## © Statistical Measures

### 9.1 Introduction to statistics

### 9.2 Mean

## 9,3 Measures of Center

9.4 Measures of Variation
9.5 Mean Absolute Deviation

"Mom, my owner, and Fluffy have agreed to participate in my survey. Will you be my fourth participant?"

"Please hold still. I am trying to find the mean of 6, 8, and 10 by dividing their sum into three equal piles."

## What You Learned Before

## Ordering Decimals



Example 1 Use a number line to order 8, 6.5, 7.25, 5.5, 4.25, and 7 from least to greatest.


## Try It Yourself

Use a number line to order the numbers from least to greatest.

1. $7.25,4.5,6.5,6,5.5,8.75$
2. $4,2.5,3.25,5.5,4.5,6.75$
3. $6.25,3,2.5,3.5,5.75,5$
4. $1.25,5.5,4.75,4.5,3.5,2.25$

## - Analyzing Double Bar Graphs

## Example 2 How many more male athletes than female athletes participated in the 1992 Summer Olympics?


¿-• 3948 more male athletes participated.


## Example 3 How many athletes participated in the 2000 Summer Olympics?

$\therefore 6582+4069=10,651$ athletes participated in the 2000 Summer Olympics.

## Try It Yourself

5. How many more female athletes participated in the 2012 Summer Olympics than in the 1992 Summer Olympics?
6. Describe the relationship between the number of athletes in the 2000 Summer Olympics and the number of athletes in the 2004 Summer Olympics.

### 9.1 Intiroduction to Statistics

## Essential Question How can you tell whether a question

## is a statistical question?

Your heart rate is the number of times your heart beats in a certain time period, such as 1 minute. To measure your heart rate, you can check your pulse. The illustration shows how to check your pulse by pressing lightly on your wrist.


Here are other places to check your pulse:

- inside your elbow
- side of your neck
- top of your foot


## Statistics

In this lesson, you will

- recognize statistical questions.
- use dot plots to display numerical data.


## Work with a partner.

a. Find your pulse by counting the number of beats in 10 seconds. Have your partner keep track of the time. Write a rate to describe your result.
b. Complete the ratio table. What is your heart rate in beats per minute?

| Time (seconds) | 10 | 30 | 60 |
| :--- | :---: | :---: | :---: |
| Number of Beats |  |  |  |

c. Collect the recorded heart rates (in beats per minute) of the students in your class, including yourself. Compare the heart rates.
d. MODELING Make a line plot of your data. Then answer the following questions:

- How many values are in your data set?
- Do the heart rates cluster around a particular value or values?
- Are there any peaks or gaps in the data?
- Are there any unusual heart rates that are far removed from the other values?
e. REASONING How would you answer the following question by using only one value? Explain your reasoning.
"What is the heart rate of sixth grade students?"
f. REASONING Read and compare the following questions. How did you answer each question? Could the answer be the same for both questions? Explain.
- What is your heart rate?
- What is the heart rate of sixth grade students?


## 2 ACTIVITY: Identifying Types of Questions

## Work with a partner.

a. Answer each question below on your own. Then compare your answers with your partner's answers. For which questions should your answers be the same? For which questions might your answers be different?

1. What is your shoe size?
2. How many states are in the United States?
3. How many brothers and sisters do you have?
4. How many U.S. presidents have been in office?
5. What is your favorite type of movie?
6. How tall are you?
b. CONJECTURE Some of the questions above are considered statistical questions. Which ones do you think they are? Why?

## 3 ACTIVITY: Analyzing a Question in a Survey

Work with a partner. A student asks the following question in a survey:


## "Do you prefer salty potato chips or healthy granola bars to be sold in the school's vending machines?"


a. Do you think this is a fair question to ask in a survey? Explain.
b. LOGIC Identify the words in the question that may influence someone's response. Then explain how you can reword the question.
c. How might the results of the survey differ when the student asks the original question and your reworded question in part (b)?

## What is Your Answer?

4. REASONING What do you think "statistics" means?
5. IN YOUR OWN WORDS How can you tell whether a question is a statistical question? Give examples to support your explanation.
6. Find the least and the greatest heart rates in your class. How can you use these two values to answer the question in Activity 1 (e)?
7. Create a one-question survey. Explain why your question is a statistical question. Then conduct your survey and organize your results in a line plot. Make three observations about your data set.

## Practice

Use what you learned about different types of questions to complete Exercises 4-7 on page 394.

## Key Vocabulary

 statistics, p. 392statistical question, p. 392

Statistics is the science of collecting, organizing, analyzing, and interpreting data. A statistical question is one for which you do not expect to get a single answer. Instead, you expect a variety of answers, and you are interested in the distribution and tendency of those answers.
Recall that a dot plot uses a number line to show the number of times each value in a data set occurs. Dot plots show the spread and the distribution of a data set.

## EXAMPLE

## Answering a Statistical Question

You conduct a science experiment on house mice. Your teacher asks you, "What is the weight of a mouse?"
a. Is this a statistical question? Explain.
$\therefore$ Because you can anticipate that the weights of mice will vary, it is a statistical question.

| Weights (grams) |  |  |  |
| :---: | :---: | :---: | :---: |
| 20 | 19 | 21 | 20 |
| 18 | 20 | 27 | 21 |
| 28 | 23 | 20 | 19 |
| 20 | 21 | 18 | 27 |
| 19 | 22 | 21 | 20 |

b. You weigh some mice and record the weights (in grams) in the table. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

Draw a number line that includes the least value, 18 , and greatest value, 28 . Then place a dot above the number line for each data value.


Most of the data are clustered around 20. There is a peak at 20 and a gap between 23 and 27 .
c. Use the distribution of the data to answer the question.
$\therefore$ Most mice weigh about 20 grams.

## On Your Own

Now You're Ready
Exercises 8-16

1. The table shows the ages of some people who retired early. You are asked, "How old are people who retire early?"
a. Is this a statistical question? Explain.
b. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
c. Use the distribution of the data to answer the question.

You record the high temperature every day while at summer camp in August. Then you create the vertical dot plot.
a. How many weeks were you at summer camp?

Because there are 28 data values on the dot plot, you were at camp 28 days.
28 days $\cdot \frac{1 \text { week }}{7 \text { days }}=4$ weeks
$\therefore$ So, you were at summer camp for 4 weeks.
b. How can you collect these data? What are the units?
$\therefore$ You can collect these data with a thermometer. The units are degrees Fahrenheit ( ${ }^{\circ} \mathrm{F}$ ).
c. Write a statistical question that you can answer using the dot plot. Then answer the question.


One possible statistical question is:

## What is the daily high temperature in August?

$\therefore$ The high temperatures are spread out with about half of the temperatures around $81^{\circ} \mathrm{F}$ and half of the temperatures around $86^{\circ}$.

## On Your Own


2. The dot plot shows the times of sixth grade students in a 100-meter race.

a. How many students ran in the race?
b. How can you collect these data? What are the units?
c. Write a statistical question that you can answer using the dot plot. Then answer the question.

## Vocabulary and Concept Check

1. VOCABULARY What is a statistical question? Give an example.
2. CRITICAL THINKING What process can you use to answer a statistical question?
3. NUMBER SENSE The results of a survey are shown in the table. Did the survey ask a statistical question? Explain.

| Miles |  |  |  |
| :---: | :---: | :---: | :---: |
| 6 | 1 | 9 | 2 |
| 2 | 5 | 4 | 9 |
| 8 | 10 | 6 | 6 |
| 5 | 1 | 8 | 1 |

## Practice and Problem Solving

Answer the question. Tell whether your answer would be the same as your classmates'.
4. How many inches are in 1 foot?
6. On what day of the month were you born?
5. How many pets do you have?
7. How many senators are in Congress?

## Determine whether the question is a statistical question. Explain.

8. What is the eye color of sixth grade students?
9. At what temperature (in degrees Fahrenheit) does water freeze?
10. How many pages are in the favorite books of students your age?
11. How many hours do sixth grade students use the Internet each week?

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
12.

| Number of Fouls |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1 | 2 | 0 | 0 | 2 |
| 2 | 1 | 6 | 1 | 1 | 0 |

13. 

| Camper Registrations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 21 | 25 | 25 | 22 | 21 |
| 23 | 24 | 26 | 25 | 16 |
| 24 | 26 | 22 | 25 | 22 |

14. 

| Years |  |  |  |
| :--- | :--- | :--- | :--- |
| 2011 | 2008 | 2013 | 2009 |
| 2009 | 2010 | 2010 | 2009 |
| 2010 | 2012 | 2009 | 2010 |

15. 

| Test Scores |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 85 | 80 | 83 | 90 | 88 |
| 82 | 83 | 81 | 80 | 89 |
| 89 | 84 | 86 | 87 | 83 |

16. SURVEY You conduct a survey to answer: "How many

| $\|$Hours of <br> Homework    <br> 2 4 3 2 <br> 1 2 2 1 <br> 2 3 5 2 |
| :--- | hours does a sixth grade student spend on homework during a school night?" The table shows the results.

a. Is this a statistical question? Explain.
b. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
c. Use the distribution of the data to answer the question.
17. EARTHWORMS The dot plot shows the lengths of earthworms.

a. How many earthworms does it represent?
b. How can you collect these data? What are the units?
c. Write a statistical question that you can answer using the dot plot. Then answer the question.
18. BASKETBALL The vertical dot plot shows the heights of the players on a recent NBA championship team.
a. How many players were on the team?
b. How can you collect these data? What are the units?
c. Write a statistical question that you can answer using the dot plot. Then answer the question.


Use the Internet to research and identify the method of measurement and the units used when collecting data about the topic.
19. wind speed
20. amount of rainfall
21. earthquake intensity

The dot plot shows the speeds of cars in a traffic study. Estimate the speed limit. Explain your reasoning.
22.

23.

24. REASONING "How many letters are in the English alphabet?" is not a statistical question. Write a question about letters that is a statistical question. Explain your reasoning.
 sense to describe the distribution of these data? Explain.

## Fair Game Review what you learned in previous grades \& lessons

Tell whether the ordered pair is a solution of the equation. (Section 7.4)
26. $y=4 x ;(2,8)$
27. $y=3 x+5$; $(3,15)$
28. $y=6 x-15 ;(4,9)$
29. MULTIPLE CHOICE A point is reflected in the $x$-axis. The reflected point is $(4,-3)$. What is the original point? (Section 6.5)
(A) $(-3,4)$
(B) $(-4,3)$
(C) $(-4,-3)$
(D) $(4,3)$

## ESSentilad Question How can you find an average value of a

 data set?1. ACTIVIIVY: Finding a Balance Point

Work with a partner. Discuss the distribution of the data. Where on the number line do you think the data set is balanced? Is this a good representation of the average? Explain.
a. number of quarters brought to a batting cage

b. annual income of recent graduates (in thousands of dollars)


## 2 ACIJV/JY: Finding a fair share

## Statistics

In this lesson, you will

- understand the concept of the mean of data sets.
- find the mean of data sets.
- compare and interpret the means of data sets.

Work with a partner. It costs $\$ 0.25$ to hit 12 baseballs in a batting cage. The table shows the numbers of quarters six friends bring to the batting cage. They want to group the quarters so that everyone has the same amount.

| Quarters |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| John | Lisa | Miguel | Matt | Cheryl | Jean |
| 6 | 3 | 4 | 5 | 2 | 4 |

Use counters to represent each number in the table. How can you use the counters to determine how many times each friend can use the batting cage? Explain how this procedure results in a "fair share."

## 3 ACJIVIJY: Finding an Average

Work with a partner. Use the information in Activity 2.
a. What is the total number of quarters the group of friends brought to the batting cage?
b. REASONING How can you use math to find the average number of quarters that each friend brought to the batting cage? Find the average number of quarters. Why do you think this average represents a fair share?

## 4 ACJIVIJY: Answering a Statistical Question

Work with a partner. The table shows the numbers of quarters several people bring to a batting cage. You want to answer the question:

## Math Practice

## Use Clear Definitions

What does it mean for data to have an average? How does this help you answer the question?
"How many quarters do people bring to a batting cage?"
a. Explain why this question is a statistical question.
b. MODELING Make a dot plot of the data. Use the distribution of the data to answer the question. Explain your reasoning.
c. REASONING Use an average to answer the question. Explain your reasoning.


## What Is Your Answer?

5. IN YOUR OWN WORDS How can you find an average value of a data set?
6. Give two real-life examples of averages.
7. Explain what it means to say the average of a data set is the point on a number line where the data set is balanced. 96 inches tall.
8. There are 5 students in the cartoon. Four of the students are 66 inches tall. One is
a. How do you think the students decided that their average height is 6 feet?
b. Does a height of 6 feet seem like a good representation of the average height of the 5 students? Explain why or why not.


Use what you learned about averages to complete Exercises 4 and 5 on page 400 .

## Key Vocabulary

 mean, p. 398outlier, p. 399

A mean is a type of average.

## GO Key Idea

## Mean

Words The mean of a data set is the sum of the data divided by the number of data values.
Numbers Data: $\underbrace{8,5,6,9}_{\hookrightarrow 4 \text { data values }}$ Mean: $\frac{8+5+6+9}{4}=\frac{28}{4}=7$

## EXAMPLE (1) Finding the Mean

## Text Messages Sent

Mark: 120
Laura: 95
Stacy: 101
Josh: 125
Kevin: 82
Maria: 108
Manny: 90

The table shows the number of text messages sent by a group of friends over 1 week. What is the mean number of messages sent?
(A) 100
(B) 102
(C) 103
(D) 104

$$
\begin{aligned}
& \text { mean }=\frac{120+95+101+125+82+108+90}{7} \longleftarrow \\
&=\frac{721}{7} \text {, or } 103 \quad \text { Sum of the data } \\
& \text { Number of values }
\end{aligned}
$$

$\therefore$ The mean number of text messages sent is 103 . The correct answer is (C).

## EXAMPLE 2 Comparing Means

The double bar graph shows the monthly rainfall amounts for two cities over a six-month period. Compare the mean monthly rainfalls.


City 1 mean: $\frac{3.5+2.2+1.9+2.1+2.5+3.4}{6}=\frac{15.6}{6}$, or 2.6
City 2 mean: $\frac{1.7+1.6+2.2+2.1+2.7+1.7}{6}=\frac{12}{6}$, or 2
$\therefore$ - Because 2.6 is greater than 2, City 1 averaged more rainfall.

On Your Own

## Find the mean of the data.

1. $49,62,52,54,61,70,55,53$
2. $7.2,8.5,7.0,8.1,6.7$

An outlier is a data value that is much greater or much less than the other values. When included in a data set, it can affect the mean.

## EXAMPLE (3) Finding the Mean With and Without an Outlier

| Shetland Pony Heights (inches) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 40 | 37 | 39 | 40 | 42 |
| 38 | 38 | 37 | 28 | 40 |

The table shows the heights of several Shetland ponies.
a. Identify the outlier.
b. Find the mean with and without the outlier.
c. Describe how the outlier affects the mean.
a. Display the data in a dot plot.


The height of 28 inches is much less than the other heights.
So, it is an outlier.
b. Mean with outlier:

$$
\frac{40+37+39+40+42+38+38+37+28+40}{10}=\frac{379}{10}, \text { or } 37.9
$$

Mean without outlier:

$$
\frac{40+37+39+40+42+38+38+37+40}{9}=\frac{351}{9}, \text { or } 39
$$

c. With the outlier, the mean is less than all but three of the heights. Without the outlier, the mean better represents the heights.

## On Your Own

Now You're Ready
Exercise 14

For each data set, identify the outlier. Then describe how it affects the mean.
3. Weights (in pounds) of dogs at a kennel:
$48,50,55,60,8,37,50$
4. Prices for flights from Miami, Florida, to San Juan, Puerto Rico:
\$456, \$512, \$516, \$900, \$436, \$516

## Vocabulary and Concept Check

1. VOCABULARY Arrange the words to explain how to find a mean.
the data values divide by the number of data values add then
2. NUMBER SENSE Is the mean always equal to a value in the data set? Explain.
3. REASONING Can you use the mean to answer a statistical question? Explain.

## Practice and Problem Solving

## Describe an average value of the data.

4. Ages in a class: $11,12,12,12,12,12,13$

## Find the mean of the data.

6. 

| Pets Owned |  |
| :--- | :---: |
| Brandon | I |
| Jill | III |
| Mark | II |
| Nicole | IIII |
| Steve | 0 |

8. 

| Sit-ups |  |  |
| :---: | :---: | :---: |
| 108 | 85 | 94 |
| 103 | 112 | 115 |
| 98 | 119 | 126 |
| 105 | 82 | 89 |

7. 

| Brothers and Sisters |  |
| :--- | :---: |
| Amanda | 옷 |
| Eve | 옷옻옷옷옷 |
| Joseph | 옷옷옷옷 |
| Michael | 옷옷 |

5. Movies seen this week: $0,0,0,1,1,2,3$
6. 


10. GOLF The table shows tournament finishes for a golfer.
a. What was the golfer's mean finish?

| Tournament Finishes |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 2 | 1 | 1 | 12 | 6 | 2 |
| 15 | 37 | 1 | 2 | 1 | 26 | 9 | 1 |

b. Identify two outliers for the data.
11. COMMERCIALS You and your friends are watching a television show. One of your friends asks, "How long are the commercial breaks during this show?"
a. Is this a statistical question? Explain.
b. Use the mean of the values in the table to answer the question.

| Month | Rainfall <br> (inches) | Month | Rainfall <br> (inches) |
| :---: | :---: | :---: | :---: |
| Jan | 2.22 | Jul | 3.27 |
| Feb | 1.51 | Aug | 5.40 |
| Mar | 1.86 | Sep | 5.45 |
| Apr | 2.06 | Oct | 4.34 |
| May | 3.48 | Nov | 2.64 |
| Jun | 4.57 | Dec | 2.14 |

12. RAINFALL The table shows the monthly rainfall at a measuring station. What is the mean monthly rainfall?
13. OPEN-ENDED Create two different sets of data that have six values and a mean of 21 .
(3) 14. CELL PHONE The bar graph shows your cell phone usage for five months.
a. Which data value is an outlier? Explain.
b. Find the mean with and without the outlier. Then describe how the outlier affects the mean.
c. Describe a situation that could have caused the outlier in this problem.

Cell Phone Usage

15. HEIGHT The table shows the heights of the volleyball players from two schools. What is the difference between the mean heights of the two teams? Do outliers affect either mean? Explain.

|  | Player Height (inches) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dolphins | 59 | 65 | 53 | 56 | 58 | 61 | 64 | 68 | 51 | 56 | 54 | 57 |  |  |  |
| Tigers | 63 | 68 | 66 | 58 | 54 | 55 | 61 | 62 | 53 | 70 | 64 | 64 | 62 | 67 | 69 |

16. REASONING Make a dot plot of the data set $11,13,17,15,12,18$, and 12 . Use the dot plot to explain how the mean is the point where the data set is balanced.
17. ALLOWANCE In your class, 7 students do not receive a weekly allowance, 5 students receive $\$ 3,7$ students receive $\$ 5,3$ students receive $\$ 6$, and 2 students receive $\$ 8$. What is the mean weekly allowance? Explain how you found your answer.
18. 3 recision A collection of 8 backpacks has a mean weight of 14 pounds. A different collection of 12 backpacks has a mean weight of 9 pounds. What is the mean weight of the 20 backpacks? Explain how you found your answer.

## Fair Game Review what you learned in previous grades \& lessons

Evaluate the expression. (Section 1.3)
19. $\frac{8+10}{2}$
20. $\frac{26+34}{2}$
21. $\frac{18+19}{2}$
22. $\frac{14+17}{2}$
23. MULTIPLE CHOICE $60 \%$ of what number is 105? (Section 5.6)
(A)
63
(B) 175
(C) 630
(D) 1750

## 9.3 Measures of Center

## ESSential Question in what other ways can you describe an average of a data set?

## (1) ACTIVIIY: Finding a Median

## Work with a partner.

a. Write the total number of letters in the first and last names of 19 celebrities, historical figures, or people you know. Organize your data in a table. One person is already listed for you.

b. Order the values in your data set from least to greatest. Then write the data on a strip of grid
 paper with 19 boxes.

c. Place a finger on the square at each end of the strip. Move your fingers toward the center of the ordered data set until your fingers touch. On what value do your fingers touch?

## Statistics

In this lesson, you will

- understand the concept of measures of center.
- find the median and mode of data sets.

d. Now take your strip of grid paper and fold it in half. On what number is the crease? What do you notice? This value is called the median. How would you describe to another student what the median of a data set represents?
e. How many values are greater than the median? How many are less than the median?
f. Why do you think the median is considered an average of a data set?


## 2 ACIIVIJY: Adding a Value to a Data Set

## Work with a partner.

a. How many total letters are in your first name and last name? Add this value to the ordered data set in Activity 1. How many values are now in your data set?
b. Write the ordered data, including your new value from part (a), on a strip of grid paper.

 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

c. Repeat parts (c) and (d) from Activity 1. Explain your findings. How do you think you can find the median of this data set?
d. Compare the medians in Activities 1 and 2. Then answer the following questions. Explain your reasoning.

- Do you think the median always has to be a value in the data set?
- Do you think the median always has to be a whole number?


## 3 ACIIVIIY: Finding a Mode

## Work with a partner.

## Math <br> Practice

Use a Graph
How can you use the dot plot to find the mode?
a. Make a dot plot for the data set in Activity 2. Describe the distribution of the data.
b. Which value occurs most often in the data set? This value is called the mode.
c. Do you think a data set can have no mode or more than one mode? Explain.
d. Do you think the mode always has to be a value in the data set? Explain.
e. Why do you think the mode is considered an average of a data set?

## What is Your Answer?

4. IN YOUR OWN WORDS In what other ways can you describe an average of a data set?
5. Find the mean of your data set in Activity 2 . Then compare the mean, median, and mode. Is there one measure that you think best represents your data set? Explain your reasoning.

## Practice

Use what you learned about the median of a data set to complete Exercises 5 and 6 on page 407.

## Key Vocabulary (i)

measure of center, p. 404<br>median, p. 404<br>mode, p. 404

A measure of center is a measure that describes the typical value of a data set. The mean is one type of measure of center. Here are two others.

## Study Tip

The mode is the only measure of center that you can use to describe a set of data that is not made up of numbers.

## 50 Key Ideas

## Median

Words Order the data. For a set with an odd number of values, the median is the middle value. For a set with an even number of values, the median is the mean of the two middle values.
Numbers Data: 5, 8, 9, 12, 14 The median is 9 .
Data: 2, 3, 5, 7, 10, 11


## Mode

Words The mode of a data set is the value or values that occur most often. Data can have one mode, more than one mode, or no mode. When all values occur only once, there is no mode.

Numbers Data: 11, 13, 15, 15, 18, 21, 24, 24


## EXAMPLE (1) Finding the Median and Mode

## Bowling Scores

| 120 | 135 | 160 | 125 | 90 |
| :--- | :--- | :--- | :--- | :--- |


| 205 | 160 | 175 | 105 | 145 |
| :--- | :--- | :--- | :--- | :--- |

Find the median and mode of the bowling scores.
$90,105,120,125,135,145,160,160,175,205$ Order the data.

Median: $\frac{135+145}{2}=\frac{280}{2}$, or $140 \quad$ Add the two middle values and divide by 2 .
Mode: 90, 105, 120, 125, 135, 145, 160, 160, 175, 205

## The value 160 occurs most often.

$\therefore$ The median is 140 . The mode is 160 .

## On Your Own

Find the median and mode of the data.

1. $20,4,17,8,12,9,5,20,13$
2. $100,75,90,80,110,102$

EXAMPLE
2 Finding the Mode
The list shows the favorite types of movies for students in a class. Organize the data in a frequency table. Then find the mode.


Comedy received the most votes.
$\therefore$ So, the mode is comedy.

## On Your Own

Now You're Ready
Exercises 14-15
3. One member of the class was absent and ends up voting for horror. Does this change the mode? Explain.

EXAMPLE


## 3 Choosing the Best Measure of Center

Find the mean, median, and mode of the sneaker prices. Which measure best represents the data?

Mean: $\frac{20+31+122+48+37+20+45+65}{8}=\frac{388}{8}$, or 48.5
Median: 20, 20, 31, 37, 45, 48, 65, 122
Order from least to greatest.

$$
\frac{37+45}{2}=\frac{82}{2}, \text { or } 41
$$

Mode: 20, 20, 31, 37, 45, 48, 65, 122
The value 20 occurs most often.

$\because$ The median best represents the data. The mode is less than most of the data, and the mean is greater than most of the data.

## On Your Own



Find the mean, median, and mode of the data. Choose the measure that best represents the data. Explain your reasoning.
4. $1,93,46,48,34,194,67,55$
5. $96,150,102,87,150,75$

## EXAMPLE (4) Removing an Outlier

Identify the outlier in Example 3. Find the mean, median, and mode without the outlier. Which measure does the outlier affect the most?
The price of $\$ 122$ is much greater than any other price. So, it is the outlier.

|  | Mean | Median | Mode |
| :--- | :---: | :---: | :---: |
| With Outlier (Example 3) | 48.5 | 41 | 20 |
| Without Outlier | 38 | 37 | 20 |

$\therefore$ The mean is affected the most by the outlier.

## On Your Own

Now You're Ready Exercises 21-22
6. The times (in minutes) it takes six students to travel to school are $8,10,10,15,20$, and 45 . Identify the outlier. Find the mean, median, and mode with and without the outlier. Which measure does the outlier affect the most?

## EXAMPLE 5 Changing the Values of a Data Set

The prices of six video games at an online store are shown in the table. The price of each game increases by $\$ 4.98$ when a shipping charge is included. How does this increase affect the mean, median, and mode?

Make a new table by adding $\$ 4.98$ to each price.

| Video Game <br> Prices |  |
| :---: | :---: |
| $\$ 53.42$ | $\$ 35.69$ |
| $\$ 18.99$ | $\$ 25.13$ |
| $\$ 27.97$ | $\$ 53.42$ | Then find the mean, median, and mode of both data sets.


| Video Game <br> Prices with |  |
| :---: | :---: |
| Shipping Charge |  |
| $\$ 58.40$ | $\$ 40.67$ |
| $\$ 23.97$ | $\$ 30.11$ |
| $\$ 32.95$ | $\$ 58.40$ |


|  | Mean | Median | Mode |
| :--- | :---: | :---: | :---: |
| Original Price | 35.77 | 31.83 | 53.42 |
| Price with <br> Shipping Charge | 40.75 | 36.81 | 58.4 |

Compare:
Mean: $40.75-35.77=4.98$
Median: $36.81-31.83=4.98$
Mode: $58.4-53.42=4.98$
$\therefore$ By increasing each video game price by $\$ 4.98$ for shipping, the mean, median, and mode all increase by $\$ 4.98$.

## On Your Own

7. WHAT IF? The store decreases the price of each video game by $\$ 3$. How does this decrease affect the mean, median, and mode?

## Vocabulary and Concept Check

1. NUMBER SENSE Give an example of a data set that has no mode.
2. WRITING Which is affected most by an outlier: the mean, median, or mode? Explain.
3. WHICH ONE DOESN'T BELONG Which word does not belong with the other three? Explain.
median outlier mode mean
4. NUMBER SENSE A data set has a mean of 7 , a median of 5 , and a mode of 8 . Which of the numbers 7,5 , and 8 must be in the data set? Explain.

## Practice and Problem Solving

## Use grid paper to find the median of the data.

5. $9,7,2,4,3,5,9,6,8,0,3,8$
6. $16,24,13,36,22,26,22,28,25$

Find the median and mode(s) of the data.
7. $3,5,7,9,11,3,8$
8. $14,19,16,13,16,14$
9. $93,81,94,71,89,92,94,99$
10. $44,13,36,52,19,27,33$
11. $12,33,18,28,29,12,17,4,2$
12. $55,44,40,55,48,44,58,67$
13. ERROR ANALYSIS Describe and correct the error in finding the median of the data.


The median is 58 .
$63,55,49,58,50,59,51$

## Find the mode(s) of the data.

(2) 14.

| Shirt Color |  |  |
| :---: | :---: | :---: |
| Black | Blue | Red |
| Pink | Black | Black |
| Gray | Green | Blue |
| Blue | Blue | Red |
| Yellow | Blue | Blue |
| Black | Orange | Black |
| Black |  |  |

15. 

Talent Show Acts

| Singing | Dancing | Comedy |
| :---: | :---: | :---: |
| Singing | Singing | Dancing |
| Juggling | Dancing | Singing |
| Singing | Poetry | Dancing |
| Comedy | Magic | Dancing |
| Poetry | Singing | Singing |

16. REASONING In Exercises 14 and 15, can you find the mean and median of the data? Explain.

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning.
(3)
17. $48,12,11,45,48,48,43,32$
18. $12,13,40,95,88,7,95$
19. $2,8,10,12,56,9,5,2,4$
20. 126, 62, 144, 81, 144, 103

Find the mean, median, and mode(s) of the data with and without the outlier. Describe the effect of the outlier on the measures of center.
21. $45,52,17,63,57,42,54,58$
22. $85,77,211,88,91,84,85$

Find the mean, median, and mode(s) of the data.
23. 4.7, 8.51, 6.5, 7.42, 9.64, 7.2, 9.3
24. $8 \frac{1}{2}, 6 \frac{5}{8}, 3 \frac{1}{8}, 5 \frac{3}{4}, 6 \frac{5}{8}, 5 \frac{1}{4}, 10 \frac{5}{8}, 4 \frac{1}{2}$
25. WEATHER The weather forecast for a week is shown.

|  | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ¢ை |  | B | స్ర్ర్హ్ర్ | [ | 囫 |  |
| High | $90^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ | $89^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $101^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| Low | $74^{\circ} \mathrm{F}$ | $78^{\circ} \mathrm{F}$ | $77^{\circ} \mathrm{F}$ | $77^{\circ} \mathrm{F}$ | $83^{\circ} \mathrm{F}$ | $78^{\circ} \mathrm{F}$ | $72^{\circ} \mathrm{F}$ |

a. Find the mean, median, and mode(s) of the high temperatures. Which measure best represents the data? Explain your reasoning.
b. Repeat part (a) for the low temperatures.
26. RESEARCH Find the unit costs of 10 different kinds of cereal. Choose one cereal whose unit cost will be an outlier.
a. Find the mean, median, and mode(s) of the data. Which measure best represents the data? Explain your reasoning.
b. Identify the outlier in the data set. Find the mean, median, and mode(s) of the data set without the outlier. Which measure does the outlier affect the most?

27. PROBLEM SOLVING The bar graph shows the numbers of hours you volunteered at an animal shelter. What is the minimum number of hours you need to work in the seventh week to justify that you worked an average of 10 hours for the 7 weeks? Explain your answer using measures of center.
28. REASONING Why do you think the mode is the least frequently used measure to describe a data set? Explain.
29. MOTOCROSS The ages of the racers in a bicycle motocross race are $14,22,20,25,26,17,21,30,27,25,14$, and 29 . The 30 -year-old drops out of the race and is replaced with a 15 -year-old. How are the mean, median, and mode of the ages affected?

30. CAMERAS The data are the prices of several digital cameras at a store.


| $\$ 130$ | $\$ 170$ | $\$ 230$ | $\$ 130$ |
| :--- | :--- | :--- | :--- |
| $\$ 250$ | $\$ 275$ | $\$ 130$ | $\$ 185$ |

a. Does the price shown in the advertisement represent the prices well? Explain.
b. Why might the store use this advertisement?
c. In this situation, why might a person want to know the mean? the median? the mode? Explain.
31. SALARIES The table shows the monthly salaries for employees at a company.
a. Find the mean, median, and mode of the data.
b. Each employee receives a $5 \%$ raise. Find the mean, median, and mode of the data with the raise. How does this increase affect the mean, median, and mode of the data?
c. Use the original monthly salaries to calculate the annual salaries. Find the mean, median, and mode of the annual salaries. How are these values related to the mean, median, and mode of the monthly salaries?
32. Thrink Assume $x>0$.
a. Find the mean, median, and mode.
b. Is there an outlier? If so, what is it?

## Fair Game Review what you learned in previous grades \& lessons

Find the value of the expression. (Section 1.1)
33. 48 - 35
34. $188-123$
35. $416-297$
36. $6249-3374$
37. MULTIPLE CHOICE A shelf in your room can hold at most 30 pounds. There are 12 pounds of books already on it. Which inequality represents the number of pounds you can add to the shelf? (Section 7.6)
(A) $x<18$
(B) $x \geq 18$
(C) $x \leq 42$
(D) $x \leq 18$

You can use a concept circle to organize information about a concept. Here is an example of a concept circle for a statistical question.

## Statistical Question



## On Your Own

Make concept circles to help you study these topics.

1. mean
2. measures of center
3. mode

After you complete this chapter, make concept circles for the following topics.
6. measures of variation
7. range
8. quartiles
9. interquartile range
10. mean absolute deviation

Display the data in a dot plot．Identify any clusters，peaks，or gaps in the data．（Section 9．1）
1.

| Weight（grams） |  |  |  |
| :---: | :---: | :---: | :---: |
| 42 | 40 | 37 | 42 |
| 43 | 41 | 42 | 43 |
| 37 | 41 | 41 | 42 |

2. 

| Time（seconds） |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 63 | 66 | 65 | 60 | 59 |
| 59 | 64 | 58 | 65 | 58 |
| 64 | 60 | 59 | 64 | 63 |

Find the mean of the data．（Section 9．2）

| Tour Dates |  |
| :---: | :---: |
| May | III |
| June | \＃\＃肘 |
| July | \＃\＃\＃\＃IIII |
| August | 册 册 \＃\＃III |
| September | \＃ |

4. 

| Scores |  |
| :---: | :--- |
| Judge 1 | 8.9 |
| Judge 2 | 9.4 |
| Judge 3 | 8.6 |
| Judge 4 | 9.1 |

Find the median and the mode（s）of the data．（Section 9．3）
5． $3,5,9,11,3$
6． $24,4,37,56,6,56,45$

Find the mean，median，and mode（s）of the data．Choose the measure that best represents the data．Explain your reasoning．（Section 9．3）

7． $47,147,24,47,38,42$
8． $34,57,58,56,21$

| Hours of Exercise |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 5 | 1 | 5 | 3 | 5 |
| 4 | 5 | 2 | 5 | 4 |
| 3 | 4 | 6 | 5 | 6 |

9．EXERCISE You conduct a survey to answer：＂How many hours does a sixth－grade student spend exercising during a week？＂The table shows the results．（Section 9．1）
a．Is this a statistical question？Explain．
b．Display the data in a dot plot．Identify any clusters，peaks，or gaps in the data．
c．Use the distribution of the data to answer the question．
10．EMAILS The number of emails you received in 5 days is shown．What is the mean number of emails you received per day？（Section 9．2）

11．QUIZZES The data are your quiz scores for a class． Find the median and the mode of the data．（Section 9．3）

$$
18,19,17,14,20,20,15,21
$$

12．MUSIC The data are the lengths of the songs（in minutes）on your new CD．Which measure of center best represents the data with and without the outlier？Explain．（Section 9．3）

$$
2.2,2.2,2.4,2.6,2.8,3.0,3.2,3.4,14.2
$$

### 9.4 Measures of Variation

## ESSentilad Question How can you describe the spread of a

 data set?
## ACJIVIIY: Interpreting Statements

Work with a partner. There are 24 students in your class. Your teacher makes the following statements:

- "The exam scores range from 75\% to 96\%."
- "Most of the students received high scores."

a. What do you think the first statement means? Explain.
b. In the first statement, is your teacher describing the center of the data set? If not, what do you think your teacher is describing?
c. What do you think the scores are for most of the students in the class? Explain your reasoning.
d. Use your teacher's statements to make a dot plot that can represent the distribution of the exam scores of the class.

2 ACTIVITY: Grouping Data

## Statistics

In this lesson, you will

- find the range of data sets.
- find the interquartile range of data sets.
- check for outliers in data sets.

Work with a partner. The numbers of U.S. states visited by each student in a sixth grade class are shown.
a. Between what values do the data range?
b. Write the ordered data values on a strip of grid paper and fold it to find the median. How many values are greater than the median? How many are less than the median?
c. REPEATED REASONING Fold the strip in half again. On what values are the two new creases? What do you think these values represent?
d. Into how many parts did you divide the data set? How many data values are in each part?
e. Graph the median and the values you found in parts (a) and (c) on a number line. Are the distances the same between these points?
f. How can you use these values to describe the spread of the data?

## 3 ACJIVIJY: Adding a Value to a Data Set

Work with a partner. A new student joins the class in Activity 2. The new student has visited 41 states.
a. Add this value to the ordered data set in Activity 2. Does your answer to part (a) change? Explain.
b. How does the distribution of the data change when this value is added? Explain your reasoning.
c. How does adding this value affect the values on your number line in part (e) of Activity 2?

## 4 ACTIVITY: Analyzing Data Sets

Work with a partner. Identify the data set that is the least spread out and the data set that is the most spread out. Explain your reasoning.
a.

b.

c.

d.


## What Is Your Answer?

5. IN YOUR OWN WORDS How can you describe the spread of a data set?
6. Make a dot plot of the data set in Activity 2. Describe any similarities between the dot plot and the number line in part (e).

A measure of variation is a measure that describes the distribution of a data set. A simple measure of variation to find is the range. The range of a data set is the difference between the greatest value and the least value.

## EXAMPLE (1) Finding the Range

## Key Vocabulary

measure of variation,

$$
\text { p. } 414
$$

range, p. 414
quartiles, p. 414
first quartile, p. 414
third quartile, p. 414
interquartile range, p. 414

The table shows the lengths of several Burmese pythons captured for a study. Find and interpret the range of their lengths.
To find the least and the greatest values, order the lengths from least to greatest.

$$
5,6.25,8,10,11,12.5,14,15.5,16.25,18.5
$$

| Lengths (feet) |  |
| :---: | :---: |
| 18.5 | 8 |
| 11 | 10 |
| 14 | 15.5 |
| 12.5 | 6.25 |
| 16.25 | 5 |

The least value is 5 . The greatest value is 18.5 .
$\because$ So, the range of the lengths is $18.5-5$, or 13.5 feet. This means that the lengths vary by no more than 13.5 feet.

## On Your Own

Now You're Ready
Exercises 6-9

1. The ages of people in line for a roller coaster are $15,17,21,32,41,30$, $25,52,16,39,11$, and 24 . Find and interpret the range of their ages.

## ©0 Key Ideas

## Quartiles

The quartiles of a data set divide the data into four equal parts. Recall that the median (second quartile) divides the data set into two halves.


## Interquartile Range (IQR)

The difference between the third quartile and the first quartile is called the interquartile range. The IQR represents the range of the middle half of the data and is another measure of variation.


The dot plot shows the top speeds of 12 sports cars. Find and interpret the interquartile range of the data.


Order the speeds from slowest to fastest. Find the quartiles.

$\therefore$ So, the interquartile range is $255-235=20$. This means that the middle half of the speeds vary by no more than 20 miles per hour.

You can use the quartiles and the interquartile range to check for outliers.


## EXAMPLE 3 Checking for Outliers

Check for outliers in the data set in Example 2.

$$
\begin{array}{ccc}
Q_{1}-1.5(\mathrm{IQR}) & \text { Outlier boundaries } & Q_{3}+1.5(\mathrm{IQR}) \\
235-1.5(20) & \text { Substitute values. } & 255+1.5(20) \\
205 & \text { Simplify. } & 285
\end{array}
$$

$\therefore$ There are no speeds less than 205 miles per hour or greater than 285 miles per hour. So, the data set has no outliers.

## On Your Own

Now You're Ready
Exercises 11-14 and 17
2. The number of pages in each of an author's novels is shown.

$$
356,364,390,468,400,382,376,396,350
$$

a. Find and interpret the interquartile range of the data.
b. Does this data set contain any outliers? Justify your answer.

## Vocabulary and Concept Check

1. VOCABULARY How are measures of center different from measures of variation?
2. VOCABULARY How many quartiles does a data set have?
3. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

$$
53,47,60,45,62,59,65,50,56,48
$$

What is the interquartile range of the data?

What is the range of the middle half of the data?

What is the range of the data?

What is the difference between the third quartile and the first quartile?

## Practice and Problem Solving

Use grid paper to find the median of the data. Then find the median of the lower half and the median of the upper half of the data. Describe the spread of the data.
4. $5,8,10,1,7,6,15,8,6$
5. $82,62,95,81,89,51,72,56,97,98,79,85$

Find the range of the data.
6. $26,21,27,33,24,29$
7. $52,40,49,48,62,54,44,58,39$
8. $133,117,152,127,168,146,174$
9. $4.8,5.5,4.2,8.9,3.4,7.5,1.6,3.8$
10. ERROR ANALYSIS Describe and correct the error in finding the range of the data.


Find the median, first quartile, third quartile, and interquartile range of the data.
11. $40,33,37,54,41,34,27,39,35$
13. $132,127,106,140,158,135,129,138$
15. PAPER AIRPLANE The table shows the distances traveled by a paper airplane. Find and interpret the range and the interquartile range of the distances.
12. $84,75,90,87,99,91,85,88,76,92,94$
14. $38,55,61,56,46,67,59,75,65,58$

| Distances (feet) |  |  |  |
| :---: | :---: | :---: | :---: |
| $13 \frac{1}{2}$ | $21 \frac{1}{2}$ | 21 | $16 \frac{3}{4}$ |
| $10 \frac{1}{4}$ | 19 | 32 | $26 \frac{1}{2}$ |
| 29 | $16 \frac{1}{4}$ | $28 \frac{1}{2}$ | $18 \frac{1}{2}$ |

16. WRITING Consider a data set that has no mode. Which measure of variation is greater, the range or the interquartile range? Explain your reasoning.
(3) 17. OUTLIERS Use the interquartile range to identify any outliers in Exercises 11-14.
17. REASONING How does an outlier affect the range of a data set? Explain.
18. BASKETBALL The table shows the numbers of points scored by players on a basketball team.

| Points Scored |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 53 | 74 | 82 | 84 | 93 |
| 103 | 108 | 116 | 122 | 193 |  |

a. Find the range and the interquartile range of the data.
b. Use the interquartile range to identify the outlier(s) in the data set. Find the range and the interquartile range of the data set without the outlier(s). Which measure did the outlier(s) affect more?
20. STRUCTURE Two data sets have the same range. Can you assume that the interquartile ranges of the two data sets are about the same? Give an example to justify your answer.

21. SINGING The tables show the ages of the finalists for two reality singing competitions.
a. Find the mean, median, range, and interquartile range of the ages for each show. Compare the results.

| Ages for <br> Show |  | Ages for <br> Show |  |
| :---: | :---: | :---: | :---: |
| 18 | 17 | 21 | 20 |
| 15 | 21 | 23 | 13 |
| 22 | 16 | 15 | 18 |
| 18 | 28 | 17 | 22 |
| 24 | 21 | 36 | 25 |

22. Qpen-Ended Create a set of data with 7 values that has a mean of 30 , a median of 26 , a range of 50 , and an interquartile range of 36 .

## Fair Game Review what you learned in previous grades \& lessons

Find the mean of the data. (Section 9.2)
23. $8,14,22,7,2,11,25,7,5,9$
24. $55,64,58,43,49,67$
25. MULTIPLE CHOICE What is the surface area of the rectangular prism? (Section 8.2)
(A) $62 \mathrm{~m}^{2}$
(B) $72 \mathrm{~m}^{2}$
(C) $88 \mathrm{~m}^{2}$
(D) $124 \mathrm{~m}^{2}$


## 9.5 <br> Mean Absolute Deviation

ESSentiad Qusestion How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?

## The Meaning of a Word Deviate

When you deviate from something,

you stray or depart from the normal course of action.


## ACJIV/IY: Finding Distances From the Mean

Work with a partner. The table shows the exam scores of 14 students in your class.

| Exam Scores |  |  |  |
| :--- | :--- | :--- | :--- |
| Ben | 89 | Mike | 95 |
| Emma | 86 | Hong | 96 |
| Jeremy | 80 | Rob | 92 |
| Pete | 80 | Amy | 90 |
| Ryan | 96 | Sue | 76 |
| Dan | 94 | Kim | 84 |
| Lucy | 89 | Heather | 85 |

## Statistics

In this lesson, you will

- understand the meaning of mean absolute deviation.
- find the mean absolute deviation of data sets.
a. What is the mean exam score?
b. Make a dot plot of the data. Place an " X " on the number line to represent the mean.
c. Is the number of exam scores that are greater than the mean equal to the number of exam scores that are less than the mean? Explain.
d. Which exam score deviates the most from the mean? Which exam score deviates the least from the mean? Explain how you found your answers.
e. Overall, do you think the exam scores are close to the mean or far away from the mean? Explain your reasoning.


## 2 ACJIV/IJ: Using Distances from the Mean

## Work with a partner. Use the information in Activity 1.

a. Complete the table below. Add rows if needed. Be sure to find the sum of

## Math Practice

Use Operations
What operation can you use to find the distance from the mean? Explain. the values in the last column of the table.

| Student with Score <br> Less Than the Mean | Exam Score | Distance from <br> the Mean |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  | Sum: |  |

b. Create a table similar to the one above for students with scores greater than the mean.
c. LOGIC What do you notice about the sums you found in your tables? Why do you think this happens?

## 3 ACIIVIJY: Interpreting Distances from the Mean

## Work with a partner.

a. LOGIC Add the sums you found in your tables in Activity 2. Divide that amount by the total number of students. Round your result to the nearest tenth.
In your own words, what do you think this value represents?
b. REASONING In a data set, what do you think it means when the value you found in part (a) is close to 0 ? Explain.

## What Is Your Answer?

4. IN YOUR OWN WORDS How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?
5. REASONING Find the range and the interquartile range of the data set in Activity 1 . What do you think it means when these values are close to 0 ? Explain.

## Key Vocabulary

mean absolute
deviation, p. 420

Another measure of variation is the mean absolute deviation. The mean absolute deviation is an average of how much data values differ from the mean.

## © 50 Key Idea

## Finding the Mean Absolute Deviation (MAD)

Step 1: Find the mean of the data.
Step 2: Find the distance between each data value and the mean.
Step 3: Find the sum of the distances in Step 2.
Step 4: Divide the sum in Step 3 by the total number of data values.

## EXAMPLE Finding the Mean Absolute Deviation

You record the numbers of raisins in 8 scoops of cereal. Find and interpret the mean absolute deviation of the data.

$$
1,2,2,2,4,4,4,5
$$

Step 1: $\quad$ Mean $=\frac{1+2+2+2+4+4+4+5}{8}=\frac{24}{8}=3$
Step 2: You can use a dot plot to organize the data. Replace each dot with its distance from the mean.


Step 3: The sum of the distances is $2+1+1+1+1+1+1+2=10$.
Step 4: The mean absolute deviation is $\frac{10}{8}=1.25$.
$\therefore$ So, the data values differ from the mean by an average of 1.25 raisins.

## -

On Your Own

Now You're Ready
Exercises 5-8

1. Find and interpret the mean absolute deviation of the data.

$$
5,8,8,10,13,14,16,22
$$

The smartphones show the numbers of runs allowed
 by two pitchers in their last 10 starts.
a. Find the mean, median, and mean absolute deviation of the numbers of runs allowed for each pitcher.

Order the runs allowed for Mendoza:
$0,0,0,2,4,4,5,6,6,8$.
Mean $=\frac{35}{10}=3.5 \quad$ Median $=\frac{4+4}{2}=4$
Mean absolute deviation:


## Study Tip

The greater the mean absolute deviation, the greater the variation of the data.

The mean absolute deviation is $\frac{24}{10}=2.4$
Order the runs allowed for Rodriguez: 0, 2, 2, 3, 4, 4, 4, 5, 5, 6 .
Mean $=\frac{35}{10}=3.5$
Median $=\frac{4+4}{2}=4$
Mean absolute deviation:


The mean absolute deviation is $\frac{14}{10}=1.4$.
b. Which measure can you use to distinguish the data? What can you conclude about the pitchers from this measure?
You cannot use the measures of center to distinguish the data because they are the same for each data set. The measure of variation, MAD, is 2.4 for Mendoza and 1.4 for Rodriguez. This indicates that the data for Rodriguez has less variation.
$\therefore$ Using the MAD to distinguish the data, you can conclude that Rodriguez is more consistent than Mendoza.

## On Your Own

2. WHAT IF? Mendoza allows 4 runs in the next game. How would you expect the mean absolute deviation to change? Explain.

## Vocabulary and Concepł Check

1. REASONING Describe a data set that has a mean absolute deviation of 0 .
2. WHICH ONE DOESN'T BELONG? Which one does not belong with the other three? Explain your reasoning.
range interquartile range mean mean absolute deviation

## Practice and Problem Solving

Find the average distance each data value in the set is from the mean. Round your answer to the nearest tenth, if necessary.
3. Model years of used cars on a lot: 2010, 2002, 2005, 2007, 2001
4. Prices of kites at a shop: $\$ 7, \$ 20, \$ 9, \$ 35, \$ 12, \$ 15, \$ 7, \$ 10, \$ 20, \$ 25$

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth, if necessary.
5.

| Prices of Microphones <br> (dollars) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 25 | 28 | 20 | 22 | 32 |
| 28 | 35 | 34 | 30 | 36 |

7. 

Capacities of Stadiums
(thousands of people)

| 101.5 | 95.4 | 109.8 |
| :---: | :---: | :---: |
| 98.7 | 92.3 | 104.7 |

6. 

| Heights of <br> Octuplets (inches) |  |  |  |
| :---: | :---: | :---: | :---: |
| 61 | 61 | 61 | 61 |
| 61 | 61 | 61 | 61 |

8. 

| Numbers of Visitors to a <br> Website During a Week <br> 103$\| 115$ |  |  |  |
| :---: | :---: | :---: | :---: |
| 171 | 165 | 170 |  |

9. ERROR ANALYSIS Describe and correct the error in finding the mean absolute deviation of the data set $35,40,38,32,42$, and 41.

$$
\begin{aligned}
& \text { mean }=\frac{35+40+38+32+42+41}{6}=38 \\
& M A D=\frac{3+2+6+4+3}{5}=3.6 \\
& \text { So, the values differ from the mean by an } \\
& \text { average of } 3.6 \text {. }
\end{aligned}
$$

10. MUSEUMS The data set shows the admission prices at several museums.

$$
\$ 20, \$ 20, \$ 16, \$ 12, \$ 15, \$ 25, \$ 11
$$

Find and interpret the range, interquartile range, and mean absolute deviation of the data.

11. MENU The table shows the prices of the five most-expensive and least-expensive dishes on a menu. Find the MAD of each data set.

| Five Most-Expensive <br> Dishes |  |  |  | Five <br> Least-Expensive <br> Dishes |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\$ 28$ | $\$ 30$ | $\$ 28$ | $\$ 39$ | $\$ 25$ | $\$ 7$ | $\$ 7$ | $\$ 10$ | $\$ 8$ | $\$ 12$ | Then compare their variations.

12. COINS The data sets show the years of the coins in two collections. Derek's collection: 1950, 1952, 1908, 1902, 1955, 1954, 1901, 1910

Paul's collection: 1929, 1935, 1928, 1930, 1925, 1932, 1933, 1920
Find the measures of center and the measures of variation for each data set. Compare the measures. What can you conclude?

13. PROBLEM SOLVING You survey students in your class about the number of movies they watched last month. The results are shown in the table.

| Movies Watched |  |  |  |
| :---: | :---: | :---: | :---: |
| 7 | 5 | 14 | 5 |
| 6 | 9 | 10 | 12 |
| 15 | 4 | 5 | 8 |
| 11 | 10 | 9 | 2 |

a. Find the measures of center and the measures of variation for the data.
b. A new student joins the class who watched 21 movies last month. Is 21 an outlier? How does including this value affect the measures of center and the measures of variation? Explain.

REASONING Which data set do you think would have the greater mean absolute deviation? Explain your reasoning.
14. guesses for number of gumballs in a jar guesses for number of baseballs in a jar
15. monthly rainfall amounts in a city monthly amounts of water used in a home
16. REASONING The MAD of a data set is considered a more reliable measure of variation than the range or the interquartile range. Why do you think this is true?
17. Brippeal

Add and subtract the MAD from the mean in the original data set in Exercise 13.
a. What percent of the values are within one MAD of the mean? two MADs of the mean? Which values are more than twice the MAD from the mean?
b. What do you notice as you get more and more MADs away from the mean? Explain.

## Fair Game Review what you learned in previous grades \& lessons

Find the mean, median, and mode(s) of the data. (Section 9.2 and Section 9.3)
18. $4,6,7,9,6,4,5,6,8,10$
19. $1.2,1.7,1.7,2.1,1.4,1.2,1.9$
20. MULTIPLE CHOICE What is the surface area of the square pyramid? (Section 8.3)
(A) $100.8 \mathrm{yd}^{2}$
(B) $147.2 \mathrm{yd}^{2}$
(C) $211.2 \mathrm{yd}^{2}$
(D) $368 \mathrm{yd}^{2}$


Find the range of the data. (Section 9.4)

1. $35,76,43,58,34,67$
2. $19,21,22,22,19,25,24,23,24$

Find the median, first quartile, third quartile, and interquartile range of the data.
(Section 9.4)
3. $56,48,72,37,35,42,48,33,28$
4. $95,14,86,62,55,46,28,37,33,70,31$

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary. (Section 9.5)
5.

| Ages of Television Show <br> Viewers (years) |  |  |  |
| :---: | :---: | :---: | :---: |
| 29 | 18 | 26 | 33 |
| 33 | 22 | 34 | 26 |

6. 

| Prices of Houses <br> (thousands of dollars) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 80 | 120 | 95 | 240 | 140 |
| 75 | 135 | 110 | 90 | 125 |

7. AMUSEMENT PARKS The data set shows the admission prices at several amusement parks.

$$
\$ 65, \$ 70, \$ 40, \$ 55, \$ 35, \$ 40, \$ 60
$$

Find and interpret the range, interquartile range, and mean absolute deviation of the data. (Section 9.4 and Section 9.5)
8. TEACHING EXPERIENCE The tables show the years of teaching experience of faculty members at two schools. (Section 9.4)
a. Find the mean, median, range, and interquartile range of the years of experience for each school. Compare the results.
b. The teacher with 11 years of experience leaves School A, and the teacher with 33 years of experience retires from School B. How does

| School A: <br> Teaching <br> Experience <br> (years) | School B: <br> Teaching <br> Experience <br> (years) |  |  |
| :---: | :---: | :---: | :---: |
| 5 | 11 | 4 | 15 |
| 10 | 22 | 6 | 12 |
| 7 | 8 | 10 | 33 |
| 8 | 6 | 12 | 20 |
| 10 | 35 | 8 | 7 | this affect the measures in part (a)? Explain.

9. BOOK CLUB You survey the students in your book club about the number of books they read last summer. The results are shown in the table.
(Section 9.4 and Section 9.5)

| Books Read |  |  |  |
| :---: | :---: | :---: | :---: |
| 8 | 14 | 15 | 9 |
| 6 | 12 | 9 | 13 |
| 11 | 11 | 7 | 5 |
| 12 | 6 | 10 | 8 |

a. Find the measures of center and the measures of variation for the data.
b. A new student who read 18 books last summer joins the club. Is 18 an outlier? How does adding this value to the data set affect the measures of center and variation? Explain.

## Review Key Vocabulary

statistics, p. 392
statistical question, p. 392
mean, p. 398
outlier, p. 399
measure of center, p. 404
median, p. 404
mode, p. 404
measure of variation, p. 414
range, p. 414
quartiles, p. 414

Check If Oust
first quartile, p. 414
third quartile, p. 414
interquartile range, p. 414 mean absolute deviation, p. 420

## Review Examples and Exercises

### 9.1 Introduction to Statistics (pp. 390-395)

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.


| Heights (inches) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 70 | 71 | 70 | 72 | 69 |
| 68 | 69 | 71 | 64 | 70 |
| 64 | 63 | 72 | 70 | 67 |

## Exercises

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
1.

| Distance (feet) |  |  |  |
| :---: | :---: | :---: | :---: |
| 56 | 55 | 56 | 57 |
| 58 | 54 | 51 | 55 |
| 51 | 56 | 49 | 56 |

2. 

| Weight (pounds) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 83 | 88 | 89 | 90 | 89 |
| 91 | 89 | 84 | 90 | 92 |
| 90 | 88 | 89 | 83 | 88 |

### 9.2 Mean (pp. 396-401)

Find the mean of 5, 9, 10, 6, 6, and 12.

$$
\begin{array}{rlrl}
\text { mean } & =\frac{5+9+10+6+6+12}{6} \longleftarrow & \text { sum of the data } \\
& =\frac{48}{6} \text {, or } 8 & & \text { number of values } \\
\text { Simplify. }
\end{array}
$$

## Exercises

Find the mean of the data.
3. $4,5,7,14,17,12,18$
4. $15,5,8,12,5,9,4,10,2,11$

## Q.3 Measures of Center (pp. 402-409)

Find the median and the mode of the movie lengths in the table.
Order the data from least to greatest.

| Movie Lengths <br> (minutes) |  |  |
| :---: | :---: | :---: |
| 91 | 112 | 126 |
| 142 | 122 | 112 |
| 92 | 144 |  |

## Median:

91, 92, 112, 112, 122, 126, 142, 144

$$
\frac{112+122}{2}=\frac{234}{2}, \text { or } 117
$$

## Mode:

91, 92, 112, 112, 122, 126, 142, 144


The value 112 occurs most often.
$\therefore$ The median is 117 minutes, and the mode is 112 minutes.

## Exercises

Find the median and the mode(s) of the data.
5. $8,8,6,8,4,5,6$
6. $24,74,61,29,38,27,68,54$

## (9) 4 Measures of Variation (pp. 412-417)

The table shows the weights of several adult emperor penguins. (a) Find and interpret the range. (b) Find and interpret the interquartile range. (c) Check for outliers.
a. Ordered from least to greatest, the weights are 23.5 , $24,25,27,29.25,30.75,31.25,32,33.5$, and 36.
$\because$ So, the range of the weights is $36-23.5$, or 12.5 kilograms. The weights vary by no more than 12.5 kilograms.
b. Find the quartiles.

$\therefore$ So, the interquartile range is $32-25=7$. This means that the middle half of the weights vary by no more than 7 kilograms.
c. Calculate the outlier boundaries.

$$
\begin{aligned}
& Q_{1}-1.5(\mathrm{IQR})=25-1.5(7)=14.5 \\
& Q_{3}+1.5(\mathrm{IQR})=32+1.5(7)=42.5
\end{aligned}
$$

$\therefore$ - There are no weights less than 14.5 kilograms or greater than 42.5 kilograms. So, the data set has no outliers.

## Exercises

Find the range of the data.
7. $45,76,98,21,52,39$
8. $95,63,52,8,93,16,42,37,62$

Find the median, first quartile, third quartile, and interquartile range of the data.
9. $28,46,25,76,18,25,47,83,44$
10. $14,25,97,55,66,28,92,38,94$

### 9.5 Mean Absolute Deviation (pp. 418-423)

You record the prices of 8 printers. Find and interpret the mean absolute deviation of the data.
\$120, \$150, \$90, \$110,
\$140, \$120, \$140, \$90


Step 1: Mean $=\frac{120+150+90+110+140+120+140+90}{8}=\frac{960}{8}=120$
Step 2: Use a dot plot to organize the data. Replace each dot with its distance from the mean.


Step 3: The sum of the distances is $30+30+10+0+0+20+20+30=140$.
Step 4: The mean absolute deviation is $\frac{140}{8}=17.50$.
$\therefore$ The data values differ from the mean by an average of $\$ 17.50$.

## Exercises

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary.
11.

| Shoe Sizes |  |  |  |
| :---: | :---: | :---: | :---: |
| 6 | 8.5 | 6 | 9 |
| 10 | 7 | 8 | 9.5 |

12. 

| Prices of Monitors (dollars) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 130 | 150 | 190 | 100 | 175 |
| 120 | 165 | 140 | 180 | 190 |

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
1.

| Time (minutes) |  |  |  |
| :--- | :--- | :--- | :--- |
| 33 | 40 | 32 | 40 |
| 39 | 38 | 40 | 39 |
| 38 | 39 | 39 | 33 |

2. 

| Temperature ( $\left.{ }^{\circ} \mathrm{F}\right)$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 81 | 81 | 80 | 82 | 81 |
| 83 | 76 | 83 | 76 | 80 |
| 75 | 83 | 82 | 82 | 81 |

Find the mean, median, and mode(s) of the data.
3. $2,7,7,12,4$
4. $4,5,7,5,9,9,10,6$

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning.
5. $5,6,4,24,18$
6. $46,27,94,56,53,65,43$

Find the range of the data.
7. $24,56,9,83,77,14$
8. $43,12,55,91,25,86,84,23,1$

Find the median, first quartile, third quartile, and interquartile range of the data.
9. $32,58,19,36,44,57,11,26,74$
10. $36,24,49,32,37,28,38,40,39$

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary.
11.

| Distances Driven (miles) |  |  |  |
| :---: | :---: | :---: | :---: |
| 312 | 286 | 196 | 201 |
| 158 | 225 | 206 | 192 |

12. 

| Prices of Sunglasses <br> (dollars) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 15 | 8 | 19 | 20 | 18 |
| 20 | 22 | 14 | 10 | 15 |

13. HOTEL The table shows the numbers of guests at a hotel on different days.
a. Find the range and the interquartile range of

| Numbers of Guests |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 66 | 58 | 90 | 57 | 63 | 55 |
| 60 | 62 | 56 | 54 | 72 |  | the data.

b. Use the interquartile range to identify the outlier(s) in the data set. Find the range and the interquartile range of the data set without the outlier(s). Which measure did the outlier(s) affect more?
14. JOBS The data sets show the numbers of hours worked each week by two friends for several weeks.

Greg's hours: 9, 18, 12, 6, 9, 21, 3, 12
Tom's hours: $12,18,15,16,14,12,15,18$
Find the measures of center and the measures of variation for each data set.
Compare the measures. What can you conclude?

1. What is the value of the expression below?

$$
8 \frac{4}{9} \div 4 \frac{2}{3}
$$

A. $1 \frac{17}{21}$
B. $2 \frac{2}{3}$
C. $32 \frac{8}{27}$
D. $39 \frac{11}{27}$
2. What is the value of the expression below?

$$
4.18+6.225+5.7
$$

F. 15.005
G. 15.105
H. 16.005
I. 16.105

3. One number is missing from the data set in the box below.

$18,24,22,30,26$, $\qquad$ 25

The median of the data set is 24 . What is the greatest possible value of the missing number?
4. The number of hours that each of 6 students spent reading last week is shown in the bar graph below.


For the data in the bar graph, which measure is the least?
A. mean
C. mode
B. median
D. range
5. You go to a beach and collect buckets of shells. Of the many shells you have collected, you notice the following.

- $9 \%$ of the seashells are auger shells.
- $\frac{1}{8}$ of the seashells are coquina shells.
- $11 \%$ of the seashells are rough scallop shells.
- 0.1 of the seashells are fighting conch shells.

Which list correctly shows the types of shells in order from least to greatest?
F. auger, coquina, rough scallop, fighting conch
G. fighting conch, coquina, auger, rough scallop
H. fighting conch, auger, rough scallop, coquina
I. auger, fighting conch, rough scallop, coquina
6. What is the mean absolute deviation of the data shown in the line plot, rounded to the nearest tenth?

A. 1.4
B. 3
C. 3.2
D. 5
7. A family wants to buy tickets to a theme park. There are separate ticket prices for adults and children.

## Rollercoaster WorldI <br> Thickets: <br> ${ }^{\text {s }} 30$ for adults <br> \$20 for children

Which expression represents the total cost, in dollars, for $a$ adult tickets and $c$ child tickets?
F. $600(a+c)$
G. $50(a \times c)$
H. $30 a+20 c$
I. $30 a \times 20 c$
8. What is the value of the expression below?


$$
52.8 \div 0.16
$$

9. What is the value of the expression below when $a=6$ and $b=14$ ?

$$
0.8 a+0.02 b
$$

A. 0.4828
B. 0.8814
C. 5.08
D. 16.4
10. Which property was not used in the box below to simplify the expression?

$$
\begin{aligned}
0.3 \times 53+53 \times 0.7 & =53 \times 0.3+53 \times 0.7 \\
& =53 \times(0.3+0.7) \\
& =53 \times 1 \\
& =53
\end{aligned}
$$

F. Distributive Property
G. Associative Property of Addition
H. Identity Property of Multiplication
I. Commutative Property of Multiplication
11. Determine a data set of 5 numbers that has the following measures:

- a mean of 7 and
- a median of 9 .

Explain how you determined your data set. Then demonstrate that the mean of your data set is 7 and the median is 9 .
12. What is the value of the expression below?
$25 \%$ of 400
A. 16
B. 100
C. 1000
D. 10,000

