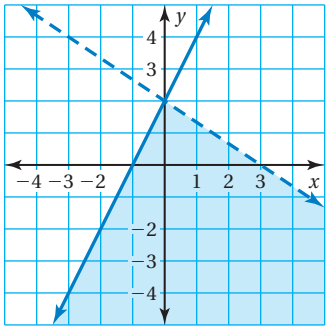
F.



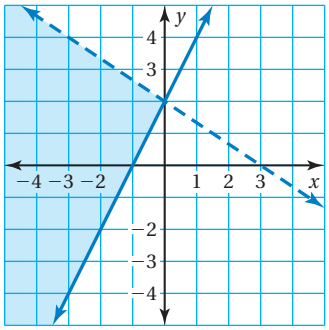
H.



G.



I.



12. Andy was factoring the polynomial in the box below. *(A.SSE.2)*

$$16t^2 - 49 = 4t^2 - 7^2$$
$$= (2t + 7)(2t - 7)$$

What should Andy do to correct the error that he made?

A. Rewrite $16t^2$ as $(16t)^2$.

B. Rewrite $4t^2 - 7^2$ as $(2t - 7)^2$.

C. Rewrite $16t^2$ as $(4t)^2$.

D. Rewrite $16t^2 - 49$ as $(4t - 7)^2$.

13. What is the common ratio of the sequence $243, -81, 27, -9, \ldots$? *(F.LE.2)*

F. $\frac{1}{3}$

G. $-\frac{1}{3}$

H. 3

I. -3