Statistical Measures

9.1	Introduction to Statistics
9.2	Mean
9.3	Measures of Center
9.4	Measures of Variation
9.5	Mean Absolute Deviation



"Mom, my owner, and Fluffy have agreed to participate in my survey. Will you be my fourth participant?"



Help! I've got an itch. Can someone please scratch me?

"Please hold still. I am trying to find the mean of 6, 8, and 10 by dividing their sum into three equal piles."

What You **Learned Before**

Ordering Decimals

One of us is eating too many dog biscuits. WOIGHT Our mean weight is 18 pounds.

Example 1 Use a number line to order 8, 6.5, 7.25, 5.5, 4.25, and 7 from least to greatest.



Try It Yourself

Use a number line to order the numbers from least to greatest.

- **1.** 7.25, 4.5, 6.5, 6, 5.5, 8.75
- **2.** 4, 2.5, 3.25, 5.5, 4.5, 6.75

3. 6.25, 3, 2.5, 3.5, 5.75, 5

4. 1.25, 5.5, 4.75, 4.5, 3.5, 2.25

Analyzing Double Bar Graphs

Example 2 How many more male athletes than female athletes participated in the 1992 Summer Olympics?



- athletes in 1992
- 3948 more male athletes participated.



Example 3 How many athletes participated in the 2000 Summer Olympics?

6582 + 4069 = 10,651 athletes participated in the 2000 Summer Olympics.

Try It Yourself

- 5. How many more female athletes participated in the 2012 Summer Olympics than in the 1992 Summer Olympics?
- 6. Describe the relationship between the number of athletes in the 2000 Summer Olympics and the number of athletes in the 2004 Summer Olympics.

9.1 Introduction to Statistics

Essential Question How can you tell whether a question

is a statistical question?

Your heart rate is the number of times your heart beats in a certain time period, such as 1 minute. To measure your heart rate, you can check your pulse. The illustration shows how to check your pulse by pressing lightly on your wrist.



1 ACTIVITY: Using Data to Answer Questions

Work with a partner.

- **a.** Find your pulse by counting the number of beats in 10 seconds. Have your partner keep track of the time. Write a rate to describe your result.
- **b.** Complete the ratio table. What is your heart rate in beats per minute?

Time (seconds)	10	30	60
Number of Beats			

- **c.** Collect the recorded heart rates (in beats per minute) of the students in your class, including yourself. Compare the heart rates.
- **d. MODELING** Make a *line plot* of your data. Then answer the following questions:
 - How many values are in your data set?
 - Do the heart rates *cluster* around a particular value or values?
 - Are there any *peaks* or *gaps* in the data?
 - Are there any unusual heart rates that are far removed from the other values?
- e. **REASONING** How would you answer the following question by using only one value? Explain your reasoning.

"What is the heart rate of sixth grade students?"

- **f. REASONING** Read and compare the following questions. How did you answer each question? Could the answer be the same for both questions? Explain.
 - What is your heart rate?
 - What is the heart rate of sixth grade students?

In this lesson, you will

- recognize statistical questions.
 use dot plots to display
- use dot plots to display numerical data.

ACTIVITY: Identifying Types of Questions

Work with a partner.

- **a.** Answer each question below on your own. Then compare your answers with your partner's answers. For which questions should your answers be the same? For which questions might your answers be different?
 - **1.** What is your shoe size?
 - 2. How many states are in the United States?
 - 3. How many brothers and sisters do you have?
 - 4. How many U.S. presidents have been in office?
 - 5. What is your favorite type of movie?
 - 6. How tall are you?
- **b. CONJECTURE** Some of the questions above are considered *statistical* questions. Which ones do you think they are? Why?

3 ACTIVITY: Analyzing a Question in a Survey





"Do you prefer salty potato chips or healthy granola bars to be sold in the school's vending machines?"



- **a.** Do you think this is a fair question to ask in a survey? Explain.
- **b. LOGIC** Identify the words in the question that may influence someone's response. Then explain how you can reword the question.
- **c.** How might the results of the survey differ when the student asks the original question and your reworded question in part (b)?

-What Is Your Answer?

- 4. **REASONING** What do you think "statistics" means?
- **5. IN YOUR OWN WORDS** How can you tell whether a question is a statistical question? Give examples to support your explanation.
- **6.** Find the least and the greatest heart rates in your class. How can you use these two values to answer the question in Activity 1(e)?
- **7.** Create a one-question survey. Explain why your question is a statistical question. Then conduct your survey and organize your results in a line plot. Make three observations about your data set.



Use what you learned about different types of questions to complete Exercises 4–7 on page 394.

Build Arguments How can comparing your answers help you support your conjecture?

Math Practice

9.1 Lesson



Key Vocabulary () statistics, *p. 392* statistical question, *p. 392*

EXAMPLE

Statistics is the science of collecting, organizing, analyzing, and interpreting data. A **statistical question** is one for which you do not expect to get a single answer. Instead, you expect a variety of answers, and you are interested in the distribution and tendency of those answers.

Recall that a dot plot uses a number line to show the number of times each value in a data set occurs. Dot plots show the *spread* and the *distribution* of a data set.

1 Answering a Statistical Question

You conduct a science experiment on house mice. Your teacher asks you, "What is the weight of a mouse?"

- a. Is this a statistical question? Explain.
 - Because you can anticipate that the weights of mice will vary, it is a statistical question.

b. You weigh some mice and record the weights

(in grams) in the table. Display the data in a

dot plot. Identify any clusters, peaks, or gaps

Weights (grams) 20 19 21 20 18 20 27 21 28 23 20 19 20 21 18 27 22 21 19 20



called *line plots*. It is easy to see clusters, peaks, and gaps in a dot plot.

Now You're Ready

Exercises 8–16



Draw a number line that includes the least value, 18, and greatest

value, 28. Then place a dot above the number line for each data value.

Most of the data are clustered around 20. There is a peak at 20 and a gap between 23 and 27.

c. Use the distribution of the data to answer the question.

Most mice weigh about 20 grams.

On Your Own

in the data.

- 1. The table shows the ages of some people who retired early. You are asked, "How old are people who retire early?"
 - **a.** Is this a statistical question? Explain.
 - **b.** Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
 - **c.** Use the distribution of the data to answer the question.

Ages					
60	61	59	60		
62	56	64	59		
58	60	61	60		
59	60	58	61		

EXAMPLE

2

Using a Dot Plot

You record the high temperature every day while at summer camp in August. Then you create the vertical dot plot.

a. How many weeks were you at summer camp?

Because there are 28 data values on the dot plot, you were at camp 28 days.

$$28 \text{ days} \cdot \frac{1 \text{ week}}{7 \text{ days}} = 4 \text{ weeks}$$

So, you were at summer camp for 4 weeks.

b. How can you collect these data? What are the units?

You can collect these data with a thermometer. The units are degrees Fahrenheit (°F).

c. Write a statistical question that you can answer using the dot plot. Then answer the question.

One possible statistical question is:

What is the daily high temperature in August?

The high temperatures are spread out with about half of the temperatures around 81°F and half of the temperatures around 86°F.

On Your Own

2. The dot plot shows the times of sixth grade students in a 100-meter race.



- **a.** How many students ran in the race?
- b. How can you collect these data? What are the units?
- **c.** Write a statistical question that you can answer using the dot plot. Then answer the question.



You're Ready

Exercises 17 and 18



9.1 Exercises



\checkmark

Vocabulary and Concept Check

- **1. VOCABULARY** What is a statistical question? Give an example.
- **2. CRITICAL THINKING** What process can you use to answer a statistical question?
- **3. NUMBER SENSE** The results of a survey are shown in the table. Did the survey ask a statistical question? Explain.

Miles					
6	1	9	2		
2	5	4	9		
8	10	6	6		
5	1	8	1		

Practice and Problem Solving

Answer the question. Tell whether your answer would be the same as your classmates'.

- **4.** How many inches are in 1 foot? **5.** How many pets do you have?
- 6. On what day of the month were you born? 7. How many senators are in Congress?

Determine whether the question is a statistical question. Explain.

- **8.** What is the eye color of sixth grade students?
 - 9. At what temperature (in degrees Fahrenheit) does water freeze?
 - **10.** How many pages are in the favorite books of students your age?
 - 11. How many hours do sixth grade students use the Internet each week?

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

12.		Nur	nber	of Fo	ouls	
	2	1	2	0	0	2
	2	1	6	1	1	0

14.		Yea	ars	
	2011	2008	2013	2009
	2009	2010	2010	2009
	2010	2012	2009	2010

13.	Ca	Camper Registrations					
	21	25	25	22	21		
	23	24	26	25	16		
	24	26	22	25	22		

Test Scores 85 80 83 90 88 82 83 81 80 89				
85	80	83	90	88
82	83	81	80	89
89	84	86	87	83
	85 82 89	Tes 85 80 82 83 89 84	Test Sco 85 80 83 82 83 81 89 84 86	Test Scoves 85 80 83 90 82 83 81 80 89 84 86 87

Hours of Homework				
2	4	3	2	
1	2	2	1	
2	3	5	2	

- **16. SURVEY** You conduct a survey to answer: "How many hours does a sixth grade student spend on homework during a school night?" The table shows the results.
 - **a.** Is this a statistical question? Explain.
 - **b.** Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
 - **c.** Use the distribution of the data to answer the question.

2 17. EARTHWORMS The dot plot shows the lengths of earthworms.



- **a.** How many earthworms does it represent?
- b. How can you collect these data? What are the units?
- **c.** Write a statistical question that you can answer using the dot plot. Then answer the question.
- **18. BASKETBALL** The vertical dot plot shows the heights of the players on a recent NBA championship team.
 - a. How many players were on the team?
 - **b.** How can you collect these data? What are the units?
 - **c.** Write a statistical question that you can answer using the dot plot. Then answer the question.

Use the Internet to research and identify the method of measurement and the units used when collecting data about the topic.

19.	wind speed	20. amount of rainfall	21. earthquake intensity
-----	------------	-------------------------------	---------------------------------

The dot plot shows the speeds of cars in a traffic study. Estimate the speed limit. Explain your reasoning.



- **24. REASONING** "How many letters are in the English alphabet?" is *not* a statistical question. Write a question about letters that is a statistical question. Explain your reasoning.
- **25.** Reasoning A bar graph shows the favorite colors of 30 people. Does it make sense to describe the distribution of these data? Explain.

B	Fair Game Revi	EW What you lear	ned in previous grades &	lessons
Tell	whether the ordered pa	air is a solution of th	ne equation. (Section 7	7.4)
26.	y = 4x; (2, 8)	27. $y = 3x + 5$	i ; (3, 15) 28. <i>y</i>	= 6x - 15; (4, 9)
29.	MULTIPLE CHOICE A potential $(4, -3)$. What is the original formula $(4, -3)$.	oint is reflected in th ginal point? (Section	e <i>x</i> -axis. The reflected p on 6.5)	oint is
	(-3, 4)	B (-4, 3)	(−4, −3)	D (4, 3)

85

84

83 82 81

80

79 78

77 76

75

74 73

72

Essential Question How can you find an average value of a data set?

ACTIVITY: Finding a Balance Point

Work with a partner. Discuss the distribution of the data. Where on the number line do you think the data set is *balanced*? Is this a good representation of the average? Explain.

a. number of quarters brought to a batting cage



b. annual income of recent graduates (in thousands of dollars)



c. hybrid fuel economy (miles per gallon)

21 24 27 30 33 36 39

ACTIVITY: Finding a Fair Share

Statistics

- In this lesson, you will
- understand the concept of the mean of data sets.
- find the mean of data sets.
- compare and interpret the means of data sets.

Work with a partner. It costs \$0.25 to hit 12 baseballs in a batting cage. The table shows the numbers of quarters six friends bring to the batting cage. They want to group the quarters so that everyone has the same amount.

65

45

42

0

Quarters					
John Lisa Miguel Matt Cheryl Jean					
6	3	4	5	2	4

Use counters to represent each number in the table. How can you use the counters to determine how many times each friend can use the batting cage? Explain how this procedure results in a "fair share."

2

ACTIVITY: Finding an Average

Work with a partner. Use the information in Activity 2.

- **a.** What is the total number of quarters the group of friends brought to the batting cage?
- **b. REASONING** How can you use math to find the average number of quarters that each friend brought to the batting cage? Find the average number of quarters. Why do you think this average represents a fair share?

ACTIVITY: Answering a Statistical Question

Work with a partner. The table shows the numbers of quarters several people bring to a batting cage. You want to answer the question:

Math Practice

Use Clear Definitions

What does it mean for data to have an average? How does this help you answer the question?

"How many quarters do people bring to a batting cage?"

- **a.** Explain why this question is a statistical question.
- **b. MODELING** Make a dot plot of the data. Use the distribution of the data to answer the question. Explain your reasoning.
- **c. REASONING** Use an average to answer the question. Explain your reasoning.



-What Is Your Answer?

- 5. IN YOUR OWN WORDS How can you find an average value of a data set?
- 6. Give two real-life examples of averages.
- **7.** Explain what it means to say the average of a data set is the point on a number line where the data set is balanced.
- **8.** There are 5 students in the cartoon. Four of the students are 66 inches tall. One is 96 inches tall.
 - **a.** How do you think the students decided that their average height is 6 feet?
 - **b.** Does a height of 6 feet seem like a good representation of the average height of the 5 students? Explain why or why not.



Yup, the average height in our class is 6 feet."



Use what you learned about averages to complete Exercises 4 and 5 on page 400.



Because 2.6 is greater than 2, City 1 averaged more rainfall.



On Your Own Find the mean of the data.

1. 49, 62, 52, 54, 61, 70, 55, 53

2. 7.2, 8.5, 7.0, 8.1, 6.7

An **outlier** is a data value that is much greater or much less than the other values. When included in a data set, it can affect the mean.

EXAMPLE 3

Shetland Pony Heights (inches)							
40	37	39	40	42			
38	38	37	28	40			

- The table shows the heights of several Shetland ponies.
- a. Identify the outlier.
- b. Find the mean with and without the outlier.

Finding the Mean With and Without an Outlier



c. Describe how the outlier affects the mean.



low You're Ready

Exercise 14

a. Display the data in a dot plot.



The height of 28 inches is much less than the other heights. So, it is an outlier.

b. Mean with outlier:

$$\frac{40+37+39+40+42+38+38+37+28+40}{10} = \frac{379}{10}, \text{ or } 37.9$$

Mean without outlier:

$$\frac{40+37+39+40+42+38+38+37+40}{9} = \frac{351}{9}, \text{ or } 39$$

c. With the outlier, the mean is less than all but three of the heights. Without the outlier, the mean better represents the heights.

🕨 On Your Own

For each data set, identify the outlier. Then describe how it affects the mean.

3. Weights (in pounds) of dogs at a kennel:

48, 50, 55, 60, 8, 37, 50

4. Prices for flights from Miami, Florida, to San Juan, Puerto Rico:

\$456, \$512, \$516, \$900, \$436, \$516



9.2 Exercises



Vocabulary and Concept Check

1. VOCABULARY Arrange the words to explain how to find a mean.

the data values divide by	the number of data values	add	then
---------------------------	---------------------------	-----	------

- 2. NUMBER SENSE Is the mean always equal to a value in the data set? Explain.
- **3. REASONING** Can you use the mean to answer a statistical question? Explain.

Practice and Problem Solving

Describe an average value of the data.

- **4.** Ages in a class: 11, 12, 12, 12, 12, 12, 13
- **5.** Movies seen this week: 0, 0, 0, 1, 1, 2, 3

Find the mean of the data.

6.	Pets Own	Pets Owned			
	Brandon	Ι			
	Jill	III			
	Mark	II			
	Nicole	1111			
	Steve	0			
8					

Sit-ups						
108	85	94				
103	112	115				
98	119	126				
105	82	89				

7. **Brothers and Sisters** £ Amanda Eve Joseph Michael



- **10. GOLF** The table shows tournament finishes for a golfer.
 - **a.** What was the golfer's mean finish?
 - **b.** Identify two outliers for the data.

Time (minutes)							
4.2	3.5	4.55	2.75	2.25			

1	1	2	1	1	12	6	2
15	37	1	2	1	26	9	1

Tournament Finishes

- 11. COMMERCIALS You and your friends are watching a television show. One of your friends asks, "How long are the commercial breaks during this show?"
 - **a.** Is this a statistical question? Explain.
 - **b.** Use the mean of the values in the table to answer the question.

Nicole	1111
Steve	0

Month	Rainfall (inches)	Month	Rainfall (inches)
Jan	2.22	Jul	3.27
Feb	1.51	Aug	5.40
Mar	1.86	Sep	5.45
Apr	2.06	Oct	4.34
May	3.48	Nov	2.64
Jun	4.57	Dec	2.14

- **12. RAINFALL** The table shows the monthly rainfall at a measuring station. What is the mean monthly rainfall?
- **13. OPEN-ENDED** Create two different sets of data that have six values and a mean of 21.



3 14. CELL PHONE The bar graph shows your cell phone usage for five months.

- a. Which data value is an outlier? Explain.
- **b.** Find the mean with and without the outlier. Then describe how the outlier affects the mean.
- **c.** Describe a situation that could have caused the outlier in this problem.



15. HEIGHT The table shows the heights of the volleyball players from two schools. What is the difference between the mean heights of the two teams? Do outliers affect either mean? Explain.

	Player Height (inches)														
Dolphins	59	65	53	56	58	61	64	68	51	56	54	57			
Tigers	63	68	66	58	54	55	61	62	53	70	64	64	62	67	69

- **16. REASONING** Make a dot plot of the data set 11, 13, 17, 15, 12, 18, and 12. Use the dot plot to explain how the mean is the point where the data set is balanced.
- **17. ALLOWANCE** In your class, 7 students do not receive a weekly allowance, 5 students receive \$3, 7 students receive \$5, 3 students receive \$6, and 2 students receive \$8. What is the mean weekly allowance? Explain how you found your answer.
- **18.** Precision A collection of 8 backpacks has a mean weight of 14 pounds. A different collection of 12 backpacks has a mean weight of 9 pounds. What is the mean weight of the 20 backpacks? Explain how you found your answer.

A	Fair Game	2 Review What yo	u learned in previous gra	des & lessons
I	Evaluate the expres	ssion. (Section 1.3)		
	19. $\frac{8+10}{2}$	20. $\frac{26+34}{2}$	21. $\frac{18+19}{2}$	22. $\frac{14+17}{2}$
	23. MULTIPLE CHO	ICE 60% of what numb	oer is 105? (Section 5.6))
	A 63	B 175	© 630	D 1750

Essential Question In what other ways can you describe an

average of a data set?

ACTIVITY: Finding a Median

Work with a partner.

a. Write the total number of letters in the first and last names of 19 celebrities, historical figures, or people you know. Organize your data in a table. One person is already listed for you.

Person	Number of letters in first and last name
Abraham Lincoln	14

b. Order the values in your data set from least to greatest. Then write the data on a strip of grid paper with 19 boxes.





c. Place a finger on the square at each end of the strip. Move your fingers toward the center of the ordered data set until your fingers touch. On what value do your fingers touch?



- **d.** Now take your strip of grid paper and fold it in half. On what number is the crease? What do you notice? This value is called the *median*. How would you describe to another student what the median of a data set represents?
- **e.** How many values are greater than the median? How many are less than the median?
- f. Why do you think the median is considered an average of a data set?

Statistics In this lesson, you will

- understand the concept of measures of center.
- find the median and mode of data sets.

ACTIVITY: Adding a Value to a Data Set

Work with a partner.

- **a.** How many total letters are in your first name and last name? Add this value to the ordered data set in Activity 1. How many values are now in your data set?
- **b.** Write the ordered data, including your new value from part (a), on a strip of grid paper.



- **c.** Repeat parts (c) and (d) from Activity 1. Explain your findings. How do you think you can find the median of this data set?
- **d.** Compare the medians in Activities 1 and 2. Then answer the following questions. Explain your reasoning.
 - Do you think the median always has to be a value in the data set?
 - Do you think the median always has to be a whole number?

ACTIVITY: Finding a Mode

Work with a partner.

- **a.** Make a dot plot for the data set in Activity 2. Describe the distribution of the data.
- **b.** Which value occurs most often in the data set? This value is called the *mode*.
- c. Do you think a data set can have no mode or more than one mode? Explain.
- **d.** Do you think the mode always has to be a value in the data set? Explain.
- e. Why do you think the mode is considered an average of a data set?

-What Is Your Answer?

- **4. IN YOUR OWN WORDS** In what other ways can you describe an average of a data set?
- **5.** Find the mean of your data set in Activity 2. Then compare the mean, median, and mode. Is there one measure that you think best represents your data set? Explain your reasoning.



Use what you learned about the median of a data set to complete Exercises 5 and 6 on page 407.

Math Practice

Use a Graph How can you use the dot plot to find the mode?

9.3 Lesson



Key Vocabulary measure of center. p. 404 median, p. 404 mode, p. 404

Study Tip

The mode is the only measure of center that you can use to describe a set of data that is not made up of numbers.

A **measure of center** is a measure that describes the typical value of a data set. The mean is one type of measure of center. Here are two others.

^O Key Ideas

Median

Order the data. For a set with an odd number of values, the Words median is the middle value. For a set with an even number of values, the **median** is the mean of the two middle values.

Numbers Data: 5, 8, 9, 12, 14

Data: 2, 3, 5, 7, 10, 11

The median is 9.

The median is $\frac{5+7}{2}$, or 6.

Mode

The **mode** of a data set is the value or values that occur most Words often. Data can have one mode, more than one mode, or no mode. When all values occur only once, there is no mode.

> The modes are 15 and 24

Numbers Data: 11, 13, 15, 15, 18, 21, 24, 24

Finding the Median and Mode

EXAMPLE 1

Bowling Scores							
120	135	160	125	90			
205	160	175	105	145			

Find the median and mode of the bowling scores.

90, 105, 120, 125, 135, 145, 160, 160, 175, 205 Order the data.

Median: $\frac{135 + 145}{2} = \frac{280}{2}$, or 140 Add the two middle values and divide by 2.

Mode: 90, 105, 120, 125, 135, 145, 160, 160, 175, 205

The value 160 occurs most often.

The median is 140. The mode is 160.

Find the median and mode of the data.

On Your Own



1. 20, 4, 17, 8, 12, 9, 5, 20, 13

2. 100, 75, 90, 80, 110, 102



EXAMPLE 2 Finding the Mode

Favorite Types of Movies

Comedy	Drama	Horror
Horror	Drama	Horror
Comedy	Comedy	Action
Action	Comedy	Action
Horror	Drama	Comedy
Comedy	Comedy	Horror
Horror	Comedy	Action
Horror	Action	Drama

The list shows the favorite types of movies for students in a class. Organize the data in a frequency table. Then find the mode.



Comedy received the most votes.

• So, the mode is comedy.

On Your Own



3. One member of the class was absent and ends up voting for horror. Does this change the mode? Explain.

EXAMPLE 3 Choosing the Best Measure of Center



Now You're Ready

Exercises 17-20

Find the mean, median, and mode of the sneaker prices. Which measure best represents the data?

Mean:
$$\frac{20+31+122+48+37+20+45+65}{8} = \frac{388}{8}$$
, or 48.5

Median: 20, 20, 31, **37**, **45**, 48, 65, 122

Order from least to greatest.

$$\frac{37+45}{2} = \frac{82}{2}$$
, or 41

Mode: 20, 20, 31, 37, 45, 48, 65, 122

The value 20 occurs most often.



The median best represents the data. The mode is less than most of the data, and the mean is greater than most of the data.

On Your Own

Find the mean, median, and mode of the data. Choose the measure that best represents the data. Explain your reasoning.

4. 1, 93, 46, 48, 34, 194, 67, 55 **5.** 96, 150, 102, 87, 150, 75

EXAMPLE 4 Removing an Outlier

Identify the outlier in Example 3. Find the mean, median, and mode without the outlier. Which measure does the outlier affect the most?

The price of \$122 is much greater than any other price. So, it is the outlier.

	Mean	Median	Mode
With Outlier (Example 3)	48.5	41	20
Without Outlier	38	37	20

The mean is affected the most by the outlier.

On Your Own



6. The times (in minutes) it takes six students to travel to school are 8, 10, 10, 15, 20, and 45. Identify the outlier. Find the mean, median, and mode with and without the outlier. Which measure does the outlier affect the most?

EXAMPLE 5 Changing the Values of a Data Set

The prices of six video games at an online store are shown in the table. The price of each game increases by \$4.98 when a shipping charge is included. How does this increase affect the mean, median, and mode?

Video Game Prices							
\$53.42	\$35.69						
\$18.99	\$25.13						
\$27.97	\$53.42						

Make a new table by adding \$4.98 to each price.

Then find the mean, median, and mode of both data sets.

	Mean	Median	Mode
Original Price	35.77	31.83	53.42
Price with Shipping Charge	40.75	36.81	58.4

Compare:

Mean: 40.75 - 35.77 = 4.98

Median: 36.81 - 31.83 = 4.98

Mode: 58.4 - 53.42 = 4.98

By increasing each video game price by \$4.98 for shipping, the mean, median, and mode all increase by \$4.98.

📄 On Your Own

7. WHAT IF? The store decreases the price of each video game by\$3. How does this decrease affect the mean, median, and mode?

Video Game Prices with Shipping Charge						
\$58.40	\$40.67					
\$23.97	\$30.11					
\$32.95	\$58.40					



Find the mode(s) of the data.

9.3

Exercises

Vocabulary and Concept Check

1. NUMBER SENSE Give an example of a data set that has no mode.

2. WRITING Which is affected most by an outlier: the mean, median, or mode? Explain.

2 14.	Shirt Color								
	Black	Blue	Red						
	Pink	Black	Black						
	Gray	Green	Blue						
	Blue	Blue	Red						
	Yellow	Blue	Blue						
	Black	Orange	Black						
	Black								

15.	Talent Show Acts							
	Singing	Comedy						
	Singing	Singing	Dancing					
	Juggling	Dancing	Singing					
	Singing	Poetry	Dancing					
	Comedy	Magic	Dancing					
	Poetry	Singing	Singing					

16. REASONING In Exercises 14 and 15, can you find the mean and median of the data? Explain.

elp with Homework BigIdeasMath Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning.

3 17.	48, 12, 11, 45, 48, 48, 43, 32	18.	12, 13, 40, 95, 88, 7, 95
19.	2, 8, 10, 12, 56, 9, 5, 2, 4	20.	126, 62, 144, 81, 144, 103

Find the mean, median, and mode(s) of the data with and without the outlier. Describe the effect of the outlier on the measures of center.

Find the mean, median, and mode(s) of the data.

23. 4.7, 8.51, 6.5, 7.42, 9.64, 7.2, 9.3	24.	$8\frac{1}{2}$,	$6\frac{5}{8}$,	$3\frac{1}{8}$,	$5\frac{3}{4}$,	$6\frac{5}{8}$,	$5\frac{1}{4}$,	$10\frac{5}{8}$,	$4\frac{1}{2}$	2
---	-----	------------------	------------------	------------------	------------------	------------------	------------------	-------------------	----------------	---

25. WEATHER The weather forecast for a week is shown.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
	ß						States and the second s
High	90° F	91° F	89° F	97° F	101° F	99° F	91° F
Low	74° F	78° F	77° F	77° F	83° F	78° F	72° F

- **a.** Find the mean, median, and mode(s) of the high temperatures. Which measure best represents the data? Explain your reasoning.
- **b.** Repeat part (a) for the low temperatures.
- **26. RESEARCH** Find the unit costs of 10 different kinds of cereal. Choose one cereal whose unit cost will be an outlier.
 - **a.** Find the mean, median, and mode(s) of the data. Which measure best represents the data? Explain your reasoning.
 - **b.** Identify the outlier in the data set. Find the mean, median, and mode(s) of the data set without the outlier. Which measure does the outlier affect the most?



- **27. PROBLEM SOLVING** The bar graph shows the numbers of hours you volunteered at an animal shelter. What is the minimum number of hours you need to work in the seventh week to justify that you worked an average of 10 hours for the 7 weeks? Explain your answer using measures of center.
- **28. REASONING** Why do you think the mode is the least frequently used measure to describe a data set? Explain.

29. MOTOCROSS The ages of the racers in a bicycle motocross race are 14, 22, 20, 25, 26, 17, 21, 30, 27, 25, 14, and 29. The 30-year-old drops out of the race and is replaced with a 15-year-old. How are the mean, median, and mode of the ages affected?



30. CAMERAS The data are the prices of several digital cameras at a store.

\$130 \$170 \$230 \$130 \$250 \$275 \$130 \$185

- **a.** Does the price shown in the advertisement represent the prices well? Explain.
- **b.** Why might the store use this advertisement?
- **c.** In this situation, why might a person want to know the mean? the median? the mode? Explain.
- **31. SALARIES** The table shows the monthly salaries for employees at a company.

Monthly Salaries (dollars)							
1940	1660	1860	2100	1720			
1540	1760	1940	1820	1600			

- **a.** Find the mean, median, and mode of the data.
- **b.** Each employee receives a 5% raise. Find the mean, median, and mode of the data with the raise. How does this increase affect the mean, median, and mode of the data?
- **c.** Use the original monthly salaries to calculate the annual salaries. Find the mean, median, and mode of the annual salaries. How are these values related to the mean, median, and mode of the monthly salaries?
- **32.** Consider the algebraic expressions 3x, 9x, 4x, 23x, 6x, and 3x. Assume x > 0.
 - **a.** Find the mean, median, and mode.
 - **b.** Is there an outlier? If so, what is it?

R	Fair Game	Review What you	learned in previous grad	es & lessons
Find	l the value of the	expression. (Section	ı 1.1)	
33.	48 - 35	34. 188 – 123	35. 416 – 297	36. 6249 - 3374
37.	MULTIPLE CHOI There are 12 po the number of p	CE A shelf in your room unds of books already pounds you can add to	n can hold at most 30 po on it. Which inequality r the shelf? <i>(Section 7.6)</i>	represents
	(A) x < 18	$(\mathbf{B}) x \ge 18$	$(\mathbf{C}) x \le 42$	$(\mathbf{D}) x \le 18$

9 Study Help



You can use a **concept circle** to organize information about a concept. Here is an example of a concept circle for a statistical question.

Statistical Question



On Your Own

Make concept circles to help you study these topics.

- **1.** mean **2.** outlier
- **3.** measures of center **4.** median
- **5.** mode

After you complete this chapter, make concept circles for the following topics.

- 6. measures of variation
- 7. range
- 8. quartiles
- **9.** interquartile range
- **10.** mean absolute deviation



"Do you think this concept circle will help my owner understand that 'Speak' and 'Sit' need motivation?"

9.1–9.3 Quiz

1.



Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data. (Section 9.1)

2.

Weight (grams)						
42	40	37	42			
43	41	42	43			
37	41	41	42			

Find the mean of the data. (Section 9.2)

3.	Tour Dates				
	May	III			
	June				
	July	JHT JHT IIII			
	August	JHT JHT JHT III			
	September	1111			

Time (seconds)						
63	66	65	60	59		
59	64	58	65	58		
64	60	59	64	63		

4.	Scores					
	Judge 1	8.9				
	Judge 2	9.4				
	Judge 3	8.6				
	Judge 4	9.1				

Find the median and the mode(s) of the data. (Section 9.3)

5. 3, 5, 9, 11, 3

6. 24, 4, 37, 56, 6, 56, 45

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning. (Section 9.3)

7. 47, 147, 24, 47, 38, 42

8. 34, 57, 58, 56, 21

Hours of Exercise						
5	1	5	3	5		
4	5	2	5	4		
3	4	6	5	6		

- **9. EXERCISE** You conduct a survey to answer: "How many hours does a sixth-grade student spend exercising during a week?" The table shows the results. *(Section 9.1)*
 - **a.** Is this a statistical question? Explain.
- **b.** Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
- **c.** Use the distribution of the data to answer the question.
- **10. EMAILS** The number of emails you received in 5 days is shown. What is the mean number of emails you received per day? (*Section 9.2*)
- **11. QUIZZES** The data are your quiz scores for a class. Find the median and the mode of the data. *(Section 9.3)*

18, 19, 17, 14, 20, 20, 15, 21

12. MUSIC The data are the lengths of the songs (in minutes) on your new CD. Which measure of center best represents the data with and without the outlier? Explain. *(Section 9.3)*

2.2, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 14.2

Emails

12

6

8

10

4

Monday

Tuesday

Thursday

Friday

Wednesday

Essential Question How can you describe the spread of a

data set?

ACTIVITY: Interpreting Statements

Work with a partner. There are 24 students in your class. Your teacher makes the following statements:

- "The exam scores range from 75% to 96%."
- "Most of the students received high scores."



- a. What do you think the first statement means? Explain.
- **b.** In the first statement, is your teacher describing the center of the data set? If not, what do you think your teacher is describing?
- **c.** What do you think the scores are for most of the students in the class? Explain your reasoning.
- **d.** Use your teacher's statements to make a dot plot that can represent the distribution of the exam scores of the class.

2

ACTIVITY: Grouping Data

Work with a partner. The numbers of U.S. states visited by each student in a sixth grade class are shown.

- a. Between what values do the data range?
- **b.** Write the ordered data values on a strip of grid paper and fold it to find the median. How many values are greater than the median? How many are less than the median
- **c. REPEATED REASONING** Fold the strip in half again. On what values are the two new creases? What do you think these values represent?
- **d.** Into how many parts did you divide the data set? How many data values are in each part?
- **e.** Graph the median and the values you found in parts (a) and (c) on a number line. Are the distances the same between these points?
- f. How can you use these values to describe the spread of the data?

es	NU	imber	of Sta	tes			
	Visited						
	1	7	5	2			
	11	6	3	20			
	4	18	1	6			
	2	7	1	8			
	10	2	12	5			
0		3	21				
an?							
LC I							

Statistics

In this lesson, you will

of data sets.

data sets.

check for outliers in

find the range of data sets.find the interquartile range

ACTIVITY: Adding a Value to a Data Set

3

Work with a partner. A new student joins the class in Activity 2. The new student has visited 41 states.

- **a.** Add this value to the ordered data set in Activity 2. Does your answer to part (a) change? Explain.
- **b.** How does the distribution of the data change when this value is added? Explain your reasoning.
- **c.** How does adding this value affect the values on your number line in part (e) of Activity 2?

4 ACTIVITY: Analyzing Data Sets

Work with a partner. Identify the data set that is the least spread out and the data set that is the most spread out. Explain your reasoning.



-What Is Your Answer?

- 5. IN YOUR OWN WORDS How can you describe the spread of a data set?
- **6.** Make a dot plot of the data set in Activity 2. Describe any similarities between the dot plot and the number line in part (e).



Use what you learned about variation to complete Exercises 4 and 5 on page 416.



A **measure of variation** is a measure that describes the distribution of a data set. A simple measure of variation to find is the *range*. The **range** of a data set is the difference between the greatest value and the least value.

EXAMPLE



low You're Ready Exercises 6–9

ฦ **Finding the Range**

The table shows the lengths of several Burmese pythons captured for a study. Find and interpret the range of their lengths.

To find the least and the greatest values, order the lengths from least to greatest.

5, 6.25, 8, 10, 11, 12.5, 14, 15.5, 16.25, **18.5**

Lengths (feet)				
18.5	8			
11	10			
14	15.5			
12.5	6.25			
16.25	5			

The least value is 5. The greatest value is 18.5.

÷. So, the range of the lengths is 18.5 - 5, or 13.5 feet. This means that the lengths vary by no more than 13.5 feet.

On Your Own

1. The ages of people in line for a roller coaster are 15, 17, 21, 32, 41, 30, 25, 52, 16, 39, 11, and 24. Find and interpret the range of their ages.

O Key Ideas

Quartiles

The **quartiles** of a data set divide the data into four equal parts. Recall that the median (second quartile) divides the data set into two halves.



Interguartile Range (IQR)

The difference between the third quartile and the first quartile is called the **interquartile range**. The IQR represents the range of the middle half of the data and is another measure of variation.

18 21 **22** 24 28 30 31 **32** 36 37

$$IQR = Q_3 - Q_1$$

 $= 32 - 22$
 $= 10$

Reading

The first quartile can also be called the lower quartile. The third quartile can also be called the *upper* quartile.

EXAMPLE 2 Finding the Interguartile Range

The dot plot shows the top speeds of 12 sports cars. Find and interpret the interquartile range of the data.



• So, the interquartile range is 255 - 235 = 20. This means that the middle half of the speeds vary by no more than 20 miles per hour.

You can use the quartiles and the interquartile range to check for outliers.



9.4 Exercises



Vocabulary and Concept Check

- **1. VOCABULARY** How are measures of center different from measures of variation?
- 2. VOCABULARY How many quartiles does a data set have?
- 3. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

53, 47, 60, 45, 62, 59, 65, 50, 56, 48

What is the interquartile range of the data?

What is the range of the data?

What is the range of the middle half of the data?

What is the difference between the third quartile and the first quartile?

> Practice and Problem Solving

Use grid paper to find the median of the data. Then find the median of the lower half and the median of the upper half of the data. Describe the spread of the data.

- **4.** 5, 8, 10, 1, 7, 6, 15, 8, 6
- Find the range of the data.
- **6.** 26, 21, 27, 33, 24, 29
 - **8.** 133, 117, 152, 127, 168, 146, 174
 - **10. ERROR ANALYSIS** Describe and correct the error in finding the range of the data.
- **5.** 82, 62, 95, 81, 89, 51, 72, 56, 97, 98, 79, 85
- **7.** 52, 40, 49, 48, 62, 54, 44, 58, 39
- **9.** 4.8, 5.5, 4.2, 8.9, 3.4, 7.5, 1.6, 3.8



Find the median, first quartile, third quartile, and interquartile range of the data.

- **2 11.** 40, 33, 37, 54, 41, 34, 27, 39, 35
 - **13.** 132, 127, 106, 140, 158, 135, 129, 138
 - **15. PAPER AIRPLANE** The table shows the distances traveled by a paper airplane. Find and interpret the range and the interquartile range of the distances.
- 84, 75, 90, 87, 99, 91, 85, 88, 76, 92, 94
 38, 55, 61, 56, 46, 67, 59, 75, 65, 58

D	oistance	es (feet))
$13\frac{1}{2}$	$21\frac{1}{2}$	21	$16\frac{3}{4}$
$10\frac{1}{4}$	19	32	$26\frac{1}{2}$
29	$16\frac{1}{4}$	$28\frac{1}{2}$	$18\frac{1}{2}$

- **16. WRITING** Consider a data set that has no mode. Which measure of variation is greater, the range or the interquartile range? Explain your reasoning.
- **3 17. OUTLIERS** Use the interquartile range to identify any outliers in Exercises 11–14.
 - **18. REASONING** How does an outlier affect the range of a data set? Explain.
 - **19. BASKETBALL** The table shows the numbers of points scored by players on a basketball team.

Points Scored						
21	53	74	82	84	93	
103 108 116 122 193						

- **a.** Find the range and the interquartile range of the data.
- **b.** Use the interquartile range to identify the outlier(s) in the data set. Find the range and the interquartile range of the data set without the outlier(s). Which measure did the outlier(s) affect more?
- **20. STRUCTURE** Two data sets have the same range. Can you assume that the interquartile ranges of the two data sets are about the same? Give an example to justify your answer.



- **21. SINGING** The tables show the ages of the finalists for two reality singing competitions.
 - **a.** Find the mean, median, range, and interquartile range of the ages for each show. Compare the results.
 - **b.** A 21-year-old is voted off Show A, and the 36-year-old is voted off Show B. How do these changes affect the measures in part (a)? Explain.

Ages for Show A		Age: Sho	s for w B
18	17	21	20
15	21	23	13
22	16	15	18
18	28	17	22
24	21	36	25

22. Open-Ended Create a set of data with 7 values that has a mean of 30, a median of 26, a range of 50, and an interquartile range of 36.

Fair Game Review What you learned in previous grades & lessons Find the mean of the data. (Section 9.2) 23. 8, 14, 22, 7, 2, 11, 25, 7, 5, 9 24. 55, 64, 58, 43, 49, 67 25. MULTIPLE CHOICE What is the surface area of the rectangular prism? (Section 8.2) (A) 62 m² (B) 72 m² 9 m

9.5 Mean Absolute Deviation

Essential Question How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?

The Meaning of a Word Deviate

When you deviate from something,

you stray or depart from the normal course of action.





ACTIVITY: Finding Distances From the Mean

Work with a partner. The table shows the exam scores of 14 students in your class.

Exam Scores						
Ben	89	Mike	95			
Emma	86	Hong	96			
Jeremy	80	Rob	92			
Pete	80	Amy	90			
Ryan	96	Sue	76			
Dan	94	Kim	84			
Lucy	89	Heather	85			

Statistics

- In this lesson, you will
- understand the meaning of *mean absolute deviation*.
- find the mean absolute deviation of data sets.
- **a.** What is the mean exam score?
- **b.** Make a dot plot of the data. Place an "X" on the number line to represent the mean.
- **c.** Is the number of exam scores that are greater than the mean equal to the number of exam scores that are less than the mean? Explain.
- **d.** Which exam score *deviates* the most from the mean? Which exam score *deviates* the least from the mean? Explain how you found your answers.
- **e.** Overall, do you think the exam scores are *close* to the mean or *far away* from the mean? Explain your reasoning.

1

ACTIVITY: Using Distances from the Mean

Work with a partner. Use the information in Activity 1.

a. Complete the table below. Add rows if needed. Be sure to find the sum of the values in the last column of the table.

Student with Score <i>Less Than</i> the Mean	Exam Score	Distance from the Mean
	Sum:	

- **b.** Create a table similar to the one above for students with scores *greater than* the mean.
- **c. LOGIC** What do you notice about the sums you found in your tables? Why do you think this happens?

3 ACTIVITY: Interpreting Distances from the Mean

Work with a partner.

a. LOGIC Add the sums you found in your tables in Activity 2. Divide that amount by the total number of students. Round your result to the nearest tenth.

In your own words, what do you think this value represents?

b. REASONING In a data set, what do you think it means when the value you found in part (a) is close to 0? Explain.

-What Is Your Answer?

- **4. IN YOUR OWN WORDS** How can you use the distances between each data value and the mean of a data set to measure the spread of a data set?
- **5. REASONING** Find the range and the interquartile range of the data set in Activity 1. What do you think it means when these values are close to 0? Explain.



Use what you learned about distances from the mean to complete Exercises 3 and 4 on page 422.

Practice Use Operations What operation can you use to find the distance from the mean? Explain.

Math

9.5 Lesson



Key Vocabulary () mean absolute deviation, *p. 420* Another measure of variation is the *mean absolute deviation*. The **mean absolute deviation** is an average of how much data values differ from the mean.

BO Key Idea

Finding the Mean Absolute Deviation (MAD)

- **Step 1:** Find the mean of the data.
- **Step 2:** Find the distance between each data value and the mean.
- **Step 3:** Find the sum of the distances in Step 2.
- **Step 4:** Divide the sum in Step 3 by the total number of data values.

EXAMPLE 1 Finding the Mean Absolute Deviation

You record the numbers of raisins in 8 scoops of cereal. Find and interpret the mean absolute deviation of the data.

1, 2, 2, 2, 4, 4, 4, 5

Step 1: Mean =
$$\frac{1+2+2+2+4+4+4+5}{8} = \frac{24}{8} = 3$$

Step 2: You can use a dot plot to organize the data. Replace each dot with its distance from the mean.



Step 3: The sum of the distances is 2 + 1 + 1 + 1 + 1 + 1 + 1 + 2 = 10.

Step 4: The mean absolute deviation is $\frac{10}{8} = 1.25$.

• So, the data values differ from the mean by an average of 1.25 raisins.

👂 On Your Own

Statistical Measures

Now You're Ready Exercises 5-8 1. Find and interpret the mean absolute deviation of the data.

5, 8, 8, 10, 13, 14, 16, 22

Chapter 9

420



EXAMPLE

Mendoza Date Win/Loss Runs Aug 8 4 Aug 3 6 Jul 29 L 6 W 0 Jul 24 Jul 13 L 8 Jul 8 4 L Jul 7 5 Jul 2 0 W Jun 27 2 Jun 22 W 0

	_	
.ul 🔊		×
	Rodriguez	
Date	Win/Loss	Runs
Aug 7	L	6
Aug 2	W	4
Jul 28	W	4
Jul 22	-	5
Jul 17	W	0
Jul 8	W	2
Jul 3	L	3
Jun 28	L	2
Jun 23	W	4
Jun 17	W	5
	0 ð 🏠	



The greater the mean absolute deviation, the greater the variation of the data. The smartphones show the numbers of runs allowed by two pitchers in their last 10 starts.

a. Find the mean, median, and mean absolute deviation of the numbers of runs allowed for each pitcher.

Order the runs allowed for Mendoza: 0, 0, 0, 2, 4, 4, 5, 6, 6, 8.

Mean
$$=\frac{35}{10}=3.5$$

Real-Life Application

Mean absolute deviation:



Median $=\frac{4+4}{2}=4$

The mean absolute deviation is $\frac{24}{10} = 2.4$

Order the runs allowed for Rodriguez: 0, 2, 2, 3, 4, 4, 4, 5, 5, 6.

Mean
$$=$$
 $\frac{35}{10} = 3.5$ Median $=$ $\frac{4+4}{2} = 4$

Mean absolute deviation:



b. Which measure can you use to distinguish the data? What can you conclude about the pitchers from this measure?

You cannot use the measures of center to distinguish the data because they are the same for each data set. The measure of variation, MAD, is 2.4 for Mendoza and 1.4 for Rodriguez. This indicates that the data for Rodriguez has less variation.

Using the MAD to distinguish the data, you can conclude that Rodriguez is more consistent than Mendoza.

On Your Own

2. WHAT IF? Mendoza allows 4 runs in the next game. How would you expect the mean absolute deviation to change? Explain.



Practice and Problem Solving

Find the average distance each data value in the set is from the mean. Round your answer to the nearest tenth, if necessary.

- 3. Model years of used cars on a lot: 2010, 2002, 2005, 2007, 2001
- **4.** Prices of kites at a shop: \$7, \$20, \$9, \$35, \$12, \$15, \$7, \$10, \$20, \$25

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth, if necessary.

1 5.		Prices of Microphones (dollars)						
	25	28	20	22	32			
	28	35	34	30	36			

7.	Capa (thou	cities of Stad usands of peo	liums ople)
	101.5	95.4	109.8
	98.7	92.3	104.7

6.	He	eights of Octuplets	10-Year-O s (inches)	ld
	61	61	61	61
	61	61	61	61

8.	Nu We	mbers of ebsite Du	Visitors to ring a We	o a ek
	103	115	124	125
	171	165	170	

9. ERROR ANALYSIS Describe and correct the error in finding the mean absolute deviation of the data set 35, 40, 38, 32, 42, and 41.

K	$mean = \frac{35 + 40 + 38 + 32 + 42 + 41}{6} = 38$
	$MAD = \frac{3+2+6+4+3}{5} = 3.6$

So, the values differ from the mean by an average of 3.6.

10. MUSEUMS The data set shows the admission prices at several museums.

\$20, \$20, \$16, \$12, \$15, \$25, \$11

Find and interpret the range, interquartile range, and mean absolute deviation of the data.



11. MENU The table shows the prices of the five most-expensive and least-expensive dishes on a menu. Find the MAD of each data set. Then compare their variations.

Five Most-Expensive Dishes			Five Least-Expensive Dishes							
\$28	\$30	\$28	\$39	\$25	\$7	\$7	\$10	\$8	\$12	

12. COINS The data sets show the years of the coins in two collections.

Derek's collection: 1950, 1952, 1908, 1902, 1955, 1954, 1901, 1910

Paul's collection: 1929, 1935, 1928, 1930, 1925, 1932, 1933, 1920

Find the measures of center and the measures of variation for each data set. Compare the measures. What can you conclude?

13. PROBLEM SOLVING You survey students in your class about the number of movies they watched last month. The results are shown in the table.

Movies Watched				
7	5	14	5	
6	9	10	12	
15	4	5	8	
11	10	9	2	

- **a.** Find the measures of center and the measures of variation for the data.
- **b.** A new student joins the class who watched 21 movies last month. Is 21 an outlier? How does including this value affect the measures of center and the measures of variation? Explain.

REASONING Which data set do you think would have the greater mean absolute deviation? Explain your reasoning.

- 14. guesses for number of gumballs in a jar guesses for number of baseballs in a jar monthly amounts of water used in a home
- **16. REASONING** The MAD of a data set is considered a more reliable measure of variation than the range or the interquartile range. Why do you think this is true?
- **17.** Add and subtract the MAD from the mean in the original data set in Exercise 13.
 - **a.** What percent of the values are within one MAD of the mean? two MADs of the mean? Which values are more than twice the MAD from the mean?
 - **b.** What do you notice as you get more and more MADs away from the mean? Explain.

Fair Game Review What you learned in previous grades & lessons

Find the mean, median, and mode(s) of the data. (Section 9.2 and Section 9.3)

18. 4, 6, 7, 9, 6, 4, 5, 6, 8, 10

- **19.** 1.2, 1.7, 1.7, 2.1, 1.4, 1.2, 1.9
- **20. MULTIPLE CHOICE** What is the surface area of the square pyramid? *(Section 8.3)*
 - (A) 100.8 yd^2 (B) 147.2 yd^2
 - **(C)** 211.2 yd^2 **(D)** 368 yd^2







Find the range of the data. (Section 9.4)

1. 35, 76, 43, 58, 34, 67 **2.** 19, 21, 22, 22, 19, 25, 24, 23, 24

Find the median, first quartile, third quartile, and interquartile range of the data. *(Section 9.4)*

3. 56, 48, 72, 37, 35, 42, 48, 33, 28 **4.** 95, 14, 86, 62, 55, 46, 28, 37, 33, 70, 31

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary. (*Section* 9.5)

6.

5.	Ages	of Tele Viewers	vision S s (years)	bow)
	29	18	26	33
	33	22	34	26

	Price (thousa	s of Ho nds of	uses dollars)	
80	120	95	240	140
75	135	110	90	125



7.	AMUSEMENT PARKS The data set
	shows the admission prices at several
	amusement parks.

\$65, \$70, \$40, \$55, \$35, \$40, \$60

Find and interpret the range, interquartile range, and mean absolute deviation of the data. *(Section 9.4 and Section 9.5)*

- **8. TEACHING EXPERIENCE** The tables show the years of teaching experience of faculty members at two schools. (*Section 9.4*)
 - **a.** Find the mean, median, range, and interquartile range of the years of experience for each school. Compare the results.
 - **b.** The teacher with 11 years of experience leaves School A, and the teacher with 33 years of experience retires from School B. How does this affect the measures in part (a)? Explain.

School A: Teaching Experience (years)			Scho Teac Exper (yea	ol B: hing rience ars)
5	11		4	15
10	22		6	12
7	8		10	33
8	6		12	20
10	35		8	7

9. BOOK CLUB You survey the students in your book club about the number of books they read last summer. The results are shown in the table. *(Section 9.4 and Section 9.5)*

Books Read						
8	14	15	9			
6	12	9	13			
11	11	7	5			
12	6	10	8			

- **a.** Find the measures of center and the measures of variation for the data.
- **b.** A new student who read 18 books last summer joins the club. Is 18 an outlier? How does adding this value to the data set affect the measures of center and variation? Explain.



Review Key Vocabulary

statistics, p. 392 statistical question, p. 392 mean, p. 398 outlier, p. 399 measure of center, p. 404 median, *p. 404* mode, *p. 404* measure of variation, *p. 414* range, *p. 414* quartiles, *p. 414* first quartile, *p. 414* third quartile, *p. 414* interquartile range, *p.414* mean absolute deviation, *p. 420*

Review Examples and Exercises

9.1 Introduction to Statistics (pp. 390–395)



Heights (inches)							
70	71	70	72	69			
68	69	71	64	70			
64	63	72	70	67			

Exercises

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

1.	D	istanc	e (fee	t)
	56	55	56	57
	58	54	51	55
	51	56	49	56

2.		Weig	ht (po	unds)	
	83	88	89	90	89
	91	89	84	90	92
	90	88	89	83	88



Mean (pp. 396–401)

Find the mean of 5, 9, 10, 6, 6, and 12.

mean =
$$\frac{5+9+10+6+6+12}{6}$$
 sum of the data
= $\frac{48}{6}$, or 8 Simplify.

Exercises

Find the mean of the data.

3. 4, 5, 7, 14, 17, 12, 18

^{4.} 15, 5, 8, 12, 5, 9, 4, 10, 2, 11

Measures of Center (pp. 402–409) 9.3

Find the median and the mode of the movie lengths in the table.

Order the data from least to greatest.

Movie Lengths (minutes)				
91	112	126		
142	122	112		
92	144			

ian:	Mode:
12, 112, 112, 122, 126, 142, 144	91, 92, 112, 112, 122, 126, 142, 144
$\frac{112 + 122}{2} = \frac{234}{2}, \text{ or } 117$	The value 112 occurs most often.

The median is 117 minutes, and the mode is 112 minutes. 20

Exercises

Median:

91, 92, 112, 112, 12

Find the median and the mode(s) of the data.

5. 8, 8, 6, 8, 4, 5, 6

6. 24, 74, 61, 29, 38, 27, 68, 54

94 Measures of Variation (pp. 412–417)

The table shows the weights of several adult emperor penguins. (a) Find and interpret the range. (b) Find and interpret the interquartile range. (c) Check for outliers.

- **a.** Ordered from least to greatest, the weights are 23.5, 24, 25, 27, 29.25, 30.75, 31.25, 32, 33.5, and 36.
 - So, the range of the weights is 36 23.5, or 12.5 kilograms. The weights vary by no more than 12.5 kilograms.
- **b.** Find the quartiles.



29.25

Weights (kilograms)

27

23.5

31.25

32

25

36

24

33.5 30.75

So, the interquartile range is 32 - 25 = 7. This means that the middle half of the weights vary by no more than 7 kilograms.

c. Calculate the outlier boundaries.

 $Q_1 - 1.5(IQR) = 25 - 1.5(7) = 14.5$

 $Q_3 + 1.5(IQR) = 32 + 1.5(7) = 42.5$

There are no weights less than 14.5 kilograms or greater than 42.5 kilograms. So, the data set has no outliers.

Exercises

Find the range of the data.

7. 45, 76, 98, 21, 52, 39

8. 95, 63, 52, 8, 93, 16, 42, 37, 62

Find the median, first quartile, third quartile, and interquartile range of the data.

9. 28, 46, 25, 76, 18, 25, 47, 83, 44 **10.** 14, 25, 97, 55, 66, 28, 92, 38, 94

9.5 Mean Absolute Deviation (pp. 418–423)

You record the prices of 8 printers. Find and interpret the mean absolute deviation of the data.

\$120, \$150, \$90, \$110,





Step 1: Mean = $\frac{120 + 150 + 90 + 110 + 140 + 120 + 140 + 90}{8} = \frac{960}{8} = 120$

Step 2: Use a dot plot to organize the data. Replace each dot with its distance from the mean.



Step 3: The sum of the distances is 30 + 30 + 10 + 0 + 0 + 20 + 20 + 30 = 140.

Step 4: The mean absolute deviation is $\frac{140}{8} = 17.50$.

The data values differ from the mean by an average of \$17.50.

Exercises

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary.

1.		Shoe	Sizes		12.		Prices	of Mo	nitors	
	6	8.5	6	9		(dollars)				
	10	7	8	9.5		130	150	190	100	175
	- 0		<u> </u>	2.70		120	165	140	180	190

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

 Image: Text of the system
 Time (minutes)

 33
 40
 32
 40

 39
 38
 40
 39

 38
 39
 39
 33

Chapter Test

2.		Temp	eratur	e (°F)	
	81	81	80	82	81
	83	76	83	76	80
	75	83	82	82	81

Find the mean, median, and mode(s) of the data.

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning.

5. 5, 6, 4, 24, 18	6. 46, 27, 94, 56, 53, 65, 43
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Find the range of the data.

7.	24, 56, 9, 83, 77, 14	8.	. 43	, 12	, 55	, 91	, 25,	86,	84,	23,	1
					-						

Find the median, first quartile, third quartile, and interquartile range of the data.

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary.

 11.
 Distances Driven (miles)
 12.

 312
 286
 196
 201

 158
 225
 206
 192

Prices of Sunglasses (dollars)								
15	8	19	20	18				
20	22	14	10	15				

13. HOTEL The table shows the numbers of guests at a hotel on different days.

Numbers of Guests									
66	58	90	57	63	55				
60	62	56	54	72					

- **a.** Find the range and the interquartile range of the data.
- **b.** Use the interquartile range to identify the outlier(s) in the data set. Find the range and the interquartile range of the data set without the outlier(s). Which measure did the outlier(s) affect more?
- **14. JOBS** The data sets show the numbers of hours worked each week by two friends for several weeks.

Greg's hours: 9, 18, 12, 6, 9, 21, 3, 12 Tom's hours: 12, 18, 15, 16, 14, 12, 15, 18

Find the measures of center and the measures of variation for each data set. Compare the measures. What can you conclude?

Cumulative Assessment Test-Taking Strategy Use Intelligent Guessing 1. What is the value of the expression below? What is the mean length of these hyena fangs: 4 in., 3 in., 3 in., 4 in., 5 in., 5 in.? $8\frac{4}{9} \div 4\frac{2}{3}$ $\mathbb{B}\frac{1}{3}$ ft $\mathbb{C}2$ in. **A.** $1\frac{17}{21}$ **C.** $32\frac{8}{27}$ **B.** $2\frac{2}{3}$ **D.** $39\frac{11}{27}$ 2. What is the value of the expression below? 4.18 + 6.225 + 5.7"The mean can't be 6 or 2 or 5 inches. So, you can use intelligent guessing to **F.** 15.005 **H.** 16.005 find that the answer is $\frac{1}{3}$ ft, or 4 in." **G.** 15.105 **I.** 16.105

3. One number is missing from the data set in the box below.



18, 24, 22, 30, 26, ____, 25

The median of the data set is 24. What is the greatest possible value of the missing number?

4. The number of hours that each of 6 students spent reading last week is shown in the bar graph below.



For the data in the bar graph, which measure is the *least*?

- A. mean C. mode
- B. median D. range

- **5.** You go to a beach and collect buckets of shells. Of the many shells you have collected, you notice the following.
 - 9% of the seashells are auger shells.
 - $\frac{1}{8}$ of the seashells are coquina shells.
 - 11% of the seashells are rough scallop shells.
 - 0.1 of the seashells are fighting conch shells.

Which list correctly shows the types of shells in order from least to greatest?

- F. auger, coquina, rough scallop, fighting conch
- G. fighting conch, coquina, auger, rough scallop
- H. fighting conch, auger, rough scallop, coquina
- I. auger, fighting conch, rough scallop, coquina
- **6.** What is the mean absolute deviation of the data shown in the line plot, rounded to the nearest tenth?



7. A family wants to buy tickets to a theme park. There are separate ticket prices for adults and children.



Which expression represents the total cost, in dollars, for *a* adult tickets and *c* child tickets?

F.	600(a+c)	Н.	30a + 20c
G.	$50(a \times c)$	I.	$30a \times 20c$

8. What is the value of the expression below?



 $52.8 \div 0.16$

9. What is the value of the expression below when a = 6 and b = 14?

0.8a + 0.02b

- **A.** 0.4828 **C.** 5.08
- **B.** 0.8814 **D.** 16.4
- **10.** Which property was *not* used in the box below to simplify the expression?

$$0.3 \times 53 + 53 \times 0.7 = 53 \times 0.3 + 53 \times 0.7$$
$$= 53 \times (0.3 + 0.7)$$
$$= 53 \times 1$$
$$= 53$$

- **F.** Distributive Property
- G. Associative Property of Addition
- H. Identity Property of Multiplication
- I. Commutative Property of Multiplication
- **11.** Determine a data set of 5 numbers that has the following measures:



- a mean of 7 and
- a median of 9.

Explain how you determined your data set. Then demonstrate that the mean of your data set is 7 and the median is 9.

12. What is the value of the expression below?

25% of 400

- **A.** 16 **C.** 1000
- **B.** 100 **D.** 10,000