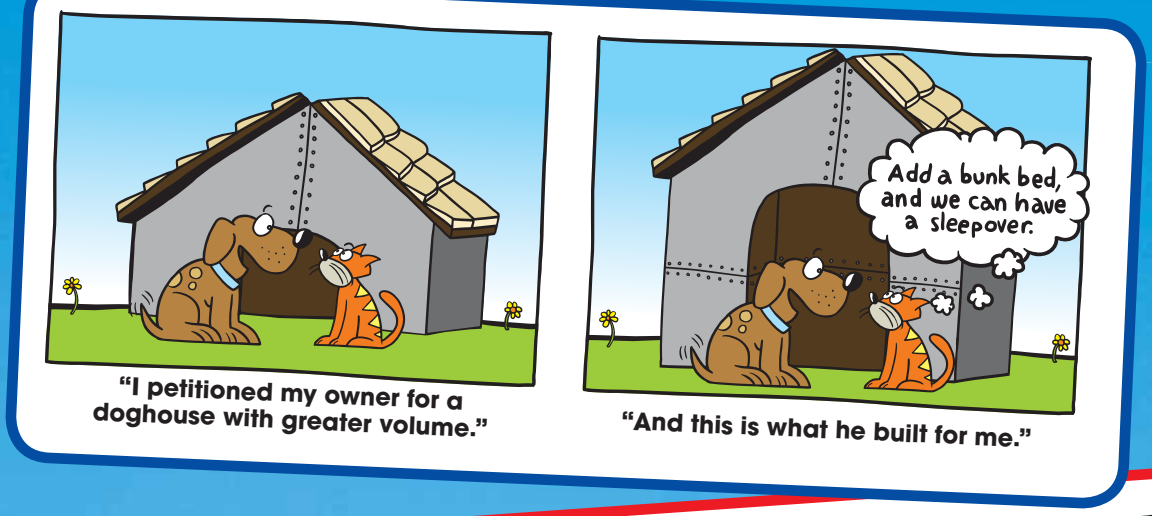
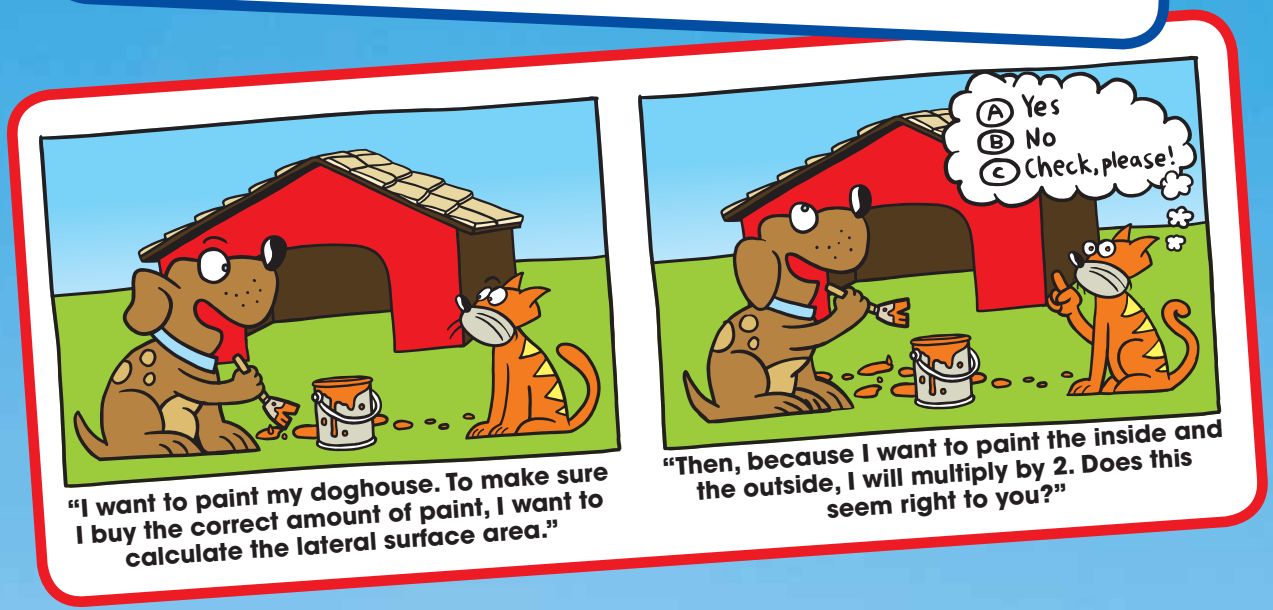
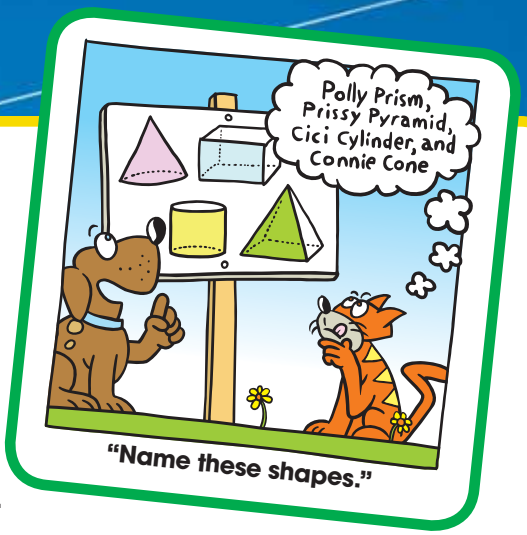
8 Surface Area and Volume





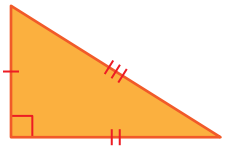
What You Learned Before



Classifying Figures

Example 1

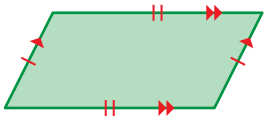
Identify the figure.



∴ Because the figure has a right angle and three sides of different lengths, it is a right scalene triangle.

Example 2

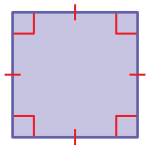
Identify the figure.

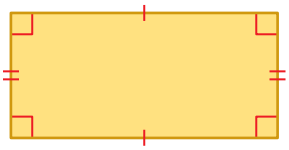


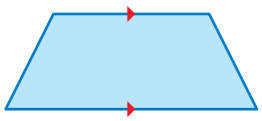
∴ Because the figure is a quadrilateral with opposite sides that are parallel, it is a parallelogram.

Try It Yourself

Identify the figure.

1. 

2. 

3. 

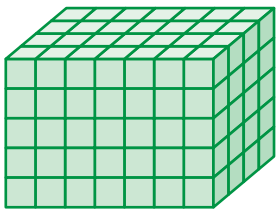
Finding Volumes of Rectangular Prisms

Example 3

Find the volume of the rectangular prism.

There are $4 \times 7 = 28$ unit cubes in each layer.

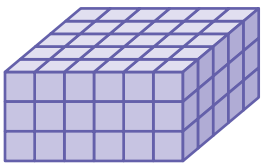
Because there are 5 layers, there are $5 \times 28 = 140$ unit cubes in the prism.

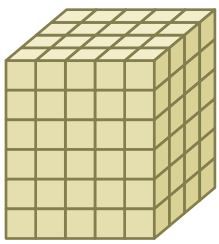


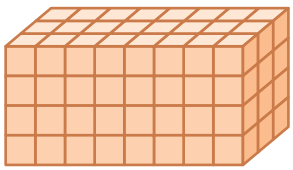
∴ So, the volume is 140 cubic units.

Try It Yourself

Find the volume of the rectangular prism.

4. 

5. 

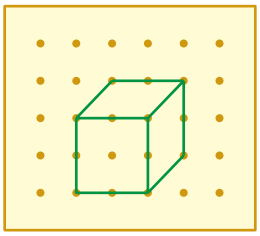
6. 

8.1 Three-Dimensional Figures

Essential Question How can you draw three-dimensional figures?

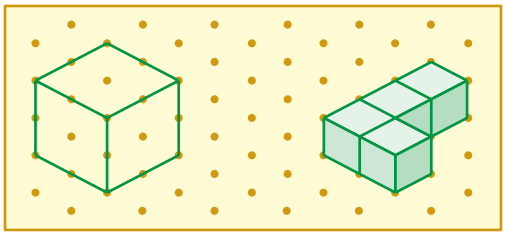
Dot paper can help you draw three-dimensional figures, or *solids*.

Square Dot Paper



Face-On view

Isometric Dot Paper

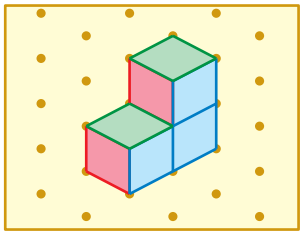


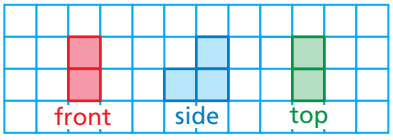
Corner view

1 ACTIVITY: Drawing Views of a Solid

Work with a partner. Draw the front, side, and top views of each stack of cubes. Then find the number of cubes in the stack.

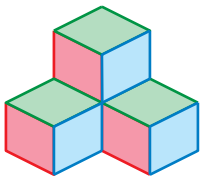
a. Sample:



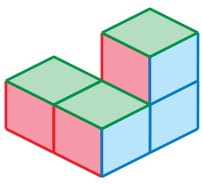


Number of cubes: 3

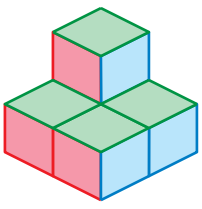
b.



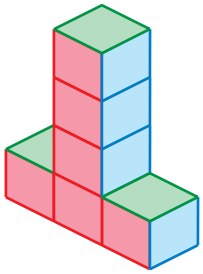
c.



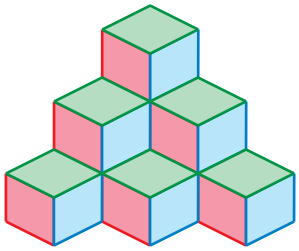
d.



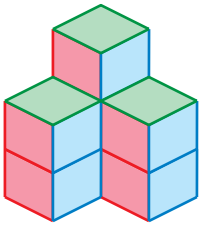
e.



f.



g.



Geometry

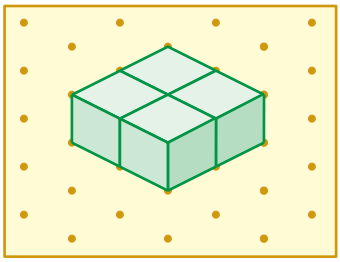
In this lesson, you will

- draw three-dimensional figures.
- find the number of faces, edges, and vertices of solids.

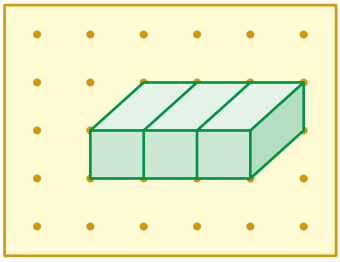
2 ACTIVITY: Drawing Solids

Work with a partner.

a. Use isometric dot paper to draw three different solids that use the same number of cubes as the solid at the right.

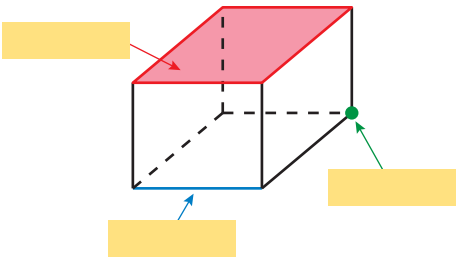


b. Use square dot paper to draw a different solid that uses the same number of *prisms* as the solid at the right.



3 ACTIVITY: Exploring Faces, Edges, and Vertices

Work with a partner. Use the solid shown.



a. Match each word to the figure. Then write a definition for each word.

face *edge* *vertex*

b. Identify the number of faces, edges, and vertices in a rectangular prism.

c. When using dot paper to draw a solid, what represents the vertices? How do you draw edges? How do you draw faces?

d. What do you think it means for lines or planes to be parallel or perpendicular in three dimensions? Use drawings to identify one pair of each of the following:

- parallel faces
- parallel edges
- edge parallel to a face
- perpendicular faces
- perpendicular edges
- edge perpendicular to a face

Math Practice

View as Components

What are the different parts of a three-dimensional object? How can dot paper help you draw the parts of the object?

What Is Your Answer?

4. IN YOUR OWN WORDS How can you draw three-dimensional figures?

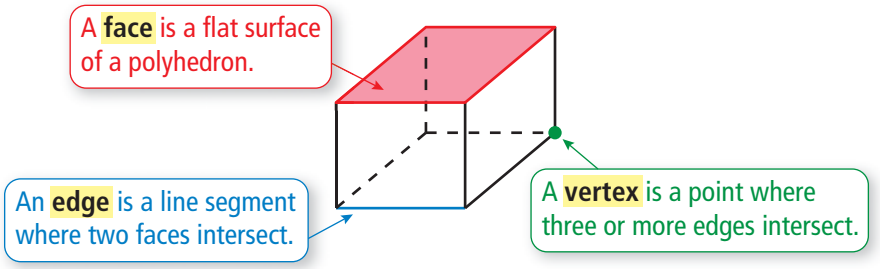
Practice Use what you learned about three-dimensional figures to complete Exercises 7–9 on page 358.

8.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

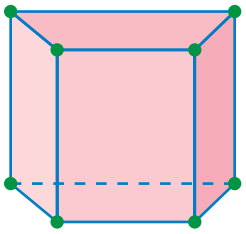
Key Vocabulary
solid, *p. 356*
polyhedron, *p. 356*
face, *p. 356*
edge, *p. 356*
vertex, *p. 356*
prism, *p. 356*
pyramid, *p. 356*

A **solid** is a three-dimensional figure that encloses a space. A **polyhedron** is a solid whose *faces* are all polygons.



EXAMPLE 1 Finding the Number of Faces, Edges, and Vertices

Find the number of faces, edges, and vertices of the solid.



The solid has 1 face on the bottom, 1 face on the top, and 4 faces on the sides.

The faces intersect at 12 different line segments.

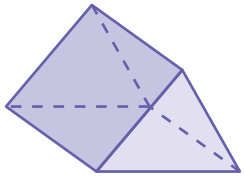
The edges intersect at 8 different points.

∴ So, the solid has 6 faces, 12 edges, and 8 vertices.

On Your Own

Now You're Ready
Exercises 10–12

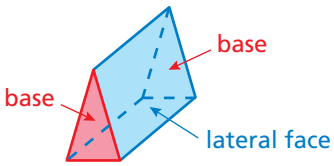
1. Find the number of faces, edges, and vertices of the solid.



Key Ideas

Prisms

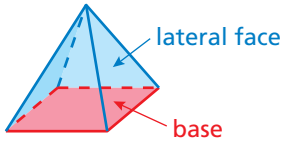
A **prism** is a polyhedron that has two parallel, identical *bases*. The *lateral faces* are parallelograms.



Triangular Prism

Pyramids

A **pyramid** is a polyhedron that has one base. The lateral faces are triangles.



Rectangular Pyramid

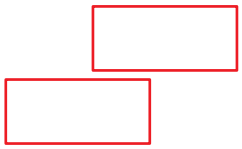
The shape of the base tells the name of the prism or the pyramid.

EXAMPLE 2 Drawing Solids

a. Draw a rectangular prism.

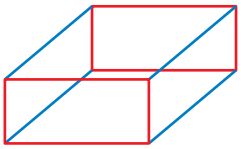
Step 1:

Draw identical rectangular bases.



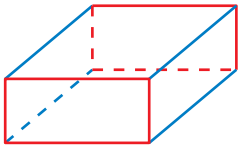
Step 2:

Connect corresponding vertices.



Step 3:

Change any *hidden* lines to dashed lines.



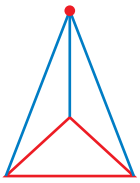
b. Draw a triangular pyramid.

Step 1:

Draw a triangular base and a point.

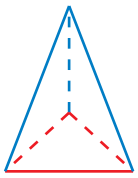
Step 2:

Connect the vertices of the triangle to the point.

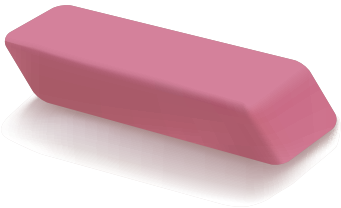


Step 3:

Change any *hidden* lines to dashed lines.



EXAMPLE 3 Drawing Views of a Solid



Draw the front, side, and top views of the eraser.

The front view is a parallelogram.

The side view is a rectangle.

The top view is a rectangle.

On Your Own

Now You're Ready
Exercises 13–22

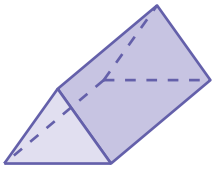
Draw the solid.

2. square prism

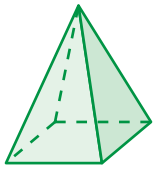
3. pentagonal pyramid

Draw the front, side, and top views of the solid.

4.



5.



8.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

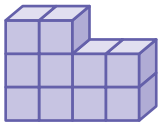
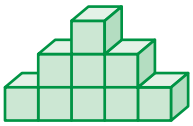
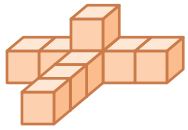
Vocabulary and Concept Check

LOGIC **Decide whether the statement is *true* or *false*. If false, explain your reasoning.**

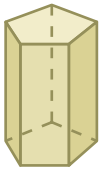
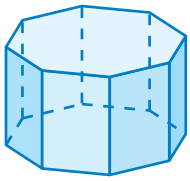
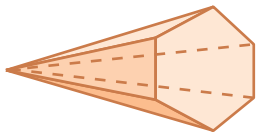
1. A triangular prism has three triangular faces.
2. A triangular prism has three rectangular faces.
3. A rectangular pyramid has one rectangular face.
4. A rectangular pyramid has three triangular faces.
5. All of the edges of a rectangular prism are parallel.
6. None of the edges of a rectangular pyramid are parallel.

Practice and Problem Solving

Draw the front, side, and top views of the stack of cubes. Then find the number of cubes in the stack.

7. 
8. 
9. 

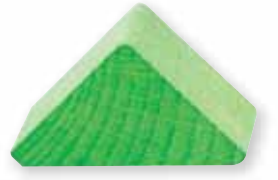
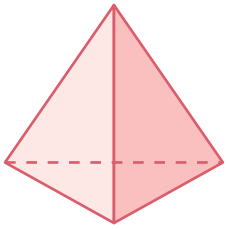
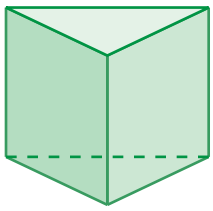
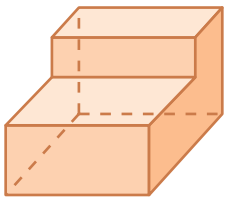
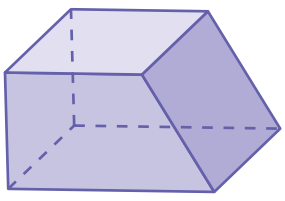
Find the number of faces, edges, and vertices of the solid.

1 10. 
11. 
12. 

Draw the solid.

2 13. triangular prism
14. pentagonal prism
15. rectangular pyramid
16. hexagonal pyramid

Draw the front, side, and top views of the solid.

3 17. 
18. 
19. 
20. 
21. 
22. 

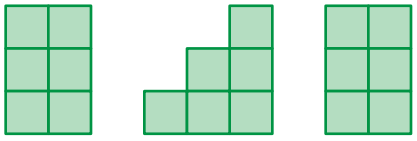
23. **PYRAMID ARENA** The Pyramid of Caius Cestius in Rome, Italy, is in the shape of a square pyramid. Draw a sketch of the pyramid.

24. **RESEARCH** Use the Internet to find a picture of the Washington Monument. Describe its shape.

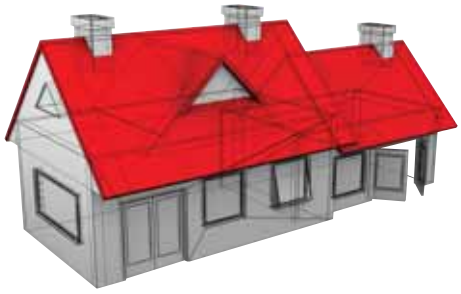
Draw a solid with the following front, side, and top views.

25.

26.



27. **PROJECT** Design and draw a house. Name the different solids that you can use to make a model of the house.

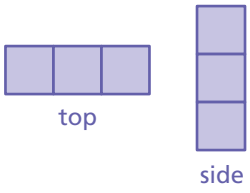


28. **REASONING** Two of the three views of a solid are shown.

a. What is the greatest number of unit cubes in the solid?

b. What is the least number of unit cubes in the solid?

c. Draw the front views of both solids in parts (a) and (b).



29. Reasoning Draw two different solids with five faces.

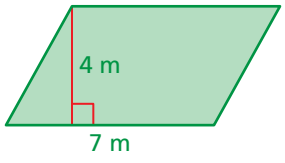
a. Write the number of vertices and edges for each solid.

b. Explain how knowing the numbers of edges and vertices helps you draw a three-dimensional figure.

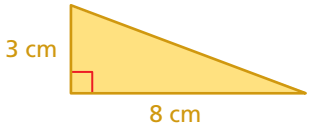
Fair Game Review What you learned in previous grades & lessons

Find the area of the figure. *(Section 4.1, Section 4.2, and Section 4.3)*

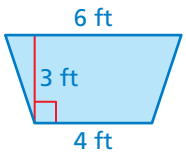
30.



31.



32.



33. **MULTIPLE CHOICE** Which statement is true when $x = -2$ and $y = |-2|$? *(Section 6.4)*

(A) $x = y$ (B) $y < 0$ (C) $x > y$ (D) $y > x$

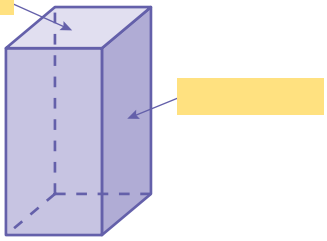
8.2 Surface Areas of Prisms

Essential Question How can you find the area of the entire surface of a prism?

1 ACTIVITY: Identifying Prisms

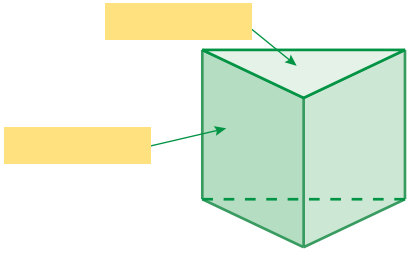
Work with a partern. Label one of the faces as a "base" and the other as a "lateral face." Use the shape of the base to identify the prism.

a.



_____ **Prism**

b.



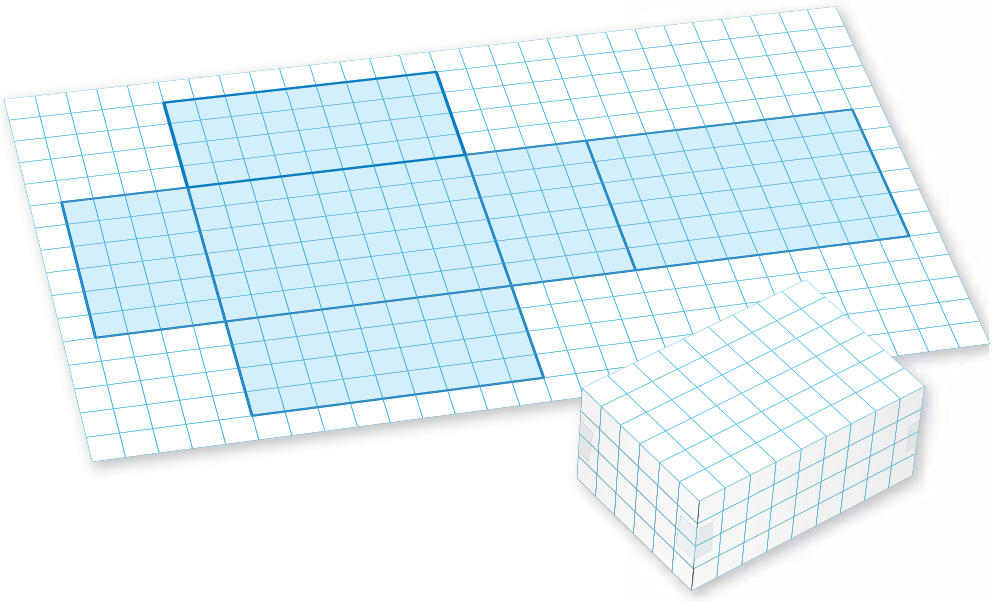
_____ **Prism**

2 ACTIVITY: Using Grid Paper to Construct a Prism

Work with a partner.

a. Copy the figure shown below onto grid paper.

b. Cut out the figure and fold it to form a prism. What type of prism does it form?



Geometry

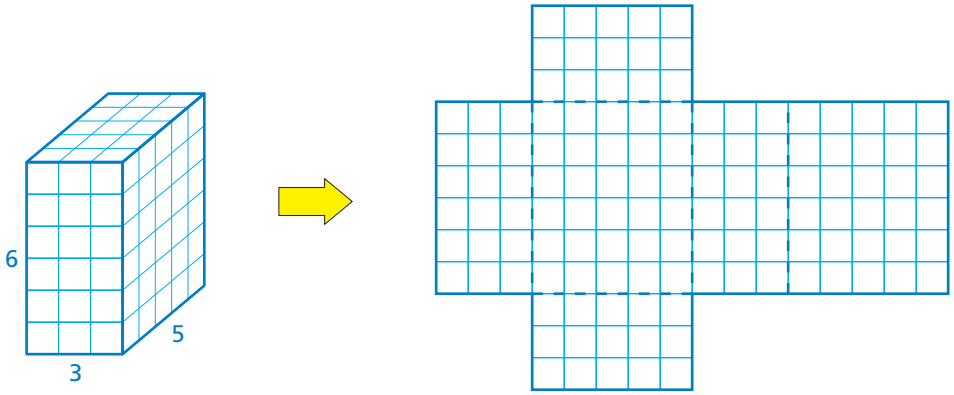
In this lesson, you will

- use nets to represent prisms.
- find the surface area of prisms.
- solve real-life problems.

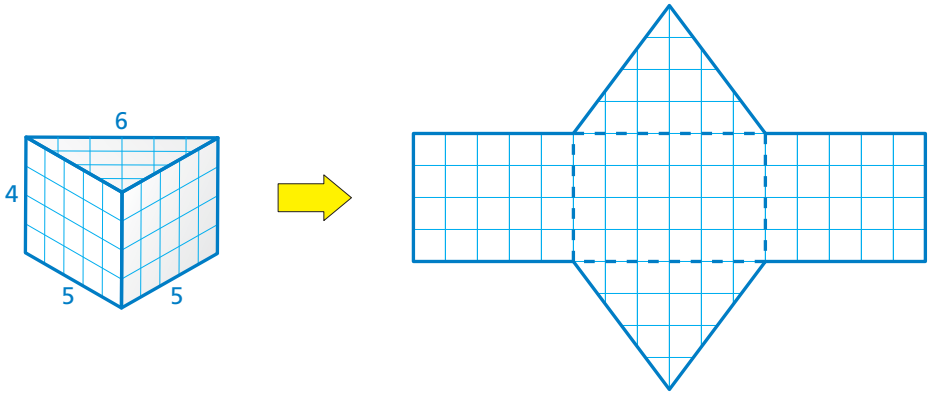
3 ACTIVITY: Finding the Area of the Entire Surface of a Prism

Work with a partner. Label each face in the two-dimensional representation of the prism as a "base" or a "lateral face." Then find the area of the entire surface of each prism.

a.



b.



Math Practice

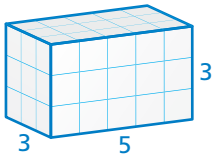
Repeat Calculations

When finding the areas of the faces, what calculations do you repeat?

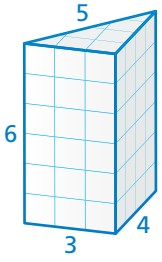
4 ACTIVITY: Drawing Two-Dimensional Representations of Prisms

Work with a partner. Draw a two-dimensional representation of each prism. Then find the area of the entire surface of each prism.

a.



b.



What Is Your Answer?

5. IN YOUR OWN WORDS How can you find the area of the entire surface of a prism?

Practice Use what you learned about the area of the entire surface of a prism to complete Exercises 3–5 on page 364.

8.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

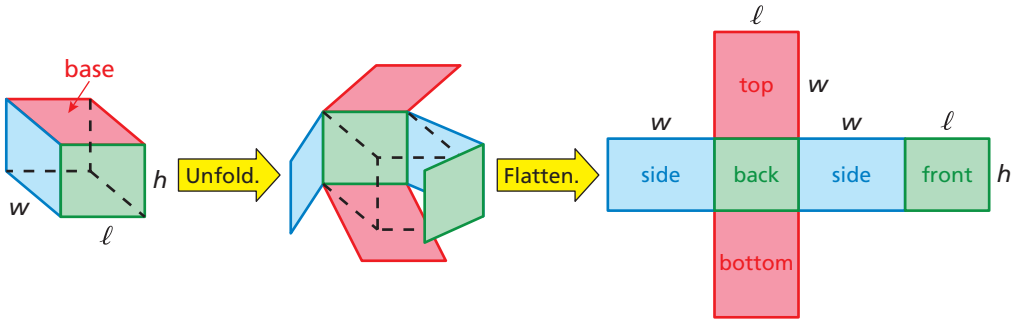
Key Vocabulary
surface area, *p. 362*
net, *p. 362*

The **surface area** of a solid is the sum of the areas of all of its faces. You can use a two-dimensional representation of a solid, called a **net**, to find the surface area of the solid. Surface area is measured in *square units*.

Key Idea

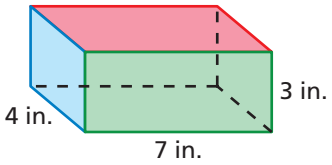
Net of a Rectangular Prism

A *rectangular prism* is a prism with rectangular bases.



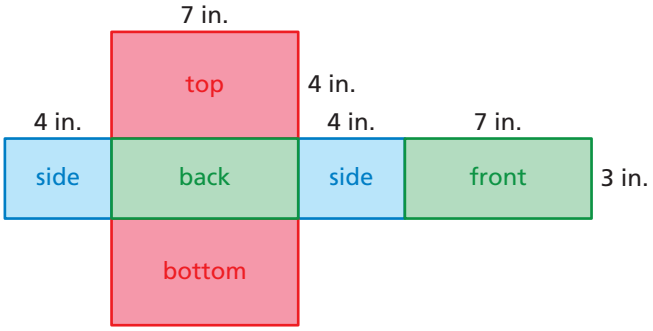
EXAMPLE 1 Finding the Surface Area of a Rectangular Prism

Find the surface area of the rectangular prism.



Use a net to find the area of each face.

Top: $7 \cdot 4 = 28$
Bottom: $7 \cdot 4 = 28$
Front: $7 \cdot 3 = 21$
Back: $7 \cdot 3 = 21$
Side: $4 \cdot 3 = 12$
Side: $4 \cdot 3 = 12$



Find the sum of the areas of the faces.

$$\begin{aligned} \text{Surface Area} &= \text{Area of top} + \text{Area of bottom} + \text{Area of front} + \text{Area of back} + \text{Area of a side} + \text{Area of a side} \\ S &= 28 + 28 + 21 + 21 + 12 + 12 \\ &= 122 \end{aligned}$$

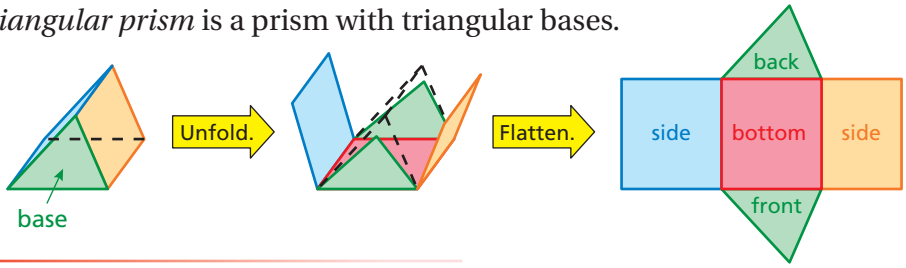
∴ So, the surface area is 122 square inches.

Multi-Language Glossary at BigIdeasMath.com

Key Idea

Net of a Triangular Prism

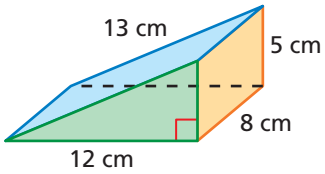
A *triangular prism* is a prism with triangular bases.



Remember

The area A of a triangle with base b and height h is $A = \frac{1}{2}bh$.

EXAMPLE 2 Finding the Surface Area of a Triangular Prism



Find the surface area of the triangular prism.

Use a net to find the area of each face.

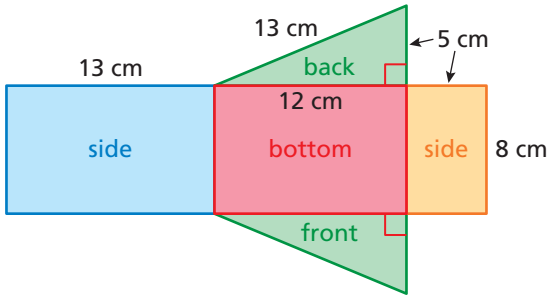
Bottom: $12 \cdot 8 = 96$

Front: $\frac{1}{2} \cdot 12 \cdot 5 = 30$

Back: $\frac{1}{2} \cdot 12 \cdot 5 = 30$

Side: $13 \cdot 8 = 104$

Side: $8 \cdot 5 = 40$



Find the sum of the areas of the faces.

$$\begin{array}{cccccccccccc} \text{Surface Area} & = & \text{Area of bottom} & + & \text{Area of front} & + & \text{Area of back} & + & \text{Area of a side} & + & \text{Area of a side} & \\ S & = & 96 & + & 30 & + & 30 & + & 104 & + & 40 & = 300 \end{array}$$

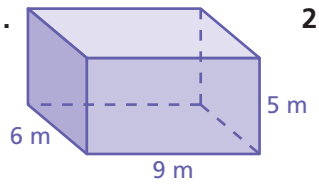
∴ So, the surface area is 300 square centimeters.

On Your Own

Now You're Ready
Exercises 6–11

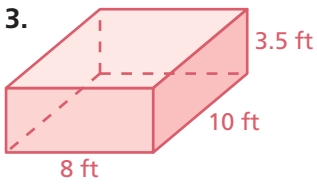
Find the surface area of the rectangular prism.

1.



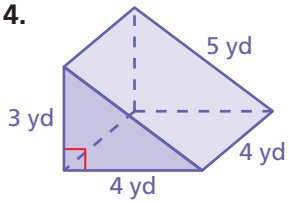
2.

3.

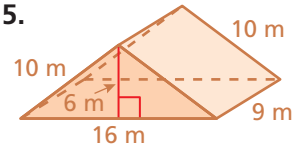


Find the surface area of the triangular prism.

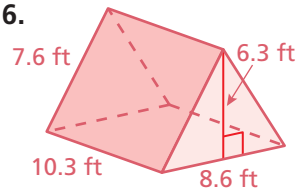
4.



5.



6.



8.2 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** Explain how to find the surface area of a prism.

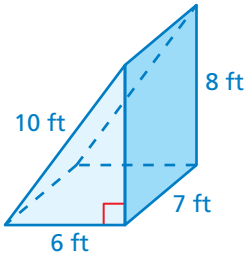
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

   What is the sum of the areas of the faces of the prism?

   What is the area of the entire surface of the prism?

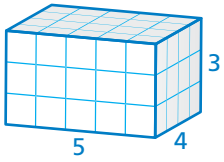
   What is the area of the triangular faces of the prism?

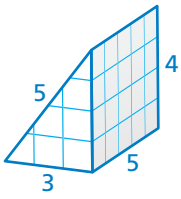
   What is the surface area of the prism?

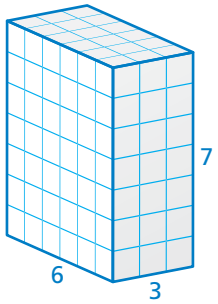


Practice and Problem Solving

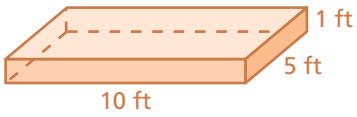
Draw a two-dimensional representation of the prism. Then find the area of the entire surface of the prism.

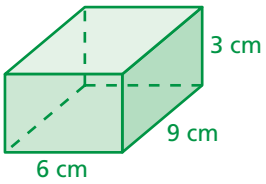
3. 

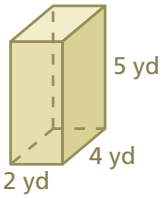
4. 

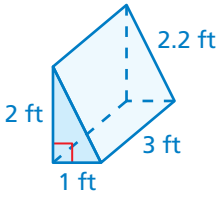
5. 

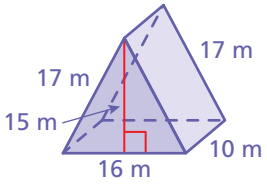
Find the surface area of the prism.

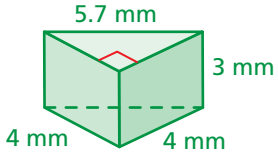
1 6. 

7. 

8. 

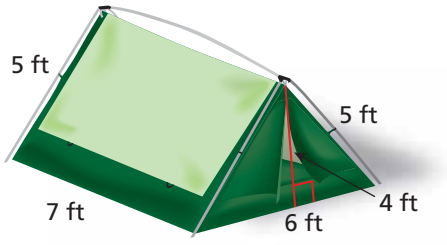
2 9. 

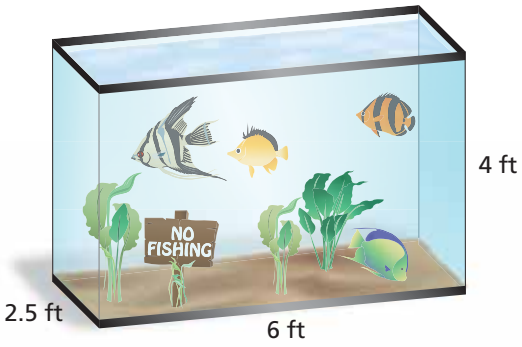
10. 

11. 

12. **GIFT BOX** A gift box in the shape of a rectangular prism measures 8 inches by 8 inches by 10 inches. What is the least amount of wrapping paper needed to wrap the gift box? Explain.

13. **TENT** What is the least amount of fabric needed to make the tent?

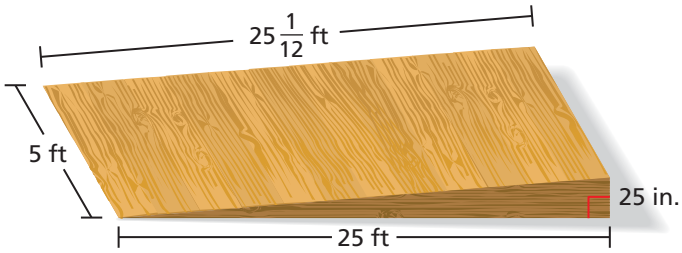




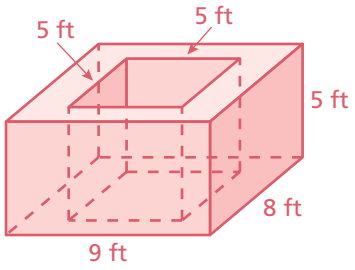
14. **AQUARIUM** A public library has an aquarium in the shape of a rectangular prism. The base is 6 feet by 2.5 feet. The height is 4 feet. How many square feet of glass were used to build the aquarium? (The top of the aquarium is open.)

15. **STORAGE BOX** The material used to make a storage box costs $1.25 per square foot. The boxes have the same volume. How much does a company save by choosing to make 50 of Box 2 instead of 50 of Box 1?

	Length	Width	Height
Box 1	20 in.	6 in.	4 in.
Box 2	15 in.	4 in.	8 in.



16. **RAMP** A quart of stain covers 100 square feet. How many quarts should you buy to stain the wheelchair ramp? (Assume you do not have to stain the bottom of the ramp.)

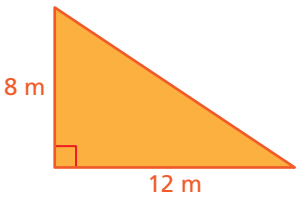


17. Critical Thinking A cube is removed from a rectangular prism. Find the surface area of the figure after removing the cube.

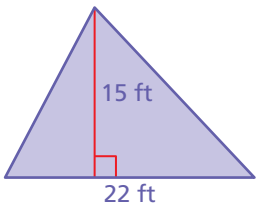
Fair Game Review What you learned in previous grades & lessons

Find the area of the triangle. *(Section 4.2)*

18.



19.



20.

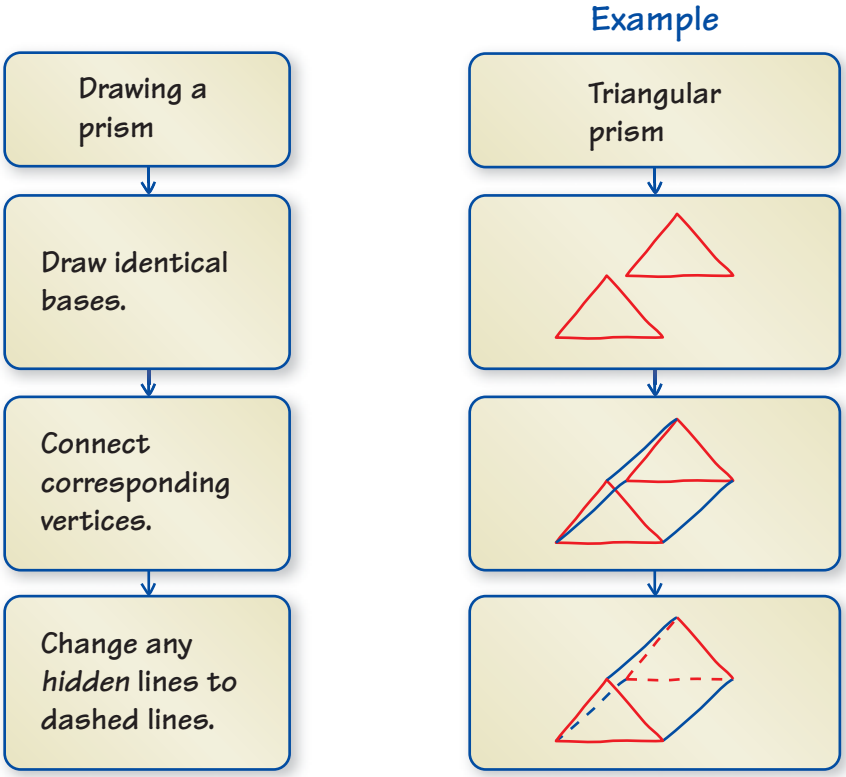
21. **MULTIPLE CHOICE** Which value is *not* a solution of the inequality $x - 4 \geq 2$? *(Section 7.5)*

(A) $x = 10$ (B) $x = 6$ (C) $x = 4$ (D) $x = 14$

8 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use a **process diagram** to show the steps involved in a procedure. Here is an example of a process diagram for drawing a prism.



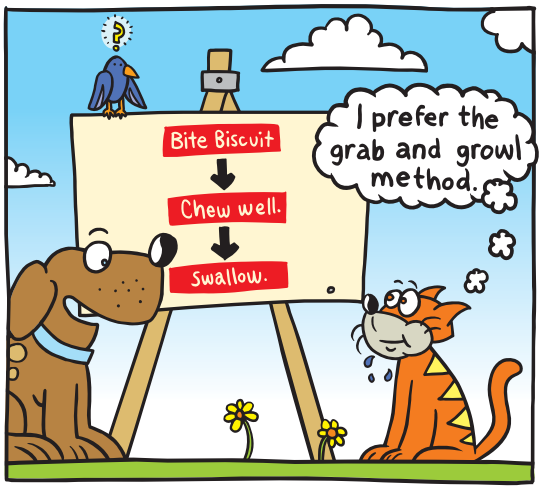
On Your Own

Make process diagrams with examples to help you study these topics.

1. drawing a pyramid
2. finding the surface area of a prism

After you complete this chapter, make process diagrams with examples for the following topics.

3. finding the surface area of a pyramid
4. finding the volume of a rectangular prism



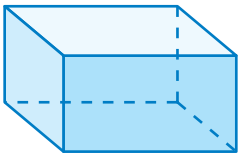
"Descartes, you should use my process diagram when you eat your treats."

8.1–8.2 Quiz

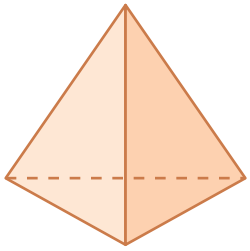
Check It Out
Progress Check
BigIdeasMath.com

Find the number of faces, edges, and vertices of the solid. *(Section 8.1)*

1.



2.



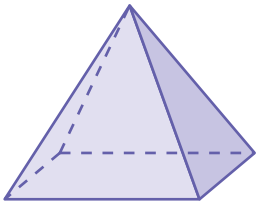
Draw the solid. *(Section 8.1)*

3. trapezoidal prism

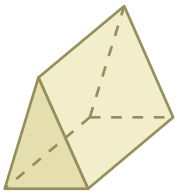
4. octagonal pyramid

Draw the front, side, and top views of the solid. *(Section 8.1)*

5.

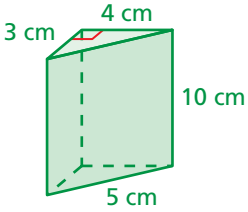


6.



Find the surface area of the prism. *(Section 8.2)*

7.



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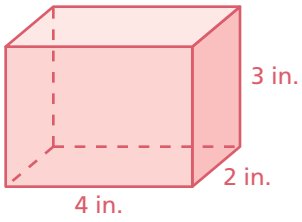


9. **CEREAL** A cereal box has the dimensions shown. *(Section 8.2)*

a. Find the surface area of the cereal box.

b. The manufacturer decides to decrease the size of the box by reducing each of the dimensions by 1 inch. Find the decrease in surface area.

10. **GIFT BOX** Find the surface area of the gift box. *(Section 8.2)*

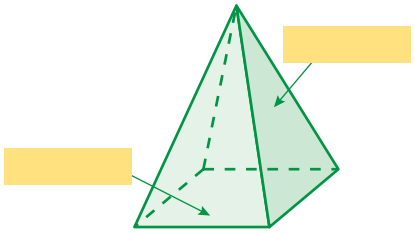
8.3 Surface Areas of Pyramids

Essential Question How can you use a net to find the surface area of a pyramid?

1 ACTIVITY: Identifying Pyramids

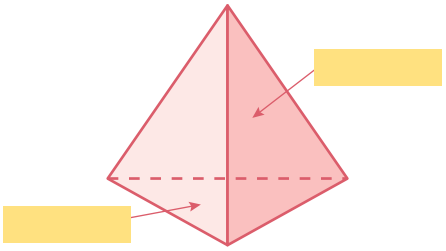
Work with a partner. Label one of the faces as a "base" and the other as a "lateral face." Use the shape of the base to identify the pyramid.

a.



_____ **Pyramid**

b.



_____ **Pyramid**

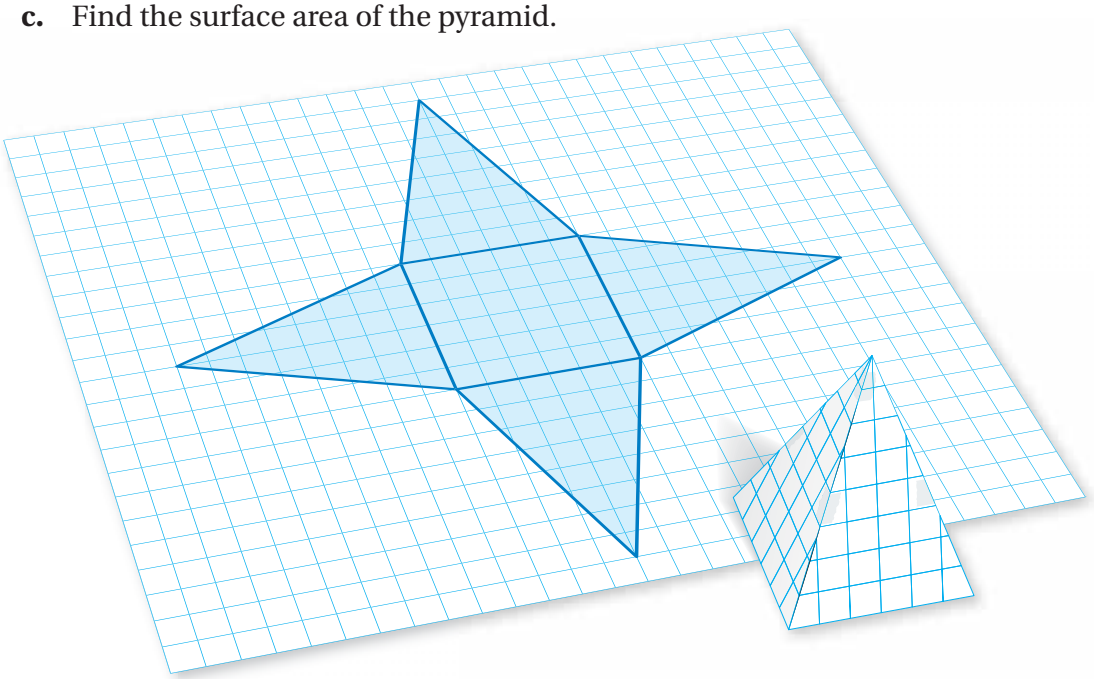
2 ACTIVITY: Using a Net

Work with a partner.

a. Copy the net shown below onto grid paper.

b. Cut out the net and fold it to form a pyramid. What type of rectangle is the base? Use this shape to name the pyramid.

c. Find the surface area of the pyramid.



Geometry

In this lesson, you will

- use nets to represent pyramids.
- find the surface area of pyramids.
- solve real-life problems.

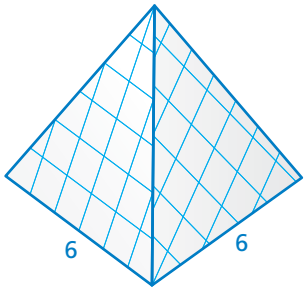
3 ACTIVITY: Estimating the Surface Area of a Triangular Pyramid

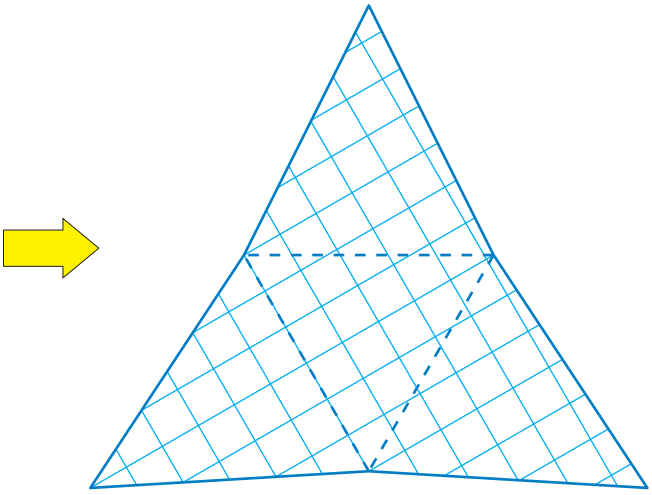
Work with a partner. Label each face in the net of the triangular pyramid as a "base" or a "lateral face." Then estimate the surface area of the pyramid.

Math Practice

Analyze Givens

What information is given in the diagram? How does this help you estimate the surface area of the pyramid?

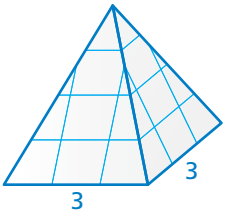




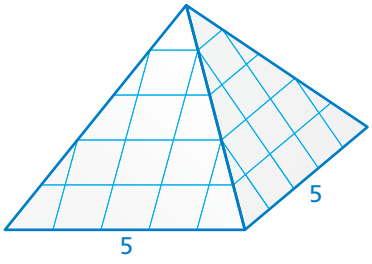
4 ACTIVITY: Finding the Surface Area of a Square Pyramid

Work with a partner. Draw a net for each square pyramid. Use the net to find the surface area of the pyramid.

a.



b.



What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use a net to find the surface area of a pyramid?

6. **CONJECTURE** Make a conjecture about the lateral faces of a pyramid when the side lengths of the base have the same measure. Explain.

Practice Use what you learned about the surface area of a pyramid to complete Exercises 3–5 on page 372.

8.3 Lesson

Check It Out
Lesson Tutorials
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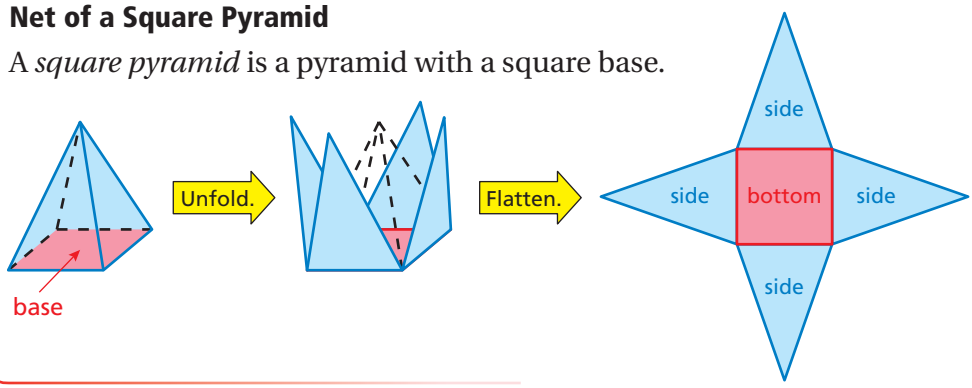
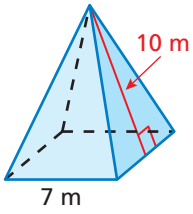
Key Idea

Net of a Square Pyramid

A *square pyramid* is a pyramid with a square base.



EXAMPLE 1 Finding the Surface Area of a Square Pyramid



Find the surface area of the square pyramid.

Use a net to find the area of each face.

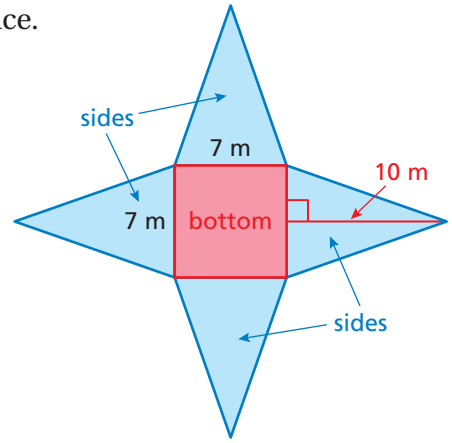
Bottom: $7 \cdot 7 = 49$

Side: $\frac{1}{2} \cdot 7 \cdot 10 = 35$

Side: $\frac{1}{2} \cdot 7 \cdot 10 = 35$

Side: $\frac{1}{2} \cdot 7 \cdot 10 = 35$

Side: $\frac{1}{2} \cdot 7 \cdot 10 = 35$



Find the sum of the areas of the faces.

$$\begin{array}{rcccccccccc} \text{Surface Area} & = & \text{Area of bottom} & + & \text{Area of a side} & + & \text{Area of a side} & + & \text{Area of a side} & + & \text{Area of a side} \\ S & = & 49 & + & 35 & + & 35 & + & 35 & + & 35 = 189 \end{array}$$

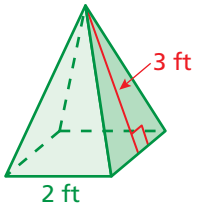
∴ So, the surface area is 189 square meters.

On Your Own

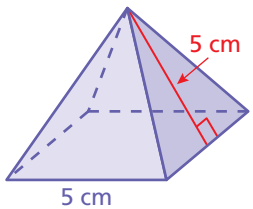
Now You're Ready
Exercises 6–8

Find the surface area of the square pyramid.

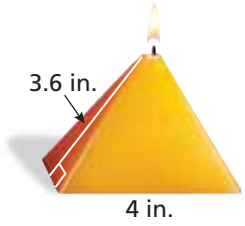
1.



2.



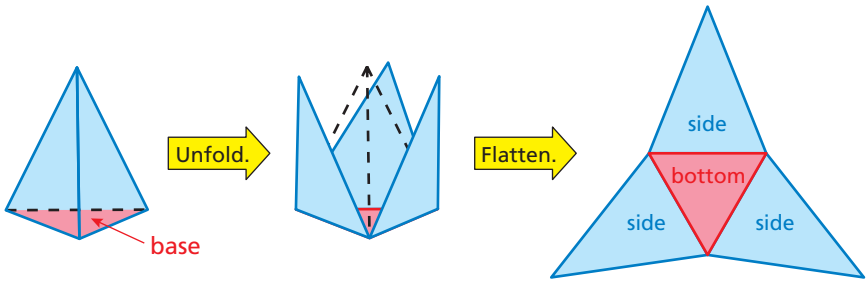
3.



Key Idea

Net of a Triangular Pyramid

A *triangular pyramid* is a pyramid with a triangular base.

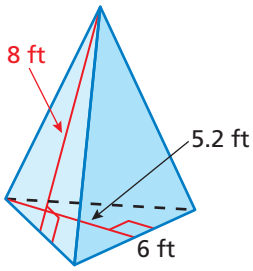


Study Tip

The base of each triangular pyramid in this section is an equilateral triangle.

EXAMPLE 2 Finding the Surface Area of a Triangular Pyramid

Find the surface area of the triangular pyramid.



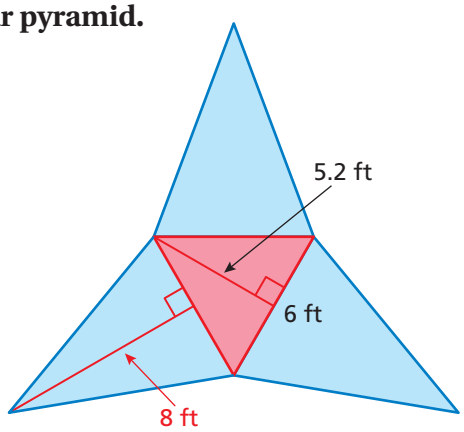
Use a net to find the area of each face.

Bottom: $\frac{1}{2} \cdot 6 \cdot 5.2 = 15.6$

Side: $\frac{1}{2} \cdot 6 \cdot 8 = 24$

Side: $\frac{1}{2} \cdot 6 \cdot 8 = 24$

Side: $\frac{1}{2} \cdot 6 \cdot 8 = 24$



Find the sum of the areas of the faces.

$$\text{Surface Area} = \text{Area of bottom} + \text{Area of a side} + \text{Area of a side} + \text{Area of a side}$$

$$S = 15.6 + 24 + 24 + 24$$

$$= 87.6$$

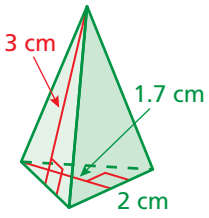
∴ So, the surface area is 87.6 square feet.

On Your Own

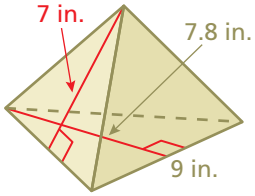
Now You're Ready
Exercises 9–11

Find the surface area of the triangular pyramid.

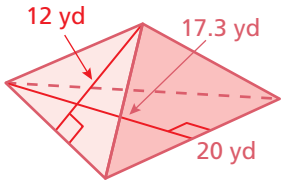
4.



5.



6.



8.3 Exercises

Check It Out
Help with Homework
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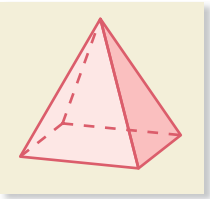
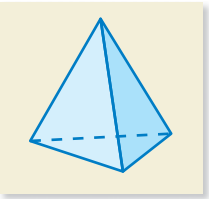
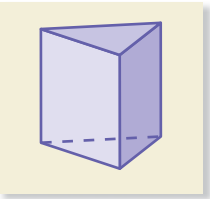
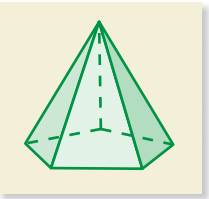
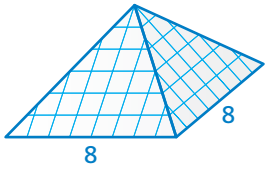
Vocabulary and Concept Check

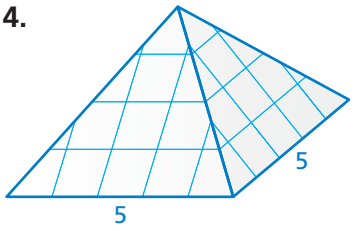
1. **PRECISION** Explain how to find the surface area of a pyramid.
2. **WHICH ONE DOESN'T BELONG?** Which figure does *not* belong with the other three? Explain your reasoning.

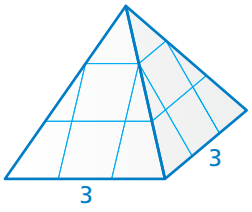





Practice and Problem Solving

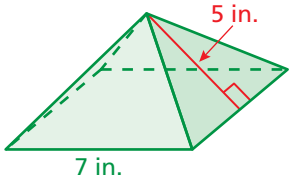
Draw a net of the square pyramid. Then find the surface area of the pyramid.

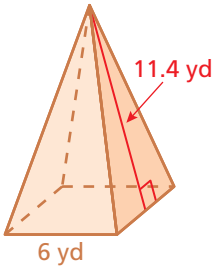
3. 

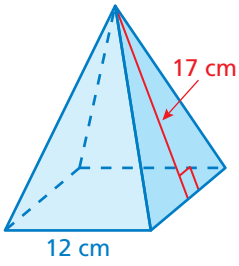
4. 

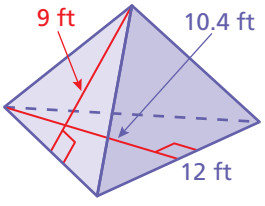
5. 

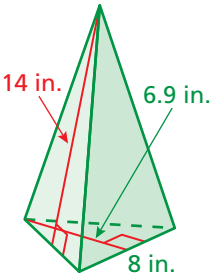
Find the surface area of the pyramid. The side lengths of the base are equal.

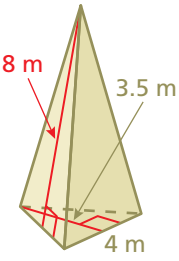
1 6. 

7. 

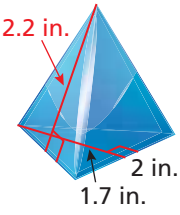
8. 

2 9. 

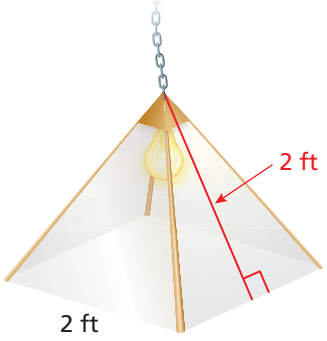
10. 

11. 

12. **PAPERWEIGHT** A paperweight is shaped like a triangular pyramid. The base is an equilateral triangle. Find the surface area of the paperweight.

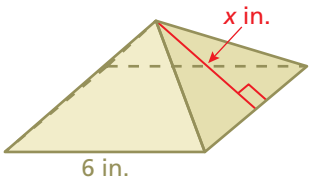


13. LOUVRE The entrance to the Louvre Museum in Paris, France, is a square pyramid. The side length of the base is 116 feet, and the height of one of the triangular faces is 91.7 feet. Find the surface area of the four triangular faces of the entrance to the Louvre Museum.

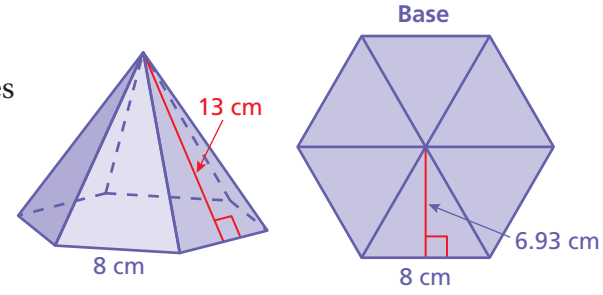


14. LIGHT COVER A hanging light cover made of glass is shaped like a square pyramid. The cover does not have a bottom. One square foot of the glass weighs 2.45 pounds. The chain can support 35 pounds. Will the chain support the light cover? Explain.

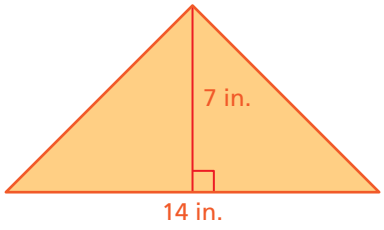
15. GEOMETRY The surface area of a square pyramid is 84 square inches. The side length of the base is 6 inches. What is the value of x?



16. STRUCTURE In the diagram of the base of the hexagonal pyramid, all the triangles are the same. Find the surface area of the hexagonal pyramid.



17. Critical Thinking Can you form a square pyramid using four of the triangles shown? Explain your reasoning.



Fair Game Review What you learned in previous grades & lessons

Find the missing values in the ratio table. Then write the equivalent ratios. *(Section 5.2)*

18.

Frogs	7		28
Turtles	3	6	

19.

Apples	10	5	
Oranges	4		12

20. MULTIPLE CHOICE Which ordered pair is in Quadrant III? *(Section 6.5)*

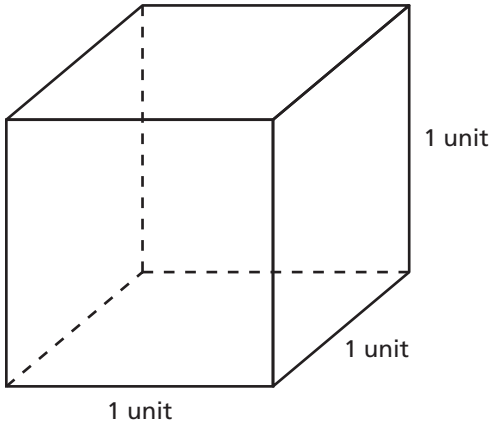
Ⓐ $(5, -1)$ Ⓑ $(-2, -3)$ Ⓒ $(2, 4)$ Ⓓ $(-7, 1)$

8.4 Volumes of Rectangular Prisms

Essential Question How can you find the volume of a rectangular prism with fractional edge lengths?

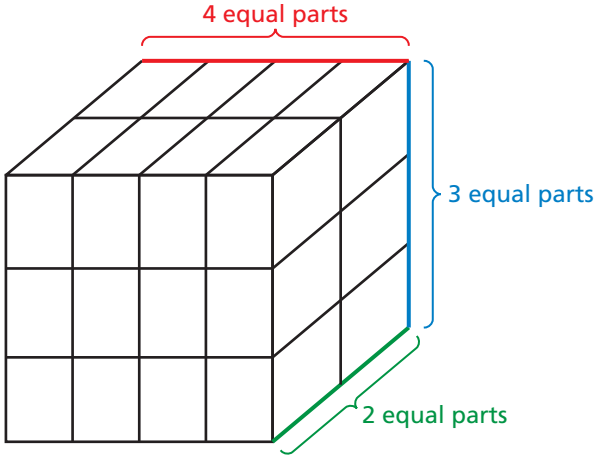
Recall that the **volume** of a three-dimensional figure is a measure of the amount of space that it occupies. Volume is measured in *cubic units*.

A *unit cube* is a cube with an edge length of 1 unit.



1 ACTIVITY: Using a Unit Cube

Work with a partner. The parallel edges of the unit cube have been divided into 2, 3, and 4 equal parts to create smaller rectangular prisms that are identical.



a. Draw one of these identical prisms and label its dimensions.

b. What fraction of the volume of the unit cube does one of these identical prisms represent? Use this value to find the volume of one of the identical prisms. Explain your reasoning.

Geometry

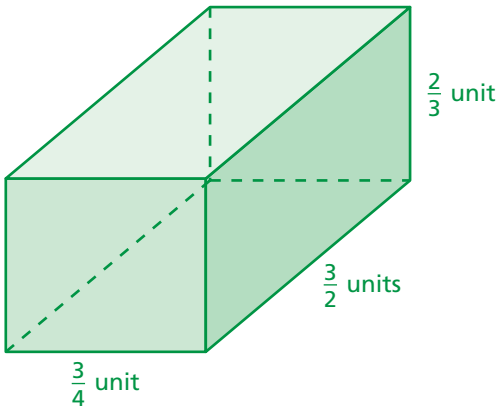
In this lesson, you will

- find the volume of prisms with fractional edge lengths by using models.
- find the volume of prisms by using formulas.

2 ACTIVITY: Finding the Volume of a Rectangular Prism

Work with a partner.

a. How many of the identical prisms in Activity 1(a) does it take to fill the rectangular prism below? Support your answer with a drawing.



b. Use the volume of one of the identical prisms in Activity 1(a) to find the volume of the rectangular prism above. Explain your reasoning.

Math Practice

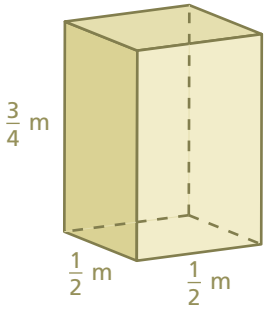
Analyze Relationships

What is the relationship between the solid shown here and the solid in the previous activity?

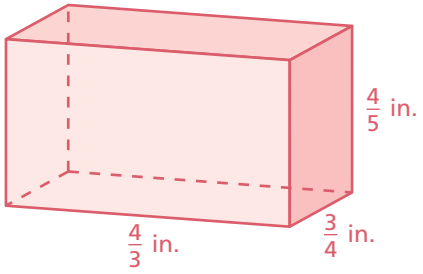
3 ACTIVITY: Finding the Volumes of Rectangular Prisms

Work with a partner. Explain how you can use the procedure in Activities 1 and 2 to find the volume of each rectangular prism. Then find the volume of each prism.

a.



b.



What Is Your Answer?

4. You have used the formulas $V = Bh$ and $V = \ell wh$ to find the volume V of a rectangular prism with whole number edge lengths. Do you think the formulas work for rectangular prisms with fractional edge lengths? Give examples with your answer.

5. **IN YOUR OWN WORDS** How can you find the volume of a rectangular prism with fractional edge lengths?

Practice Use what you learned about the volume of a rectangular prism to complete Exercises 4–6 on page 378.

8.4 Lesson

Check It Out
Lesson Tutorials
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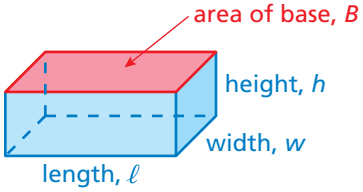
Key Vocabulary
volume, *p. 374*

Key Idea

Volume of a Rectangular Prism

Words The volume V of a rectangular prism is the product of the area of the base and the height of the prism.

Algebra $V = Bh$ or $V = \ell wh$



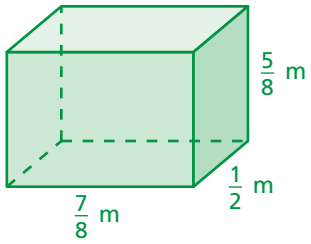
EXAMPLE 1 Finding Volumes of Rectangular Prisms

Find the volume of each prism.

Study Tip

In Example 1(b), the rectangular prism is a cube. You can use the formula $V = s^3$ to find the volume V of a cube with an edge length of s.

a.



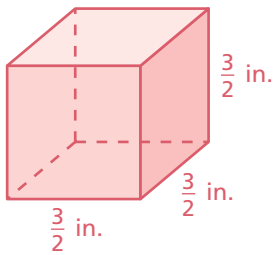
$V = \ell wh$ Write formula.

$= \frac{7}{8}\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)$ Substitute values.

$= \frac{35}{128}$ Multiply.

So, the volume is $\frac{35}{128}$ cubic meter.

b.



$V = \ell wh$ Write formula.

$= \frac{3}{2}\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)$ Substitute values.

$= \frac{27}{8}$ Multiply.

$= 3\frac{3}{8}$

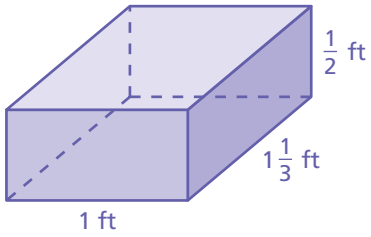
So, the volume is $3\frac{3}{8}$ cubic inches.

On Your Own

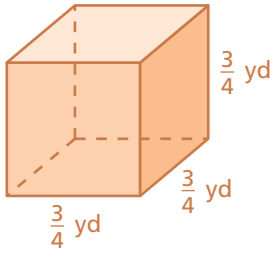
Now You're Ready
Exercises 4–9

Find the volume of the prism.

1.



2.



Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Using the Volume of a Rectangular Prism

One cubic foot of dirt weighs about 70 pounds. How many pounds of dirt can the dump truck haul when it is full?

Find the volume of dirt that the dump truck can haul when it is full.

$$\begin{aligned} V &= \ell wh && \text{Write formula for volume.} \\ &= 17(8)\left(4\tfrac{3}{4}\right) && \text{Substitute values.} \\ &= 646 && \text{Multiply.} \end{aligned}$$

So, the dump truck can haul 646 cubic feet of dirt when it is full. To find the weight of the dirt, multiply by $\frac{70 \text{ lb}}{1 \text{ ft}^3}$.

$$646 \text{ ft}^3 \times \frac{70 \text{ lb}}{1 \text{ ft}^3} = 45{,}220 \text{ lb}$$

∴ The dump truck can haul about 45,220 pounds of dirt when it is full.

EXAMPLE 3 Finding a Missing Dimension of a Rectangular Prism

Write and solve an equation to find the height of the computer tower.

Volume = 1792 in.^{3}

$$\begin{aligned} V &= \ell wh && \text{Write formula for volume.} \\ 1792 &= 16(7)h && \text{Substitute values.} \\ 1792 &= 112h && \text{Simplify.} \\ \frac{1792}{112} &= \frac{112h}{112} && \text{Division Property of Equality} \\ 16 &= h && \text{Simplify.} \end{aligned}$$

∴ So, the height of the computer tower is 16 inches.

On Your Own

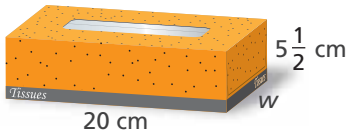
3. **WHAT IF?** In Example 2, the length of the dump truck is 20 feet. How many pounds of dirt can the dump truck haul when it is full?

Now You're Ready
Exercises 10–12

Write and solve an equation to find the missing dimension of the prism.

4. Volume = 72 in.^{3}

5. Volume = 1375 cm^3



8.4 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

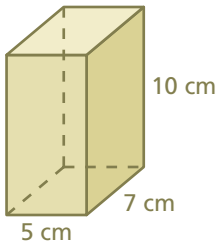
1. **CRITICAL THINKING** Explain how volume and surface area are different.
2. **REASONING** Will the formulas for volume work for rectangular prisms with decimal edge lengths? Explain.
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

How much does it take to fill the rectangular prism?

What is the capacity of the rectangular prism?

How much does it take to cover the rectangular prism?

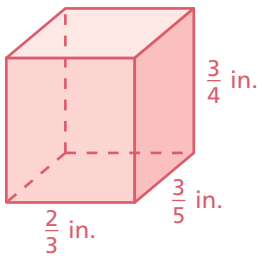
How much does the rectangular prism contain?



Practice and Problem Solving

Find the volume of the prism.

1 **4.**



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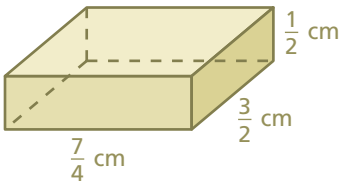
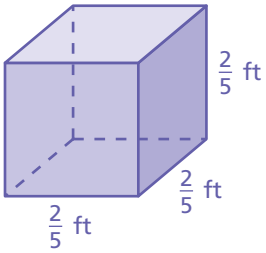


6.



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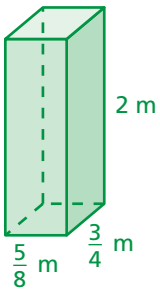

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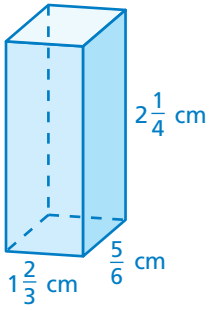
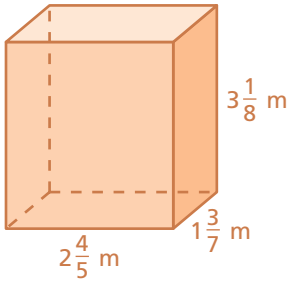
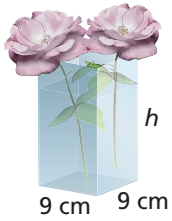


9.

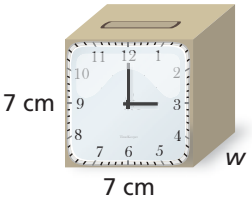


Write and solve an equation to find the missing dimension of the prism.

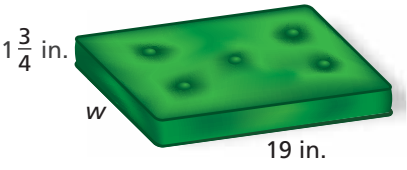
3 **10.** Volume = 1620 cm^3



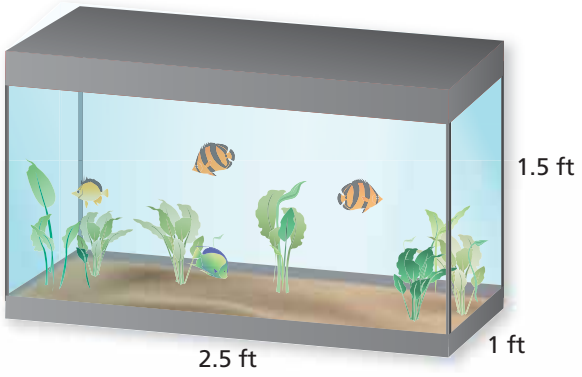
11. Volume = 220.5 cm^3



12. Volume = 532 $in.^3$



13. FISH TANK One cubic foot of water weighs about 62.4 pounds. How many pounds of water can the fish tank hold when it is full?



14. CUBE How many $\frac{3}{4}$-centimeter cubes do you need to create a cube with an edge length of 12 centimeters?

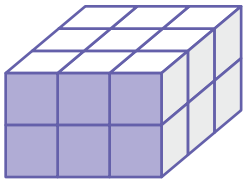
15. REASONING How many 1-inch cubes do you need to fill a cube that has an edge length of 1 foot? How can this result help you convert a volume from cubic inches to cubic feet? from cubic feet to cubic inches?

16. FOOD STORAGE

a. Estimate the amount of casserole left in the dish.

b. Will the casserole fit in the storage container? Explain your reasoning.

17. PROBLEM SOLVING The area of the shaded face is 96 square centimeters. What is the volume of the rectangular prism?



18. Project You have 1400 square feet of boards to use for a new tree house.

a. Design a tree house that has a volume of at least 250 cubic feet. Include sketches of your tree house.

b. Are your dimensions reasonable? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Tell whether the given value is a solution of the equation. *(Section 7.2)*

19. $x + 17 = 24;\ x = 7$ **20.** $\frac{x}{5} = 6;\ x = 35$ **21.** $x - 19 = 42;\ x = 21$

22. MULTIPLE CHOICE Which set of integers is ordered from least to greatest? *(Section 6.2)*

(A) $-1, 3, -5, -8, 12$ (B) $-1, 3, -5, -8, 12$

(C) $-4, -2, 1, 7, 10$ (D) $-14, -9, 6, -4, 2$

8.3–8.4 Quiz

Check It Out
Progress Check
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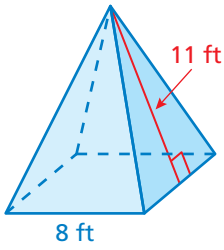
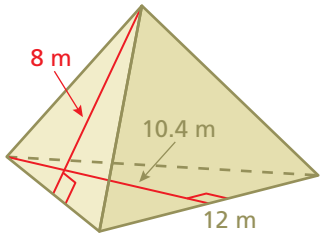
Find the surface area of the pyramid. The side lengths of the base are equal. *(Section 8.3)*

1.

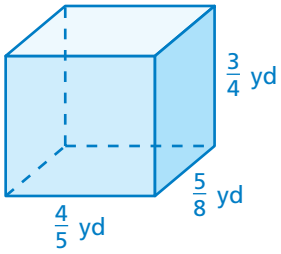


2.

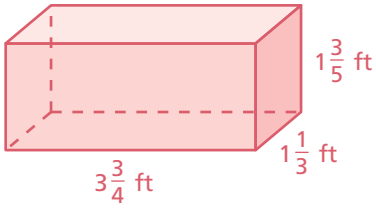


Find the volume of the prism. *(Section 8.4)*

3.

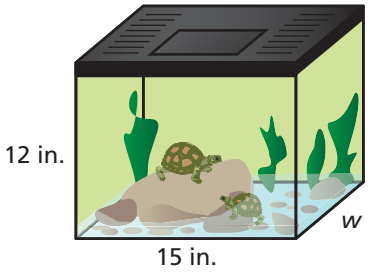


4.



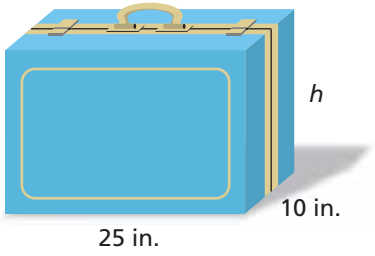
Write and solve an equation to find the missing dimension of the prism. *(Section 8.4)*

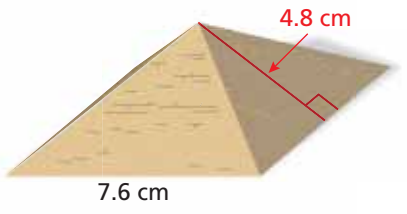
5. Volume = 1620 in.^{3}



6. Volume = 154 in.^{3}

7. Volume = 4250 in.^{3}

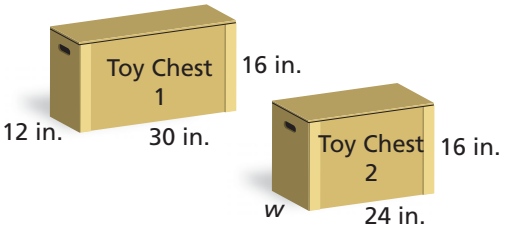




8. GREAT PYRAMID The Great Pyramid of Giza is a square pyramid. A gift shop sells miniature models of this pyramid. Find the surface area of the model shown at the left. *(Section 8.3)*

9. CUBE How many 1-inch cubes do you need to create a cube with an edge length of 7 inches? *(Section 8.4)*

10. TOY CHEST A toy company sells two different toy chests. The toy chests have different dimensions, but the same volume. What is the width w of Toy Chest 2? *(Section 8.4)*



8 Chapter Review

Check It Out
Vocabulary Help
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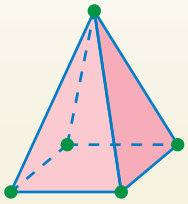
Review Key Vocabulary

solid, *p. 356*
polyhedron, *p. 356*
face, *p. 356*
edge, *p. 356*
vertex, *p. 356*
prism, *p. 356*
pyramid, *p. 356*
surface area, *p. 362*
net, *p. 362*
volume, *p. 374*

Review Examples and Exercises

8.1 Three-Dimensional Figures *(pp. 354–359)*

a. Find the number of faces, edges, and vertices of the solid.



The solid has 1 face on the bottom and 4 faces on the sides.

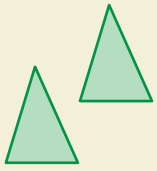
The faces intersect at 8 different line segments.

The edges intersect at 5 different points.

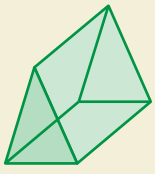
∴ So, the solid has 5 faces, 8 edges, and 5 vertices.

b. Draw a triangular prism.

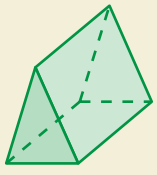
Draw identical triangular bases.



Connect corresponding vertices.



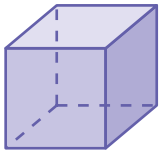
Change any *hidden* lines to dashed lines.



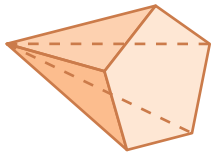
Exercises

Find the number of faces, edges, and vertices of the solid.

1.



2.



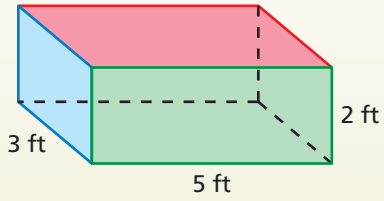
Draw the solid.

3. square pyramid

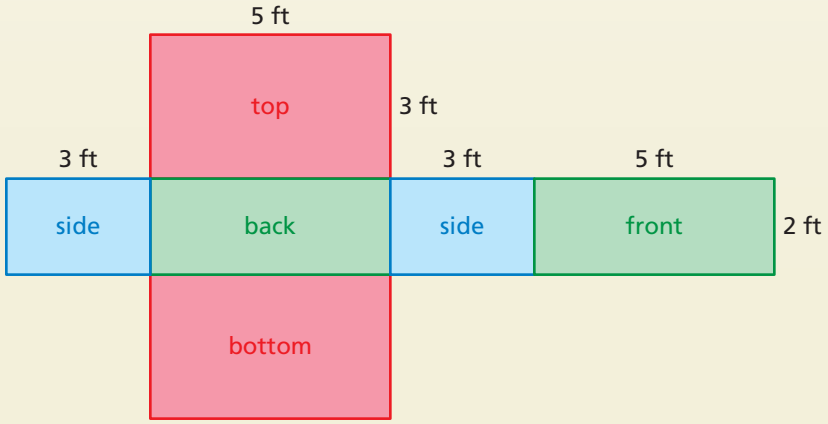
4. hexagonal prism

8.2 Surface Areas of Prisms *(pp. 360–365)*

Find the surface area of the rectangular prism.



Use a net to find the area of each face.



Top: $5 \cdot 3 = 15$ Front: $5 \cdot 2 = 10$ Side: $3 \cdot 2 = 6$

Bottom: $5 \cdot 3 = 15$ Back: $5 \cdot 2 = 10$ Side: $3 \cdot 2 = 6$

Find the sum of the areas of the faces.

$S = 15 + 15 + 10 + 10 + 6 + 6$

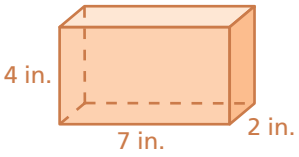
$= 62$

∴ The surface area is 62 square feet.

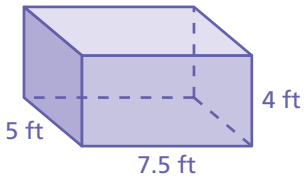
Exercises

Find the surface area of the prism.

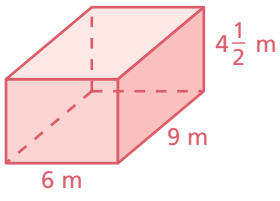
5.



6.



7.



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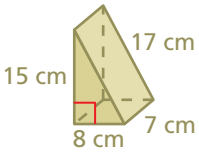
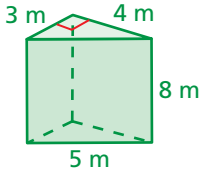
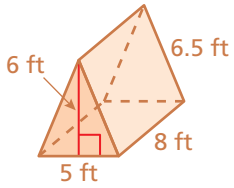


9.

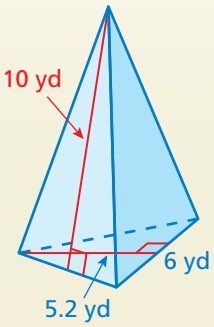


10.



8.3 Surface Areas of Pyramids *(pp. 368–373)*

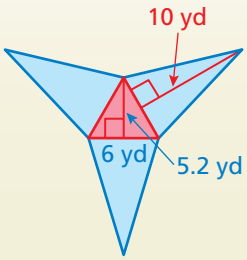
Find the surface area of the triangular pyramid.



Use a net to find the area of each face.

Bottom: $\frac{1}{2} \cdot 6 \cdot 5.2 = 15.6$ Side: $\frac{1}{2} \cdot 6 \cdot 10 = 30$

Side: $\frac{1}{2} \cdot 6 \cdot 10 = 30$ Side: $\frac{1}{2} \cdot 6 \cdot 10 = 30$



Find the sum of the areas of the faces.

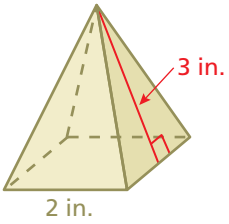
$S = 15.6 + 30 + 30 + 30 = 105.6$

∴ The surface area is 105.6 square yards.

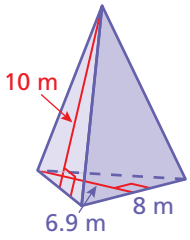
Exercises

Find the surface area of the pyramid. The side lengths of the base are equal.

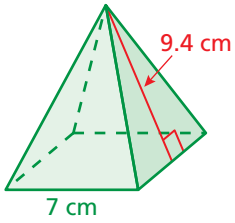
11.



12.



13.



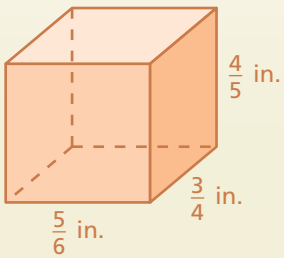
8.4 Volumes of Rectangular Prisms *(pp. 374–379)*

Find the volume of the prism.

$V = \ell wh$ Write formula for volume.

$= \frac{5}{6}\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)$ Substitute values.

$= \frac{1}{2}$ Multiply.

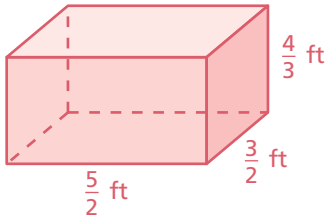


∴ The volume is $\frac{1}{2}$ cubic inch.

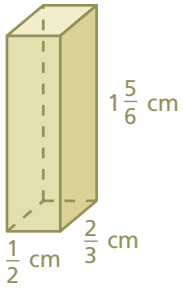
Exercises

Find the volume of the prism.

14.



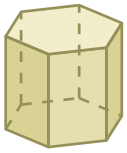
15.

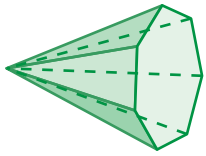


8 Chapter Test

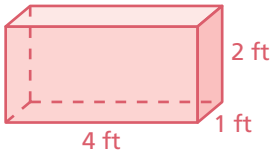
Check It Out
Test Practice
BigIdeasMath.com

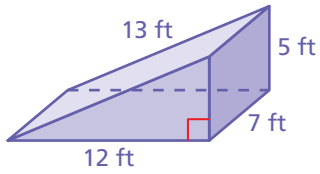
Find the number of faces, edges, and vertices of the solid.

1. 

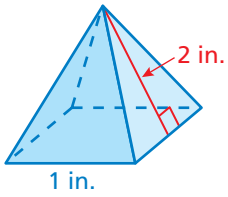
2. 

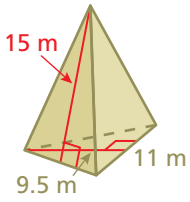
Find the surface area of the prism.

3. 

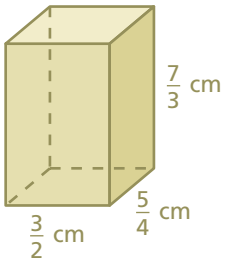
4. 

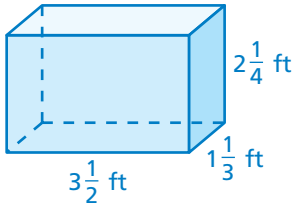
Find the surface area of the pyramid. The side lengths of the base are equal.

5. 

6. 

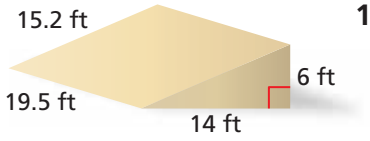
Find the volume of the prism.

7. 

8. 

9. **DRAWING A SOLID** Draw an octagonal prism.

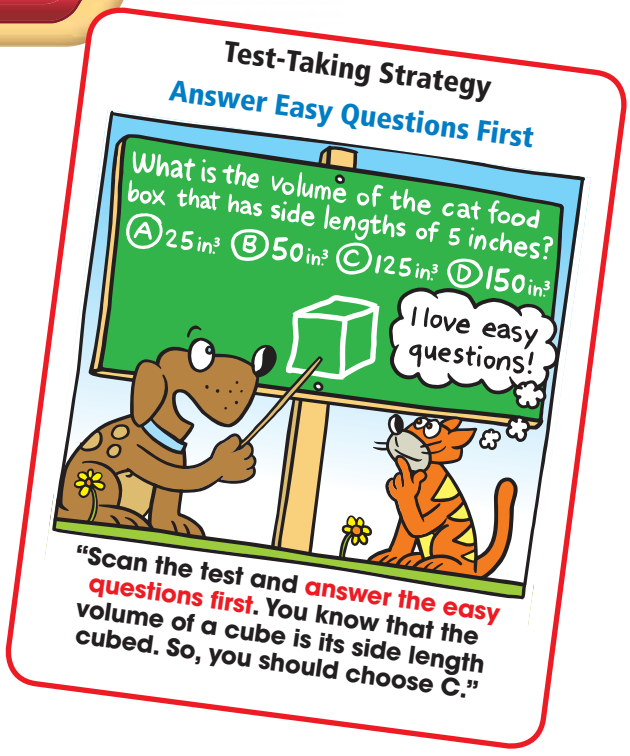
10. **DVD COLLECTION** You are wrapping the boxed DVD collection as a present. What is the least amount of wrapping paper needed to wrap the box?



11. **SKATEBOARD RAMP** A quart of paint covers 80 square feet. How many quarts should you buy to paint the ramp with two coats? (Assume you will not paint the bottom of the ramp.)

12. **CUBE** A cube has an edge length of 4 inches. You double the side lengths. How many times greater is the volume of the new cube?

8 Cumulative Assessment



1. The temperature in a town has never been above 38 degrees Fahrenheit. Let t represent the temperature, in degrees Fahrenheit. Which inequality represents the temperature in the town?

A. $t < 38$ **C.** $t > 38$

B. $t \leq 38$ **D.** $t \geq 38$

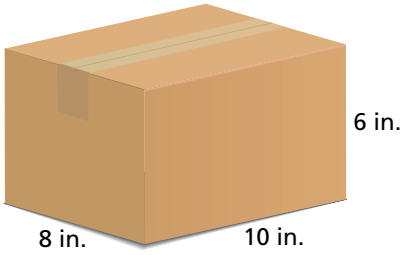
2. Which number is equivalent to the expression below?

$$3 \cdot 4^2 + 6 \div 2$$

F. 27 **H.** 51

G. 33 **I.** 75

3. What is the volume of the package shown below?



A. 240 in.^{3} **C.** 480 in.^{3}

B. 376 in.^{3} **D.** 960 in.^{3}

4. A housing community started with 60 homes. In each of the following years, 8 more homes were built. Let y represent the number of years that have passed since the first year, and let n represent the number of homes. Which equation describes the relationship between n and y?

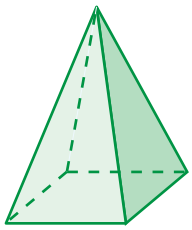
F. $n = 8y + 60$ **H.** $n = 60y + 8$

G. $n = 68y$ **I.** $n = 60 + 8 + y$

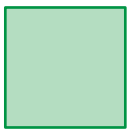
5. What is the value of m that makes the equation below true?

$$4m = 6$$

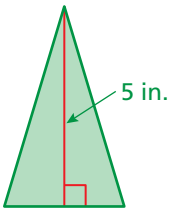
6. A square pyramid is shown below.



The square base and one of the triangular faces of the square pyramid are shown below with their dimensions.



Square Base



A Triangular Face

What is the total surface area of the square pyramid?

A. 16.5 in.^{2}

B. 31.5 in.^{2}

C. 39 in.^{2}

D. 69 in.^{2}

7. A wooden box has a length of 12 inches, a width of 6 inches, and a height of 8 inches.

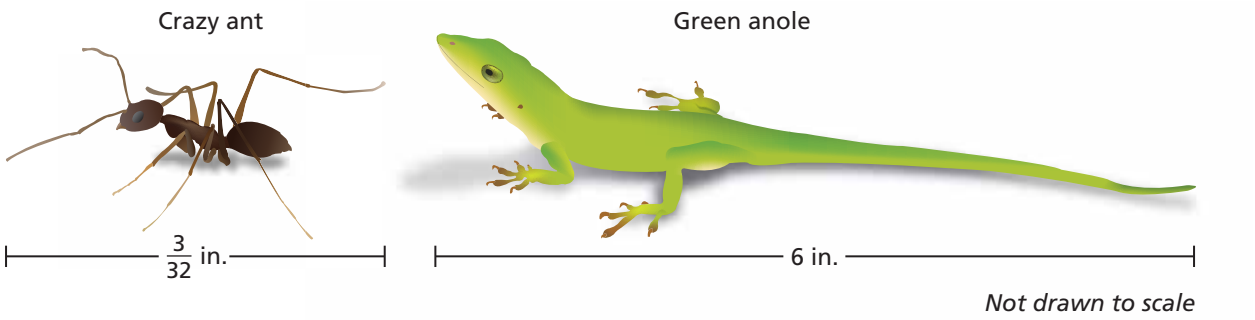
Think Solve Explain

Part A Draw and label a rectangular prism with the dimensions of the wooden box.

Part B What is the surface area, in square inches, of the wooden box? Show your work.

Part C You have a 2-ounce sample of wood stain that covers 900 square inches. Is this enough to give the entire box two coats of stain? Show your work and explain your reasoning.

8. A biologist measures the lengths of a crazy ant and a green anole that he has in his laboratory. His measurements are shown below.



The length of the green anole is how many times greater than the length of the crazy ant?

F. $\frac{9}{16}$

G. $5\frac{29}{32}$

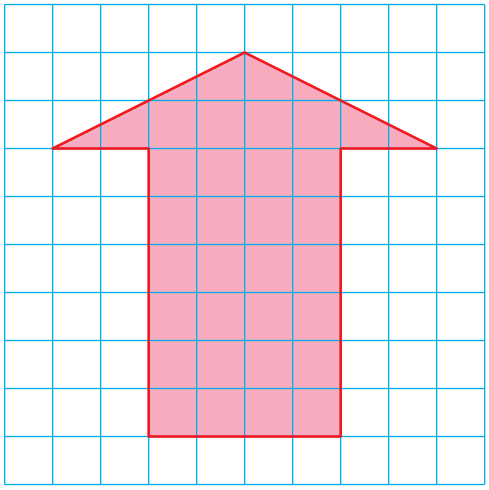
H. 16

I. 64

9. What is the missing value in the ratio table?

Castles	1	2		12
Towers	4	8	24	48

10. What is the area of the shaded figure shown below?



A. 32 units^2

B. 36 units^2

C. 40 units^2

D. 64 units^2