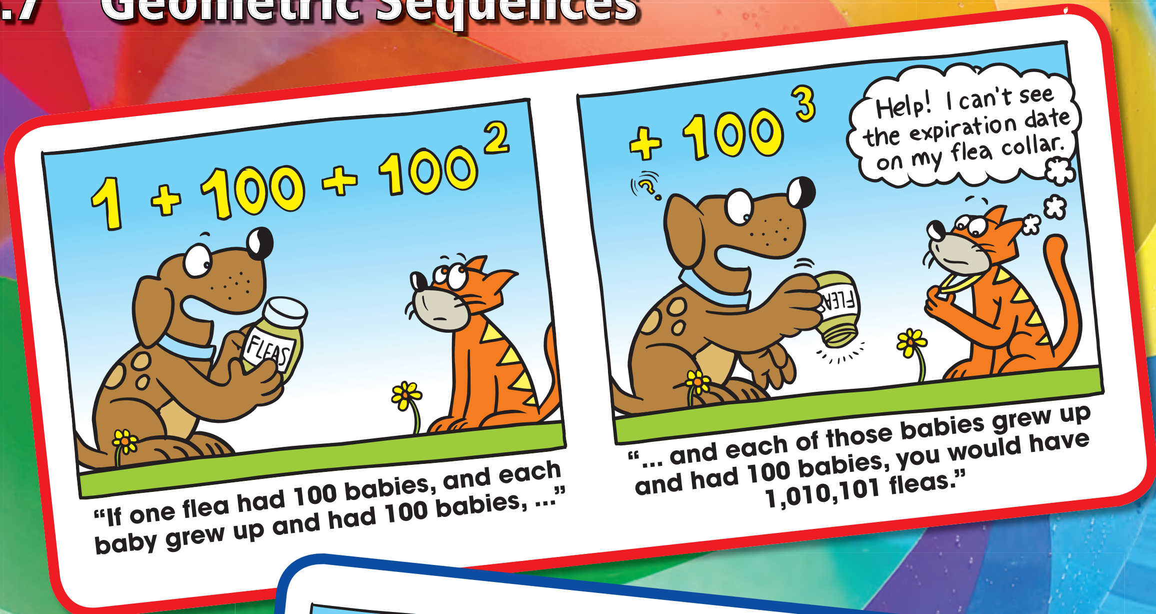
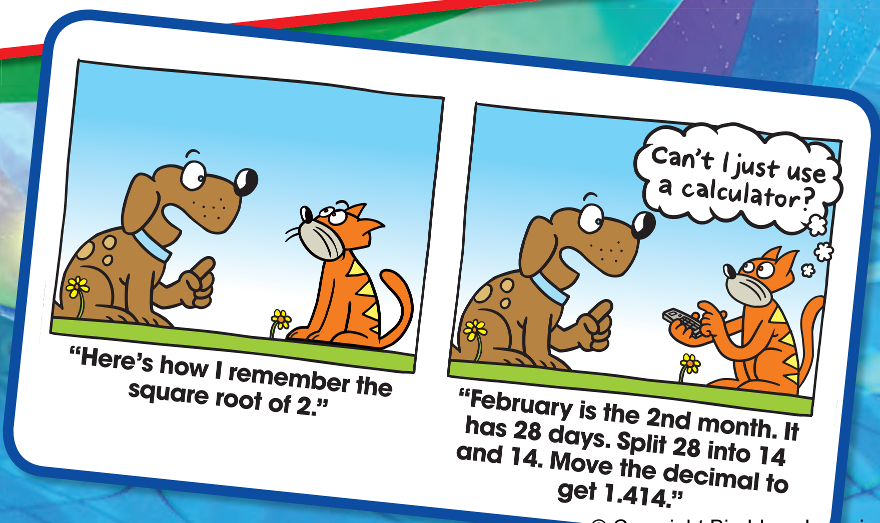
6 Exponential Equations and Functions





What You Learned Before

"It's called the Power of Negative One, Descartes!"

Using Order of Operations (8.EE.1)

Example 1 **Evaluate $10^2 \div (30 \div 3) - 4(3 - 9) + 5^0$.**

First:	**P**arentheses	$10^2 \div (30 \div 3) - 4(3 - 9) + 5^0 = 10^2 \div 10 - 4(-6) + 5^0$
Second:	**E**xponents	$= 100 \div 10 - 4(-6) + 1$
Third:	**M**ultiplication and **D**ivision (from left to right)	$= 10 + 24 + 1$
Fourth:	**A**ddition and **S**ubtraction (from left to right)	$= 35$

Try It Yourself

Evaluate the expression.

1. $12\left(\frac{14}{2}\right) - 3^3 + 15 - 2^0$ **2.** $5^2 \bullet 8 \div 2^2 + 20 \bullet 3 - 4$ **3.** $-7 + 16 \bullet 4^{-2} + (10 - 4^2)$

Finding Square Roots (8.EE.2)

Example 2 **Find $-\sqrt{81}$.**

$-\sqrt{81}$ represents the negative square root. Because $9^2 = 81$, $-\sqrt{81} = -\sqrt{9^2} = -9$.

Try It Yourself

4. Find $\sqrt{64}$. **5.** Find $\pm\sqrt{121}$. **6.** Find $-\sqrt{4}$.

Writing an Equation for an Arithmetic Sequence (F.BF.2)

Example 3 **Write an equation for the *n*th term of the arithmetic sequence 5, 15, 25, 35, ...**

The first term is 5 and the common difference is 10.

$a_n = a_1 + (n - 1)d$ Equation for an arithmetic sequence

$a_n = 5 + (n - 1)10$ Substitute 5 for a_1 and 10 for d.

$a_n = 10n - 5$ Simplify.

Try It Yourself

Write an equation for the *n*th term of the arithmetic sequence.

7. 2, 4, 6, 8, . . . **8.** 6, 3, 0, −3, . . . **9.** 22, 15, 8, 1, . . .

6.1 Properties of Square Roots

Essential Question How can you multiply and divide square roots?

Recall that when you multiply a number by itself, you square the number.

Symbol for squaring is 2nd power.

$$4^2 = 4 \cdot 4$$
$$= 16 \qquad \text{4 squared is 16.}$$

To "undo" this, take the square root of the number.

Symbol for square root is a radical sign.

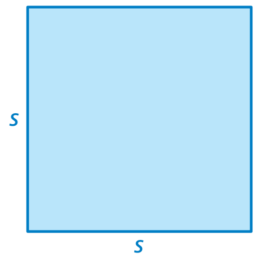
$$\sqrt{16} = \sqrt{4^2} = 4 \qquad \text{The square root of 16 is 4.}$$

1 ACTIVITY: Finding Square Roots

Work with a partner. Use a square root symbol to write the side length of the square. Then find the square root. Check your answer by multiplying.

a. Sample: $s = \sqrt{81} = 9$ ft

Area = 81 ft^2

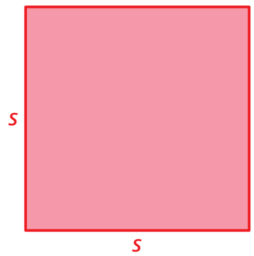


Check

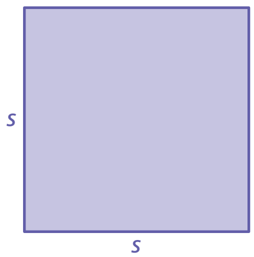
$$\begin{array}{r} 9 \\ \times\ 9 \\ \hline 81 \end{array} \checkmark$$

The side length of the square is 9 feet.

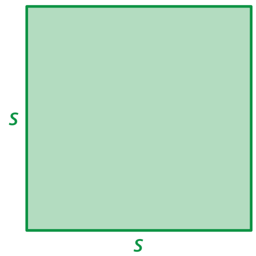
b. Area = 121 yd^2



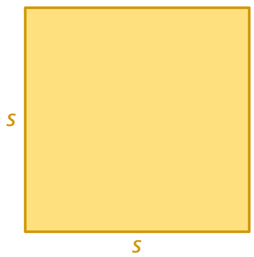
c. Area = 324 cm^2



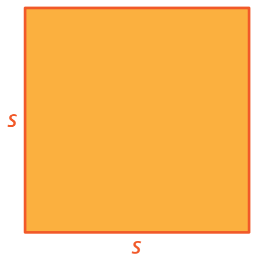
d. Area = 361 mi^2



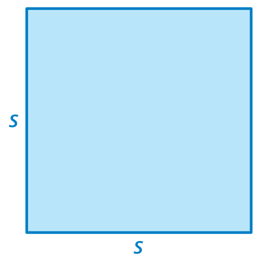
e. Area = 2.89 in.^{2}



f. Area = 6.25 m^2



g. Area = $\frac{16}{25}$ ft^2



COMMON CORE

Square Roots

In this lesson, you will

- simplify and evaluate square roots.
- simplify radical expressions.

Preparing for Standard N.RN.3

2 ACTIVITY: Operations with Square Roots

Math Practice 3

Analyze Conjectures

How do you know if your conclusion is accurate? How can you explain your conclusion?

Work with a partner. When you have an expression that involves two operations, you need to know whether you obtain the same result regardless of the order in which you perform the operations. In each of the following, compare the results obtained by the two orders. What can you conclude?

a. Square Roots and Addition

Is $\sqrt{36} + \sqrt{64}$ equal to $\sqrt{36 + 64}$?

In general, is $\sqrt{a} + \sqrt{b}$ equal to $\sqrt{a + b}$?

Explain your reasoning.

b. Square Roots and Multiplication

Is $\sqrt{4} \cdot \sqrt{9}$ equal to $\sqrt{4 \cdot 9}$?

In general, is $\sqrt{a} \cdot \sqrt{b}$ equal to $\sqrt{a \cdot b}$?

Explain your reasoning.

c. Square Roots and Subtraction

Is $\sqrt{64} - \sqrt{36}$ equal to $\sqrt{64 - 36}$?

In general, is $\sqrt{a} - \sqrt{b}$ equal to $\sqrt{a - b}$?

Explain your reasoning.

d. Square Roots and Division

Is $\dfrac{\sqrt{100}}{\sqrt{4}}$ equal to $\sqrt{\dfrac{100}{4}}$?

In general, is $\dfrac{\sqrt{a}}{\sqrt{b}}$ equal to $\sqrt{\dfrac{a}{b}}$?

Explain your reasoning.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you multiply and divide square roots? Write a rule for:

   a. The product of square roots

   b. The quotient of square roots

Practice Use what you learned about square roots to complete Exercises 3–5 on page 264.

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Ideas

Product Property of Square Roots

Algebra

$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$, where $x, y \geq 0$

Numbers

$\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Quotient Property of Square Roots

Algebra

$\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$, where $x \geq 0$ and $y > 0$

Numbers

$\sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{\sqrt{4}} = \dfrac{\sqrt{3}}{2}$

EXAMPLE 1 Simplifying Square Roots

a. $\sqrt{150} = \sqrt{25 \cdot 6}$ — Factor using the greatest perfect square factor.

$= \sqrt{25} \cdot \sqrt{6}$ — Product Property of Square Roots

$= 5\sqrt{6}$ — Simplify.

b. $\sqrt{\dfrac{15}{64}} = \dfrac{\sqrt{15}}{\sqrt{64}}$ — Quotient Property of Square Roots

$= \dfrac{\sqrt{15}}{8}$ — Simplify.

Remember

A square root is simplified when the radicand has no perfect square factors other than 1.

EXAMPLE 2 Evaluating Square Roots

Evaluate $\sqrt{b^2 - 4ac}$ when $a = 2$, $b = -8$, and $c = 4$.

$\sqrt{b^2 - 4ac} = \sqrt{(-8)^2 - 4(2)(4)}$ — Substitute.

$= \sqrt{32}$ — Simplify.

$= \sqrt{16 \cdot 2}$ — Factor.

$= \sqrt{16} \cdot \sqrt{2}$ — Product Property of Square Roots

$= 4\sqrt{2}$ — Simplify.

On Your Own

Now You're Ready
Exercises 6–17

Simplify the expression.

1. $\sqrt{\dfrac{23}{9}}$ **2.** $-\sqrt{80}$ **3.** $\sqrt{\dfrac{27}{100}}$

4. Evaluate $\sqrt{b^2 - 4ac}$ when $a = 2$, $b = -6$, and $c = -5$.

EXAMPLE 3 Simplifying Radical Expressions

Simplify $\dfrac{6+\sqrt{8}}{2}$.

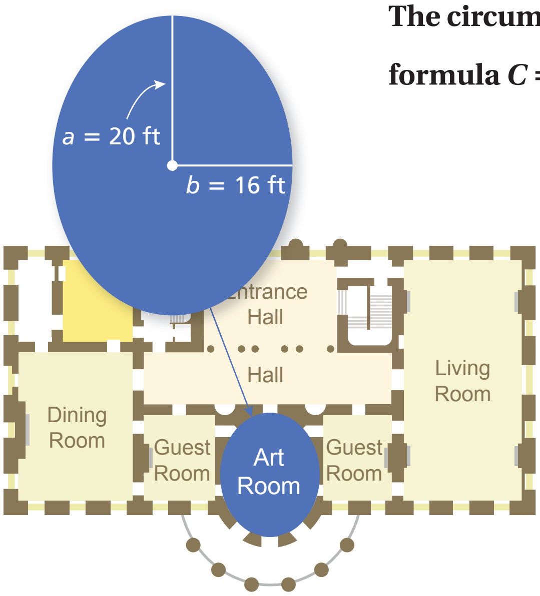
$$\frac{6+\sqrt{8}}{2} = \frac{6+\sqrt{4 \cdot 2}}{2} \qquad \text{Factor the radicand.}$$

$$= \frac{6+\sqrt{4} \cdot \sqrt{2}}{2} \qquad \text{Product Property of Square Roots}$$

$$= \frac{6+2\sqrt{2}}{2} \qquad \text{Simplify.}$$

$$= 3+\sqrt{2} \qquad \text{Divide.}$$

EXAMPLE 4 Real-Life Application



The circumference C of the art room in a mansion is given by the formula $C = 2\pi\sqrt{\dfrac{a^2+b^2}{2}}$. Find the circumference of the room.

$$C = 2\pi\sqrt{\frac{a^2+b^2}{2}} \qquad \text{Write formula.}$$

$$= 2\pi\sqrt{\frac{20^2+16^2}{2}} \qquad \text{Substitute.}$$

$$= 2\pi\sqrt{328} \qquad \text{Simplify.}$$

$$= 2\pi\sqrt{4 \cdot 82} \qquad \text{Factor.}$$

$$= 2\pi \cdot \sqrt{4} \cdot \sqrt{82} \qquad \text{Product Property of Square Roots}$$

$$= 4\pi\sqrt{82} \qquad \text{Simplify.}$$

The circumference of the room is $4\pi\sqrt{82}$, or about 114 feet.

On Your Own

Now You're Ready
Exercises 21–26

Simplify the expression.

5. $\dfrac{8+\sqrt{32}}{2}$

6. $\dfrac{-1-\sqrt{27}}{4}$

7. $\dfrac{2-\sqrt{28}}{2(3)}$

8. Use the formula in Example 4 to find the circumference of an ellipse in which $a = 14$ feet and $b = 6$ feet.

6.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

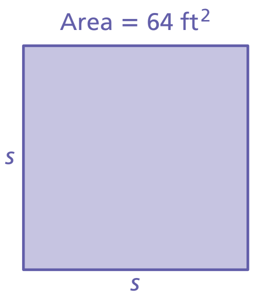
Vocabulary and Concept Check

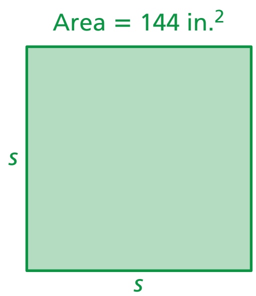
1. **WRITING** How do you know when the square root of a positive integer is simplified?

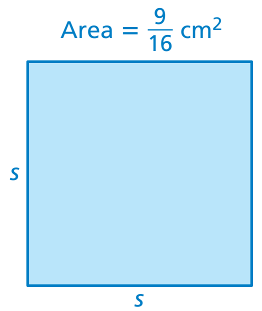
2. **WRITING** How is the Product Property of Square Roots similar to the Quotient Property of Square Roots?

Practice and Problem Solving

Find the dimensions of the square. Check your answer.

3. 

4. 

5. 

Simplify the expression.

1 6. $\sqrt{18}$

7. $-\sqrt{200}$

8. $\sqrt{12}$

9. $\sqrt{48}$

10. $\sqrt{125}$

11. $-\sqrt{\dfrac{23}{64}}$

12. $-\sqrt{\dfrac{65}{121}}$

13. $\sqrt{\dfrac{18}{49}}$

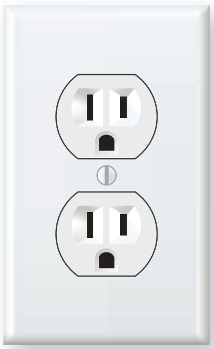
14. $\sqrt{\dfrac{25}{36}}$

Evaluate the expression when $x = -2$, $y = 8$, and $z = \dfrac{1}{2}$.

2 15. $\sqrt{x^2 + yz}$

16. $\sqrt{2x^2 + y^2}$

17. $\sqrt{y - 44xz}$



18. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

✗ $$\sqrt{\frac{20}{9}} = \frac{\sqrt{20}}{\sqrt{9}} = \frac{\sqrt{20}}{3}$$

19. **ELECTRICITY** The electric current I (in amperes) an appliance uses is given by the formula $I = \sqrt{\dfrac{P}{R}}$, where P is the power (in watts) and R is the resistance (in ohms). Find the current an appliance uses when the power is 147 watts and the resistance is 4 ohms.

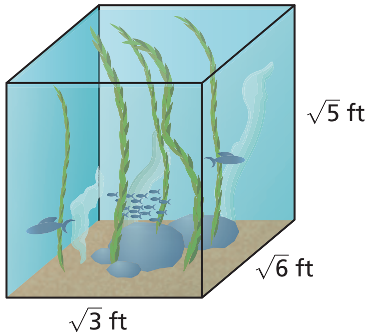
20. **BASEBALL** You drop a baseball from a height of 56 feet. Use the expression $\sqrt{\frac{h}{16}}$, where h is the height (in feet), to find the time (in seconds) it takes the baseball to hit the ground.

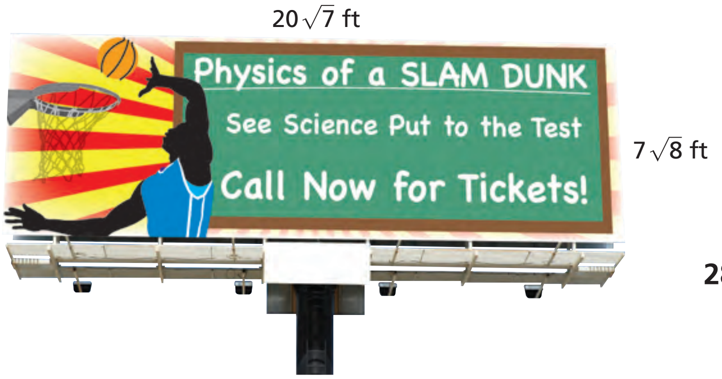
Simplify the expression.

③ **21.** $\dfrac{6 + \sqrt{44}}{2}$ **22.** $\dfrac{-7 - \sqrt{98}}{7}$ **23.** $\dfrac{10 + \sqrt{300}}{5}$

24. $\dfrac{-3 - \sqrt{80}}{6}$ **25.** $\dfrac{2 + \sqrt{28}}{4}$ **26.** $\dfrac{-4 + \sqrt{32}}{-2(5)}$

27. **VOLUME** A pet store installs a new aquarium in your teacher's classroom. What is the volume of the aquarium?



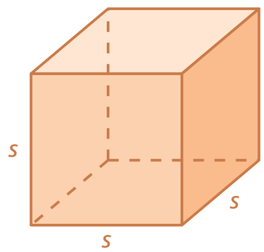


28. **BILLBOARD** What is the area of the rectangular billboard?

Simplify the expression. Assume all variables are positive.

29. $\sqrt{42x^2y^2}$ **30.** $\sqrt{25y^2z}$ **31.** $\sqrt{18x^3y^2z}$

32. **Modeling** Write an equation that represents the side length s of a cube as a function of the surface area A of the cube. Find the side length when the surface area is 72 square feet.



Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Skills Review Handbook)*

33. 3^5 **34.** 2^4 **35.** 5^3

36. **MULTIPLE CHOICE** Which value is equivalent to $6(0.2)^3$? *(Skills Review Handbook)*

Ⓐ 0.008 Ⓑ 0.048 Ⓒ 1.728 Ⓓ 3.6

Extension 6.1 Real Number Operations

Check It Out
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Key Vocabulary
closed, *p. 266*

A set of numbers is **closed** under an operation when the operation performed on any two numbers in the set results in a number that is also in the set. For example, the set of integers is closed under addition, subtraction, and multiplication. This means that if a and b are two integers, then $a + b$, $a - b$, and ab are also integers.

ACTIVITY 1: Sums and Products of Rational Numbers

Remember

A *rational number* is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$. An *irrational number* cannot be written as the ratio of two integers.

The table shows several sums and products of rational numbers. Complete the table.

Sum or Product	Answer	Rational or Irrational?
$12 + 5$		
$-4 + 9$		
$\frac{4}{5} + \frac{2}{3}$		
$0.74 + 2.1$		
3×8		
-4×6		
3.1×0.6		
$\frac{3}{4} \times \frac{5}{7}$		

ACTIVITY 2: Sums of Rational and Irrational Numbers

The table shows several sums of rational and irrational numbers. Complete the table.

Sum	Answer	Rational or Irrational?
$1 + \sqrt{5}$		
$\sqrt{2} + \frac{5}{6}$		
$4 + \pi$		
$-8 + \sqrt{10}$		

Practice

1. Using the results in Activity 1, do you think the set of rational numbers is closed under addition? under multiplication? Explain your reasoning.

2. Using the results in Activity 2, what do you notice about the sum of a rational number and an irrational number?

Multi-Language Glossary at BigIdeasMath.com

ACTIVITY 3: Products of Rational and Irrational Numbers

Common Core

Real Number Operations

In this extension, you will

- determine whether sums or products are rational or irrational.

Learning Standard
N.RN.3

The table shows several products of rational and irrational numbers. Complete the table.

Product	Answer	Rational or Irrational?
$6 \cdot \sqrt{12}$		
$-2 \cdot \pi$		
$\frac{2}{5} \cdot \sqrt{3}$		
$0 \times \sqrt{6}$		

ACTIVITY 4: Sums and Products of Irrational Numbers

The table shows several sums and products of irrational numbers. Complete the table.

Sum or Product	Answer	Rational or Irrational?
$3\sqrt{2} + 5\sqrt{2}$		
$\sqrt{12} + \sqrt{27}$		
$\sqrt{7} + \pi$		
$-\pi + \pi$		
$\pi \cdot \sqrt{7}$		
$\sqrt{5} \times \sqrt{2}$		
$4\pi \cdot \sqrt{3}$		
$\sqrt{3} \times \sqrt{3}$		

Practice

3. Using the results in Activity 3, is the product of a rational number and an irrational number always irrational? Explain.

4. Using the results in Activity 4, do you think the set of irrational numbers is closed under addition? under multiplication? Explain your reasoning.

5. **CRITICAL THINKING** Is the set of irrational numbers closed under division? If not, find a counterexample. (A *counterexample* is an example that shows that a statement is false.)

6. **STRUCTURE** The set of integers is closed under addition and multiplication. Use this information to show that the sum and product of two rational numbers are always rational numbers.

6.2 Properties of Exponents

Essential Question How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

1 ACTIVITY: Writing a Rule for Products of Powers

Work with a partner. Write the product of the two powers as a single power. Then, write a *general rule* for finding the product of two powers with the same base.

a. Sample: $(3^4)(3^3) = (3 \cdot 3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3) = 3^7$

b. $(2^2)(2^3) =$

c. $(4^1)(4^5) =$

d. $(5^3)(5^5) =$

e. $(x^2)(x^6) =$

2 ACTIVITY: Writing a Rule for Quotients of Powers

Work with a partner. Write the quotient of the two powers as a single power. Then, write a *general rule* for finding the quotient of two powers with the same base.

a. Sample: $\dfrac{3^4}{3^2} = \dfrac{3 \cdot 3 \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3}} = 3^2$

b. $\dfrac{4^3}{4^2} =$

c. $\dfrac{2^5}{2^2} =$

d. $\dfrac{x^6}{x^3} =$

e. $\dfrac{3^4}{3^4} =$

Common Core
Exponents
In this lesson, you will
- simplify expressions using the properties of exponents.

Learning Standard
N.RN.2

3 ACTIVITY: Writing a Rule for Powers of Powers

Work with a partner. Write the expression as a single power. Then, write a *general rule* for finding a power of a power.

a. Sample: $(3^2)^3 = (3 \cdot 3)(3 \cdot 3)(3 \cdot 3) = 3^6$

b. $(2^2)^4 =$

c. $(7^3)^2 =$

d. $(y^3)^3 =$

e. $(x^4)^2 =$

4 ACTIVITY: Writing a Rule for Powers of Products

Math Practice 7

View as Components

What are the different parts of the expressions? How does this help you rewrite the product?

Work with a partner. Write the expression as the product of two powers. Then, write a *general rule* for finding a power of a product.

a. Sample: $(2 \cdot 3)^3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = (2^3)(3^3)$

b. $(2 \cdot 5)^2 =$ ________

c. $(5 \cdot 4)^3 =$ ________

d. $(6a)^4 =$ ________

e. $(3x)^2 =$ ________

5 ACTIVITY: Writing a Rule for Powers of Quotients

Work with a partner. Write the expression as the quotient of two powers. Then, write a *general rule* for finding a power of a quotient.

a. Sample: $\left(\frac{3}{2}\right)^4 = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{3^4}{2^4}$

b. $\left(\frac{2}{3}\right)^2 =$ ________

c. $\left(\frac{4}{3}\right)^3 =$ ________

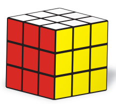
d. $\left(\frac{x}{2}\right)^3 =$ ________

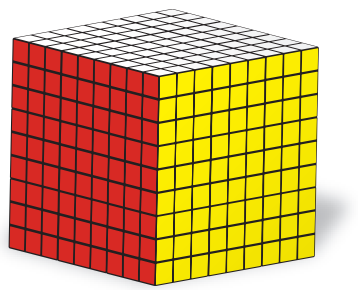
e. $\left(\frac{a}{b}\right)^4 =$ ________

What Is Your Answer?

6. IN YOUR OWN WORDS How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

7. There are 3^3 small cubes in the cube below. Write an expression for the number of small cubes in the large cube at the right.





Practice Use what you learned about exponents to complete Exercises 6–11 on page 273.

6.2 Lesson

Check It Out
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Key Ideas

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \cdot 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property

Words To divide powers with the same base, subtract their exponents.

Numbers $\dfrac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\dfrac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

Remember

For any integer n and any nonzero integer a, $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$.

EXAMPLE 1 Using Properties of Exponents

Simplify. Write your answer using only positive exponents.

a. $3^2 \cdot 3^6 = 3^{2+6}$ Product of Powers Property

$= 3^8$ Simplify.

The base is 3. Add the exponents.

b. $\dfrac{(-4)^2}{(-4)^7} = (-4)^{2-7}$ Quotient of Powers Property

$= (-4)^{-5}$ Simplify.

$= \dfrac{1}{(-4)^5}$ Definition of negative exponent

The base is -4. Subtract the exponents.

c. $(z^4)^{-3} = z^{4 \cdot (-3)}$ Power of a Power Property

$= z^{-12}$ Simplify.

$= \dfrac{1}{z^{12}}$ Definition of negative exponent

The base is z. Multiply the exponents.

On Your Own

Now You're Ready
Exercises 12–17

Simplify. Write your answer using only positive exponents.

1. $10^4 \cdot 10^{-6}$ **2.** $x^9 \cdot x^{-9}$ **3.** $\dfrac{-5^8}{-5^4}$

4. $\dfrac{y^6}{y^7}$ **5.** $(6^{-2})^{-5}$ **6.** $(w^{12})^5$

Key Ideas

Power of a Product Property

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Words To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

EXAMPLE 2 Using Properties of Exponents

Simplify. Write your answer using only positive exponents.

a. $(-1.5y)^2 = (-1.5)^2 \cdot y^2$ Power of a Product Property

$= 2.25y^2$ Simplify.

b. $\left(\frac{a}{-10}\right)^3 = \frac{a^3}{(-10)^3}$ Power of a Quotient Property

$= -\frac{a^3}{1000}$ Simplify.

c. $\left(\frac{2x}{3}\right)^{-5} = \frac{(2x)^{-5}}{3^{-5}}$ Power of a Quotient Property

$= \frac{3^5}{(2x)^5}$ Definition of negative exponent

$= \frac{3^5}{2^5x^5}$ Power of a Product Property

$= \frac{243}{32x^5}$ Simplify.

On Your Own

Now You're Ready
Exercises 21–26

Simplify. Write your answer using only positive exponents.

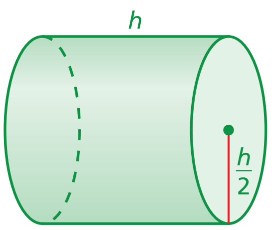
7. $(10y)^{-3}$

8. $\left(-\frac{4}{n}\right)^5$

9. $\left(\frac{1}{2k^2}\right)^5$

10. $\left(\frac{6c}{7}\right)^{-2}$

EXAMPLE 3 Simplifying an Expression



Which expression represents the volume of the cylinder?

Ⓐ $\frac{h^2}{2}$ Ⓑ $\frac{\pi h^2}{4}$ Ⓒ $\frac{\pi h^3}{2}$ Ⓓ $\frac{\pi h^3}{4}$

$V = \pi r^2 h$ — Formula for volume of a cylinder

$= \pi\left(\frac{h}{2}\right)^2(h)$ — Substitute $\frac{h}{2}$ for r.

$= \pi\left(\frac{h^2}{2^2}\right)(h)$ — Power of a Quotient Property

$= \frac{\pi h^3}{4}$ — Simplify.

The correct answer is Ⓓ.

EXAMPLE 4 Real-Life Application

A jellyfish emits about 1.25×10^8 particles of light, or photons, in 6.25×10^{-4} second. How many photons does the jellyfish emit each second? Write your answer in scientific notation and in standard form.

Divide to find the unit rate.

$\frac{1.25 \times 10^8}{6.25 \times 10^{-4}}$ — photons / seconds — Write the rate.

$= \frac{1.25}{6.25} \times \frac{10^8}{10^{-4}}$ — Rewrite.

$= 0.2 \times 10^{12}$ — Simplify.

$= 2 \times 10^{11}$ — Write in scientific notation.

The jellyfish emits 2×10^{11}, or 200,000,000,000 photons per second.

Remember

A number is written in scientific notation when it is of the form $a \times 10^b$, where $1 \le a < 10$ and b is an integer.

On Your Own

11. In Example 3, which expression represents the area of a base of the cylinder?

12. It takes the Sun about 2.3×10^8 years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^2 years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the Milky Way? Write your answer in scientific notation.

6.2 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

MATCHING **Match the property with its example.**

1. Quotient of Powers Property
2. Power of a Power Property
3. Power of a Quotient Property
4. Power of a Product Property

A. $(4^5)^2 = 4^{5 \cdot 2}$ **B.** $\left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4}$ **C.** $(5 \cdot 2)^4 = 5^4 \cdot 2^4$ **D.** $\frac{4^5}{4^2} = 4^{5-2}$

5. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Simplify $3^3 \cdot 3^6$.	Simplify 3^{3+6}.	Simplify 3^{6-3}.	Simplify $3^6 \cdot 3^3$.

Practice and Problem Solving

Simplify the expression.

6. $(n^4)(n^3)$
7. $\frac{x^5}{x^3}$
8. $(c^5)^3$
9. $(4b)^3$
10. $\left(\frac{k}{3}\right)^5$
11. $\frac{(2a)^6}{a^2}$

Simplify. Write your answer using only positive exponents.

1 12. $8^{-2} \cdot 8^7$
13. $b^4 \cdot b^7$
14. $\frac{12^7}{12^2}$
15. $\frac{d^5}{d^8}$
16. $(5^5)^4$
17. $(x^3)^{-2}$

ERROR ANALYSIS **Describe and correct the error in simplifying the expression.**

18.
$$\begin{aligned} x^5 \cdot x^{-2} &= x^{5 \cdot (-2)} \\ &= x^{-10} \\ &= \frac{1}{x^{10}} \end{aligned}$$

19.
$$\begin{aligned} (m^3)^4 &= m^{3+4} \\ &= m^7 \end{aligned}$$

20. **MICROSCOPE** A microscope magnifies an object 10^5 times. The length of an object is 10^2 nanometers. What is its magnified length?

Simplify. Write your answer using only positive exponents.

2 3 **21.** $(6.2y)^2$ **22.** $\left(\frac{w}{4}\right)^4$ **23.** $\left(-\frac{6}{d}\right)^{-2}$

24. $(7p)^{-3}$ **25.** $(-5x)^5$ **26.** $\left(\frac{3n^3}{4}\right)^2$

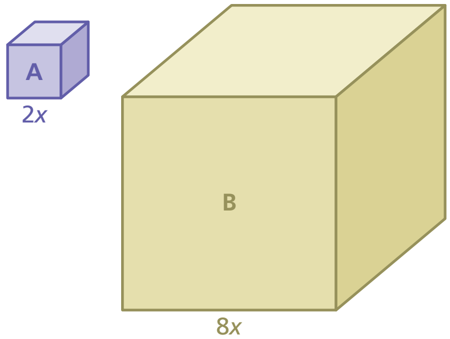
27. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

$$\left(\frac{x^3}{3}\right)^2 = \frac{(x^3)^2}{3} = \frac{x^6}{3}$$

28. **OPEN-ENDED** Use the properties of exponents to write three expressions equivalent to x^8.

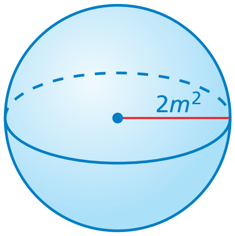
29. **REASONING** Are the expressions $(a^4)^2$ and a^{4^2} equivalent? Explain your reasoning.

30. **GEOMETRY** Consider Cube A and Cube B.



a. Which property of exponents should you use to find the volume of each cube?

b. How can you use the Power of a Quotient Property to find how many times greater the volume of Cube B is than the volume of Cube A?



31. **SPHERE** The volume V of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius. What is the volume of the sphere in terms of m and π?

32. **PROBABILITY** The probability of rolling a 6 on a number cube is $\frac{1}{6}$. The probability of rolling a 6 twice in a row is $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$.

a. Write an expression that represents the probability of rolling a 6 n times in a row.

b. What is the probability of rolling a 6 five times in a row?

c. What is the probability of flipping heads on a coin five times in a row?

Evaluate the expression. Write your answer in scientific notation.

4 **33.** $(3.4 \times 10^2)(1.5 \times 10^{-5})$ **34.** $(6.1 \times 10^{-3})(8 \times 10^9)$ **35.** $(4.8 \times 10^{-4})(7.2 \times 10^{-6})$

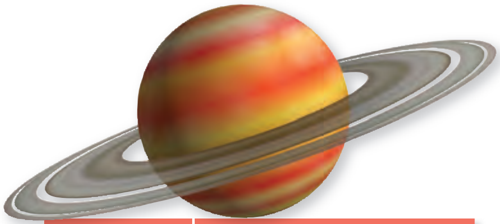
36. $\dfrac{(3 \times 10^3)}{(4 \times 10^5)}$ **37.** $\dfrac{(6.4 \times 10^{-7})}{(1.6 \times 10^{-5})}$ **38.** $\dfrac{(3.9 \times 10^{-5})}{(7.8 \times 10^{-8})}$

Simplify. Write your answer using only positive exponents.

39. $(6x^2y^{-4})^{-3}$ **40.** $\dfrac{(2m)^{-2}n^5}{-m^4n^{-3}}$ **41.** $\dfrac{15b^{-3}c^4}{(6b^{-4}c^{-5})^2}$

42. **REASONING** Write $8x^3y^3$ as the power of a product.

43. **COMPUTER CHIP** The area of a rectangular computer chip is $112a^3b^2$ square microns. The width is $8ab$ microns. What is the length?



44. **PROBLEM SOLVING** The speed of light is approximately 3×10^5 kilometers per second. The table shows the average distance each planet is from the Sun. How long does it take sunlight to reach Earth? Jupiter? Neptune?

Planet	Average Distance from the Sun (km)
Mercury	5.8×10^7
Venus	1.1×10^8
Earth	1.5×10^8
Mars	2.3×10^8
Jupiter	7.8×10^8
Saturn	1.4×10^9
Uranus	2.9×10^9
Neptune	4.5×10^9

45. **RICHTER SCALE** The Richter Scale is used to compare the intensities of earthquakes. An increase of 1 in magnitude on the Richter Scale represents a tenfold increase in intensity. An earthquake registers 7.4 on the Richter Scale and is followed by an aftershock that is 1000 times less intense. What is the magnitude of the aftershock?

46. Precision Find x and y when $\dfrac{k^{2x}}{k^y} = k^{13}$ and $(k^xk^{2y})^2 = k^{28}$. Explain how you found your answer.

Fair Game Review What you learned in previous grades & lessons

Simplify the expression. *(Section 6.1)*

47. $\sqrt{48}$ **48.** $\sqrt{\dfrac{70}{36}}$ **49.** $\sqrt{\dfrac{180}{121}}$

50. **MULTIPLE CHOICE** Which of the following is the solution of $\dfrac{x}{3} < -6$? *(Section 3.3)*

Ⓐ $x > -2$ Ⓑ $x < -2$ Ⓒ $x > -18$ Ⓓ $x < -18$

6.3 Radicals and Rational Exponents

Essential Question How can you write and evaluate an nth root of a number?

Recall that you cube a number as follows.

Symbol for cubing is 3rd power.

$$2^3 = 2 \cdot 2 \cdot 2$$

$$= 8 \qquad \text{2 cubed is 8.}$$

To "undo" this, take the cube root of the number.

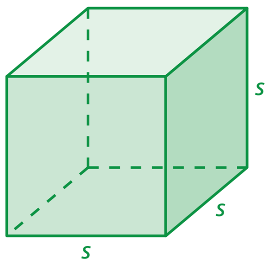
Symbol for cube root is $\sqrt[3]{}$.

$$\sqrt[3]{8} = \sqrt[3]{2^3} = 2 \qquad \text{The cube root of 8 is 2.}$$

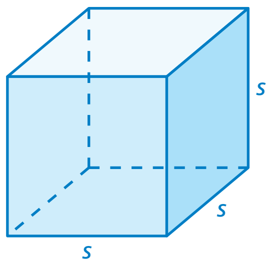
1 ACTIVITY: Finding Cube Roots

Work with a partner. Use a cube root symbol to write the side length of the cube. Then find the cube root. Check your answer by multiplying. Which cube is the largest? Which two are the same size? Explain your reasoning.

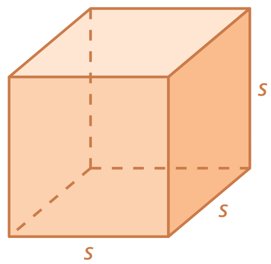
a. Volume = 27 ft^3



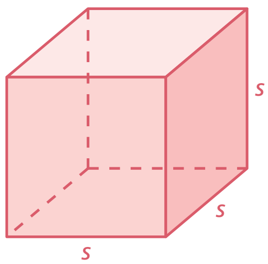
b. Volume = 125 cm^3



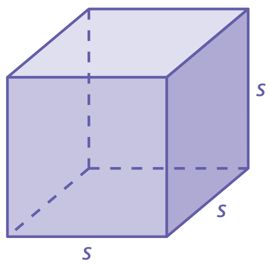
c. Volume = 3375 in.^{3}



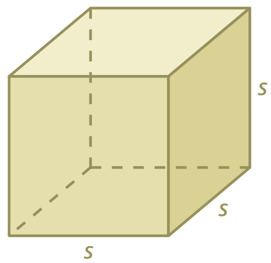
d. Volume = 3.375 m^3



e. Volume = 1 yd^3



f. Volume = $\frac{125}{8}$ mm^3



Cubes are not drawn to scale.

Common Core

Exponents

In this lesson, you will
- simplify expressions with rational exponents.

Learning Standards
N.RN.1
N.RN.2

2 ACTIVITY: Estimating *n*th Roots

Work with a partner. When you raise an *n*th root of a number to the *n*th power, you get the original number.

$$(\sqrt[n]{a})^n = a$$

Sample: The 4th root of 16 is 2 because $2^4 = 16$.

$$\sqrt[4]{16} = 2$$

Check: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ ✓

Match the *n*th root with the point on the number line. Justify your answer.

a. $\sqrt[4]{25}$ **b.** $\sqrt{0.5}$ **c.** $\sqrt[5]{2.5}$

d. $\sqrt[3]{65}$ **e.** $\sqrt[3]{55}$ **f.** $\sqrt[6]{20{,}000}$

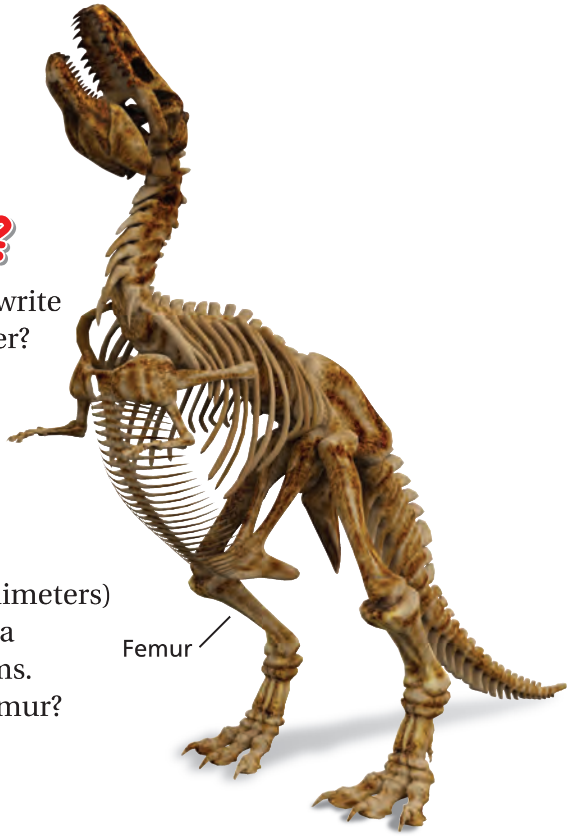
What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you write and evaluate the *n*th root of a number?

4. The body mass m (in kilograms) of a dinosaur that walked on two feet can be modeled by

$$m = (0.00016)C^{2.73}$$

where C is the circumference (in millimeters) of the dinosaur's femur. The mass of a Tyrannosaurus rex was 4000 kilograms. What was the circumference of its femur?



Math Practice 3

Justify Conclusions

What information can you use to justify your answer?

Practice Use what you learned about cube roots to complete Exercises 3–5 on page 280.

6.3 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
*n*th root, *p. 278*

When $b^n = a$ for an integer n greater than 1, b is an ***n*th root** of a.

$\sqrt[n]{a}$ *n*th root of *a*

The *n*th roots of a number may be real numbers or *imaginary numbers*. You will study imaginary numbers in a future course.

EXAMPLE 1 Finding *n*th Roots

Study Tip
In Example 1b, although $3^4 = 81$ and $(-3)^4 = 81$, $\sqrt[4]{81} = 3$ because the radical symbol indicates the positive root.

Simplify each expression.

a. $\sqrt[3]{64}$

$$\sqrt[3]{64} = \sqrt[3]{4 \cdot 4 \cdot 4}$$
$$= 4$$

b. $\sqrt[4]{81}$

$$\sqrt[4]{81} = \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3}$$
$$= 3$$

Key Idea

Rational Exponents

Words The *n*th root of a positive number a can be written as a power with base a and an exponent of $1/n$.

Numbers $\sqrt[4]{81} = 81^{1/4}$ **Algebra** $\sqrt[n]{a} = a^{1/n}$

EXAMPLE 2 Simplifying Expressions with Rational Exponents

Reading
When $n = 2$, the 2 is typically not written with the radical sign.

Simplify each expression.

a. $400^{1/2}$

$400^{1/2} = \sqrt{400}$ Write the expression in radical form.

$= \sqrt{20 \cdot 20}$ Rewrite.

$= 20$ Simplify.

b. $243^{1/5}$

$243^{1/5} = \sqrt[5]{243}$ Write the expression in radical form.

$= \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$ Rewrite.

$= 3$ Simplify.

On Your Own

Now You're Ready
Exercises 13–18

Simplify the expression.

1. $\sqrt[3]{216}$ **2.** $\sqrt[5]{32}$ **3.** $\sqrt[4]{625}$

4. $49^{1/2}$ **5.** $343^{1/3}$ **6.** $64^{1/6}$

Multi-Language Glossary at BigIdeasMath.com

You can use properties of exponents to simplify expressions involving rational exponents.

EXAMPLE 3 Using Properties of Exponents

a. $16^{3/4} = 16^{(1/4) \cdot 3}$ — Rewrite the exponent.

$= (16^{1/4})^3$ — Power of a Power Property

$= 2^3$ — Evaluate the fourth root of 16.

$= 8$ — Evaluate power.

b. $27^{4/3} = 27^{1/3 \cdot 4}$ — Rewrite the exponent.

$= (27^{1/3})^4$ — Power of a Power Property

$= 3^4$ — Evaluate the third root of 27.

$= 81$ — Evaluate power.

On Your Own

Now You're Ready
Exercises 20–25

Simplify the expression.

7. $64^{2/3}$ **8.** $9^{5/2}$ **9.** $256^{3/4}$

EXAMPLE 4 Real-Life Application

Volume = 113 cubic feet

The radius r of a sphere is given by the equation $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, where V is the volume of the sphere. Find the radius of the beach ball to the nearest foot. Use 3.14 for π.

$r = \left(\frac{3V}{4\pi}\right)^{1/3}$ — Write the equation.

$= \left[\frac{3(113)}{4(3.14)}\right]^{1/3}$ — Substitute 113 for V and 3.14 for π.

$= \left(\frac{339}{12.56}\right)^{1/3}$ — Multiply.

≈ 3 — Use a calculator.

The radius of the beach ball is about 3 feet.

On Your Own

10. **WHAT IF?** In Example 4, the volume of the beach ball is 17,000 cubic inches. Find the radius to the nearest inch. Use 3.14 for π.

6.3 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **WRITING** Explain how to simplify $81^{1/4}$.

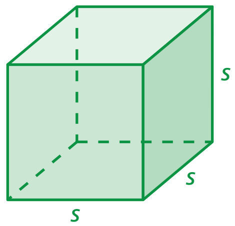
2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$(\sqrt[3]{27})^2$ $\qquad 27^{2/3}$ $\qquad 3^2$ $\qquad 27^{3/2}$

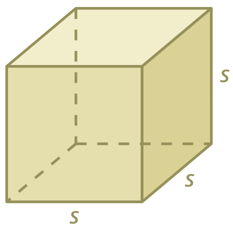
Practice and Problem Solving

Find the dimensions of the cube. Check your answer.

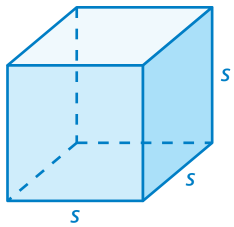
3. Volume = 64 in.^{3}



4. Volume = 216 cm^3



5. Volume = $\frac{343}{512}$ ft^3



Write the expression in rational exponent form.

6. $\sqrt[7]{5}$

7. $(\sqrt[3]{4})^2$

8. $(\sqrt[5]{8})^4$

Write the expression in radical form.

9. $15^{1/3}$

10. $140^{1/7}$

11. $78^{2/5}$

12. **ERROR ANALYSIS** Describe and correct the error in writing the expression in rational exponent form.

✗ $(\sqrt[3]{2})^4 = 2^{3/4}$

Simplify the expression.

1 13. $\sqrt[4]{256}$

14. $\sqrt[3]{125}$

15. $\sqrt[5]{1024}$

2 16. $128^{1/7}$

17. $1000^{1/3}$

18. $81^{1/2}$

19. **BAKE SALE** A math club is having a bake sale. Find the length and width of the bake sale sign.

$4^{1/2}$ ft

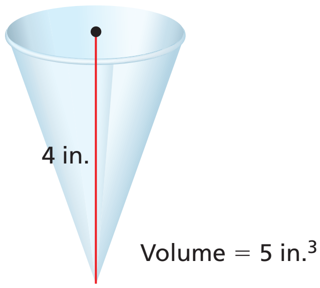
$\sqrt[6]{729}$ ft

Simplify the expression.

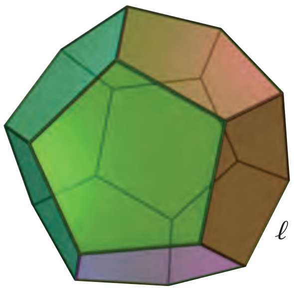
3 **20.** $32^{3/5}$ **21.** $125^{2/3}$ **22.** $36^{3/2}$

23. $243^{2/5}$ **24.** $128^{5/7}$ **25.** $343^{4/3}$

26. **PAPER CUPS** The radius r of the base of a cone is given by the equation $r = \left(\frac{3V}{\pi h}\right)^{1/2}$, where V is the volume of the cone and h is the height of the cone. Find the radius of the paper cup to the nearest inch. Use 3.14 for π.



27. **WRITING** Explain how to write $(\sqrt[n]{a})^m$ in rational exponent form.



28. **PROBLEM SOLVING** The formula for the volume of a regular dodecahedron is $V \approx 7.66\,\ell^3$, where ℓ is the length of an edge. The volume of the dodecahedron is 20 cubic feet. Estimate the edge length.

Logic **Determine whether the statement is *always*, *sometimes*, or *never* true. Let x be a nonnegative real number. Justify your answer.**

29. $(x^{1/3})^3 = x$ **30.** $x^{1/3} = x^{-3}$ **31.** $x^{1/3} = \sqrt[3]{x}$

32. $x^{1/3} = x^3$ **33.** $\frac{x^{2/3}}{x^{1/3}} = \sqrt[3]{x}$ **34.** $x = x^{1/3} \cdot x^3$

Fair Game Review What you learned in previous grades & lessons

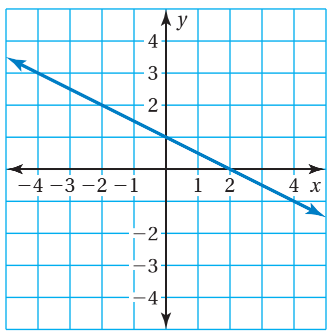
Graph the linear equation. *(Section 2.3 and Section 2.4)*

35. $y = -2x + 1$ **36.** $4x - 2y = 6$ **37.** $y = -\frac{1}{3}x - 5$

38. **MULTIPLE CHOICE** Which equation is shown in the graph? *(Section 2.1)*

Ⓐ $y = -\frac{1}{2}x + 1$ Ⓑ $y = -\frac{1}{2}x - 1$

Ⓒ $y = \frac{1}{2}x - 1$ Ⓓ $y = \frac{1}{2}x + 1$



6 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **information frame** to help you organize and remember concepts. Here is an example of an information frame for the Product of Powers Property.

Words:

To multiply powers with the same base, add their exponents.

Numbers:

$3^4 \cdot 3^3 = 3^{4+3}$
$= 3^7$

Product of Powers Property

Algebra:

$a^m \cdot a^n = a^{m+n}$

Example:

Simplify $5^7 \cdot 5^3$. Write your answer using only positive exponents.

$5^7 \cdot 5^3 = 5^{7+3} = 5^{10}$

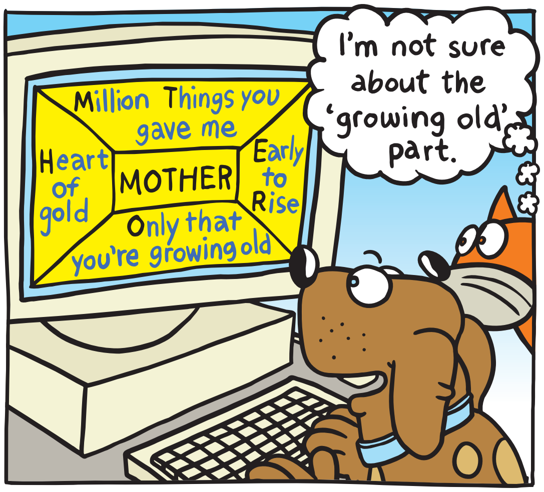
On Your Own

Make information frames to help you study these topics.

1. Product Property of Square Roots
2. Quotient Property of Square Roots
3. Quotient of Powers Property
4. Power of a Power Property
5. Power of a Product Property
6. Power of a Quotient Property
7. rational exponents

After you complete this chapter, make information frames for the following topics.

8. exponential growth functions
9. exponential decay functions
10. geometric sequences



"Dear Mom, I am sending you an **information frame** card for Mother's Day!"

6.1–6.3 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Simplify the expression. *(Section 6.1)*

1. $\sqrt{20}$

2. $\sqrt{\dfrac{11}{81}}$

3. $\dfrac{4-\sqrt{12}}{2}$

4. $\dfrac{-6+\sqrt{45}}{3}$

Evaluate the expression when $x = 2$, $y = -3$, and $z = 6$. *(Section 6.1)*

5. $\sqrt{x + y^2z}$

6. $\sqrt{3xz - y^2}$

Simplify. Write your answer using only positive exponents. *(Section 6.2)*

7. $3^2 \cdot 3^4$

8. $(k^4)^3$

9. $(4y)^{-2}$

10. $\left(\dfrac{r}{2}\right)^3$

Simplify. *(Section 6.3)*

11. $\sqrt[3]{27}$

12. $16^{1/4}$

13. $512^{2/3}$

14. $4^{5/2}$

15. CEDAR CHEST You store blankets in a cedar chest. What is the volume of the cedar chest? *(Section 6.1)*

16. CRITICAL THINKING Is the set of irrational numbers closed under subtraction? If not, find a counterexample. *(Section 6.1)*

Unit of Mass	Mass
gigagram	10^9 grams
megagram	10^6 grams
kilogram	10^3 grams
hectogram	10^2 grams
dekagram	10^1 grams
decigram	10^{-1} gram
centigram	10^{-2} gram
milligram	10^{-3} gram
microgram	10^{-6} gram
nanogram	10^{-9} gram

17. METRIC UNITS The table shows several units of mass. *(Section 6.2)*

a. How many times larger is a kilogram than a nanogram? Write your answer using only positive exponents.

b. How many times smaller is a milligram than a hectogram? Write your answer using only positive exponents.

c. Which is greater, 10,000 milligrams or 1000 decigrams? Explain your reasoning.

6.4 Exponential Functions

Essential Question What are the characteristics of an exponential function?

1 ACTIVITY: Describing an Exponential Function

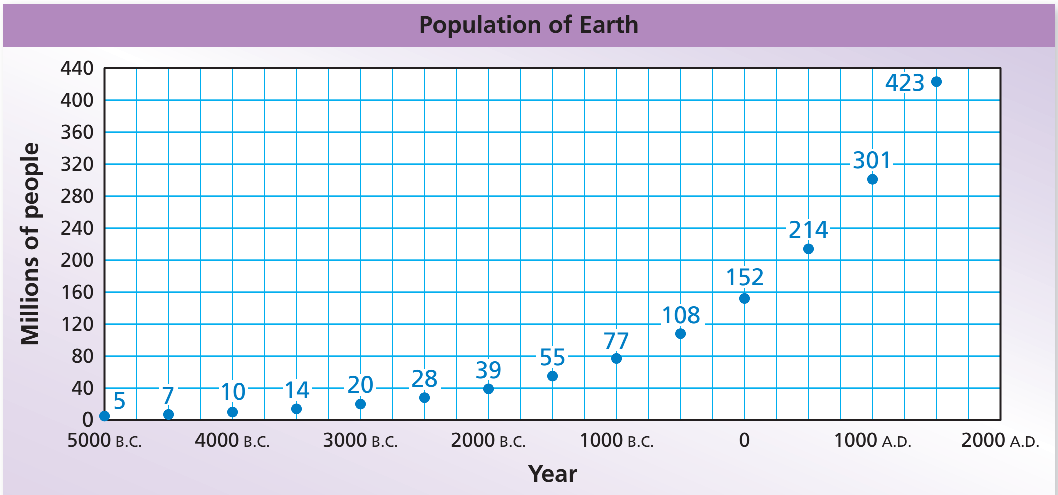
Work with a partner. The graph below shows estimates of the population of Earth from 5000 B.C. through 1500 A.D. at 500-year intervals.

a. Describe the pattern.

b. Did Earth's population increase by the same *amount* or the same *percent* for each 500-year period? Explain.

c. Assume the pattern continued. Estimate Earth's population in 2000.

d. Use the Internet to find Earth's population in 2000. Did the pattern continue? If not, why did the pattern change?



COMMON CORE

Exponential Functions

In this lesson, you will
- identify, evaluate, and graph exponential functions.

Learning Standards
A.REI.3
A.REI.11
F.BF.3
F.IF.7e
F.LE.1a
F.LE.2

4000 B.C.
Civilization begins to develop in Mesopotamia.

3000 B.C.
Stonehenge is built in England.

2000 B.C.
Middle Kingdom in Egypt

2 ACTIVITY: Modeling an Exponential Function

Math Practice 6

Calculate Accurately

How can you check the accuracy of your answers?

Work with a partner. Use the following exponential function to complete the table. Compare the results with the data in Activity 1.

$$P = 152(1.406)^{t/500}$$

1 B.C.
Augustus Caesar controls most of the Mediterranean world. (Use $t = 0$ to approximate 1 B.C.)

1000 A.D.
Song Dynasty has about one-fifth of Earth's population.

Year	t	Population from Activity 1	P
5000 B.C.	−5000		
4500 B.C.	−4500		
4000 B.C.	−4000		
3500 B.C.	−3500		
3000 B.C.	−3000		
2500 B.C.	−2500		
2000 B.C.	−2000		
1500 B.C.	−1500		
1000 B.C.	−1000		
500 B.C.	−500		
1 B.C.	0		
500 A.D.	500		
1000 A.D.	1000		
1500 A.D.	1500		

What Is Your Answer?

3. **IN YOUR OWN WORDS** What are the characteristics of an exponential function?

4. Sketch the graph of each exponential function. Does the function match the characteristics you described in Question 3? Explain.

   a. $y = 2^x$ **b.** $y = 2(3)^x$ **c.** $y = 3(1.5)^x$

Practice Use what you learned about exponential functions to complete Exercises 4 and 5 on page 289.

6.4 Lesson

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Key Vocabulary
exponential function, *p. 286*

A function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$ is an **exponential function**. The exponential function $y = ab^x$ is a nonlinear function that changes by equal factors over equal intervals.

EXAMPLE 1 Identifying Functions

Does each table represent a *linear* or an *exponential* function? Explain.

a.

x	0	1	2	3
y	2	4	6	8

$+1$ $+1$ $+1$

$+2$ $+2$ $+2$

As x increases by 1, y increases by 2. The rate of change is constant. So, the function is linear.

b.

x	y
0	4
1	8
2	16
3	32

$+1$ $+1$ $+1$

$\times 2$ $\times 2$ $\times 2$

As x increases by 1, y is multiplied by 2. So, the function is exponential.

EXAMPLE 2 Evaluating Exponential Functions

Evaluate each function for the given value of x.

a. $y = -2(5)^x$; $x = 3$

$y = -2(5)^x$ — Write the function.

$= -2(5)^3$ — Substitute for x.

$= -2(125)$ — Evaluate the power.

$= -250$ — Multiply.

b. $y = 3(0.5)^x$; $x = -2$

$y = 3(0.5)^x$

$= 3(0.5)^{-2}$

$= 3(4)$

$= 12$

On Your Own

Now You're Ready
Exercises 6–15

Does the table represent a *linear* or an *exponential* function? Explain.

1.

x	0	1	2	3
y	8	4	2	1

2.

x	y
-4	1
0	0
4	-1
8	-2

Evaluate the function when $x = -2$, 0, and $\frac{1}{2}$.

3. $y = 2(9)^x$

4. $y = 1.5(2)^x$

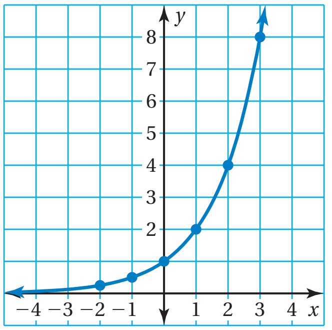
Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 3 Graphing an Exponential Function

Graph $y = 2^x$. Describe the domain and range.

Step 1: Make a table of values.

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

From the graph, you can see that the domain is all real numbers and the range is all positive real numbers.

Study Tip

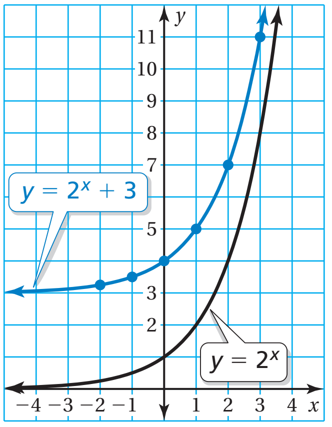
In Example 3, you can substitute any value for *x*. So, the domain is all real numbers.

EXAMPLE 4 Graphing a Vertical Translation

Graph $y = 2^x + 3$. Describe the domain and range. Compare the graph to the graph of $y = 2^x$.

Step 1: Make a table of values.

x	-2	-1	0	1	2	3
y	$\frac{13}{4}$	$\frac{7}{2}$	4	5	7	11



Step 2: Plot the ordered pairs.

Step 3: Draw a smooth curve through the points.

From the graph, you can see that the domain is all real numbers and the range is all real numbers greater than 3. The graph of $y = 2^x + 3$ is a translation 3 units up of the graph of $y = 2^x$.

Remember

In Section 5.4, you learned that the graph of $f(x) + k$ is a vertical translation of the graph of $f(x)$.

On Your Own

Now You're Ready Exercises 21–23 and 27–29

Graph the function. Describe the domain and range.

5. $y = 3^x$

6. $y = \left(\frac{1}{2}\right)^x$

7. $y = -2\left(\frac{1}{4}\right)^x$

8. Graph $y = \left(\frac{1}{2}\right)^x - 2$. Describe the domain and range. Compare the graph to the graph of $y = \left(\frac{1}{2}\right)^x$.

Study Tip

To find the y-intercept of the graph of $y = ab^x$, substitute 0 for x.

$y = ab^0$

$y = a(1)$

$y = a$

So, the y-intercept is a.

For an exponential function of the form $y = ab^x$, the y-values change by a factor of b as x increases by 1. Also notice that a is the y-intercept.

+1 +1 +1 +1

x	0	1	2	3	4
y	2	10	50	250	1250

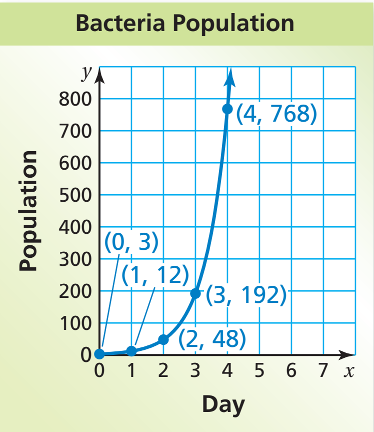
×5 ×5 ×5 ×5

$y = 2(5)^x$

EXAMPLE 5 Real-Life Application

The graph represents a bacteria population y after x days.

a. Write an exponential function that represents the population.



Use the graph to make a table of values.

+1 +1 +1 +1

x	0	1	2	3	4
y	3	12	48	192	768

×4 ×4 ×4 ×4

The y-intercept is 3 and the y-values increase by a factor of 4 as x increases by 1.

So, the population can be modeled by $y = 3(4)^x$.

b. Find the population after 12 hours and after 5 days.

Study Tip

For help with rational exponents, see Section 6.3.

12 hours $= \frac{1}{2}$ day

Population after 12 hours		*Population after 5 days*
$y = 3(4)^x$	Write the function.	$y = 3(4)^x$
$= 3(4)^{1/2}$	Substitute for x.	$= 3(4)^5$
$= 3(2)$	Evaluate the power.	$= 3(1024)$
$= 6$	Multiply.	$= 3072$

There are 6 bacteria after 12 hours and 3072 bacteria after 5 days.

On Your Own

Now You're Ready
Exercises 36–39

9. A bacteria population y after x days can be represented by an exponential function whose graph passes through (0, 100) and (1, 200).

   a. Write a function that represents the population.

   b. Find the population after 6 days. Does this bacteria population grow faster than the bacteria population in Example 5? Explain.

6.4 Exercises

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Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** Describe how linear and exponential functions change over equal intervals.

2. **OPEN-ENDED** Sketch an increasing exponential function whose graph has a y-intercept of 2.

3. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

   $y = 3^x$ $\quad$ $f(x) = 2(4)^x$ $\quad$ $f(x) = (-3)^x$ $\quad$ $y = 5(3)^x$

Practice and Problem Solving

Sketch the graph of the exponential function.

4. $y = 4^x$

5. $y = 2(2)^x$

Does the table represent a *linear* or an *exponential* function? Explain.

1 6.

x	y
0	-2
1	0
2	2
3	4

7.

x	y
1	6
2	12
3	24
4	48

8.

x	-1	0	1	2
y	0.25	1	4	16

9.

x	-3	0	3	6
y	10	1	-8	-17

Evaluate the function for the given value of x.

2 10. $y = 3^x$; $x = 2$

11. $f(x) = 3(2)^x$; $x = -1$

12. $y = -4(5)^x$; $x = 2$

13. $f(x) = 0.5^x$; $x = -3$

14. $f(x) = \frac{1}{3}(6)^x$; $x = 3$

15. $y = \frac{1}{4}(4)^x$; $x = \frac{3}{2}$

16. **ERROR ANALYSIS** Describe and correct the error in evaluating the function.

$$g(x) = 6(0.5)^x;\ x = -2$$
$$g(-2) = 6(0.5)^{-2}$$
$$= 3^{-2}$$
$$= \frac{1}{9}$$

17. **CALCULATOR** You graph an exponential function on a calculator. You zoom in repeatedly at 25% of the screen size. The function $y = 0.25^x$ represents the percent (in decimal form) of the original screen display that you see, where x is the number of times you zoom in. You zoom in twice. What percent of the original screen do you see?

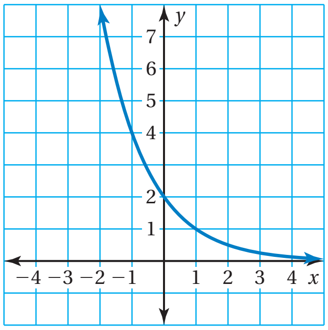
Match the function with its graph.

18. $f(x) = -3(4)^x$

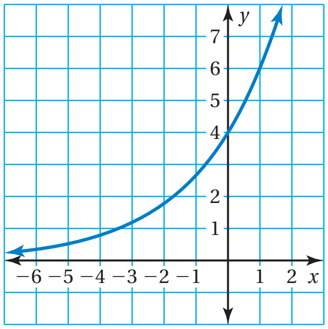
19. $y = 2(0.5)^x$

20. $y = 4(1.5)^x$

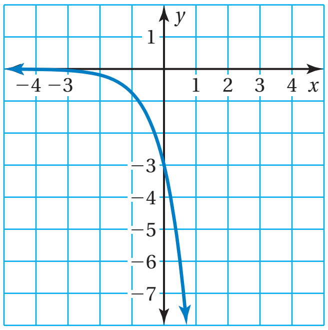
A.



B.



C.



Graph the function. Describe the domain and range.

3 **21.** $y = 9^x$

22. $f(x) = -7^x$

23. $f(x) = 4\left(\frac{1}{4}\right)^x$

24. **LOGIC** Describe the graph of $y = a(2)^x$ when a is (a) positive and (b) negative. (c) How does the graph change as a changes?

25. **NUMBER SENSE** Consider the graph of $f(x) = 2(b)^x$. How do the graphs differ when $b > 1$ and $0 < b < 1$?

26. **COYOTES** A population y of coyotes in a national park triples every 20 years. The function $y = 15(3)^x$ represents the population, where x is the number of 20-year periods.

a. Graph the function. Describe the domain and range.

b. Find and interpret the y-intercept.

c. How many coyotes are in the national park after 20 years?

Graph the function. Describe the domain and range. Compare the graph to the graph of $y = 3^x$.

4 **27.** $y = 3^x - 1$

28. $y = 3^x + 3$

29. $y = 3^x - \frac{1}{2}$

30. **REASONING** Graph the function $f(x) = -2^x$. Then graph $g(x) = -2^x - 3$.

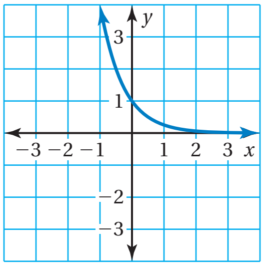
a. Describe the domain and range of each function.

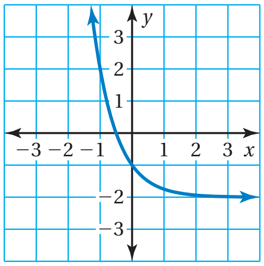
b. Find the y-intercept of the graph of each function.

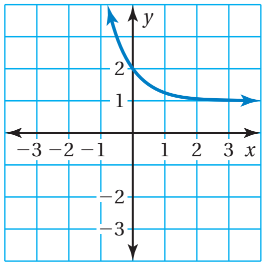
c. How are the y-intercept, domain, and range affected by the translation?

31. **REASONING** Graph $y = 3^x$ and $y = 4^x$. Use the graph to solve the inequality $3^x < 4^x$.

Given $g(x) = 0.25^x - 1$, find the value of k so that the graph is $g(x) + k$.

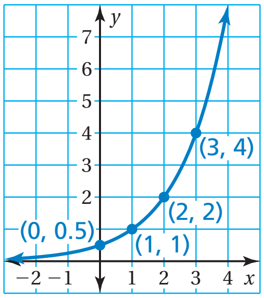
32. 

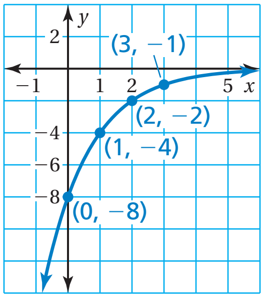
33. 

34. 

35. **REASONING** Graph $g(x) = 4^{x+2}$. Compare the graph to the graph of $f(x) = 4^x$.

Write an exponential function represented by the graph or table.

5 **36.** 

37. 

38.

x	0	1	2	3
y	2	8	32	128

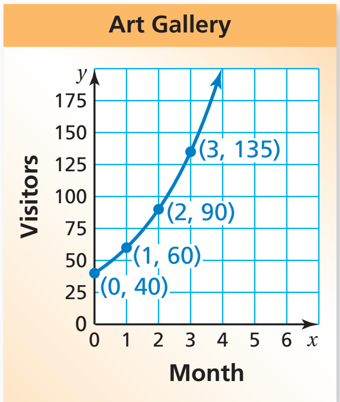
39.

x	0	1	2	3
y	−3	−15	−75	−375

40. **ART GALLERY** The graph represents the number y of visitors to a new art gallery after x months.

a. Write an exponential function that represents this situation.

b. Approximate the number of visitors after 5 months.



41. **SALES** A sales report shows that 3300 gas grills were purchased from a chain of hardware stores last year. The store expects grill sales to increase 6% each year. About how many grills does the store expect to sell in year 6? Use an equation to justify your answer.

42. **Structure** The graph of g is a translation 4 units up and 3 units right of the graph of $f(x) = 2^x$. Write an equation for g.

Fair Game Review What you learned in previous grades & lessons

Write the percent as a decimal. *(Skills Review Handbook)*

43. 23% **44.** 3% **45.** 150%

46. **MULTIPLE CHOICE** Which of the following is equivalent to 100(0.95)? *(Skills Review Handbook)*

(A) 0.95 (B) 9.5 (C) 95 (D) 950

Extension 6.4 Solving Exponential Equations

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To solve an exponential equation of the form $b^x = b^y$ when $b > 0$ and $b \neq 1$, solve the equation $x = y$.

EXAMPLE 1 Solving Exponential Equations

a. Solve $5^x = 125$.

$5^x = 125$	Write the equation.
$5^x = 5^3$	Rewrite 125 as 5^3.
$x = 3$	Equate the exponents.

b. Solve $4^x = 2^{x-3}$.

$4^x = 2^{x-3}$	Write the equation.
$(2^2)^x = 2^{x-3}$	Rewrite 4 as 2^2.
$2^{2x} = 2^{x-3}$	Power of a Power Property
$2x = x - 3$	Equate the exponents.
$x = -3$	Solve for x.

Check

$4^x = 2^{x-3}$

$4^{-3} \stackrel{?}{=} 2^{-3-3}$

$\frac{1}{4^3} \stackrel{?}{=} \frac{1}{2^6}$

$\frac{1}{64} = \frac{1}{64}$ ✓

c. Solve $9^{x+2} = 27^x$.

$9^{x+2} = 27^x$	Write the equation.
$(3^2)^{x+2} = (3^3)^x$	Rewrite 9 as 3^2 and 27 as 3^3.
$3^{2x+4} = 3^{3x}$	Power of a Power Property
$2x + 4 = 3x$	Equate the exponents.
$4 = x$	Solve for x.

Check

$9^{x+2} = 27^x$

$9^{4+2} \stackrel{?}{=} 27^4$

$531{,}441 = 531{,}441$ ✓

Practice

Solve the equation. Check your solution, if possible.

1. $3^x = 81$
2. $2^x = 32$
3. $\frac{1}{16} = 4^x$
4. $10^x = 10^{x+1}$
5. $\left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^{3x}$
6. $6^{x-5} = 36^x$
7. $100^{5x+2} = 1000^{4x-1}$
8. $32^{1-x} = 8^{2x-2}$
9. $\left(\frac{1}{8}\right)^{x-5} = 4^x$
10. **NUMBER SENSE** Explain how you can use mental math to solve the equation $8^{x-4} = 1$.
11. **REASONING** Why does this method for solving $b^x = b^y$ not work when $b = 1$? Give an example to justify your answer.

EXAMPLE 2 Solving an Equation by Graphing

COMMON CORE

Exponential Functions

In this extension, you will

- solve exponential equations algebraically and graphically.

Learning Standards
A.REI.3
A.REI.11
F.BF.3
F.IF.7e
F.LE.1a
F.LE.2

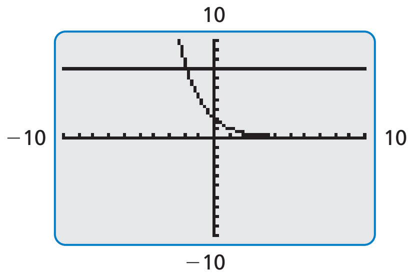
Use a graphing calculator to solve $\left(\frac{1}{2}\right)^{x-1} = 7$.

Step 1: Write a system of equations using each side of the equation.

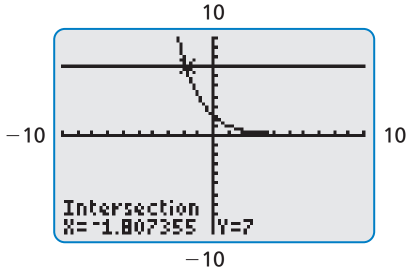
$$y = \left(\frac{1}{2}\right)^{x-1} \quad \text{Equation 1}$$

$$y = 7 \quad \text{Equation 2}$$

Step 2: Enter the equations into your calculator. Then graph the equations in a standard viewing window.



Step 3: Use the *intersect* feature to find the point of intersection. It is at about $(-1.81, 7)$.



So, the solution is $x \approx -1.81$.

Check: Check the solution algebraically.

$$\left(\frac{1}{2}\right)^{x-1} = 7 \quad \text{Write the equation.}$$

$$\left(\frac{1}{2}\right)^{-1.81-1} \stackrel{?}{=} 7 \quad \text{Substitute } -1.81 \text{ for } x.$$

$$7.01 \approx 7 \ \checkmark \quad \text{Use a calculator.}$$

Practice

Use a graphing calculator to solve the equation.

12. $4^{x+3} = 6$

13. $2^x = 1.8$

14. $4 = 8^x$

15. $\left(\frac{3}{4}\right)^{x+2} = 10$

16. $2^{-x-3} = 3^{x+1}$

17. $5^x = -4^{x+4}$

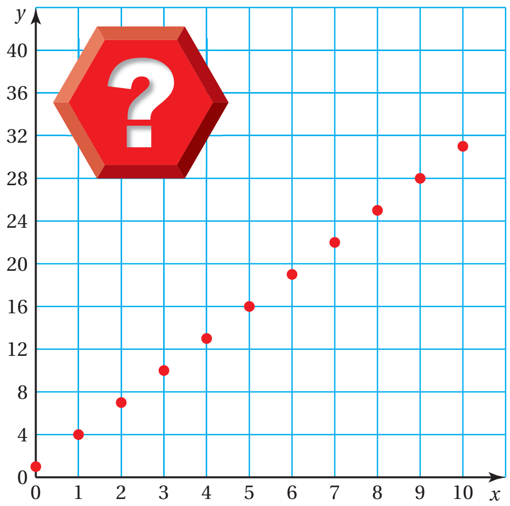
6.5 Exponential Growth

Essential Question What are the characteristics of exponential growth?

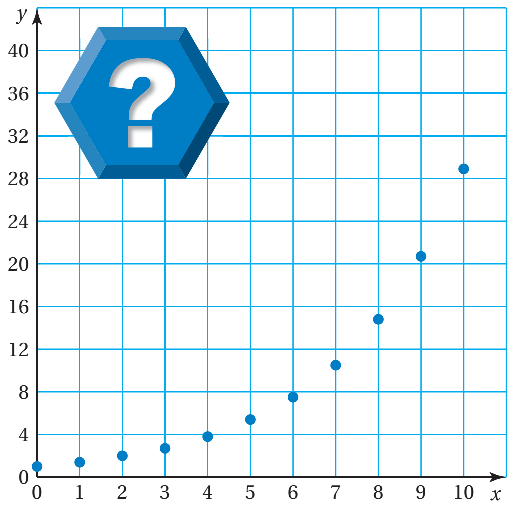
1 ACTIVITY: Comparing Types of Growth

Work with a partner. Describe the pattern of growth for each sequence and graph. How many of the patterns represent exponential growth? Explain your reasoning.

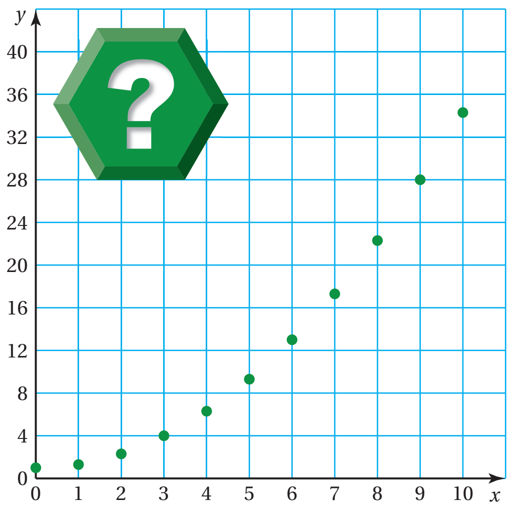
a. 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31



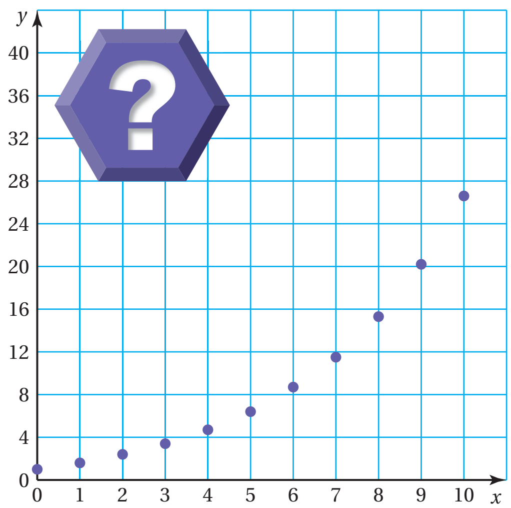
b. 1.0, 1.4, 2.0, 2.7, 3.8, 5.4, 7.5, 10.5, 14.8, 20.7, 28.9



c. 1.0, 1.3, 2.3, 4.0, 6.3, 9.3, 13.0, 17.3, 22.3, 28.0, 34.3



d. 1.0, 1.6, 2.4, 3.4, 4.7, 6.4, 8.7 11.5, 15.3, 20.2, 26.6



COMMON CORE

Exponential Functions

In this lesson, you will

- write, interpret, and graph exponential growth functions.

Learning Standards
A.SSE.1a
A.SSE.1b
F.IF.7e

2 ACTIVITY: Predicting a Future Event

Math Practice 1

Consider Similar Problems

How can you use the results from the previous activity to help you solve this problem?

Work with a partner. It is estimated that in 1782 there were about 100,000 nesting pairs of bald eagles in the United States. By the 1960s, this number had dropped to about 500 nesting pairs. This decline was attributed to loss of habitat, loss of prey, hunting, and the use of the pesticide DDT.

The 1940 Bald Eagle Protection Act prohibited the trapping and killing of the birds. In 1967, the bald eagle was declared an endangered species in the United States. With protection, the nesting pair population began to increase, as shown in the graph. Finally, in 2007, the bald eagle was removed from the list of endangered and threatened species.

Describe the growth pattern shown in the graph. Is it exponential growth? Assume the pattern continues. When will the population return to the levels of the late 1700s? Explain your reasoning.

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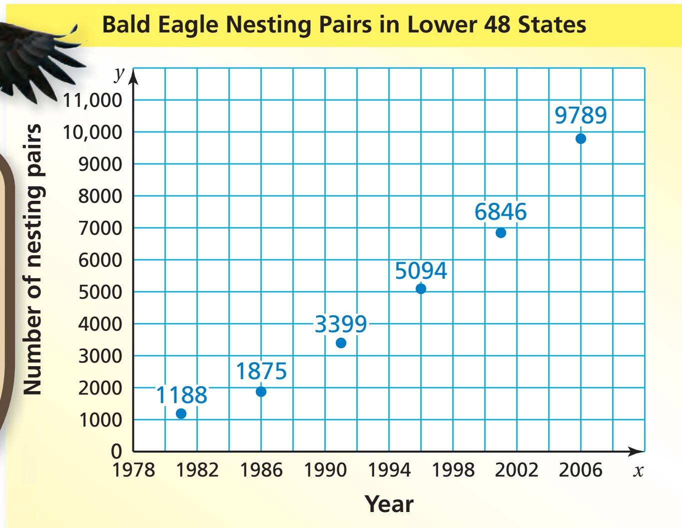

What Is Your Answer?

3. **IN YOUR OWN WORDS** What are the characteristics of exponential growth? How can you distinguish exponential growth from other growth patterns?

4. Which of the following are examples of exponential growth? Explain.

   a. Growth of the balance of a savings account

   b. Speed of the moon in orbit around Earth

   c. Height of a ball that is dropped from a height of 100 feet

Practice Use what you learned about exponential growth to complete Exercises 3 and 4 on page 298.

6.5 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
exponential growth, *p. 296*
exponential growth function, *p. 296*
compound interest, *p. 297*

Study Tip
Notice that an exponential growth function is of the form $y = ab^x$, where b is replaced by $1 + r$ and x is replaced by t.

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time.

Key Idea

Exponential Growth Functions

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.

Initial amount — Rate of growth (in decimal form)

$$\text{Final amount} \rightarrow y = a\underbrace{(1 + r)}_{\text{Growth factor}}{}^{t} \leftarrow \text{Time}$$

EXAMPLE 1 Using an Exponential Growth Function

The function $y = 150{,}000(1.1)^t$ represents the attendance y at a music festival t years after 2010.

a. By what percent does the festival attendance increase each year?

Use the growth factor $1 + r$ to find the rate of growth.

$1 + r = 1.1$ — Write an equation.

$r = 0.1$ — Subtract 1 from each side.

So, the festival attendance increases by 10% each year.

b. How many people will attend the festival in 2014? Round your answer to the nearest ten thousand.

The value $t = 4$ represents 2014.

$y = 150{,}000(1.1)^t$ — Write exponential growth function.

$= 150{,}000(1.1)^4$ — Substitute 4 for t.

$= 219{,}615$ — Use a calculator.

About 220,000 people will attend the festival in 2014.

On Your Own

Now You're Ready
Exercises 5–10

1. The function $y = 500{,}000(1.15)^t$ represents the number y of members of a website t years after 2010.
   a. By what percent does the website membership increase each year?
   b. How many members will there be in 2016? Round your answer to the nearest hundred thousand.

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Key Idea

Compound Interest

Compound interest is interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P = principal (initial amount)
r = annual interest rate (in decimal form)
t = time (in years)
n = number of times interest is compounded per year

EXAMPLE 2 Writing a Function

You deposit $100 in a savings account that earns 5% annual interest compounded yearly. Write a function for the balance after *t* years.

$y = P\left(1 + \frac{r}{n}\right)^{nt}$ — Write compound interest formula.

$y = 100\left(1 + \frac{0.05}{1}\right)^{(1)(t)}$ — Substitute 100 for P, 0.05 for r, and 1 for n.

$y = 100(1.05)^t$ — Simplify.

Study Tip

For interest compounded yearly, you can substitute 1 for n in the formula to get $y = P(1 + r)^t$.

EXAMPLE 3 Real-Life Application

The table shows the balance of a money market account over time.

Year, t	Balance
0	$100
1	$110
2	$121
3	$133.10
4	$146.41
5	$161.05

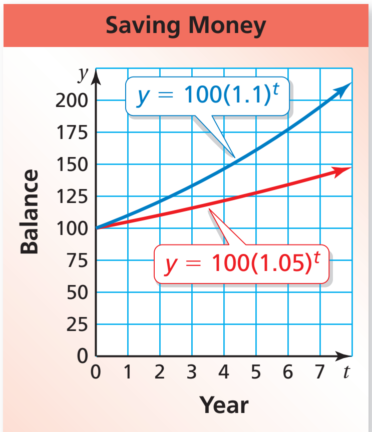
a. Write a function for the balance after *t* years.

From the table, you know the balance increases 10% each year.

$y = a(1 + r)^t$ — Write exponential growth function.

$y = 100(1 + 0.1)^t$ — Substitute 100 for a and 0.1 for r.

$y = 100(1.1)^t$ — Simplify.



b. Graph the functions from part (a) and Example 2 in the same coordinate plane. Compare the account balances.

The money market account earns 10% interest each year and the savings account earns 5% interest each year. So, the balance of the money market account increases faster.

On Your Own

Now You're Ready
Exercises 11–14

2. You deposit $500 in a savings account that earns 4% annual interest compounded yearly. Write and graph a function that represents the balance y (in dollars) after t years.

6.5 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** When does the exponential function $y = a(1 + r)^t$ represent an exponential growth function?
2. **VOCABULARY** The population of a city grows by 3% each year. What is the growth factor?

Practice and Problem Solving

Describe the pattern of growth for the sequence.

3. 1.0, 1.2, 1.4, 1.7, 2.1, 2.5, 3.0, 3.6, 4.3, 5.2, 6.2
4. 1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61

Identify the initial amount *a* and the rate of growth *r* (as a percent) of the exponential function. Evaluate the function when $t = 5$. Round your answer to the nearest tenth.

1 5. $y = 25(1.2)^t$
6. $f(t) = 12(1.05)^t$
7. $d(t) = 1500(1.074)^t$
8. $y = 175(1.028)^t$
9. $g(t) = 6.7(2)^t$
10. $h(t) = 1.8^t$

Write and graph a function that represents the situation.

2 3 11. You deposit \$800 in an account that earns 7% annual interest compounded yearly.
12. Your \$35,000 annual salary increases by 4% each year.
13. A population of 210,000 increases by 12.5% each year.
14. Sales of \$10,000 increase by 70% each year.
15. **ERROR ANALYSIS** The growth rate of a bacteria culture is 150% each hour. Initially, there are 10 bacteria. Describe and correct the error in finding the number of bacteria in the culture after 8 hours.

✗
$b(t) = 10(1.5)^t$
$b(8) = 10(1.5)^8 \approx 256.3$
After 8 hours, there are about 256 bacteria in the culture.

16. **INVESTMENT** The function $y = 7500(1.08)^t$ represents the value y of an investment after t years.
    a. What is the initial investment?
    b. What is the value of the investment after 6 years?
17. **POPULATION** The population of a city has been increasing by 2% annually. In 2000, the population was 315,000. Predict the population of the city in 2020. Round your answer to the nearest thousand.

Write a function that represents the situation. Find the balance in the account after the given time period.

18. \$2000 deposit that earns 5% annual interest compounded quarterly; 5 years

19. \$6200 deposit that earns 8.4% annual interest compounded monthly; 18 months

20. **NUMBER SENSE** During a flu epidemic, the number of sick people triples every week. What is the growth rate as a percent? Explain your reasoning.

21. **SAVINGS** You deposit \$9000 in a savings account that earns 3.6% annual interest compounded monthly. You also save \$40 per month in a safe at home. Write a function $C(t) = b(t) + h(t)$, where $b(t)$ represents the balance of your savings account and $h(t)$ represents the amount in your safe after t years. What does $C(t)$ represent?

22. **REASONING** The number of concert tickets sold doubles every hour. After 12 hours, all of the tickets are sold. After how many hours are about one-fourth of the tickets sold? Explain your reasoning.

23. **YOU BE THE TEACHER** The balance of a savings account can be modeled by the function $b(t) = 5000(1.024)^t$, where t is the time in years. To model the monthly balance, a student writes

$$b(t) = 5000(1.024)^t = 5000(1.024)^{\left(\frac{1}{12} \cdot 12\right)t} = 5000\left(1.024^{\frac{1}{12}}\right)^{12t} \approx 5000(1.002)^{12t}.$$

Is the student correct? Explain your reasoning.

24. Critical Thinking Gordon Moore stated that the number of transistors that can be placed on an integrated circuit will double every 2 years. This trend is known as Moore's Law. In 1978, the Intel® 8086 held 29,000 transistors on an integrated circuit.

a. Write a function that represents Moore's Law, where t is the number of years since 1978.

b. How many transistors could be placed on an integrated circuit in 2015?

Fair Game Review *What you learned in previous grades & lessons*

Simplify the expression. *(Section 6.2)*

25. $\left(\frac{2}{3}\right)^2$ **26.** $\left(\frac{1}{4}\right)^3$ **27.** $\left(\frac{3}{5}\right)^4$

28. **MULTIPLE CHOICE** The domain of the function $y = 4x - 3$ is 1, 4, 7, 10, and 13. Which number is *not* in the range of the function? *(Section 5.1)*

Ⓐ 1 Ⓑ 10 Ⓒ 13 Ⓓ 25

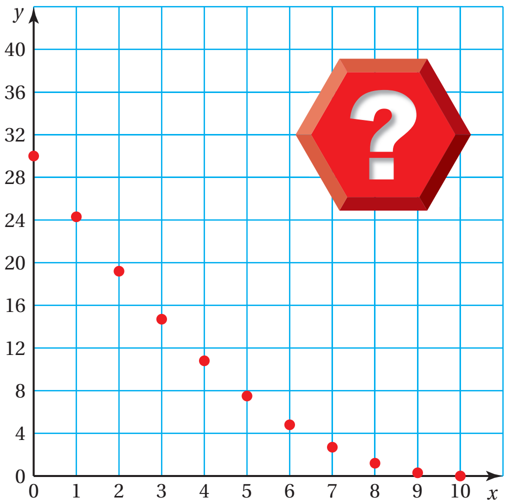
6.6 Exponential Decay

Essential Question What are the characteristics of exponential decay?

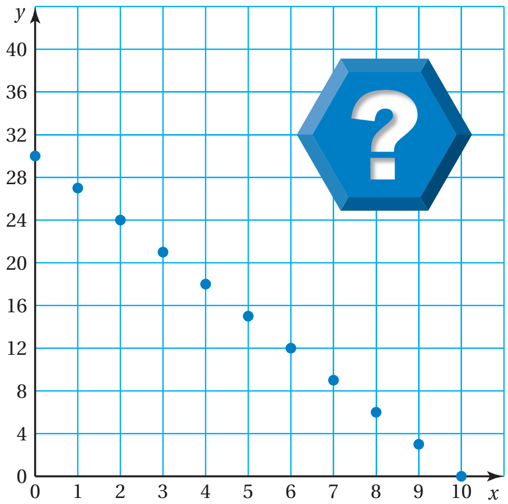
1 ACTIVITY: Comparing Types of Decay

Work with a partner. Describe the pattern of decay for each sequence and graph. Which of the patterns represent exponential decay? Explain your reasoning.

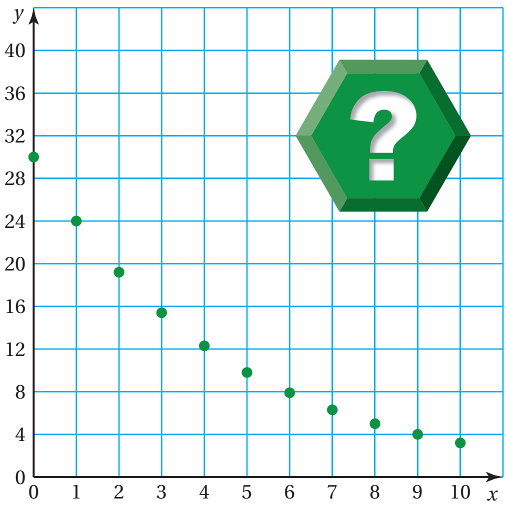
a. 30.0, 24.3, 19.2, 14.7, 10.8, 7.5, 4.8, 2.7, 1.2, 0.3, 0.0



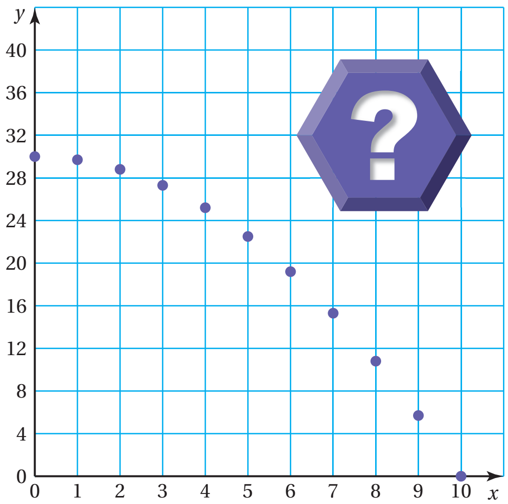
b. 30, 27, 24, 21, 18, 15, 12, 9, 6, 3, 0



c. 30.0, 24.0, 19.2, 15.4, 12.3, 9.8, 7.9, 6.3, 5.0, 4.0, 3.2



d. 30.0, 29.7, 28.8, 27.3, 25.2, 22.5, 19.2, 15.3, 10.8, 5.7, 0.0



COMMON CORE

Exponential Functions

In this lesson, you will
- identify exponential growth and decay.
- write, interpret, and graph exponential decay functions.

Learning Standards
A.SSE.1a
A.SSE.1b
F.IF.7e

2 ACTIVITY: Describing a Decay Pattern

Math Practice 4

Simplify a Situation

How can you organize the given information to simplify this problem? How is the answer affected?

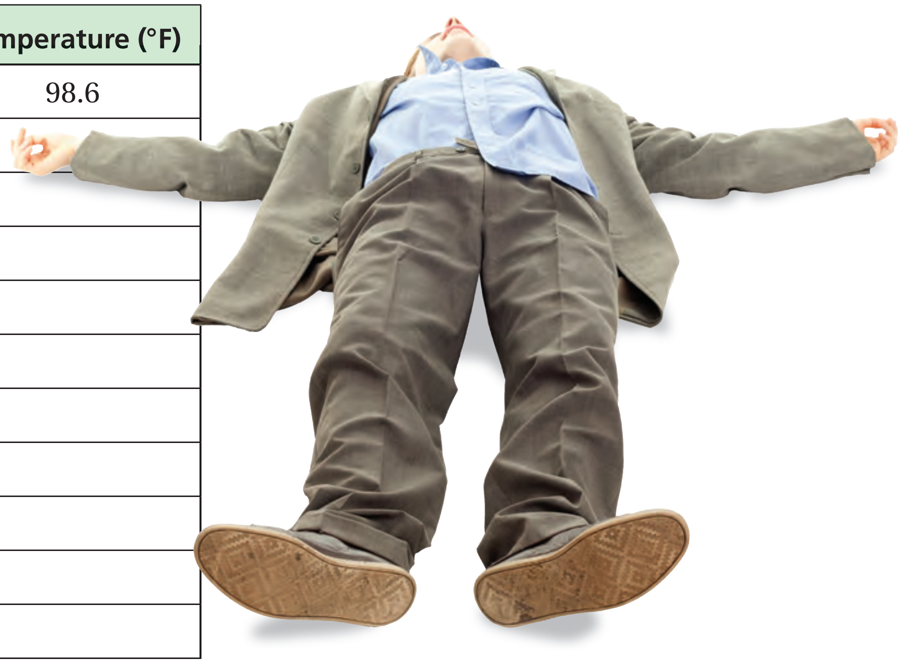
Work with a partner. Newton's Law of Cooling states that when an object at one temperature is exposed to air of another temperature, the difference in the two temperatures drops by the same percent each hour.

A forensic pathologist was called to estimate the time of death of a person. At midnight, the body temperature was 80.5°F and the room temperature was 60°F. One hour later, the body temperature was 78.5°F.

a. By what percent did the difference between the body temperature and the room temperature drop during the hour?

b. Assume that the original body temperature was 98.6°F. Use the percent decrease found in part (a) to make a table showing the decreases in body temperature. Use the table to estimate the time of death.

Time	Temperature (°F)
0	98.6
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



What Is Your Answer?

3. **IN YOUR OWN WORDS** What are the characteristics of exponential decay? How can you distinguish exponential decay from other decay patterns?

4. Sketch a graph of the data from the table in Activity 2. Do the data represent exponential decay? Explain your reasoning.

5. Suppose the pathologist arrived at 5:30 A.M. What was the body temperature at 6 A.M.?

Practice Use what you learned about exponential decay to complete Exercises 3 and 4 on page 304.

6.6 Lesson

Check It Out
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Key Vocabulary
exponential decay, *p. 302*
exponential decay function, *p. 302*

Study Tip
Notice that an exponential decay function is of the form $y = ab^x$, where b is replaced by $1 - r$ and x is replaced by t.

Exponential decay occurs when a quantity decreases by the same factor over equal intervals of time.

Key Idea

Exponential Decay Functions

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an **exponential decay function**.

Initial amount
Rate of decay (in decimal form)

$$\text{Final amount} \rightarrow y = a\underbrace{(1 - r)}_{\text{Decay factor}}{}^{t} \leftarrow \text{Time}$$

For exponential growth, the value inside the parentheses is greater than 1 because r is added to 1. For exponential decay, the value inside the parentheses is less than 1 because r is subtracted from 1.

EXAMPLE 1 Identifying Exponential Growth and Decay

Determine whether each table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

a.

x	y
0	270
1	90
2	30
3	10

$+1$, $+1$, $+1$; $\times \frac{1}{3}$, $\times \frac{1}{3}$, $\times \frac{1}{3}$

As x increases by 1, y is multiplied by $\frac{1}{3}$.

So, the table represents an exponential decay function.

b.

$+1$, $+1$, $+1$

x	0	1	2	3
y	5	10	20	40

$\times 2$, $\times 2$, $\times 2$

As x increases by 1, y is multiplied by 2.

So, the table represents an exponential growth function.

On Your Own

Now You're Ready
Exercises 8–13

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

1.

x	0	1	2	3
y	64	16	4	1

2.

x	1	3	5	7
y	4	11	18	25

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EXAMPLE 2 Interpreting an Exponential Decay Function

The function $P = 4870(0.94)^t$ represents the population P of a town after t years. By what percent does the population decrease each year?

Use the decay factor $1 - r$ to find the rate of decay.

$1 - r = 0.94$ Write an equation.

$r = 0.06$ Solve for r.

So, the population of the town decreases by 6% each year.

On Your Own

Now You're Ready
Exercises 15–17

3. The function $A = 275\left(\frac{9}{10}\right)^t$ represents the area A (in square miles) of a coral reef after t years. By what percent does the area of the coral reef decrease each year?

EXAMPLE 3 Real-Life Application

The value of a car is \$21,500. It loses 12% of its value every year.

a. Write a function that represents the value y (in dollars) of the car after t years.

$y = a(1 - r)^t$ Write exponential decay function.

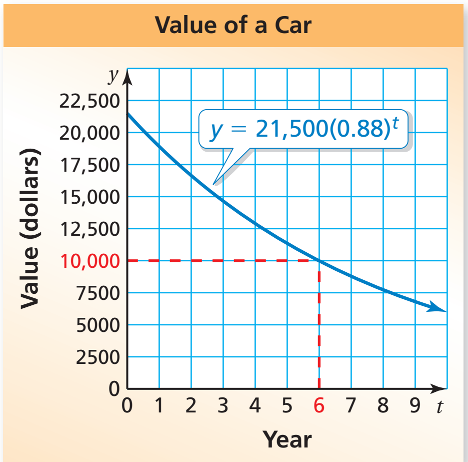
$y = 21{,}500(1 - 0.12)^t$ Substitute 21,500 for a and 0.12 for r.

$y = 21{,}500(0.88)^t$ Simplify.

b. Graph the function from part (a). Use the graph to estimate the value of the car after 6 years.

From the graph, you can see that the y-value is about 10,000 when $t = 6$.

So, the value of the car is about \$10,000 after 6 years.



Check

$y = 21{,}500(0.88)^t$

$= 21{,}500(0.88)^6$

≈ 9985 ✓

On Your Own

Now You're Ready
Exercise 22

4. **WHAT IF?** The car loses 9% of its value every year.

   a. Write a function that represents the value y (in dollars) of the car after t years.

   b. Graph the function from part (a). Estimate the value of the car after 12 years. Round your answer to the nearest thousand.

6.6 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

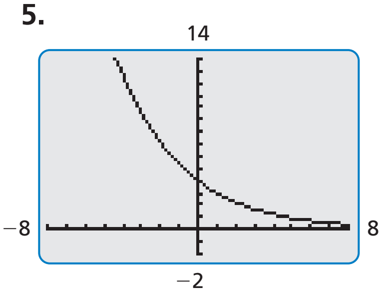
1. **WRITING** When does the function $y = ab^x$ represent exponential growth? exponential decay?
2. **VOCABULARY** What is the decay factor in the function $y = a(1 - r)^t$?

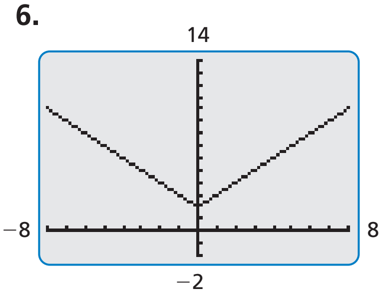
Practice and Problem Solving

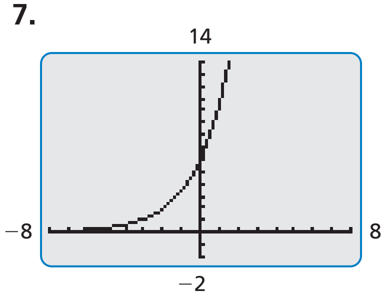
Describe the pattern of decay for the sequence.

3. 28, 26, 24, 22, 20, 18, 16, 14, 12, 10, 8
4. 256, 192, 144, 108, 81, 60.8, 45.6, 34.2, 25.6, 19.2, 14.4

Determine whether the graph represents an *exponential growth function*, an *exponential decay function*, or *neither*.

5.


6.


7.


Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

1 8.

x	0	1	2	3
y	17	51	153	459

9.

x	1	2	3	4
y	32	28	24	20

10.

x	1	2	3	4
y	625	125	25	5

11.

x	2	4	6	8
y	256	64	16	4

12.

x	2	4	6	8
y	35	42	49	42

13.

x	3	5	7	9
y	6	216	7776	279,936

14. **CAMPER** The table shows the value of a camper t years after it is purchased.
   a. Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*.
   b. What is the value of the camper after 5 years?

t	Value
1	$24,000
2	$19,200
3	$15,360
4	$12,288

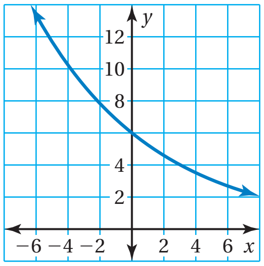
Write the rate of decay of the function as a percent.

2 **15.** $y = 4(0.8)^t$ **16.** $f(t) = 30(0.95)^t$ **17.** $g(t) = \left(\frac{3}{4}\right)^t$

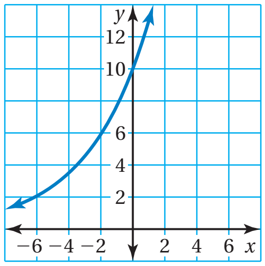
Match the exponential function with its graph.

18. $y = 10(1.3)^t$ **19.** $h(t) = 6\left(\frac{7}{8}\right)^t$ **20.** $y = 2(0.6)^t$

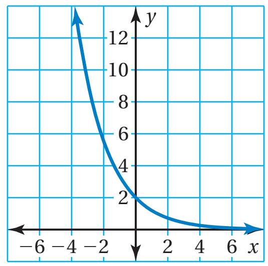
A.



B.



C.



21. **CHOOSE TOOLS** When would you graph an exponential decay function by hand? When would you use a graphing calculator? Explain your reasoning.

3 **22.** **POPULATION** A city has a population of 250,000. The population is expected to decrease by 1.5% annually for the next decade. Write a function that represents this situation. Then predict the population in 10 years.

23. **TIRE PRESSURE** At noon on Monday, the air pressure of a tire is 32 pounds per square inch (psi). The tire loses 8% of its air every day. The tire pressure monitoring system (TPMS) will alert the driver when the tire pressure is less than or equal to 24 psi. On what day of the week will the TPMS alert the driver? Use the *trace* feature of a graphing calculator to help find the answer.

24. **Structure** The graph of an exponential function passes through $\left(2, \frac{3}{2}\right)$ and $\left(4, \frac{3}{8}\right)$.

a. Do the y-values increase or decrease as x increases? How do you know?

b. Find the y-intercept of the graph.

c. Write an exponential function that represents the graph.

Fair Game Review What you learned in previous grades & lessons

Write an equation for the *n*th term of the arithmetic sequence. Then find a_{15}. *(Section 5.6)*

25. 9, 12, 15, 18, . . . **26.** 3, 1, −1, −3, . . . **27.** −7, −11, −15, −19, . . .

28. **MULTIPLE CHOICE** What is the solution of the linear system? *(Section 4.3)*

$$2x - 5y = 11$$
$$5x - 3y = -1$$

Ⓐ (−2, −3) Ⓑ (−2, 3)

Ⓒ (2, −3) Ⓓ (2, 3)

6.7 Geometric Sequences

Essential Question How are geometric sequences used to describe patterns?

1 ACTIVITY: Describing Calculator Patterns

Work with a partner.

- **Enter the keystrokes on a calculator and record the results in the table.**
- **Describe the pattern.**

a. Step 1   2   =

Step 2   ×   2   =

Step 3   ×   2   =

Step 4   ×   2   =

Step 5   ×   2   =

Step	1	2	3	4	5
Calculator Display					

b. Step 1   6   4   =

Step 2   ×   .   5   =

Step 3   ×   .   5   =

Step 4   ×   .   5   =

Step 5   ×   .   5   =

Step	1	2	3	4	5
Calculator Display					

c. Use a calculator to make your own sequence. Start with any number and multiply by 3 each time. Record your results in the table.

Step	1	2	3	4	5
Calculator Display					

COMMON CORE

Geometric Sequences

In this lesson, you will

- extend and graph geometric sequences.
- write equations for geometric sequences.
- solve real-life problems.

Learning Standards
F.BF.2
F.IF.3
F.LE.2

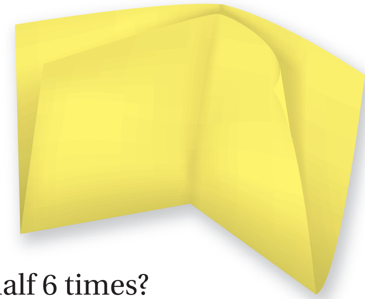
2 ACTIVITY: Folding a Sheet of Paper

Math Practice 8

Repeat Calculations

What calculations are repeated? How does this help you answer the question?

Work with a partner. A sheet of paper is about 0.1 mm thick.



a. How thick would it be if you folded it in half once?

b. How thick would it be if you folded it in half a second time?

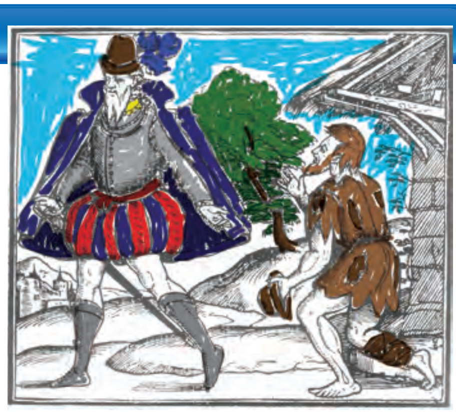
c. How thick would it be if you folded it in half 6 times?

d. What is the greatest number of times you can fold a sheet of paper in half? How thick is the result?

e. Do you agree with the statement below? Explain your reasoning.

"If it were possible to fold the paper 15 times, it would be taller than you."

3 ACTIVITY: Writing a Story



The King and the Beggar

A king offered a beggar fabulous meals for one week. Instead, the beggar asked for a single grain of rice the first day, 2 grains the second day, and double the amount each day after for one month. The king agreed. But, as the month progressed, he realized that he would lose his entire kingdom.

Work with a partner.

- **Why does the king think he will lose his entire kingdom?**
- **Write your own story about doubling or tripling a small object many times.**
- **Draw pictures for your story.**
- **Include a table to organize the amounts.**
- **Write your story so that one of the characters is surprised by the size of the final number.**

What Is Your Answer?

4. **IN YOUR OWN WORDS** How are geometric sequences used to describe patterns? Give an example from real life.

Practice Use what you learned about geometric sequences to complete Exercise 4 on page 310.

6.7 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
geometric sequence, *p. 308*
common ratio, *p. 308*

Key Idea

Geometric Sequence

In a **geometric sequence**, the ratio between consecutive terms is the same. This ratio is called the **common ratio**. Each term is found by multiplying the previous term by the common ratio.

1, 5, 25, 125, . . . Terms of a geometric sequence

×5 ×5 ×5 ← Common ratio

EXAMPLE 1 Extending a Geometric Sequence

Write the next three terms of the geometric sequence 3, 6, 12, 24, ...

Use a table to organize the terms and extend the pattern.

Position	1	2	3	4	5	6	7
Term	3	6	12	24	48	96	192

×2 ×2 ×2 ×2 ×2 ×2

Each term is twice the previous term. So, the common ratio is 2.

Multiply a term by 2 to find the next term.

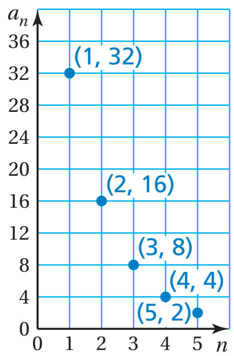
∴ The next three terms are 48, 96, and 192.

EXAMPLE 2 Graphing a Geometric Sequence

Graph the geometric sequence 32, 16, 8, 4, 2, ... What do you notice?

Make a table. Then plot the ordered pairs (n, a_n).

Position, n	1	2	3	4	5
Term, a_n	32	16	8	4	2



∴ The points of the graph appear to lie on an exponential curve.

On Your Own

Now You're Ready
Exercises 11–16

Write the next three terms of the geometric sequence. Then graph the sequence.

1. 1, 3, 9, 27, . . .

2. 64, 16, 4, 1, . . .

3. 80, −40, 20, −10, . . .

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Because consecutive terms of a geometric sequence change by equal factors, the points of any geometric sequence with a positive common ratio lie on an exponential curve. You can use the first term and the common ratio to write an exponential function that describes a geometric sequence.

Position, n	Term, a_n	Written using a_1 and r	Numbers
1	first term, a_1	a_1	1
2	second term, a_2	$a_1 r$	$1 \cdot 5 = 5$
3	third term, a_3	$a_1 r^2$	$1 \cdot 5^2 = 25$
4	fourth term, a_4	$a_1 r^3$	$1 \cdot 5^3 = 125$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	nth term, a_n	$a_1 r^{n-1}$	$1 \cdot 5^{n-1}$

Key Idea

Study Tip

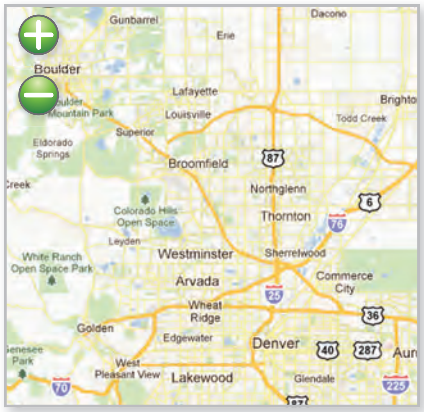
Notice that $a_n = a_1 r^{n-1}$ is of the form $y = ab^x$.

Equation for a Geometric Sequence

Let a_n be the nth term of a geometric sequence with first term a_1 and common ratio r. The nth term is given by

$$a_n = a_1 r^{n-1}.$$

EXAMPLE 3 Real-Life Application



Clicking the *zoom-out* button on a mapping website doubles the side length of the square map.

Zoom-out Clicks	1	2	3
Map Side Length (miles)	5	10	20

a. Write an equation for the nth term of the geometric sequence.

The first term is 5 and the common ratio is 2.

$a_n = a_1 r^{n-1}$ Equation for a geometric sequence

$a_n = 5(2)^{n-1}$ Substitute 5 for a_1 and 2 for r.

b. Find and interpret a_8.

Use the equation to find the 8th term.

$a_n = 5(2)^{n-1}$ Write the equation.

$= 5(2)^{8-1}$ Substitute 8 for n.

$= 640$ Simplify.

The side length of the square map after 8 clicks is 640 miles.

On Your Own

Now You're Ready
Exercises 25–28

4. **WHAT IF?** After how many clicks on the *zoom-out* button is the side length of the map 2560 miles?

6.7 Exercises

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Vocabulary and Concept Check

1. **WRITING** How are arithmetic sequences and geometric sequences different?

2. **REASONING** Compare and contrast the two sequences.

   2, 4, 6, 8, 10, . . .   2, 4, 8, 16, 32, . . .

3. **CRITICAL THINKING** Why do the points of a geometric sequence lie on an exponential curve only when the common ratio is positive?

Practice and Problem Solving

4. Enter 4 on a calculator. Multiply by 6 four times. Record your results in the table. Describe the pattern.

Step	1	2	3	4	5
Calculator Display					

Find the common ratio of the geometric sequence.

5. 3, -12, 48, -192, . . .
6. 200, 100, 50, 25, . . .
7. 7640, 764, 76.4, 7.64, . . .
8. 9, -18, 36, -72, . . .
9. 0.1, 0.9, 8.1, 72.9, . . .
10. 5, 1, $\frac{1}{5}$, $\frac{1}{25}$, . . .

Write the next three terms of the geometric sequence. Then graph the sequence.

1 2 11. 2, 10, 50, 250, . . .
12. -7, 14, -28, 56, . . .
13. 81, -27, 9, -3, . . .
14. -375, -75, -15, -3, . . .
15. 36, 6, 1, $\frac{1}{6}$, . . .
16. $\frac{1}{49}$, $\frac{1}{7}$, 1, 7, . . .

17. **ERROR ANALYSIS** Describe and correct the error in writing the next three terms of the geometric sequence.

$$-8, \quad 4, \quad -2, \quad 1, \ldots$$
$$\times(-2) \quad \times(-2) \quad \times(-2)$$

The next three terms are -2, 4, and -8.

18. **BADMINTON** A badminton tournament begins with 128 teams. After the first round, 64 teams remain. After the second round, 32 teams remain. How many teams remain after the third, fourth, and fifth rounds?

Tell whether the sequence is *geometric*, *arithmetic*, or *neither*.

19. -8, 0, 8, 16, . . .
20. -1, 3, -5, 7, . . .
21. 1, 4, 9, 16, . . .
22. $\frac{3}{49}$, $\frac{3}{7}$, 3, 21, . . .
23. 192, 24, 3, $\frac{3}{8}$, . . .
24. -25, -18, -12, -7, . . .

Write an equation for the nth term of the geometric sequence. Then find a_7.

3 **25.** $1, -5, 25, -125, \ldots$

26. $2, 8, 32, 128, \ldots$

27.

n	1	2	3	4
a_n	5	15	45	135

28.

n	1	2	3	4
a_n	2	14	98	686

29. CHAIN EMAIL You start a chain email and send it to 6 friends. The process continues and each of your friends forwards the email to 6 people.

a. Write an equation for the nth term of the geometric sequence.

b. Describe the domain. Is the domain discrete or continuous?

30. REASONING What is the 9th term of a geometric sequence where $a_3 = 81$ and $r = 3$?

31. PRECISION Are the terms of a geometric sequence independent or dependent? Explain your reasoning.

32. ROOM AND BOARD A college student makes a deal with her parents to live at home instead of living on campus. She will pay her parents \$0.01 for the first day of the month, \$0.02 for the second day, \$0.04 for the third day, and so on.

a. Write an equation for the nth term of the geometric sequence.

b. What will she pay on the 25th day?

c. Did the student make a good choice or should she have chosen to live on campus? Explain.

33. Repeated Reasoning A soup kitchen makes 16 gallons of soup. Each day, a quarter of the soup is served and the rest is saved for the next day.

a. Write the first five terms of the sequence of the number of fluid ounces of soup left each day.

b. Write an equation to represent the sequence.

c. When is all the soup gone? Explain.

Fair Game Review What you learned in previous grades & lessons

Simplify the expression. *(Skills Review Handbook)*

34. $2n - 6n$

35. $2(4x - 5) + x$

36 $4(y - 1) - (y + 2)$

37. MULTIPLE CHOICE What is the solution of $6(3 - x) = -4x + 12$? *(Section 1.3)*

Ⓐ $x = -3$ Ⓑ $x = -2$ Ⓒ $x = 3$ Ⓓ $x = 6$

Extension 6.7 Recursively Defined Sequences

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Key Vocabulary
recursive rule, *p. 312*

In Sections 5.6 and 6.7, you wrote *explicit* equations for sequences. Now, you will write *recursive* equations for sequences. A **recursive rule** gives the beginning term(s) of a sequence and an equation that indicates how any term a_n in the sequence relates to the previous term.

Key Idea

Recursive Equation for an Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference.

Recursive Equation for a Geometric Sequence

$a_n = r \cdot a_{n-1}$, where r is the common ratio.

EXAMPLE 1 Writing Terms of Recursively Defined Sequences

Write the first six terms of each sequence. Then graph each sequence.

a. $\boldsymbol{a_1 = 2, a_n = a_{n-1} + 3}$

$a_1 = 2$

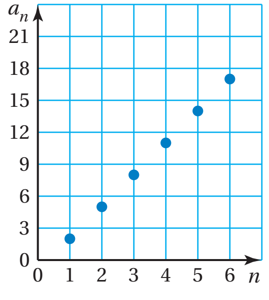
$a_2 = a_1 + 3 = 2 + 3 = 5$

$a_3 = a_2 + 3 = 5 + 3 = 8$

$a_4 = a_3 + 3 = 8 + 3 = 11$

$a_5 = a_4 + 3 = 11 + 3 = 14$

$a_6 = a_5 + 3 = 14 + 3 = 17$



b. $\boldsymbol{a_1 = 1, a_n = 3a_{n-1}}$

$a_1 = 1$

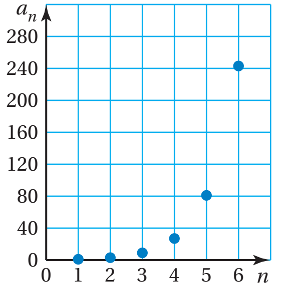
$a_2 = 3a_1 = 3(1) = 3$

$a_3 = 3a_2 = 3(3) = 9$

$a_4 = 3a_3 = 3(9) = 27$

$a_5 = 3a_4 = 3(27) = 81$

$a_6 = 3a_5 = 3(81) = 243$



Practice

Write the first six terms of the sequence. Then graph the sequence.

1. $a_1 = 0, a_n = a_{n-1} - 8$

2. $a_1 = -7.5, a_n = a_{n-1} + 2.5$

3. $a_1 = -36, a_n = \frac{1}{2}a_{n-1}$

4. $a_1 = 0.7, a_n = 10a_{n-1}$

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Writing Recursive Rules

COMMON CORE

Recursive Sequences

In this extension, you will
- write the terms of recursively defined sequences.
- write recursive equations for sequences.

Learning Standards
F.BF.2
F.IF.3
F.LE.2

Write a recursive rule for each sequence.

a. $-30, -18, -6, 6, 18, \ldots$

Use a table to organize the terms and find the pattern.

Position	1	2	3	4	5
Term	-30	-18	-6	6	18

$+12 \quad +12 \quad +12 \quad +12$

The sequence is arithmetic with first term -30 and common difference 12.

$a_n = a_{n-1} + d$ Recursive equation (arithmetic)

$a_n = a_{n-1} + 12$ Substitute 12 for d.

So, a recursive rule for the sequence is $a_1 = -30$, $a_n = a_{n-1} + 12$.

b. $500, 100, 20, 4, 0.8, \ldots$

Use a table to organize the terms and find the pattern.

Position	1	2	3	4	5
Term	500	100	20	4	0.8

$\times\frac{1}{5} \quad \times\frac{1}{5} \quad \times\frac{1}{5} \quad \times\frac{1}{5}$

The sequence is geometric with first term 500 and common ratio $\frac{1}{5}$.

$a_n = r \cdot a_{n-1}$ Recursive equation (geometric)

$a_n = \frac{1}{5}a_{n-1}$ Substitute $\frac{1}{5}$ for r.

So, a recursive rule for the sequence is $a_1 = 500$, $a_n = \frac{1}{5}a_{n-1}$.

Practice

Write a recursive rule for the sequence.

5. $8, 3, -2, -7, -12, \ldots$

6. $1.3, 2.6, 3.9, 5.2, 6.5, \ldots$

7. $4, 20, 100, 500, 2500, \ldots$

8. $1600, -400, 100, -25, 6.25, \ldots$

9. **SUNFLOWERS** Write a recursive rule for the height of the sunflower over time.

1 month: 2 feet | 2 months: 3.5 feet | 3 months: 5 feet | 4 months: 6.5 feet

EXAMPLE 3 Translating Recursive Rules into Explicit Equations

Write an explicit equation for each recursive rule.

a. $a_1 = 25, a_n = a_{n-1} - 10$

The recursive rule represents an arithmetic sequence with first term 25 and common difference -10.

$a_n = a_1 + (n-1)d$ — Equation for an arithmetic sequence

$a_n = 25 + (n-1)(-10)$ — Substitute 25 for a_1 and -10 for d.

$a_n = -10n + 35$ — Simplify.

b. $a_1 = 19.6, a_n = -0.5a_{n-1}$

The recursive rule represents a geometric sequence with first term 19.6 and common ratio -0.5.

$a_n = a_1 r^{n-1}$ — Equation for a geometric sequence

$a_n = 19.6(-0.5)^{n-1}$ — Substitute 19.6 for a_1 and -0.5 for r.

EXAMPLE 4 Translating Explicit Equations into Recursive Rules

Write a recursive rule for each explicit equation.

a. $a_n = -2n + 3$

The explicit equation represents an arithmetic sequence with first term $-2(1) + 3 = 1$ and common difference -2.

$a_n = a_{n-1} + d$ — Recursive equation (arithmetic)

$a_n = a_{n-1} + (-2)$ — Substitute -2 for d.

So, a recursive rule for the sequence is $a_1 = 1, a_n = a_{n-1} - 2$.

b. $a_n = -3(2)^{n-1}$

The explicit equation represents a geometric sequence with first term -3 and common ratio 2.

$a_n = r \cdot a_{n-1}$ — Recursive equation (geometric)

$a_n = 2a_{n-1}$ — Substitute 2 for r.

So, a recursive rule for the sequence is $a_1 = -3, a_n = 2a_{n-1}$.

Practice

Write an explicit equation for the recursive rule.

10. $a_1 = -45, a_n = a_{n-1} + 20$

11. $a_1 = 13, a_n = -3a_{n-1}$

Write a recursive rule for the explicit equation.

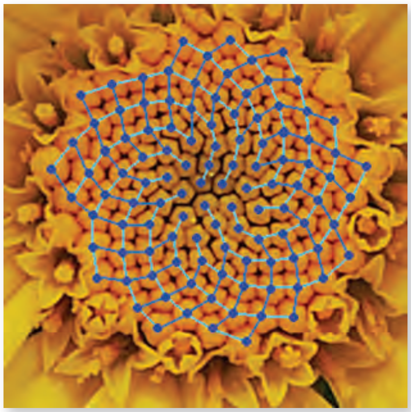
12. $a_n = -n + 1$

13. $a_n = -2.5(2)^{n-1}$

You can write recursive rules for sequences that are neither arithmetic nor geometric. One way is to look for patterns in the sums of consecutive terms.

EXAMPLE 5 Writing Recursive Rules for Other Sequences

Write a recursive rule for the sequence 1, 1, 2, 3, 5, 8, ... Then write the next 3 terms of the sequence.



The sequence in Example 5 is called the *Fibonacci sequence.* This pattern is naturally occurring in many objects, such as flowers.

The sequence does not have a common difference or a common ratio. Find the sums of consecutive terms.

$a_1 + a_2 = 1 + 1 = 2$ 2 is the third term.

$a_2 + a_3 = 1 + 2 = 3$ 3 is the fourth term.

$a_3 + a_4 = 2 + 3 = 5$ 5 is the fifth term.

$a_4 + a_5 = 3 + 5 = 8$ 8 is the sixth term.

So, a recursive equation for the sequence is $a_n = a_{n-2} + a_{n-1}$. Use the equation to find the next three terms.

$$a_7 = a_5 + a_6 = 5 + 8 = 13 \qquad a_8 = a_6 + a_7 = 8 + 13 = 21 \qquad a_9 = a_7 + a_8 = 13 + 21 = 34$$

A recursive rule for the sequence is $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-2} + a_{n-1}$. The next three terms are 13, 21, and 34.

Practice

Write a recursive rule for the sequence. Then write the next 3 terms of the sequence.

14. 5, 6, 11, 17, 28, . . .

15. $-3, -4, -7, -11, -18, \ldots$

16. $1, 1, 0, -1, -1, 0, 1, 1, \ldots$

17. $4, 3, 1, 2, -1, 3, -4, \ldots$

Use a pattern in the products of consecutive terms to write a recursive rule for the sequence. Then write the next 2 terms of the sequence.

18. 2, 3, 6, 18, 108, . . .

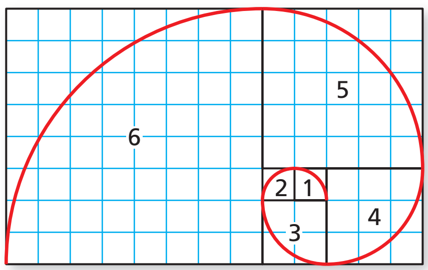
19. $-2, 2.5, -5, -12.5, 62.5, \ldots$

20. GEOMETRY Consider squares 1–6 in the diagram.

a. Write a sequence in which each term a_n is the side length of square n.

b. What is the name of this sequence? What is the next term of this sequence?

c. Use the term in part (b) to add another square to the diagram and extend the spiral.



6.4–6.7 Quiz

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Progress Check
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Does the table represent a *linear* or an *exponential* function? Explain. *(Section 6.4)*

1.

x	1	2	3	4
y	5	10	15	20

2.

x	2	4	6	8
y	5	10	20	40

Graph the function. Describe the domain and range. *(Section 6.4)*

3. $y = 5^x$

4. $y = -2\left(\frac{1}{6}\right)^x$

Solve the equation. Check your solution, if possible. *(Section 6.4)*

5. $8^{x+2} = 64^{4x+1}$

6. $7^{2x-6} = 49^{3x-11}$

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. *(Section 6.6)*

7.

x	0	1	2	3
y	7	21	63	189

8.

x	1	2	3	4
y	14,641	1331	121	11

Write the next three terms of the geometric sequence. Then graph the sequence. *(Section 6.7)*

9. 15, −45, 135, −405, . . .

10. 768, 192, 48, 12, . . .

Write a recursive rule for the sequence. *(Section 6.7)*

11. 5, 11, 17, 23, . . .

12. −14, 28, −56, 112, . . .

13. SAVINGS ACCOUNT You deposit \$2500 in a savings account that earns 6% annual interest compounded yearly. *(Section 6.5)*

a. Write and graph a function that represents the balance y (in dollars) after t years.

b. What is the balance after 5 years?

14. CURRENCY A country's base unit of currency is valued at US\$2. The country's base unit of currency loses about 3.9% of its value every month. *(Section 6.6)*

a. Write a function that represents the value y (in U.S. dollars) of the base unit of currency after t months.

b. What is the value of the country's base unit of currency after 1.5 years?

6 Chapter Review

Check It Out
Vocabulary Help
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Review Key Vocabulary

closed, *p. 266*
*n*th root, *p. 278*
exponential function, *p. 286*
exponential growth, *p. 296*
exponential growth function, *p. 296*
compound interest, *p. 297*
exponential decay, *p. 302*
exponential decay function, *p. 302*
geometric sequence, *p. 308*
common ratio, *p. 308*
recursive rule, *p. 312*

Review Examples and Exercises

6.1 Properties of Square Roots *(pp. 260–267)*

Evaluate $\sqrt{b^2 - 4ac}$ when $a = -2$, $b = 2$, and $c = 5$.

$$\sqrt{b^2 - 4ac} = \sqrt{2^2 - 4(-2)(5)} \quad \text{Substitute.}$$
$$= \sqrt{44} \quad \text{Simplify.}$$
$$= \sqrt{4 \cdot 11} \quad \text{Factor.}$$
$$= \sqrt{4} \cdot \sqrt{11} \quad \text{Product Property of Square Roots}$$
$$= 2\sqrt{11} \quad \text{Simplify.}$$

Exercises

Evaluate the expression when $x = 3$, $y = 4$, and $z = 2$.

1. $\sqrt{xy^2z}$
2. $\sqrt{2z + y}$
3. $\dfrac{8 + \sqrt{xy}}{z}$

6.2 Properties of Exponents *(pp. 268–275)*

Simplify $\left(\dfrac{3x}{4}\right)^{-4}$. Write your answer using only positive exponents.

$$\left(\frac{3x}{4}\right)^{-4} = \frac{(3x)^{-4}}{4^{-4}} \quad \text{Power of a Quotient Property}$$
$$= \frac{4^4}{(3x)^4} \quad \text{Definition of negative exponent}$$
$$= \frac{4^4}{3^4x^4} \quad \text{Power of a Product Property}$$
$$= \frac{256}{81x^4} \quad \text{Simplify.}$$

Exercises

Simplify. Write your answer using only positive exponents.

4. $y^3 \cdot y^{-3}$ **5.** $\dfrac{x^4}{x^7}$ **6.** $(xy^2)^3$ **7.** $\left(\dfrac{2x}{5y}\right)^{-2}$

6.3 Radicals and Rational Exponents (pp. 276–281)

Simplify each expression.

a. $\sqrt[3]{512} = \sqrt[3]{8 \cdot 8 \cdot 8} = 8$ Rewrite and simplify.

b. $900^{1/2} = \sqrt{900}$ Write the expression in radical form.

$= \sqrt{30 \cdot 30}$ Rewrite.

$= 30$ Simplify.

Exercises

Simplify the expression.

8. $\sqrt[3]{8}$ **9.** $64^{1/2}$ **10.** $625^{3/4}$

6.4 Exponential Functions (pp. 284–293)

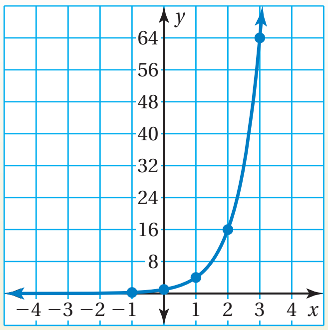
a. Graph $y = 4^x$.

Step 1: Make a table of values.

x	−1	0	1	2	3
y	0.25	1	4	16	64

Step 2: Plot the ordered pairs.

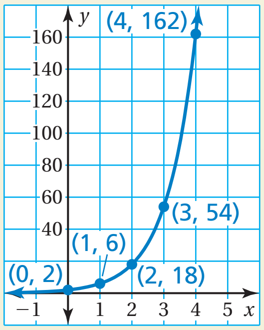
Step 3: Draw a smooth curve through the points.



b. Write an exponential function represented by the graph.

Use the graph to make a table of values.

	+1	+1	+1	+1	
x	0	1	2	3	4
y	2	6	18	54	162
	×3	×3	×3	×3	



The *y*-intercept is 2 and the *y*-values increase by a factor of 3 as *x* increases by 1.

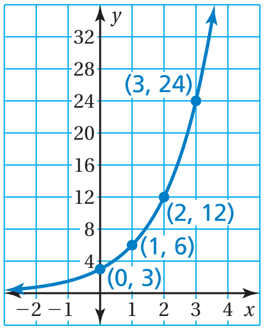
So, the exponential function is $y = 2(3)^x$.

Exercises

11. Graph $y = -2(4)^x + 3$. Describe the domain and range. Compare the graph to the graph of $y = -2(4)^x$.

Write an exponential function represented by the graph or table.

12.



13.

x	0	1	2	3
y	2	1	0.5	0.25

Solve the equation. Check your solution, if possible.

14. $3^x = 27$ **15.** $5^x = 5^{x-2}$ **16.** $2^{5x} = 8^{2x-4}$

6.5 Exponential Growth *(pp. 294–299)*

The enrollment at a high school increases by 4% each year. In 2010, there were 800 students enrolled at the school.

a. Write a function that represents the enrollment y of the high school after t years.

$y = a(1 + r)^t$ Write exponential growth function.

$y = 800(1 + 0.04)^t$ Substitute 800 for a and 0.04 for r.

$y = 800(1.04)^t$ Simplify.

b. How many students will be enrolled at the high school in 2020?

The value $t = 10$ represents 2020.

$y = 800(1.04)^t$ Write exponential growth function.

$y = 800(1.04)^{10}$ Substitute 10 for t.

≈ 1184 Use a calculator.

Exercises

17. PLUMBER A plumber charges $22 per hour. The hourly rate increases by 3% each year.

a. Write a function that represents the plumber's hourly rate y (in dollars) after t years.

b. What is the plumber's hourly rate after 8 years?

6.6 Exponential Decay *(pp. 300–305)*

The table shows the value of a boat over time.

Year, t	0	1	2	3
Value, y	\$6000	\$4800	\$3840	\$3072

a. Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

+1 +1 +1

Year, t	0	1	2	3
Value, y	\$6000	\$4800	\$3840	\$3072

× 0.8 × 0.8 × 0.8

As x increases by 1, y is multiplied by 0.8. So, the table represents an exponential decay function.

b. The boat loses 20% of its value every year. Write a function that represents the value y (in dollars) of the boat after t years.

$y = a(1 - r)^t$ — Write exponential decay function.

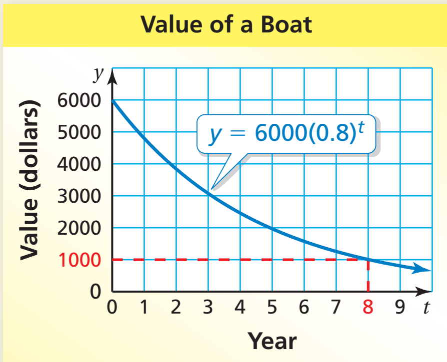
$y = 6000(1 - 0.2)^t$ — Substitute 6000 for a and 0.2 for r.

$y = 6000(0.8)^t$ — Simplify.

c. Graph the function from part (b). Use the graph to estimate the value of the boat after 8 years.

From the graph, you can see that the y-value is about 1000 when $t = 8$.

So, the value of the boat is about \$1000 after 8 years.



Exercises

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

18.

x	0	1	2	3
y	3	6	12	24

19.

x	1	2	3	4
y	162	108	72	48

20. DISCOUNT The price of a TV is \$1500. The price decreases by 6% each month. Write and graph a function that represents the price y (in dollars) of the TV after t months. Use the graph to estimate the price of the TV after 1 year.

6.7 Geometric Sequences *(pp. 306–315)*

a. Write the next three terms of the geometric sequence 2, 6, 18, 54, ...

Use a table to organize the terms and extend the pattern.

Position	1	2	3	4	5	6	7
Term	2	6	18	54	162	486	1458

× 3   × 3   × 3   × 3   × 3   × 3

Each term is 3 times the previous term. So, the common ratio is 3.

Multiply a term by 3 to find the next term.

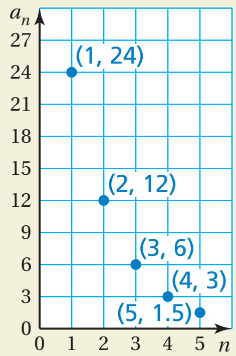
The next three terms are 162, 486, and 1458.

b. Graph the geometric sequence 24, 12, 6, 3, 1.5, ... What do you notice?

Make a table. Then plot the ordered pairs (n, a_n).

Position, n	1	2	3	4	5
Term, a_n	24	12	6	3	1.5

The points of the graph appear to lie on an exponential curve.



Exercises

Write the next three terms of the geometric sequence. Then graph the sequence.

21. −3, 9, −27, 81, . . .

22. 48, 12, 3, $\frac{3}{4}$, . . .

Write an equation for the *n*th term of the geometric sequence.

23.

n	1	2	3	4
a_n	1	4	16	64

24.

n	1	2	3	4
a_n	5	−10	20	−40

Write a recursive rule for the sequence.

25. 3, 8, 13, 18, 23, . . .

26. 3, 6, 12, 24, 48, . . .

6 Chapter Test

Check It Out
Test Practice
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Simplify the expression.

1. $\sqrt{98}$

2. $\sqrt{\frac{19}{25}}$

3. $\frac{6 - \sqrt{48}}{2}$

Simplify. Write your answer using only positive exponents.

4. $z^{-2} \cdot z^4$

5. $\frac{b^{-5}}{b^{-8}}$

6. $\left(\frac{2c^4}{5}\right)^{-3}$

Simplify the expression.

7. $\sqrt[4]{16}$

8. $729^{1/6}$

9. $32^{7/5}$

10. Graph $y = 7^x + 1$. Describe the domain and range. Compare the graph to the graph of $y = 7^x$.

Write an exponential function represented by the table.

11.

x	0	1	2	3
y	-1	-2	-4	-8

12.

x	0	1	2	3
y	3	-12	48	-192

Solve the equation. Check your solution, if possible.

13. $2^x = 128$

14. $256^{x+2} = 16^{3x-1}$

Write and graph a function that represents the situation.

15. Your \$42,500 annual salary increases by 3% each year.

16. You deposit \$500 in an account that earns 6.5% annual interest compounded yearly.

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

17.

x	0	1	2	3
y	15	30	60	120

18.

x	0	1	2	3
y	400	100	25	6.25

19. TRAINING You follow the training schedule from your coach.

a. Write an equation for the nth term of the geometric sequence.

b. Write a recursive rule for the explicit equation in part (a).

c. On what day do you run approximately 3 kilometers?

Training On Your Own

Day 1: Run 1 km.

Each day after Day 1: Run 20% farther than the previous day.

1. Which point is a solution of the system of inequalities shown below? *(A.REI.12)*

$$y \geq 4x - 3$$
$$3x - 2y < 4$$

A. $(-2, -7)$ **C.** $(-4, -8)$

B. $(1, 1)$ **D.** $(4, 5)$

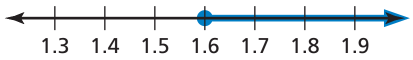
2. What is the value of the function $y = -10(5)^x$ when $x = -3$? *(F.IF.7e)*

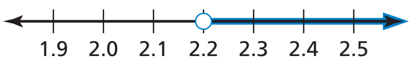
F. $-\frac{2}{25}$ **H.** $\frac{2}{25}$

G. $-\frac{2}{125}$ **I.** 30

3. Which graph shows the solution of $x - 1.9 \geq 0.3$? *(A.CED.1)*

A.

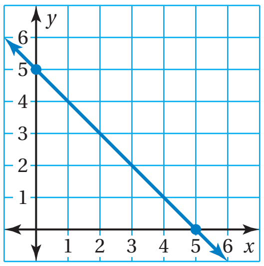

C.


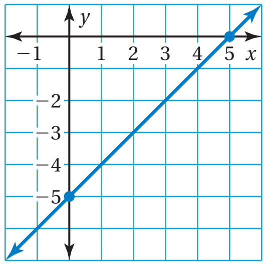
B.


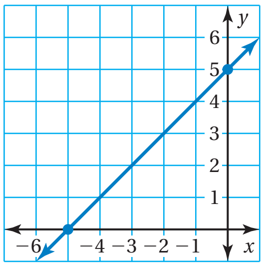
D.

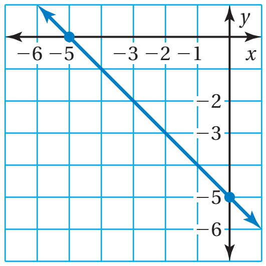

4. What is the value of $27^{4/3}$? *(N.RN.2)*

5. Which graph represents the equation $-5x - 5y = 25$? *(A.REI.10)*

F.


H.


G.


I.


6. A system of two linear equations has infinitely many solutions. What can you conclude about the graphs of the two equations? *(8.EE.8b)*

A. The lines have the same slope and the same y-intercept.

B. The lines have the same slope and different y-intercepts.

C. The lines have different slopes and the same y-intercept.

D. The lines have different slopes and different y-intercepts.

7. The domain of the function $y = -5x + 19$ is 0, 2, 4, and 6. What is the range of the function? *(F.IF.1)*

F. $-19, -9, 1, 11$

G. $-11, -1, 9, 19$

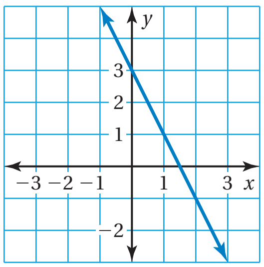
H. $6, 4, 2, 0$

I. $-19, -11, -9, 1$

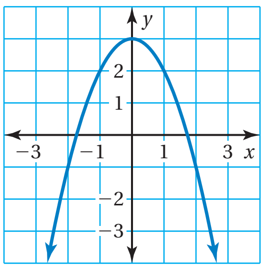
8. What is the 50th term of the sequence $20, 9, -2, -13, \ldots$? *(F.LE.2)*

9. Which graph shows an exponential decay function? *(F.IF.7E)*

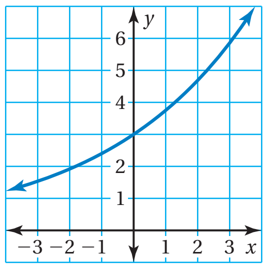
A.



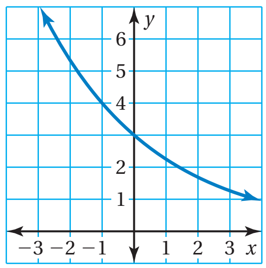
B.



C.



D.



10. The lowest temperature ever recorded on Earth is $-129°$ Fahrenheit. The highest temperature ever recorded on Earth is $136°$ Fahrenheit. Let t represent the temperature, in degrees Fahrenheit. Which inequality represents all temperatures ever recorded on Earth? *(A.CED.1)*

F. $-129 < t < 136$

G. $-129 \le t < 136$

H. $-129 \le t \le 136$

I. $-129 < t \le 136$

11. The graph of which equation is perpendicular to the line that passes through the points $(-3, -6)$ and $(5, -2)$? *(F.IF.6)*

A. $y = \frac{1}{2}x + 3$

B. $y = -2x + 7$

C. $y = -\frac{1}{2}x - 3$

D. $y = 2x + 1$

12. Which of the following is true about the graph of the linear equation $y = -7x + 5$? *(F.IF.4)*

F. The slope is 5 and the y-intercept is -7.

G. The slope is -5 and the y-intercept is -7.

H. The slope is -7 and the y-intercept is -5.

I. The slope is -7 and the y-intercept is 5.

13. At the beginning of a tennis tournament, there are 256 players. After each round, one-half of the remaining players are eliminated. *(F.IF.7e)*

Think
Solve
Explain

Part A Write a function that represents the number of players left in the tournament after each round.

Part B Does the function in part (a) represent exponential growth or exponential decay? Explain your reasoning.

Part C Graph the function in part (a).

Part D How many tennis matches does a player have to win to win the tournament? Explain your reasoning.

14. Which sequence is neither arithmetic nor geometric? *(F.IF.3)*

A. 1, 50, 2500, 125,000, . . .

B. 10, 0, -10, -20, . . .

C. 4, -4, 4, -4, . . .

D. 0, 1, 3, 6, . . .

15. For $f(x) = -3x - 10$, what value of x makes $f(x) = -7$? *(F.IF.2)*

F. -7 **G.** -1 **H.** 1 **I.** 11

16. Which expression is equivalent to $20\sqrt{200}$? *(N.RN.3)*

A. $40\sqrt{2}$ **B.** $40\sqrt{10}$ **C.** $200\sqrt{2}$ **D.** 400