7 Equations and Inequalities

"What is 7*Q* plus 3*Q*?"

"Ten *Q*"

"You're welcome."

"With the help of your twin brother, I think I have figured it out."

"You weigh 36 dog biscuits."

"Can someone call the society for cruelty to cats?"

What You Learned Before

Evaluating Expressions

Example 1 Evaluate $7x + 3y$ when $x = 2$ and $y = 4$.

$$7x + 3y = 7 \cdot 2 + 3 \cdot 4 \qquad \text{Substitute 2 for } x \text{ and 4 for } y.$$
$$= 14 + 12 \qquad \text{Using order of operations, multiply from left to right.}$$
$$= 26 \qquad \text{Add 14 and 12.}$$

Example 2 Evaluate $5x^2 - 2(y + 1) + 9$ when $x = 2$ and $y = 1$.

$$5x^2 - 2(y + 1) + 9 = 5(2)^2 - 2(1 + 1) + 9 \qquad \text{Substitute 2 for } x \text{ and 1 for } y.$$
$$= 5(2)^2 - 2 \cdot 2 + 9 \qquad \text{Using order of operations, evaluate within the parentheses.}$$
$$= 5 \cdot 4 - 2 \cdot 2 + 9 \qquad \text{Using order of operations, evaluate the exponent.}$$
$$= 20 - 4 + 9 \qquad \text{Using order of operations, multiply from left to right.}$$
$$= 25 \qquad \text{Subtract 4 from 20. Add the result to 9.}$$

Try It Yourself

Evaluate the expression when $a = \dfrac{1}{2}$ and $b = 7$.

1. $6ab$ **2.** $16a - b$ **3.** $3b - 2a - 9$ **4.** $b^2 - 16a + 5$

Writing Expressions

Example 3 Write the phrase as an expression.

a. the sum of twice a number n and five

$$2n + 5$$

b. twelve less than four times a number y

$$4y - 12$$

Try It Yourself

Write the phrase as an expression.
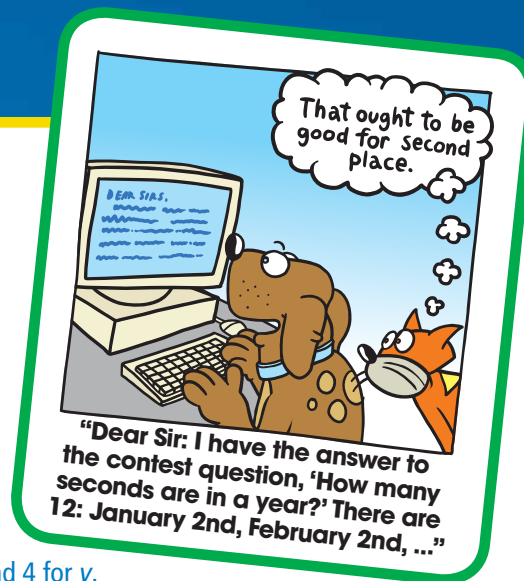
5. six more than three times a number w

6. the quotient of seven and a number p

7. two less than a number t

8. the product of a number x and five

9. five more than six divided by a number r

10. four less than three times a number b

Essential Question How does rewriting a word problem help you solve the word problem?

1 ACTIVITY: Rewriting a Word Problem

Work with a partner. Read the problem several times. Think about how you could rewrite the problem. Leave out information that you do not need to solve the problem.

Given Problem (63 words)

Your minivan has a flat, rectangular area in the back. When you fold down the rear seats of the van and move them forward, the width of the rectangular area in the van is increased by 2 feet, as shown in the diagram.
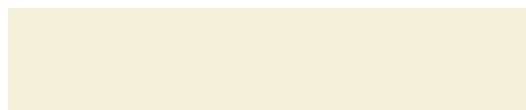
5 ft

⊢2 ft ⊢ 3 ft ⊣
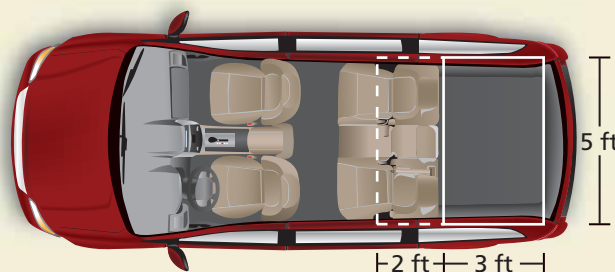
By how many square feet does the rectangular area increase when the rear seats are folded down and moved forward?

Rewritten Problem (28 words)

When you fold down the back seats of a minivan, the added area is a 5-foot by 2-foot rectangle. What is the area of this rectangle?

Can you make the problem even simpler?
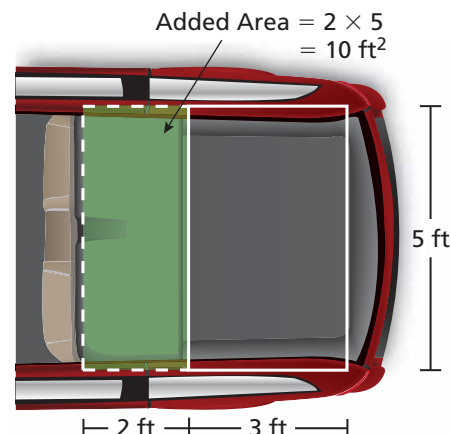
Rewriting Equations
In this lesson, you will
● write word sentences as equations.

Rewritten Problem (　words)

Added Area = 2 × 5 = 10 ft²

5 ft

⊢ 2 ft ⊢ 3 ft ⊣

Explain why your rewritten problem is easier to read.

Math
Practice

Analyze Givens
What information
do you need to
solve the problem?

2 **ACTIVITY: Rewriting a Word Problem**

Work with a partner. Rewrite each problem using fewer words. Leave out information that you do not need to solve the problem. Then solve the problem.

a. **(63 words)**

A supermarket is having its grand opening on Saturday morning. Every fifth customer will receive a $10 coupon for a free turkey. Every seventh customer will receive a $3 coupon for 2 gallons of ice cream. You are the manager of the store and you expect to have 400 customers. How many of each type of coupon should you plan to give away?

b. **(71 words)**

You and your friend are at a football game. The stadium is 4 miles from your home. You each brought $5 to spend on refreshments. During the third quarter of the game, you say, "I read that the greatest distance that a baseball has been thrown is 445 feet 10 inches." Your friend says, "That's about one and a half times the length of the football field." Is your friend correct?

c. **(90 words)**

You are visiting your cousin who lives in the city. To get back home, you take a taxi. The taxi charges $2.10 for the first mile and $0.90 for each additional mile. After riding 13 miles, you decide that the fare is going to be more than the $20 you have with you. So, you tell the driver to stop and let you out. Then you call a friend and ask your friend to come and pick you up. After paying the driver, how much of your $20 is left?

What Is Your Answer?

3. **IN YOUR OWN WORDS** How does rewriting a word problem help you solve the word problem? Make up a word problem that has more than 50 words. Then show how you can rewrite the problem using at most 25 words.

"Solving a math word problem is like making maple syrup."

"You need to boil down 40 gallons of sap from a sugar maple tree to get 1 gallon of syrup."

Sweet!

Practice

Use what you learned about writing equations to complete Exercises 4 and 5 on page 298.

Key Vocabulary
equation, *p. 296*

An **equation** is a mathematical sentence that uses an equal sign, =, to show that two expressions are equal.

Expressions	*Equations*
$4 + 8$	$4 + 8 = 12$
$x + 8$	$x + 8 = 12$

To write a word sentence as an equation, look for key words or phrases such as *is, the same as*, or *equals* to determine where to place the equal sign.

EXAMPLE 1 Writing Equations

Write the word sentence as an equation.

a. The sum of a number n and 7 is 15.

The sum of a number n and 7 is 15.

$$n + 7 \qquad = 15$$

Sum of means *addition*.

∴ An equation is $n + 7 = 15$.

b. A number y decreased by 4 is 3.

A number y decreased by 4 is 3.

$$y - 4 \qquad = 3$$

Decreased by means *subtraction*.

∴ An equation is $y - 4 = 3$.

c. 12 times a number p equals 48.

12 times a number p equals 48.

$$12p \qquad = \qquad 48$$

Times means *multiplication*.

∴ An equation is $12p = 48$.

On Your Own

Now You're Ready
Exercises 6–13

Write the word sentence as an equation.

1. 9 less than a number b equals 2.

2. The product of a number g and 5 is 30.

3. A number k increased by 10 is the same as 24.

4. The quotient of a number q and 4 is 12.

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EXAMPLE **2**

Writing an Equation

Ten servers decorate 25 tables for a wedding. Each table is decorated as shown. Let c be the total number of white and purple candles. Which equation can you use to find c?

(A) $c = 25 + (4 \times 6)$ (B) $c = 25(4 + 6)$

(C) $c = 10(25 + 4 + 6)$ (D) $c = 10(4 + 6)$

Words	The total number of candles	is	the number of tables	times	the number of candles on each table.

Variable   Let c be the total number of candles.

Equation	c	$=$	25	$\times$	$(4 + 6)$

⋮ The correct answer is (B).

EXAMPLE **3**

Real-Life Application

After two rounds, 24 students are eliminated from a spelling bee. There are 96 students remaining. Write an equation you can use to find the number of students that started the spelling bee.

Words	The number of students that started	minus	the number of students eliminated	is	the number of students remaining.

Reading

The word *eliminated* means *subtraction*.

Variable   Let s be the number of students that started.

Equation	s	$-$	24	$=$	96

⋮ An equation is $s - 24 = 96$.

On Your Own

5. You enter an elevator and go down 7 floors. You exit on the 10th floor. Write an equation you can use to find the floor where you entered the elevator.

6. Together you and a friend have $52. Your friend has $28. Write an equation you can use to find how much money you have.

7. A typical person takes about 24,000 breaths each day. Write an equation you can use to find the number of breaths a typical person takes each minute.

✓ Vocabulary and Concept Check

1. **VOCABULARY** How are expressions and equations different?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" equations.

   4 less than a number n is 8.   A number n is 4 less than 8.

   A number n minus 4 equals 8.   4 subtracted from a number n is 8.

3. **OPEN-ENDED** Write a word sentence for the equation $28 - n = 5$.

Practice and Problem Solving

Rewrite the problem using fewer words. Leave out information that you do not need to solve the problem. Then solve the problem.

4. In a cross-country race you run at a steady rate of 7 minutes per mile. After 21 minutes, you finish in fourth place. How long is the race?

5. For a science project, you record the high temperature each day. The high temperature on Day 1 was 6° less than on Day 4 and 4° less than on Day 10. The high temperature on Day 10 was 62°F. What was the high temperature on Day 1?

**Write the word sentence as an equation.**

6. The sum of a number x and 4 equals 12.

7. A number y decreased by 9 is 8.

8. 9 times a number b is 36.

9. A number w divided by 5 equals 6.

10. 54 equals 9 more than a number t.

11. 5 is one-fourth of a number c.

12. 11 is the quotient of a number y and 6.

13. 9 less than a number n equals 27.

14. **ERROR ANALYSIS** Describe the error in writing the sentence as an equation.

    ✗ A number n is 5 more than 12.
    $n + 5 = 12$

15. **FUNDRAISING** Students and faculty raised $6042 for band uniforms. The faculty raised $1780. Write an equation you can use to find the amount a raised by the students.

16. **GOLF** You hit a golf ball 90 yards. It travels three-fourths of the distance to the hole. Write an equation you can use to find the distance d from the tee to the hole.

90 yd

d

tee
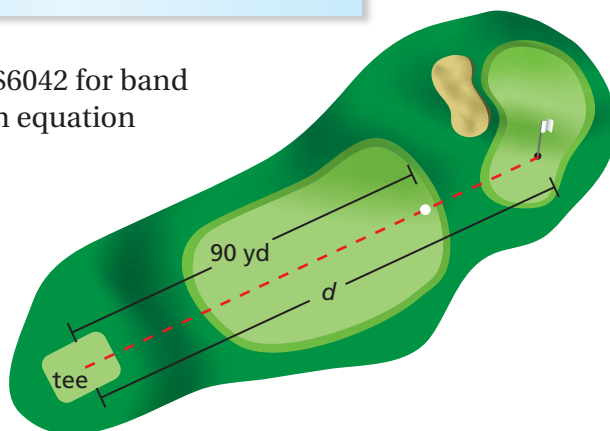
GEOMETRY Write an equation that you can use to find the value of *x*.

17. Perimeter of triangle: 16 in.


18. Perimeter of square: 30 mm
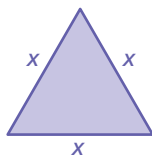
19. MUSIC You sell instruments at a Caribbean music festival. You earn $326 by selling 12 sets of maracas, 6 sets of claves, and *x djembe* drums. Write an equation you can use to find the number of *djembe* drums you sold.

20. SALES TAX Find a sales receipt from a store that shows the total price and the total amount paid including sales tax.

  a. Write an equation you can use to find the sales tax rate *r*.

  b. Can you use *r* to find the *percent* for the sales tax? Explain.

21. STRAWBERRIES You buy a basket of 24 strawberries. You eat them as you walk to the beach. It takes the same amount of time to walk each block. When you are halfway there, half of the berries are gone. After walking 3 more blocks, you still have 5 blocks to go. You reach the beach 28 minutes after you began. One-sixth of your strawberries are left.

  a. Is there enough information to find the time it takes to walk each block? Explain.

  b. Is there enough information to find how many strawberries you ate while walking the last block? Explain.

22. 🔺 **Geometry** A triangle is cut from a rectangle. The height of the triangle is half of the unknown side length *s*. The area of the shaded region is 84 square inches. Write an equation you can use to find the side length *s*.
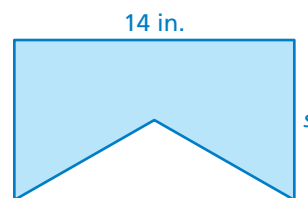
14 in.

s

Solving Equations Using Addition or Subtraction

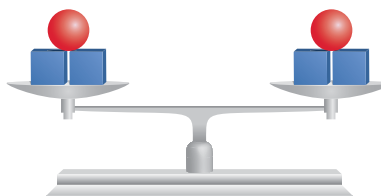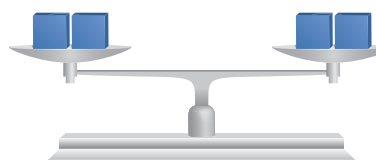**Essential Question** How can you use addition or subtraction to solve an equation?

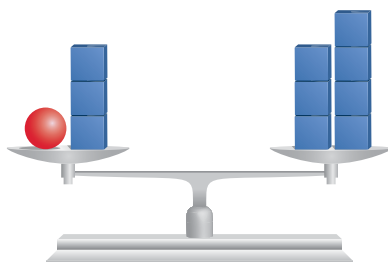When two sides of a scale weigh the same, the scale will balance.

When you add or subtract the same amount on each side of the scale, it will still balance.

1 ACTIVITY: Solving an Equation

Work with a partner.

a. Use a model to solve $n + 3 = 7$.

- Explain how the model represents the equation $n + 3 = 7$.

- How much does one ● weigh? How do you know?

⋮ The solution is $n = \boxed{}$.

b. Describe how you could check your answer in part (a).

c. Which model below represents the solution of $n + 1 = 9$? How do you know?
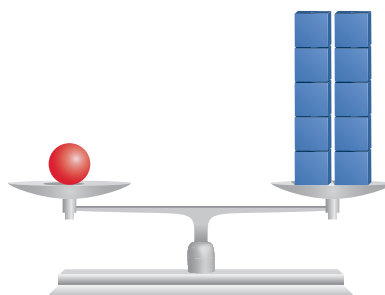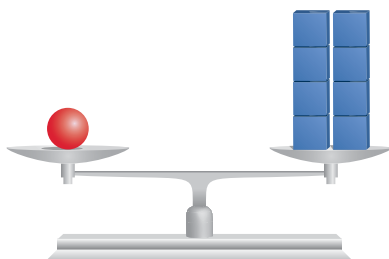
Solving Equations

In this lesson, you will
- use addition or subtraction to solve equations.
- use substitution to check answers.
- solve real-life problems.

2 ACTIVITY: Solving Equations

Work with a partner. Solve the equation using the method in Activity 1.

a. $n + 5 = 10$

b. $x + 2 = 11$

c. $6 = y + 3$

d. $8 = m + 8$

3 ACTIVITY: Solving Equations Using Mental Math

Work with a partner. Write a question that represents the equation. Use mental math to answer the question. Then check your solution.

Equation	Question	Solution	Check
a. $x + 1 = 5$			
b. $4 + m = 11$			
c. $8 = a + 3$			
d. $x - 9 = 21$			
e. $13 = p - 4$			

What Is Your Answer?

4. **REPEATED REASONING** In Activity 3, how are parts (d) and (e) different from parts (a)–(c)? Did your process to find the solution change? Explain.

5. Decide whether the statement is *true* or *false*. If false, explain your reasoning.

   a. In an equation, you can use any letter as a variable.

   b. The goal in solving an equation is to get the variable by itself.

   c. In the solution, the variable must always be on the left side of the equal sign.

   d. If you add a number to one side, you should subtract it from the other side.

6. **IN YOUR OWN WORDS** How can you use addition or subtraction to solve an equation? Give two examples to show how your procedure works.

7. Are the following equations equivalent? Explain your reasoning.

$$x - 5 = 12 \qquad \text{and} \qquad 12 = x - 5$$

Practice

Use what you learned about solving equations to complete Exercises 12–17 on page 305.

Check It Out
Lesson Tutorials
BigIdeasMath⯆com

Key Vocabulary 🔊
solution, *p. 302*
inverse operations, *p. 303*

Equations may be true for some values and false for others. A **solution** of an equation is a value that makes the equation true.

Value of x	$x + 3 = 7$	Are both sides equal?
3	$3 + 3 \overset{?}{=} 7$ $6 \neq 7$ ✗	no
4	$4 + 3 \overset{?}{=} 7$ $7 = 7$ ✓	yes
5	$5 + 3 \overset{?}{=} 7$ $8 \neq 7$ ✗	no

Reading

The symbol $\neq$ means *is not equal to*.

So, the value $x = 4$ is a solution of the equation $x + 3 = 7$.

EXAMPLE 1 **Checking Solutions**

Tell whether the given value is a solution of the equation.

a. $p + 10 = 38;\ p = 18$

$$18 + 10 \overset{?}{=} 38 \qquad \text{Substitute 18 for } p.$$
$$28 \neq 38 \quad ✗ \qquad \text{Sides are } not \text{ equal.}$$

28   38

∴ So, $p = 18$ is *not* a solution.

b. $4y = 56;\ y = 14$

$$4(14) \overset{?}{=} 56 \qquad \text{Substitute 14 for } y.$$
$$56 = 56 \quad ✓ \qquad \text{Sides are equal.}$$

56   56

∴ So, $y = 14$ is a solution.
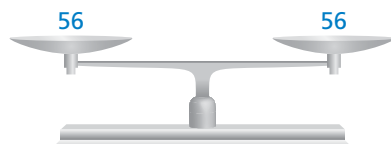
🔴 **On Your Own**

Now You're Ready
Exercises 6–11

Tell whether the given value is a solution of the equation.

1. $a + 6 = 17;\ a = 9$

2. $9 - g = 5;\ g = 3$

3. $35 = 7n;\ n = 5$

4. $\dfrac{q}{2} = 28;\ q = 14$

🔊 Multi-Language Glossary at BigIdeasMath⯆com

You can use *inverse operations* to solve equations. **Inverse operations** "undo" each other. Addition and subtraction are inverse operations.

🔑 Key Ideas

Addition Property of Equality

Words When you add the same number to each side of an equation, the two sides remain equal.

Numbers

$$\begin{array}{r} 8 = 8 \\ +5 \quad +5 \\ \hline 13 = 13 \end{array}$$

Algebra

$$\begin{array}{r} x - 4 = 5 \\ +4 \quad +4 \\ \hline x = 9 \end{array}$$

Subtraction Property of Equality

Words When you subtract the same number from each side of an equation, the two sides remain equal.

Numbers

$$\begin{array}{r} 8 = 8 \\ -5 \quad -5 \\ \hline 3 = 3 \end{array}$$

Algebra

$$\begin{array}{r} x + 4 = 5 \\ -4 \quad -4 \\ \hline x = 1 \end{array}$$

EXAMPLE 2   **Solving Equations Using Addition**

a. Solve $x - 2 = 6$.

$$x - 2 = 6 \qquad \text{Write the equation.}$$

Undo the subtraction. →
$$+2 \quad +2 \qquad \text{Addition Property of Equality}$$
$$x = 8 \qquad \text{Simplify.}$$

⋮ The solution is $x = 8$.

Check
$$x - 2 = 6$$
$$8 - 2 \overset{?}{=} 6$$
$$6 = 6 \checkmark$$

b. Solve $18 = x - 7$.

$$18 = x - 7 \qquad \text{Write the equation.}$$
$$+7 \quad +7 \qquad \text{Addition Property of Equality}$$
$$25 = x \qquad \text{Simplify.}$$

⋮ The solution is $x = 25$.

Check
$$18 = x - 7$$
$$18 \overset{?}{=} 25 - 7$$
$$18 = 18 \checkmark$$

Study Tip

You can check your solution by substituting it for the variable in the original equation.

🔴 On Your Own

Now You're Ready
Exercises 18–20

Solve the equation. Check your solution.

5. $k - 3 = 1$ **6.** $n - 10 = 4$ **7.** $15 = r - 6$

EXAMPLE 3 Solving Equations Using Subtraction

a. Solve $x + 2 = 9$.

$$x + 2 = 9 \quad \text{Write the equation.}$$

Undo the addition. → $\underline{-2 \quad -2}$ Subtraction Property of Equality

$$x = 7 \quad \text{Simplify.}$$

∴ The solution is $x = 7$.

Check
$$x + 2 = 9$$
$$7 + 2 \overset{?}{=} 9$$
$$9 = 9 \checkmark$$

b. Solve $26 = 11 + x$.

$$26 = 11 + x \quad \text{Write the equation.}$$

$$\underline{-11 \quad -11} \quad \text{Subtraction Property of Equality}$$

$$15 = x \quad \text{Simplify.}$$

∴ The solution is $x = 15$.

Check
$$26 = 11 + x$$
$$26 \overset{?}{=} 11 + 15$$
$$26 = 26 \checkmark$$

EXAMPLE 4 Real-Life Application

Your parents give you $20 to help buy the new pair of shoes shown. After you buy the shoes, you have $5.50 left. Write and solve an equation to find how much money you had before your parents gave you $20.

Words   The starting amount plus the amount your parents gave you minus the cost of the shoes is the amount left.

Variable   Let s be the starting amount.

Equation s $+$ 20 $-$ 59.95 $=$ 5.50

$$s + 20 - 59.95 = 5.50 \quad \text{Write the equation.}$$
$$s + 20 - 59.95 + 59.95 = 5.50 + 59.95 \quad \text{Addition Property of Equality}$$
$$s + 20 = 65.45 \quad \text{Simplify.}$$
$$s + 20 - 20 = 65.45 - 20 \quad \text{Subtraction Property of Equality}$$
$$s = 45.45 \quad \text{Simplify.}$$

∴ You had $45.45 before your parents gave you money.

Study Tip

In Example 4, you can solve the problem arithmetically by working backwards from $5.50.

$5.50 + 59.95 - 20$
$= 45.45$

So, your answer is reasonable.

On Your Own

Now You're Ready
Exercises 21–23

Solve the equation. Check your solution.

8. $s + 8 = 17$ **9.** $9 = y + 6$ **10.** $13 + m = 20$

11. You eat 8 blueberries and your friend eats 11 blueberries from a package. There are 23 blueberries left. Write and solve an equation to find the number of blueberries in a full package.

✓ Vocabulary and Concept Check

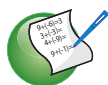**1. WRITING** How can you check the solution of an equation?

Name the inverse operation you can use to solve the equation.

2. $x - 8 = 12$ **3.** $n + 3 = 13$ **4.** $b + 14 = 33$

5. WRITING When solving $x + 5 = 16$, why do you subtract 5 from the left side of the equation? Why do you subtract 5 from the right side of the equation?

Practice and Problem Solving

Tell whether the given value is a solution of the equation.

① 6. $x + 42 = 85;\ x = 43$ **7.** $8b = 48;\ b = 6$

8. $19 - g = 7;\ g = 15$ **9.** $\dfrac{m}{4} = 16;\ m = 4$

10. $w + 23 = 41;\ w = 28$ **11.** $s - 68 = 11;\ s = 79$

Use a scale to model and solve the equation.

12. $n + 7 = 9$ **13.** $t + 4 = 5$ **14.** $c + 2 = 8$

Write a question that represents the equation. Use mental math to answer the question. Then check your solution.

15. $a + 5 = 12$ **16.** $v + 9 = 18$ **17.** $20 = d - 6$

Solve the equation. Check your solution.

② 18. $y - 7 = 3$ **19.** $z - 3 = 13$ **20.** $8 = r - 14$

③ 21. $p + 5 = 8$ **22.** $k + 6 = 18$ **23.** $64 = h + 30$

24. $f - 27 = 19$ **25.** $25 = q + 14$ **26.** $\dfrac{3}{4} = j - \dfrac{1}{2}$

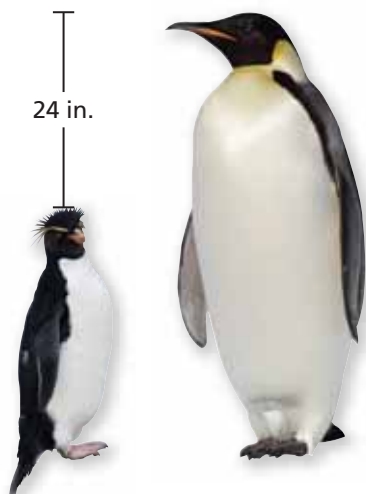**27.** $x + \dfrac{2}{3} = \dfrac{9}{10}$ **28.** $1.2 = m - 2.5$ **29.** $a + 5.5 = 17.3$

ERROR ANALYSIS Describe and correct the error in solving the equation.

30.

$$\begin{aligned} x + 7 &= 13 \\ \underline{+\,7} \quad &\underline{+\,7} \\ x &= 20 \end{aligned}$$

31.

$$\begin{aligned} 34 &= y - 12 \\ \underline{-\,12} \quad &\underline{+\,12} \\ 22 &= y \end{aligned}$$

32. PENGUINS An emperor penguin is 45 inches tall. It is 24 inches taller than a rockhopper penguin. Write and solve an equation to find the height of a rockhopper penguin. Is your answer reasonable? Explain.

24 in.

33. ELEVATOR You get in an elevator and go down 8 floors. You exit on the 16th floor. Write and solve an equation to find what floor you got on the elevator.

HAITI
DOMINICAN REPUBLIC
JAMAICA
Kingston
Port-au-Prince
Santo Domingo
Area = 4181 mi²

34. AREA The area of Jamaica is 6460 square miles less than the area of Haiti. Write and solve an equation to find the area of Haiti.
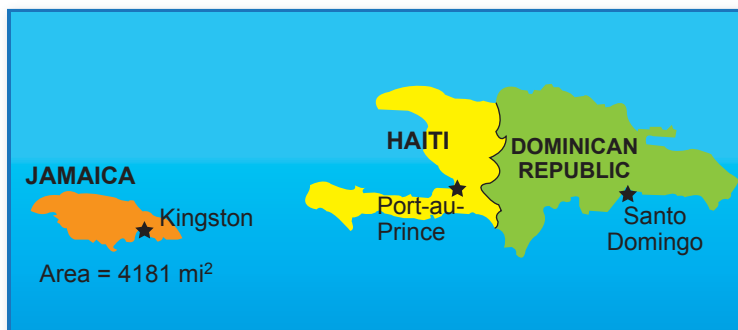
35. REASONING The solution of the equation $x + 3 = 12$ is shown. Explain each step. Use a property, if possible.

$$x + 3 = 12$$
$$x + 3 - 3 = 12 - 3$$
$$x + 0 = 9$$
$$x = 9$$

Write the equation.

Write the word sentence as an equation. Then solve the equation.

36. 13 subtracted from a number w is 15.

37. A number k increased by 7 is 34.

38. 9 is the difference of a number n and 7.

39. 93 is the sum of a number g and 58.

Solve the equation. Check your solution.

40. $b + 7 + 12 = 30$

41. $y + 4 - 1 = 18$

42. $m + 18 + 23 = 71$

43. $v - 7 = 9 + 12$

44. $5 + 44 = 2 + r$

45. $22 + 15 = d - 17$

GEOMETRY Write and solve an addition equation to find x.

46. Perimeter = 48 ft

x
20 ft
12 ft

47. Perimeter = 132 in.

16 in.
34 in.   34 in.
x

48. Perimeter = 93 ft

18 ft   18 ft
15 ft   15 ft
x
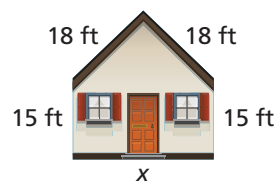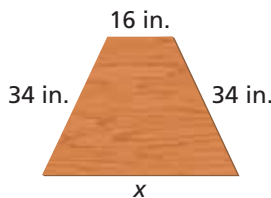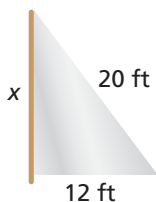
49. REASONING Explain why the equations $x + 4 = 13$ and $4 + x = 13$ have the same solution.

50. REASONING Explain why the equations $x - 13 = 4$ and $13 - x = 4$ do *not* have the same solution.

51. **SIMPLIFYING AND SOLVING** Compare and contrast the two problems.

Simplify the expression $2(x + 3) - 4$.
$2(x + 3) - 4 = 2x + 6 - 4$
$= 2x + 2$

Solve the equation $x + 3 = 4$.
$x + 3 = \quad 4$
$\underline{-3 \quad -3}$
$x = \quad 1$

52. **PUZZLE** In a *magic square*, the sum of the numbers in each row, column, and diagonal is the same. Write and solve equations to find the values of *a*, *b*, and *c*.

a	37	16
19	25	b
34	c	28

53. **FUNDRAISER** You participate in a dance-a-thon fundraiser. After your parents pledge $15.50 and your neighbor pledges $8.75, you have $66.55. Write and solve an equation to find how much money you had before your parents and neighbor pledged.

54. **MONEY** On Saturday, you spend $33, give $15 to a friend, and receive $20 for mowing your neighbor's lawn. You have $21 left. Use two methods to find how much money you started with that day.

Bumper Cars: $1.75
Roller Coaster: $1.25 more than Ferris Wheel
Giant Slide: $0.50 less than Bumper Cars
Ferris Wheel: $1.50 more than Giant Slide

55. **AMUSEMENT PARK** You have $15.

a. How much money do you have left if you ride each ride once?

b. Do you have enough money to ride each ride twice? Explain.

56. **Critical Thinking** Consider the equation $x + y = 15$. The value of x increases by 3. What has to happen to the value of y so that $x + y = 15$ remains true?

Fair Game Review What you learned in previous grades & lessons

Find the value of the expression. Use estimation to check your answer. *(Section 1.1)*

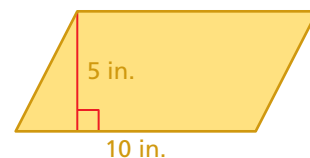57. 12×8

58. 13×16

59. $75 \div 15$

60. $72 \div 3$

61. **MULTIPLE CHOICE** What is the area of the parallelogram? *(Section 4.1)*

Ⓐ 25 in.^{2}

Ⓑ 30 in.^{2}

Ⓒ 50 in.^{2}

Ⓓ 100 in.^{2}

5 in.
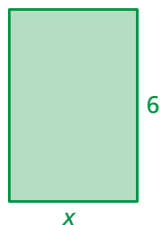
10 in.

Essential Question How can you use multiplication or division to solve an equation?

1 ACTIVITY: Finding Missing Dimensions

Work with a partner. Describe how you would find the value of x. Then find the value and check your result.
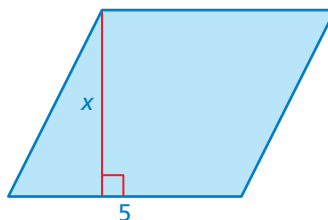
a. rectangle

Area = 24 square units

6

x

b. parallelogram

Area = 20 square units

x

5

c. triangle

Area = 28 square units

x

8
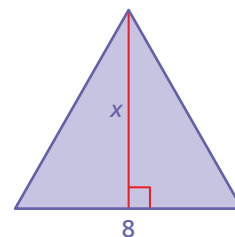
2 ACTIVITY: Using an Equation to Model a Story

Work with a partner.

a. Use a model to solve the problem.

> Three people go out to lunch. They decide to share the $12 bill evenly. How much does each person pay?

● What equation does the model represent? Explain how this represents the problem.

Solving Equations

In this lesson, you will

● use multiplication or division to solve equations.
● use substitution to check answers.
● solve real-life problems.
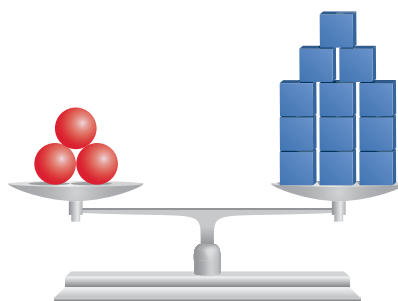
● How much does one ● weigh? How do you know?

⋮ Each person pays ☐ .

b. Describe how you can check your answer in part (a).

Work with a partner.

- What is the unknown?
- Write an equation that represents each problem.
- What does the variable in your equation represent?
- Explain how you can solve the equation.
- Answer the question.

Problem   Equation

a.

Three robots go out to lunch. They decide to share the $11.91 bill evenly. How much does each robot pay?

b.

On Earth, objects weigh 6 times what they weigh on the Moon. A robot weighs 96 pounds on Earth. What does it weigh on the Moon?

Math Practice

Interpret Results

What does the solution represent? Does the answer make sense?

c.

At maximum speed, a robot runs 6 feet in 1 second. How many feet does the robot run in 1 minute?

d.

Four identical robots lie on the ground head-to-toe and measure 14 feet. How tall is each robot?

What Is Your Answer?

4. Complete each sentence by matching.

- The inverse operation of addition
- The inverse operation of subtraction
- The inverse operation of multiplication
- The inverse operation of division

- is multiplication.
- is subtraction.
- is addition.
- is division.

5. IN YOUR OWN WORDS How can you use multiplication or division to solve an equation? Give two examples to show how your procedure works.

Practice
Use what you learned about solving equations to complete Exercises 15–18 on page 312.

🔑 Key Ideas

Remember

Inverse operations "undo" each other. Multiplication and division are inverse operations.

Multiplication Property of Equality

Words When you multiply each side of an equation by the same nonzero number, the two sides remain equal.

Numbers $\dfrac{8}{4} = 2$ **Algebra** $\dfrac{x}{4} = 2$

$\dfrac{8}{4} \cdot 4 = 2 \cdot 4$ $\dfrac{x}{4} \cdot 4 = 2 \cdot 4$

$8 = 8$ $x = 8$

Multiplicative Inverse Property

Words The product of a nonzero number n and its reciprocal, $\dfrac{1}{n}$, is 1.

Numbers $5 \cdot \dfrac{1}{5} = 1$ **Algebra** $n \cdot \dfrac{1}{n} = \dfrac{1}{n} \cdot n = 1, \, n \neq 0$

EXAMPLE ① **Solving Equations Using Multiplication**

a. Solve $\dfrac{w}{4} = 12$.

$\dfrac{w}{4} = 12$ Write the equation.

Undo the division. ⟶ $\dfrac{w}{4} \cdot 4 = 12 \cdot 4$ Multiplication Property of Equality

$w = 48$ Simplify.

⋮ The solution is $w = 48$.

Check

$\dfrac{w}{4} = 12$

$\dfrac{48}{4} \overset{?}{=} 12$

$12 = 12$ ✓

b. Solve $\dfrac{2}{7}x = 6$.

$\dfrac{2}{7}x = 6$ Write the equation.

Use the Multiplicative Inverse Property. ⟶ $\dfrac{7}{2} \cdot \left(\dfrac{2}{7}x\right) = \dfrac{7}{2} \cdot 6$ Multiplication Property of Equality

$x = 21$ Simplify.

⋮ The solution is $x = 21$.

🔴 On Your Own

Now You're Ready
Exercises 7–10

Solve the equation. Check your solution.

1. $\dfrac{a}{8} = 6$ **2.** $14 = \dfrac{2y}{5}$ **3.** $3z \div 2 = 9$

🔑 Key Idea

Division Property of Equality

Words When you divide each side of an equation by the same nonzero number, the two sides remain equal.

Numbers $8 \cdot 4 = 32$ **Algebra** $4x = 32$

$$8 \cdot 4 \div 4 = 32 \div 4$$ $$\frac{4x}{4} = \frac{32}{4}$$

$$8 = 8$$ $$x = 8$$

EXAMPLE 2 Solving an Equation Using Division

Solve $5b = 65$.

$5b = 65$	Write the equation.	

Undo the multiplication. → $\dfrac{5b}{5} = \dfrac{65}{5}$ Division Property of Equality
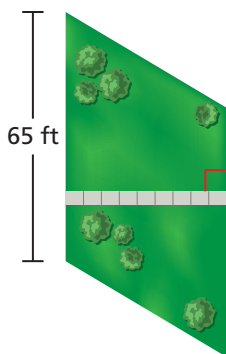
$b = 13$ Simplify.

Check

$$5b = 65$$
$$5(13) \stackrel{?}{=} 65$$
$$65 = 65 \checkmark$$

∴ The solution is $b = 13$.

EXAMPLE 3 Real-Life Application

The area of the parallelogram-shaped courtyard is 2730 square feet. What is the length of the sidewalk?

The height of the parallelogram represents the length of the sidewalk.

65 ft

$A = bh$ Use the formula for area of a parallelogram.

$2730 = 65h$ Substitute 2730 for A and 65 for b.

$\dfrac{2730}{65} = \dfrac{65h}{65}$ Division Property of Equality

$42 = h$ Simplify.

∴ So, the sidewalk is 42 feet long.

🔴 On Your Own

Now You're Ready
Exercises 11–14

Solve the equation. Check your solution.

4. $p \cdot 3 = 18$ **5.** $12q = 60$ **6.** $81 = 9r$

7. You and four friends buy tickets to a baseball game. The total cost is $70. Write and solve an equation to find the cost of each ticket.

✓ Vocabulary and Concept Check

1. **NUMBER SENSE** What number divided by 12 equals 1?

2. **WRITING** What property of equality would you use to solve $\frac{x}{6} = 7$? Explain how you would use the property.

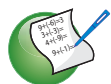**Copy and complete the first step in the solution.**

3. $4x = 24$

$$\frac{4x}{\boxed{}} = \frac{24}{\boxed{}}$$

4. $\frac{x}{3} = 11$

$$\frac{x}{3} \cdot \boxed{} = 11 \cdot \boxed{}$$

5. $8 = n \div 3$

$$8 \cdot \boxed{} = (n \div 3) \cdot \boxed{}$$

6. **OPEN-ENDED** Write an equation that can be solved using the Division Property of Equality.

Practice and Problem Solving

Solve the equation. Check your solution.

1 7. $\frac{s}{10} = 7$

8. $6 = \frac{t}{5}$

9. $5x \div 6 = 20$

10. $24 = \frac{3r}{4}$

2 11. $3a = 12$

12. $5 \cdot z = 35$

13. $40 = 4y$

14. $42 = 7k$

15. $7x = 105$

16. $75 = 6 \cdot w$

17. $13 = d \div 6$

18. $9 = v \div 5$

19. $\frac{2c}{15} = 8.8$

20. $7b \div 12 = 4.2$

21. $12.5 \cdot n = 32$

22. $3.4m = 20.4$

23. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

✗
$$x \div 4 = 28$$
$$\frac{x \div 4}{4} = \frac{28}{4}$$
$$x = 7$$

24. **ANOTHER WAY** Show how you can solve the equation $3x = 9$ by multiplying each side by the reciprocal of 3.

25. **BASKETBALL** Forty-five basketball players participate in a tournament. Write and solve an equation to find the number of 3-person teams that they can form.

26. **THEATER** A theater has 1200 seats. Each row has 20 seats. Write and solve an equation to find the number of rows in the theater.
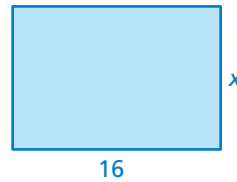
Solve for *x*. Check your answer.
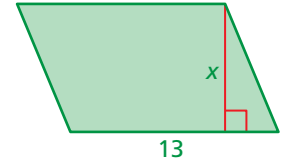
27. rectangle

Area = 45 square units

5

x

28. rectangle

Area = 176 square units

x

16

29. parallelogram

Area = 104 square units

x

13

30. TEST SCORE On a test, you correctly answer six 5-point questions and eight 2-point questions. You earn 92% of the possible points on the test. How many points *p* is the test worth?

Your Cards

48

← 12 mm

31. CARD GAME You use index cards to play a homemade game. The object is to be the first to get rid of all your cards. How many cards are in your friend's stack?

Friend's Cards

← 5 mm
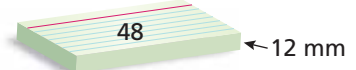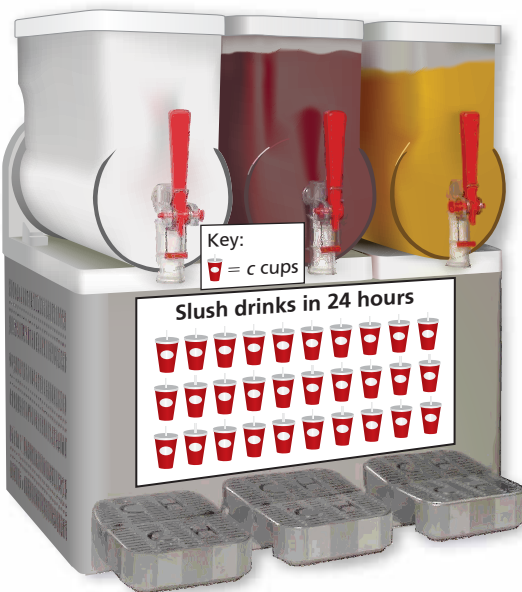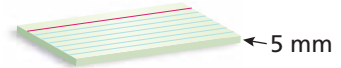
32. SLUSH DRINKS A slush drink machine fills 1440 cups in 24 hours.

Key:
🥤 = *c* cups

Slush drinks in 24 hours

  a. Write and solve an equation to find the number *c* of cups each symbol represents.

  b. To lower costs, you replace the cups with paper cones that hold 20% less. Write and solve an equation to find the number *n* of paper cones that the machine can fill in 24 hours.

STRUCTURE Solve the equation. Explain how you found your answer.

33. $5x + 3x = 5x + 18$ **34.** $8y + 2y = 2y + 40$

35. **Number Sense** The area of the picture is 100 square inches. The length is 4 times the width. Find the length and width of the picture.
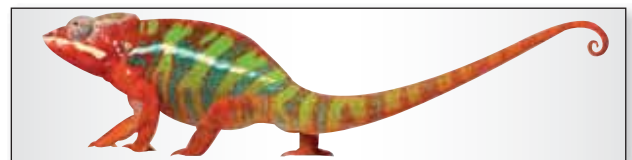
A **Fair Game Review** What you learned in previous grades & lessons

Write the word sentence as an equation. *(Section 7.1)*

36. The sum of a number *b* and 8 is 17.

37. A number *t* divided by 3 is 7.

38. MULTIPLE CHOICE What is the value of a^3 when $a = 4$? *(Section 3.1)*

  Ⓐ 12   **Ⓑ** 43   **Ⓒ** 64   **Ⓓ** 81

Essential Question How can you write an equation in two variables?

1 ACTIVITY: Writing an Equation in Two Variables

Work with a partner. You earn $8 per hour working part-time at a store.

a. Complete the table.

Hours Worked	Money Earned (dollars)
1	
2	
3	
4	
5	

b. Use the values from the table to complete the graph. Then answer each question below.

● What does the horizontal axis represent? What variable did you use to identify it?

● What does the vertical axis represent? What variable did you use to identify it?

● How are the ordered pairs in the graph related to the values in the table?

● How are the horizontal and vertical distances shown on the graph related to the values in the table?

c. How can you write an equation that shows how the two variables are related?

d. What does the green line in the graph represent?
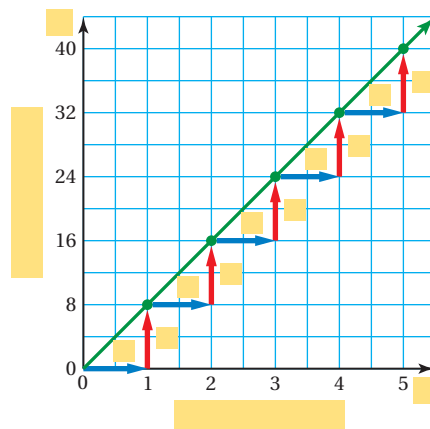
Writing Equations

In this lesson, you will
● identify independent and dependent variables.
● write equations in two variables.
● use tables and graphs to analyze the relationship between two variables.

ACTIVITY: Describing Variables

Work with a partner. Use the equation you wrote in Activity 1.

a. How is this equation different from the equations earlier in this chapter?

b. One of the variables in this equation *depends* on the other variable. Determine which variable is which by answering the following questions:

  ● Does the amount of money you earn *depend* on the number of hours you work?

  ● Does the number of hours you work *depend* on the amount of money you earn?

  What do you think is the significance of having two types of variables? How do you think you can use these types of variables in real life?

3 **ACTIVITY: Describing a Formula in Two Variables**

Math Practice

Look for Patterns

What pattern do you notice in the table for the perimeter of the square?

Work with a partner. Recall that the perimeter of a square is 4 times its side length.

a. Write the formula for the perimeter of a square. Tell what each variable represents.

b. Describe how the perimeter of a square changes as its side length increases by 1 unit. Use a table and a graph to support your answer.

c. In your formula, which variable depends on which?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you write an equation in two variables?

5. The equation $y = 7.75x$ shows how the number of movie tickets is related to the total amount of money spent. Describe what each part of the equation represents.

6. **CHOOSE TOOLS** In Activity 1, you want to know the amount of money you earn after working 30.5 hours during a week. Would you use the table, the graph, or the equation to find your earnings? What are your earnings? Explain your reasoning.

7. Give an example of another real-life situation that you can model by an equation in two variables.

Practice

Use what you learned about equations in two variables to complete Exercises 4 and 5 on page 319.

An **equation in two variables** represents two quantities that change in relationship to one another. A **solution of an equation in two variables** is an ordered pair that makes the equation true.

EXAMPLE 1 — **Identifying Solutions of Equations in Two Variables**

Key Vocabulary

equation in two variables, *p. 316*

solution of an equation in two variables, *p. 316*

independent variable, *p. 316*

dependent variable, *p. 316*

Tell whether the ordered pair is a solution of the equation.

a. $y = 2x$; (3, 6)

$6 \overset{?}{=} 2(3)$ Substitute.

$6 = 6$ ✓   Compare.

∴ So, (3, 6) is a solution.

b. $y = 4x - 3$; (4, 12)

$12 \overset{?}{=} 4(4) - 3$

$12 \neq 13$ ✗

∴ So, (4, 12) is *not* a solution.

You can use equations in two variables to represent situations involving two related quantities. The variable representing the quantity that can change freely is the **independent variable**. The other variable is called the **dependent variable** because its value *depends* on the independent variable.

EXAMPLE 2 — **Using an Equation in Two Variables**

The equation $y = 128 - 8x$ gives the amount y (in fluid ounces) of milk remaining in a gallon jug after you pour x cups.

a. Identify the independent and dependent variables.

∴ Because the amount y remaining depends on the number x of cups you pour, y is the dependent variable and x is the independent variable.

b. How much milk remains in the jug after you pour 10 cups?

Use the equation to find the value of y when $x = 10$.

$y = 128 - 8x$ Write the equation.

$= 128 - 8(10)$ Substitute 10 for *x*.

$= 48$ Simplify.

∴ There are 48 fluid ounces remaining.

On Your Own

Now You're Ready
Exercises 6–11
and 13–17

Tell whether the ordered pair is a solution of the equation.

1. $y = 7x$; (2, 21)

2. $y = 5x + 1$; (3, 16)

3. The equation $y = 10x + 25$ gives the amount y (in dollars) in your savings account after x weeks.

a. Identify the independent and dependent variables.

b. How much is in your savings account after 8 weeks?

Multi-Language Glossary at BigIdeasMath com

🔑 Key Idea

Tables, Graphs, and Equations

You can use tables and graphs to represent equations in two variables. The table and graph below represent the equation $y = x + 2$.

Independent Variable, *x*	Dependent Variable, *y*	Ordered Pair, (*x*, *y*)
1	3	(1, 3)
2	4	(2, 4)
3	5	(3, 5)
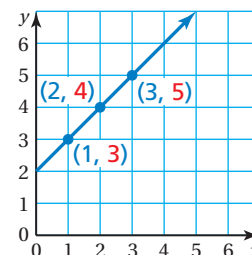
EXAMPLE 3 — Writing and Graphing an Equation in Two Variables

An athlete burns 200 calories weight lifting. The athlete then works out on an elliptical trainer and burns 10 calories for every minute. Write and graph an equation in two variables that represents the total number of calories burned during the workout.

Words   The total number of calories burned   equals   calories burned weight lifting   plus   calories burned per minute   times   the number of minutes.

Variables   Let *c* be the total number of calories burned, and let *m* be the number of minutes on the elliptical trainer.

Equation $c = 200 + 10 \cdot m$

To graph the equation, first make a table. Then plot the ordered pairs and draw a line through the points.

Minutes, *m*	*c* = 200 + 10*m*	Calories, *c*	Ordered Pair, (*m*, *c*)
10	$c = 200 + 10(10)$	300	(10, 300)
20	$c = 200 + 10(20)$	400	(20, 400)
30	$c = 200 + 10(30)$	500	(30, 500)
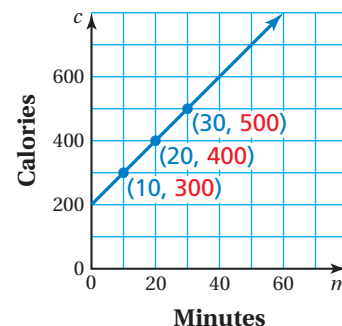
🔴 On Your Own

4. It costs $25 to rent a kayak plus $8 for each hour. Write and graph an equation in two variables that represents the total cost of renting the kayak.

You can model many rate problems by using the *distance formula d = rt,* where *d* is the distance traveled, *r* is the speed, and *t* is the time. When you are given a speed, you can use the formula to write an equation in two variables that represents the situation.

🔑 Key Idea

Distance Formula

Words To find the distance traveled *d,* multiply the speed *r* by the time *t.*

Algebra $d = rt$

EXAMPLE 4 — Real-Life Application

A train averages 40 miles per hour between two cities. Use a graph to show the relationship between the time and the distance traveled.

Method 1: Use a ratio table.
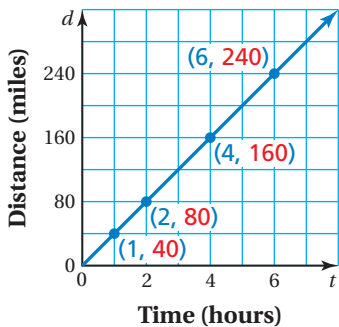
You can use a ratio table and multiplication to find equivalent rates. Then plot the ordered pairs (time, distance) from the table and draw a line through the points.

	× 2	× 4	× 6
Time (hours)	1	2	4	6
Distance (miles)	40	80	160	240

Method 2: Use an equation in two variables.

Use the distance formula to write the equation $d = 40t$. Use the equation to make a table. Then plot the ordered pairs and draw a line through the points, as shown in the graph above.

Time (hours), *t*	$d = 40t$	Distance (miles), *d*	Ordered Pair, (*t, d*)
1	$d = 40(1)$	40	(1, 40)
2	$d = 40(2)$	80	(2, 80)
4	$d = 40(4)$	160	(4, 160)
6	$d = 40(6)$	240	(6, 240)

🔴 On Your Own

5. WHAT IF? The train averages 50 miles per hour. Use a graph to show the relationship between the time and the distance traveled.

✓ Vocabulary and Concept Check

1. **VOCABULARY** How are independent variables and dependent variables different?

2. **PRECISION** Explain how to graph an equation in two variables.

3. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

| $y = 12x + 25$ | $c = 10t - 5$ | $a = 7b + 11$ | $n = 4n - 6$ |

✓ Practice and Problem Solving

Write a formula for the given measure. Tell what each variable represents. Identify which variable depends on which in the formula.

4. the perimeter of a rectangle with a length of 5 inches

5. the area of a trapezoid with base lengths of 7 feet and 11 feet

Tell whether the ordered pair is a solution of the equation.

① 6. $y = 4x$; (0, 4) 　　7. $y = 3x$; (2, 6) 　　8. $y = 5x - 10$; (3, 5)

9. $y = x + 7$; (1, 6) 　10. $y = 7x + 2$; (2, 0) 　11. $y = 2x - 3$; (4, 5)

12. **ERROR ANALYSIS** Describe and correct the error in finding a solution of the equation in two variables.

> ✗ $y = 3x + 2$; (5, 1)
> $5 \overset{?}{=} 3(1) + 2$
> $5 = 5$
> So, (5, 1) is a solution.

Identify the independent and dependent variables.

② 13. The equation $A = 25w$ gives the area A (in square feet) of a rectangular dance floor with a width of w feet.

14. The equation $c = 0.09s$ gives the amount c (in dollars) of commission a salesperson receives for making a sale of s dollars.

15. The equation $t = 12p + 12$ gives the total cost t (in dollars) of a meal with a tip of p percent (in decimal form).

16. The equation $h = 60 - 4m$ gives the height h (in inches) of the water in a tank m minutes after it starts to drain.

17. **DRUM SET** The equation $b = 540 - 30m$ gives the balance b (in dollars) that you owe on a drum set after m monthly payments. What is the balance after 9 monthly payments?

OPEN-ENDED Complete the table by describing possible independent or dependent variables.

	Independent Variable	Dependent Variable
18.	The number of hours you study for a test	
19.	The speed you are pedaling a bike	
20.		Your monthly cell phone bill
21.		The amount of money you earn

③ 22. PIZZA A cheese pizza costs $5. Additional toppings cost $1.50 each. Write and graph an equation in two variables that represents the total cost of a pizza.

23. GYM MEMBERSHIP It costs $35 to join a gym. The monthly fee is $25. Write and graph an equation in two variables that represents the total cost of a gym membership.

24. TEXTING The maximum size of a text message is 160 characters. A space counts as one character.

a. Write an equation in two variables that represents the remaining (unused) characters in a text message as you type.

b. Identify the independent and dependent variables.

c. How many characters remain in the message shown?

Jackie

Please call me.

④ 25. CHOOSE TOOLS A car averages 60 miles per hour on a road trip. Use a graph to show the relationship between the time and the distance traveled. What method did you use to create your graph?

Write and graph an equation in two variables that shows the relationship between the time and the distance traveled.

26.

Moves 2 meters every 3 hours.

27.

Rises 5 stories every 6 seconds.

28.

Moves 660 feet every 10 seconds.

29.

Moves 960 kilometers every 4 minutes.

Fill in the blank so that the ordered pair is a solution of the equation.

30. $y = 8x + 3$; $(1, \boxed{})$

31. $y = 12x + 2$; $(\boxed{}, 14)$

32. $y = 22 - 9x$; $(\boxed{}, 4)$

33. CRITICAL THINKING Can the dependent variable cause a change in the independent variable? Explain.

34. OPEN-ENDED Write an equation in two variables that has (3, 4) as a solution.

35. WALKING You walk 5 city blocks in 12 minutes. How many city blocks can you walk in 2 hours?

36. ANT How fast should the ant walk to go around the rectangle in 4 minutes?

12 in.

16 in.

37. LIGHTNING To estimate how far you are from lightning (in miles), count the number of seconds between a lightning flash and the thunder that follows. Then divid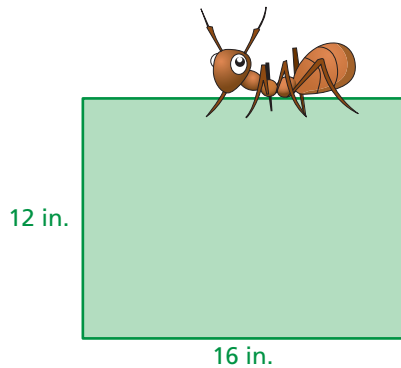e the number of seconds by 5. Use a graph to show the relationship between the time and the distance. Describe the method you used to create your graph.

38. PROBLEM SOLVING You and a friend start biking in opposite directions from the same point. You travel 108 feet every 8 seconds. Your friend travels 63 feet every 6 seconds.

    a. How far apart are you and your friend after 15 minutes?

    b. After 20 minutes, you take a 5-minute rest, but your friend does not. How far apart are you and your friend after 40 minutes? Explain your reasoning.

39. Reasoning The graph represents the cost c (in dollars) of buying n tickets to a baseball game.

    a. Should the points be connected with a line to show all the solutions? Explain your reasoning.

    b. Write an equation in two variables that represents the graph.

Dollars / Tickets — points (1, 10), (2, 20), (3, 30)
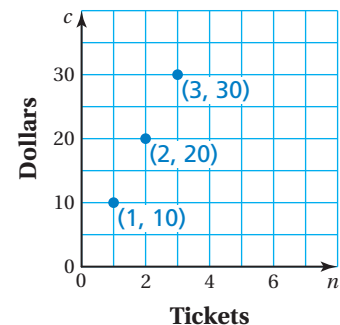
Write the fraction as a percent. *(Section 5.5)*

40. $\dfrac{3}{10}$

41. $\dfrac{4}{5}$

42. $\dfrac{9}{20}$

43. $\dfrac{17}{25}$

44. MULTIPLE CHOICE What is the area of the triangle? *(Section 4.2)*

    A 36 cm^2

    B 68 cm^2

    C 72 cm^2

    D 76.5 cm^2

17 cm   8 cm   9 cm
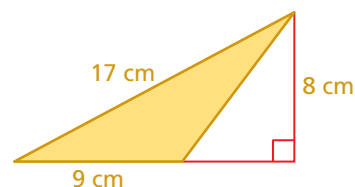
You can use an **example and non-example chart** to list examples and non-examples of a vocabulary word or term. Here is an example and non-example chart for equations.

Equations

Examples	Non-Examples
$x = 5$	5
$2a = 16$	$2a$
$x + 4 = 19$	$x + 4$
$5 = x + 3$	$x + 3$
$12 - 7 = 5$	$12 - 7$
$\frac{3}{4}y = 6$	$\frac{3}{4}$

On Your Own

Make example and non-example charts to help you study these topics.

1. inverse operations

2. equations solved using addition or subtraction

3. equations solved using multiplication or division

4. equations in two variables

After you complete this chapter, make example and non-example charts for the following topics.

5. inequalities

6. graphs of inequalities

7. inequalities solved using addition or subtraction

8. inequalities solved using multiplication or division
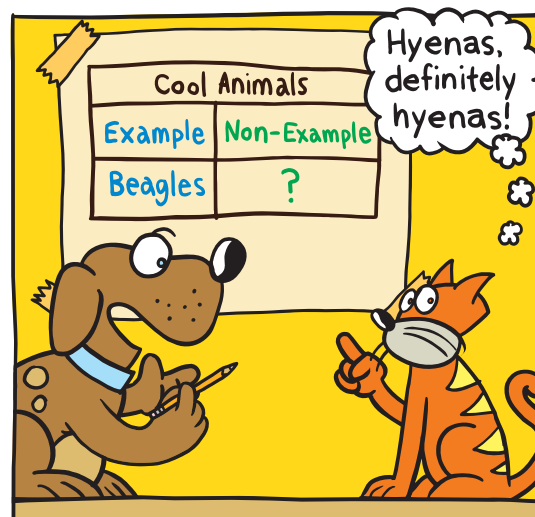
"I need a good non-example of a cool animal for my **example and non-example chart**."

Write the word sentence as an equation. *(Section 7.1)*

1. A number x decreased by 3 is 5.

2. A number a divided by 7 equals 14.

Solve the equation. Check your solution. *(Section 7.2 and Section 7.3)*

3. $4 + k = 14$

4. $3.5 = m - 2.2$

5. $8 = \dfrac{4w}{3}$

6. $31 = 6.2 \cdot y$

Tell whether the ordered pair is a solution of the equation. *(Section 7.4)*

7. $y = 6x$; $(3, 24)$

8. $y = 3x + 4$; $(4, 16)$

Write and graph an equation in two variables that shows the relationship between the time and the distance traveled. *(Section 7.4)*

9.

Rises 4 feet in 9 seconds.

10.

Moves 900 feet every 10 seconds.

11. **RIBBON** The length of the blue ribbon is two-thirds the length of the red ribbon. Write an equation you can use to find the length r of the red ribbon. *(Section 7.1)*

r

30 cm
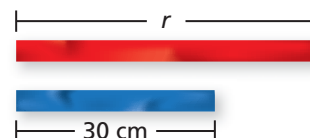
12. **BRIDGES** The main span of the Sunshine Skyway Bridge is 360 meters long. The Skyway's main span is 30 meters shorter than the main span of the Dames Point Bridge. Write and solve an equation to find the length ℓ of the main span of the Dames Point Bridge. *(Section 7.2)*

13. **SHOPPING** At a farmer's market, you buy 4 pounds of tomatoes and 2 pounds of sweet potatoes. You spend 80% of the money in your wallet. Write and solve an equation to find how much money is in your wallet before you pay. *(Section 7.3)*

Tomatoes
$3.00/pound

Sweet Potatoes
$2.00/pound
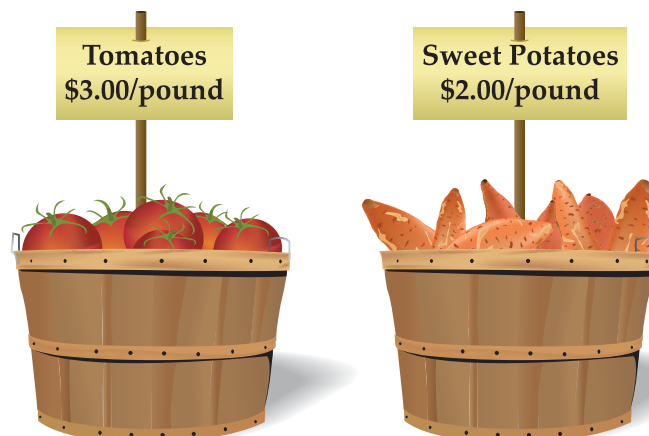
14. **SUNDAE** A sundae costs $2. Additional toppings cost $0.50 each. Write and graph an equation in two variables that represents the total cost of a sundae. *(Section 7.4)*

Essential Question How can you use a number line to represent solutions of an inequality?

1 ACTIVITY: Understanding Inequality Statements

Work with a partner. Read the statement. Circle each number that makes the statement true, and then answer the questions.

a. "Your friend is **more than** 3 minutes late."

$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

- What do you notice about the numbers that you circled?
- Is the number 3 included? Why or why not?
- Write four other numbers that make the statement true.

b. "The temperature is **at most** 2 degrees."

$$-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

- What do you notice about the numbers that you circled?
- Can the temperature be exactly 2 degrees? Explain.
- Write four other numbers that make the statement true.

c. "You need **at least** 4 pieces of paper for your math homework."

$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

- What do you notice about the numbers that you circled?
- Can you have exactly 4 pieces of paper? Explain.
- Write four other numbers that make the statement true.

Writing Inequalities

In this lesson, you will

- write word sentences as inequalities.
- use a number line to graph the solution set of inequalities.
- use inequalities to represent real-life situations.

d. "After playing a video game for 20 minutes, you have **fewer than** 6 points."

$$-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$

- What do you notice about the numbers that you circled?
- Is the number 6 included? Why or why not?
- Write four other numbers that make the statement true.

2 ACTIVITY: Understanding Inequality Symbols

Work with a partner.

a. Consider the statement "*x* is a number such that *x* < 2."

- Can the number be exactly 2? Explain.

- Circle each number that makes the statement true.

   $-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

- Write four other numbers that make the statement true.

b. Consider the statement "*x* is a number such that *x* ≥ 1."

- Can the number be exactly 1? Explain.

- Circle each number that makes the statement true.

   $-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

- Write four other numbers that make the statement true.

Math Practice

State the Meaning of Symbols

What do the symbols < and ≥ mean?

3 ACTIVITY: How Close Can You Come to 0?

Work with a partner.

a. Which number line shows *x* > 0? Which number line shows *x* ≥ 0? Explain your reasoning.
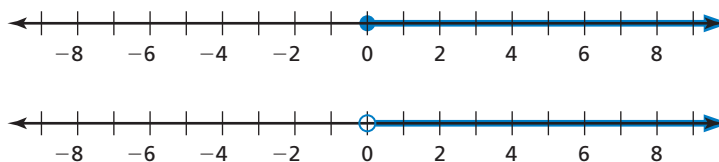
b. Write the least positive number you can think of that is still a solution of the inequality *x* > 0. Explain your reasoning.

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use a number line to represent solutions of an inequality?

5. Write an inequality. Graph all solutions of your inequality on a number line.

6. Graph the inequalities *x* > 9 and 9 < *x* on different number lines. What do you notice?

Practice

Use what you learned about graphing inequalities to complete Exercises 17–20 on page 329.

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Key Vocabulary 🔊
inequality, *p. 326*
solution of an
  inequality, *p. 327*
solution set, *p. 327*
graph of an
  inequality, *p. 328*

An **inequality** is a mathematical sentence that compares expressions. It contains the symbols $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine where to place the inequality symbol.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 — Writing Inequalities

Write the word sentence as an inequality.

a. A number c is less than -4.

A number c is less than -4.

$$c \qquad < \qquad -4$$

∴ An inequality is $c < -4$.

b. A number k plus 5 is greater than or equal to 8.

A number k plus 5 is greater than or equal to 8.

$$k + 5 \qquad\qquad \geq \qquad\qquad 8$$

∴ An inequality is $k + 5 \geq 8$.

c. Four times a number q is at most 16.

Four times a number q is at most 16.

$$4q \qquad\qquad \leq \qquad 16$$

∴ An inequality is $4q \leq 16$.

On Your Own

Now You're Ready
Exercises 5–10

Write the word sentence as an inequality.

1. A number n is greater than 1.

2. Twice a number p is fewer than 7.

3. A number w minus 3 is less than or equal to 10.

4. A number z divided by 2 is at least -6.

🔊 Multi-Language Glossary at BigIdeasMath ✓com

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 3 \leq 7$	Is the inequality true?
3	$3 + 3 \overset{?}{\leq} 7$ $6 \leq 7$ ✓	yes
4	$4 + 3 \overset{?}{\leq} 7$ $7 \leq 7$ ✓	yes
5	$5 + 3 \overset{?}{\leq} 7$ $8 \nleq 7$ ✗	no

Reading

The symbol $\nleq$ means *is not less than or equal to*.

EXAMPLE 2 **Checking Solutions**

Tell whether the given value is a solution of the inequality.

a. $x + 1 > 7$; $x = 8$

$x + 1 > 7$ Write the inequality.

$8 + 1 \overset{?}{>} 7$ Substitute 8 for x.

$9 > 7$ ✓   Add. 9 is greater than 7.

∴ So, 8 is a solution of the inequality.

b. $7y < 27$; $y = 4$

$7y < 27$ Write the inequality.

$7(4) \overset{?}{<} 27$ Substitute 4 for y.

$28 \nless 27$ ✗   Multiply. 28 is *not* less than 27.

∴ So, 4 is *not* a solution of the inequality.

c. $\dfrac{z}{3} \geq 5$; $z = 15$

$\dfrac{z}{3} \geq 5$ Write the inequality.

$\dfrac{15}{3} \overset{?}{\geq} 5$ Substitute 15 for z.

$5 \geq 5$ ✓   Divide. 5 is greater than or equal to 5.

∴ So, 15 is a solution of the inequality.

On Your Own

Now You're Ready
Exercises 11–16

Tell whether 3 is a solution of the inequality.

**5.** $b + 4 < 6$ **6.** $9 - n \geq 6$ **7.** $18 \div x \leq 10$

The **graph of an inequality** shows all the solutions of the inequality on a number line. An open circle ○ is used when a number is *not* a solution. A closed circle ● is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 — Graphing an Inequality

Graph $g > 2$.

Reading

The inequality $g > 2$ is the same as $2 < g$.

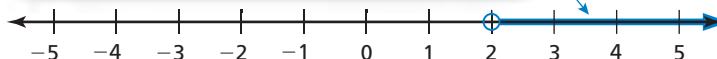Use an open circle because 2 is *not* a solution.
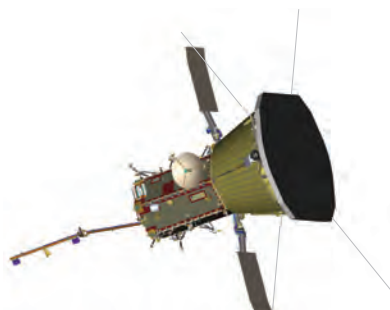
| −5 −4 −3 −2 −1 0 1 2 3 4 5 |
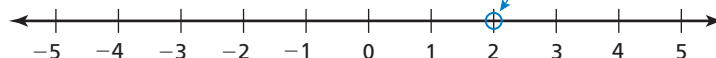
Test a number to the left of 2.
$g = 0$ is *not* a solution.

Test a number to the right of 2.
$g = 3$ is a solution.

Shade the number line on the side where you found the solution. The graph shows there are *infinitely many* solutions.

| −5 −4 −3 −2 −1 0 1 2 3 4 5 |

EXAMPLE 4 — Real-Life Application

The NASA Solar Probe Plus can withstand temperatures up to and including 2600°F. Write and graph an inequality that represents the temperatures the probe can withstand.

Words   temperatures   up to and including   2600°F

Variable   Let t be the temperatures the probe can withstand.

Inequality t $\leq$ 2600

∴   An inequality is $t \leq 2600$.

| 2000   2200   2400   2600   2800   3000 |

On Your Own

Now You're Ready
Exercises 25–36

Graph the inequality on a number line.

8. $a < 4$ **9.** $f \leq 7$ **10.** $n > 0$ **11.** $p \geq -3$

Write and graph an inequality for the situation.

12. A cruise ship can carry at most 3500 passengers.

13. A board game is designed for ages 12 and up.

✓ Vocabulary and Concept Check

1. **VOCABULARY** How are *greater than* and *greater than or equal to* similar? How are they different?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

A number n is at most 3.	A number n is at least 3.
A number n is less than or equal to 3.	A number n is no more than 3.

3. **WRITING** Explain how the graph of $x \leq 6$ is different from the graph of $x < 6$.

4. **WRITING** Are the graphs of $x \leq 5$ and $5 \geq x$ the same or different? Explain.
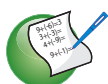
Practice and Problem Solving

Write the word sentence as an inequality.

1 **5.** A number k is less than 10.

6. A number a is more than 6.

7. A number z is fewer than $\dfrac{3}{4}$.

8. A number b is at least -3.

9. One plus a number y is no more than -13.

10. A number x divided by 3 is at most 5.

Tell whether the given value is a solution of the inequality.

2 **11.** $x - 1 \leq 7$; $x = 6$

12. $y + 5 < 13$; $y = 17$

13. $3z > 6$; $z = 3$

14. $\dfrac{b}{2} \geq 6$; $b = 10$

15. $c + 2.5 < 4.3$; $c = 1.8$

16. $a \leq 0$; $a = -5$

Match the inequality with its graph.

17. $x \geq 2$

18. $x < 2$

19. $x > -2$

20. $x \leq -2$

A.

B.

C.

D.
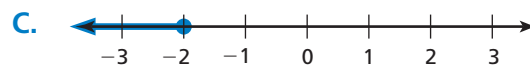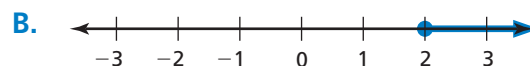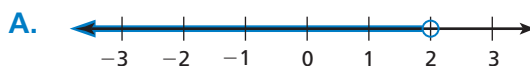
Write an inequality and a word sentence that represent the graph.

21.

22.

23.

24.

Graph the inequality on a number line.

③ 25. $a > 4$

26. $n \geq 8$

27. $3 \geq x$

28. $y < \dfrac{1}{2}$

29. $x < \dfrac{2}{9}$

30. $-3 \geq c$

31. $m > -5$

32. $0 \leq b$

33. $1.5 > f$

34. $t \geq -\dfrac{1}{2}$

35. $p > -1.6$

36. $\dfrac{7}{3} \geq z$

ERROR ANALYSIS **Describe and correct the error in graphing the inequality.**

37.

$x \geq 1$

38.
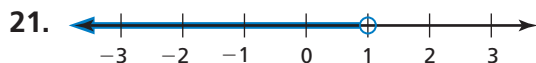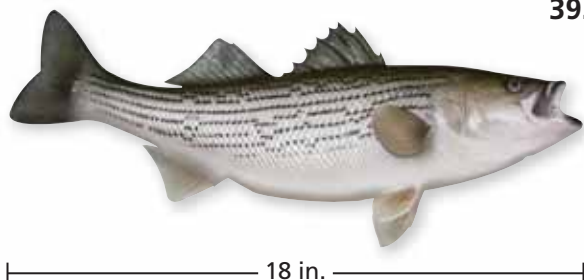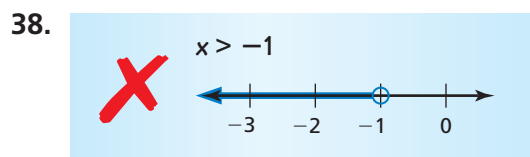

$x > -1$
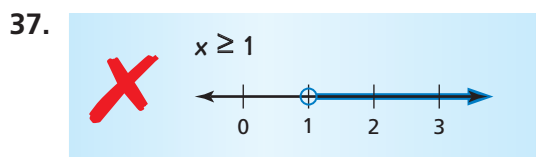
39. FISHING You are fishing and are allowed to catch at most 3 striped bass. Each striped bass must be no less than 18 inches long.

a. Write and graph an inequality to represent the number of striped bass you are allowed to catch.

b. Write and graph an inequality to represent the length of each striped bass you are allowed to catch.

18 in.

40. MODELING For a food to be labeled *low sodium*, there must be no more than 140 milligrams of sodium per serving.

a. Write and graph an inequality to represent the amount of sodium in a low-sodium serving.

b. Write and graph an inequality to represent the amount of sodium in a serving that does *not* qualify as low sodium.

c. Does the food represented by the nutrition facts label qualify as a low-sodium food? Explain.

Nutrition Facts
Serving Size ½ cup (114g)
Servings Per Container 4

Amount Per Serving

Calories 90   Calories from fat 30

% Daily Value*

Total Fat 3g		**5%**
Saturated Fat 0g		**0%**
Cholesterol 0mg		**0%**
Sodium 300mg		**13%**
Total Carbohydrate 13g		**4%**
Dietary Fiber 3g		**12%**
Sugars 3g		
Protein 3g		

Vitamin A 80% • Vitamin C 60%
Calcium 4%   •   Iron 4%

41. SHOPPING You have $33. You want to buy a necklace and one other item from the list.

    a. Write an inequality to represent the situation.

    b. Can the other item be a T-shirt? Explain.

    c. Can the other item be a book? Explain.

Item	Price (with tax)
T-shirt	$ 15
Book	$ 20
DVD	$ 13
Necklace	$ 16

Determine whether the statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

42. A number that is a solution of the inequality $x > 5$ is also a solution of the inequality $x \geq 5$.

43. A number that is a solution of the inequality $5 \leq x$ is also a solution of the inequality $x > 5$.

44. BUS RIDE A bus ride costs $1.50. A 30-day bus pass costs $36. Write an inequality to represent the number of bus rides you would need to take for the bus pass to be a better deal.

45. MOVIE THEATER Fifty people are seated in a movie theater. The maximum capacity of the theater is 425 people. Write an inequality to represent the number of additional people who can still be seated.

46. **Critical Thinking** The map shows the elevations above sea level for an area of land.
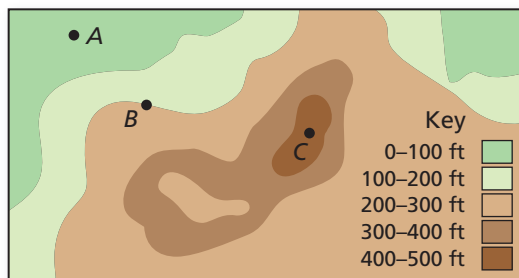
    a. Graph the possible elevations of *A*. Write the set of elevations as two inequalities.

    b. Graph the possible elevations of *C*. How can you write this set of elevations as a single inequality? Explain.

    c. What is the elevation of *B*? Explain.

Key
0–100 ft
100–200 ft
200–300 ft
300–400 ft
400–500 ft

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution.   *(Section 7.2)*

47. $x + 3 = 12$ **48.** $x - 6 = 8$ **49.** $16 + x = 44$ **50.** $7.6 = x - 6.5$

51. MULTIPLE CHOICE A stack of boards is 24 inches high. Each board is $\frac{3}{8}$ of an inch thick. How many boards are in the stack?   *(Section 2.2)*

    (A) $\frac{1}{9}$ **(B)** $\frac{1}{6}$ **(C)** 9   **(D)** 64

Solving Inequalities Using Addition or Subtraction

Essential Question How can you use addition or subtraction to solve an inequality?

1 ACTIVITY: Writing an Inequality

Work with a partner. In 3 years, your friend will still not be old enough to vote.

a. Which of the following represents your friend's situation? What does x represent? Explain your reasoning.

$x + 3 < 18$ $x + 3 \leq 18$

$x + 3 > 18$ $x + 3 \geq 18$

b. Graph the possible ages of your friend on a number line. Explain how you decided what to graph.

2 ACTIVITY: Writing an Inequality

Work with a partner. Baby manatees are about 4 feet long at birth. They grow to a maximum length of 13 feet.

a. Which of the following can represent a baby manatee's growth? What does x represent? Explain your reasoning.

$x + 4 < 13$ $x + 4 \leq 13$

$x - 4 > 13$ $x - 4 \geq 13$

b. Graph the solution on a number line. Explain how you decided what to graph.

Solving Inequalities

In this lesson, you will

- use addition or subtraction to solve inequalities.
- use a number line to graph the solution set of inequalities.
- solve real-life problems.

3 ACTIVITY: Solving Inequalities

Work with a partner. Complete the following steps for Activity 1. Then repeat the steps for Activity 2.

- Use your inequality from part (a). Replace the inequality symbol with an equal sign.
- Solve the equation.
- Replace the equal sign with the original inequality symbol.
- Graph this new inequality.
- Compare the graph with your graph in part (b). What do you notice?

4 ACTIVITY: The Triangle Inequality

Work with a partner. Draw different triangles whose sides have lengths 10 cm, 6 cm, and x cm.

a. Which of the following describes how *small x* can be? Explain your reasoning.

$6 + x < 10$ $6 + x \leq 10$

$6 + x > 10$ $6 + x \geq 10$

x 10 cm

6 cm
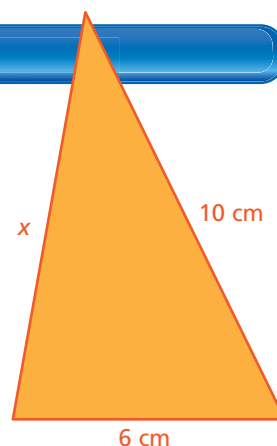
b. Which of the following describes how *large x* can be? Explain your reasoning.

$x - 6 < 10$ $x - 6 \leq 10$ $x - 6 > 10$ $x - 6 \geq 10$

c. Graph the possible values of x on a number line.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use addition or subtraction to solve an inequality?

6. Describe a real-life situation that you can represent with an inequality. Write the inequality. Graph the solution on a number line.

Practice

Use what you learned about solving inequalities to complete Exercises 5–7 on page 336.

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Study Tip

You can solve inequalities the same way you solve equations. Use inverse operations to get the variable by itself.

Key Ideas

Addition Property of Inequality

Words When you add the same number to each side of an inequality, the inequality remains true.

Numbers
$$3 < 5$$
$$\underline{+2 \quad +2}$$
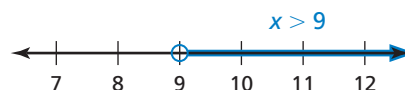$$5 < 7$$

Algebra
$$x - 4 > 5$$
$$\underline{+4 \qquad +4}$$
$$x > 9$$

Graph

$x > 9$

Subtraction Property of Inequality

Words When you subtract the same number from each side of an inequality, the inequality remains true.

Numbers
$$3 < 5$$
$$\underline{-2 \quad -2}$$
$$1 < 3$$

Algebra
$$x + 4 > 5$$
$$\underline{-4 \qquad -4}$$
$$x > 1$$

Graph

$x > 1$

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 **Solving an Inequality Using Addition**

Solve $x - 3 > 1$. Graph the solution.

$$x - 3 > 1 \qquad \text{Write the inequality.}$$

Undo the subtraction. →
$$\underline{+3 \quad +3} \qquad \text{Addition Property of Inequality}$$
$$x > 4 \qquad \text{Simplify.}$$

Check:
$x = 3$: $\quad 3 - 3 \overset{?}{>} 1$
$$0 \not> 1 \quad ✗$$
$x = 5$: $\quad 5 - 3 \overset{?}{>} 1$
$$2 > 1 \quad ✓$$

∴ The solution is $x > 4$.

$x > 4$
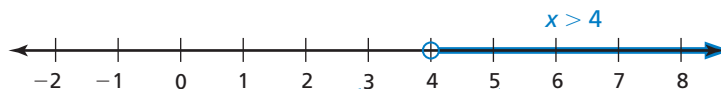
$x = 3$ is *not* a solution. $x = 5$ is a solution.

On Your Own

Solve the inequality. Graph the solution.

1. $x - 2 < 3$ **2.** $x - 6 \geq 4$ **3.** $10 \geq x - 1$

EXAMPLE 2 | Solving an Inequality Using Subtraction

Solve $15 \geq 6 + x$. Graph the solution.

$$15 \geq \quad 6 + x \qquad \text{Write the inequality.}$$

Undo the addition. → $\underline{-6 \qquad -6} \qquad$ Subtraction Property of Inequality

$$9 \geq x \qquad \text{Simplify.}$$

Reading

The inequality $x \leq 9$ is the same as $9 \geq x$.

⋮ The solution is $x \leq 9$.

$x \leq 9$

```
←———————————————————————→
 -15  -12  -9  -6  -3  0  3  6  9  12  15
```
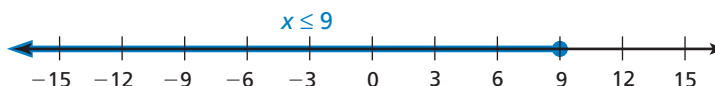
🔴 **On Your Own**

Now You're Ready
Exercises 5–16

Solve the inequality. Graph the solution.

4. $x + 3 > 7$ **5.** $y + 2 < 17$ **6.** $16 \leq m + 9$

EXAMPLE 3 | Real-Life Application

A flea market advertises that it has more than 250 vending booths. Of these, 184 are currently filled. Write and solve an inequality to represent the number of vending booths still available.

Words | The number of booths filled | plus | the number of remaining booths | is greater than | the total number of booths.

Variable Let b be the number of remaining booths.

Inequality | 184 | + | b | > | 250

$$184 + b > \quad 250 \qquad \text{Write the inequality.}$$
$$\underline{-184 \qquad\quad -184} \qquad \text{Subtraction Property of Inequality}$$
$$b > \quad 66 \qquad \text{Simplify.}$$

⋮ More than 66 vending booths are still available.

🔴 **On Your Own**

7. You have already spent $24 shopping online for clothes. Write and solve an inequality to represent the additional amount you must spend to get free shipping.

✓ Vocabulary and Concept Check

1. **OPEN-ENDED** Write an inequality that can be solved by subtracting 7 from each side.

2. **WRITING** Explain how to solve the inequality $x - 6 > 3$.

3. **WRITING** Describe the graph of the solution of $x + 3 \leq 4$.

4. **OPEN-ENDED** Write an inequality that the graph represents. Then use the Subtraction Property of Inequality to write another inequality that the graph represents.

Practice and Problem Solving

Solve the inequality. Graph the solution.

5. $x - 4 < 5$

6. $5 + h > 7$

7. $3 \geq y - 2$

8. $9 \leq c + 1$

9. $18 > 12 + x$

10. $37 + z \leq 54$

11. $y - 21 < 85$

12. $g - 17 \geq 17$

13. $7.2 < x + 4.2$

14. $12.7 \geq s - 5.3$

15. $\dfrac{3}{4} \leq \dfrac{1}{2} + n$

16. $\dfrac{1}{3} + b > \dfrac{3}{4}$

17. **ERROR ANALYSIS** Describe and correct the error in solving the inequality.

$$\begin{array}{c} 28 \geq t - 9 \\ \underline{-9 \qquad -9} \\ 19 \geq t \end{array}$$

18. **AIR TRAVEL** Your carry-on bag can weigh at most 40 pounds. Write and solve an inequality to represent how much more weight you can add to the bag and still meet the requirement.

19. **SHOPPING** It costs $\$x$ for a round-trip bus ticket to the mall. You have $\$24$. Write and solve an inequality to represent how much money you can spend for the bus fare and still have enough to buy the baseball cap.

$\$18^{99}$

Write the word sentence as an inequality. Then solve the inequality.

20. Five more than a number is less than 17.

21. Three less than a number is more than 15.

Solve the inequality. Graph the solution.

22. $x + 9 - 3 \leq 14$

**23.** $44 > 7 + s + 26$

24. $6.1 - 0.3 \geq c + 1$

25. **VIDEO GAME** The high score for a video game is 36,480. Your current score is 34,280. Each dragonfly you catch is worth 1 point. You also get a 1000-point bonus for reaching 35,000 points. Write and solve an inequality to represent the number of dragonflies you must catch to earn a new high score.

26. **PICKUP TRUCKS** You can register a pickup truck as a passenger vehicle if the truck is not used for commercial purposes and the weight of the truck with its contents does not exceed 8500 pounds.

    a. Your pickup truck weighs 4200 pounds. Write an inequality to represent the number of pounds your truck can carry and still qualify as a passenger vehicle. Then solve the inequality.
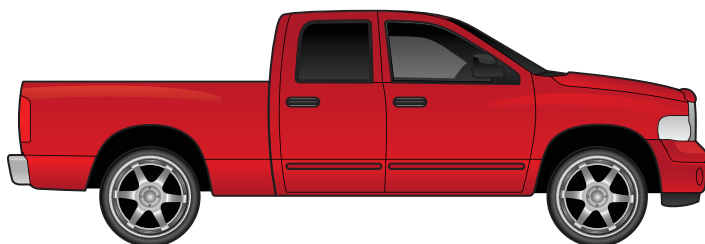
    b. A cubic yard of sand weighs about 1600 pounds. How many cubic yards of sand can you haul in your truck and still qualify as a passenger vehicle? Explain your reasoning.

27. **TRIATHLON** You complete two events of a triathlon. Your goal is to finish with an overall time of less than 100 minutes.

    a. Write and solve an inequality to represent how many minutes you can take to finish the running event and still meet your goal.

    b. The running event is 3.1 miles long. Estimate how many minutes it would take you to run 3.1 miles. Would this time allow you to reach your goal? Explain your reasoning.

Triathlon	
Event	Your Time (minutes)
Swimming	18.2
Biking	45.4
Running	?

28. **Number Sense** The possible values of x are given by $x - 3 \geq 2$. What is the least possible value of $5x$?

Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 7.3)*

29. $\dfrac{t}{12} = 4$

30. $6 = \dfrac{2s}{9}$

31. $8x = 72$

32. $9 = 1.5z$

33. **MULTIPLE CHOICE** Which brand of turkey is the best buy? *(Section 5.4)*

    A Brand A

    B Brand B

    C Brand C

    D Brand D

Brand	A	B	C	D
Cost (dollars)	10.38	13.47	21.45	34.93
Pounds	2	3	5	7

Solving Inequalities Using Multiplication or Division

Essential Question How can you use multiplication or division to solve an inequality?

1 ACTIVITY: Writing an Inequality

Work with a partner. A store has a clearance rack of shirts that each cost the same amount. You buy 2 shirts and have money left after paying with a $20 bill.

a. Which of the following represents your purchase? What does x represent? Explain your reasoning.

$$2x < 20 \qquad 2x \leq 20$$

$$2x > 20 \qquad 2x \geq 20$$

b. Graph the possible values of x on a number line. Explain how you decided what to graph.

c. Can you buy a third shirt? Explain your reasoning.

2 ACTIVITY: Writing an Inequality

Work with a partner. One of your favorite stores is having a 75% off sale. You have $20. You want to buy a pair of jeans.

a. Which of the following represents your ability to buy the jeans with $20? What does x represent? Explain your reasoning.

$$\frac{1}{4}x < 20 \qquad \frac{1}{4}x \leq 20$$

$$\frac{1}{4}x > 20 \qquad \frac{1}{4}x \geq 20$$

b. Graph the possible values of x on a number line. Explain how you decided what to graph.

c. Can you afford a pair of jeans that originally costs $100? Explain your reasoning.

Solving Inequalities
In this lesson, you will
- use multiplication or division to solve inequalities.
- use a number line to graph the solution set of inequalities.
- solve real-life problems.

ACTIVITY: Solving Inequalities

Work with a partner. Complete the following steps for Activity 1. Then repeat the steps for Activity 2.

- Use your inequality from part (a). Replace the inequality symbol with an equal sign.
- Solve the equation.
- Replace the equal sign with the original inequality symbol.
- Graph this new inequality.
- Compare the graph with your graph in part (b). What do you notice?

4 **ACTIVITY: Matching Inequalities**

Work with a partner. Match the inequality with its graph. Explain your method.

a. $3x < 9$ **b.** $3x \le 9$ **c.** $\dfrac{x}{2} \ge 1$

d. $6 < 2x$ **e.** $12 \le 4x$ **f.** $\dfrac{x}{2} < 2$

Math Practice

Make a Plan
What strategy will you use to choose the correct graph?
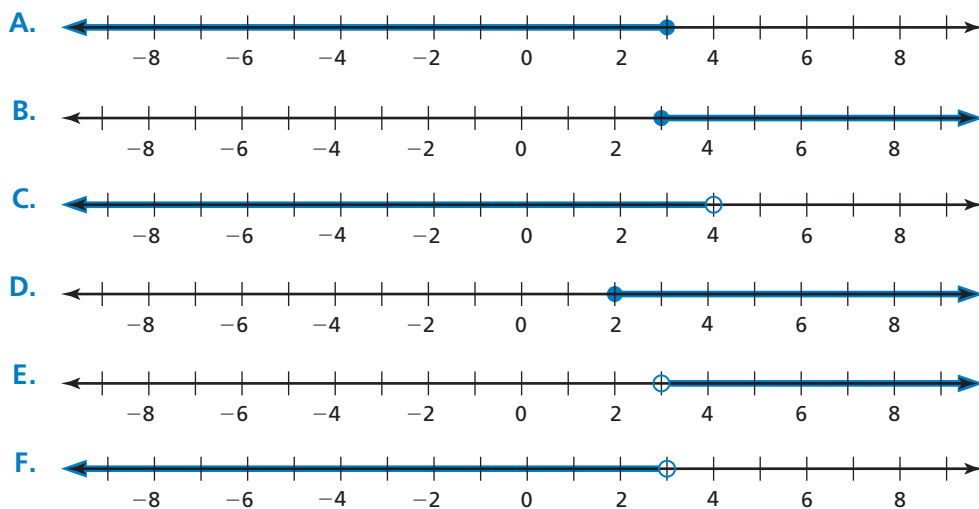
A.

B.

C.

D.

E.

F.

What Is Your Answer?

5. IN YOUR OWN WORDS How can you use multiplication or division to solve an inequality?

Practice

Use what you learned about solving inequalities to complete Exercises 8–11 on page 342.

🔑 Key Ideas

Remember

Multiplication and division are inverse operations.

Multiplication Property of Inequality

Words   When you multiply each side of an inequality by the same *positive* number, the inequality remains true.

Numbers $8 > 6$ **Algebra** $\dfrac{x}{4} < 2$

$8 \times 2 > 6 \times 2$ $\dfrac{x}{4} \cdot 4 < 2 \cdot 4$

$16 > 12$ $x < 8$

Division Property of Inequality

Words   When you divide each side of an inequality by the same *positive* number, the inequality remains true.

Numbers $8 > 6$ **Algebra** $4x < 8$

$8 \div 2 > 6 \div 2$ $\dfrac{4x}{4} < \dfrac{8}{4}$

$4 > 3$ $x < 2$

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1   Solving an Inequality Using Multiplication

Solve $\dfrac{x}{5} \leq 2$. Graph the solution.

$\dfrac{x}{5} \leq 2$ Write the inequality.

Undo the division. ⟶ $\dfrac{x}{5} \cdot 5 \leq 2 \cdot 5$ Multiplication Property of Inequality

$x \leq 10$ Simplify.

⋮ The solution is $x \leq 10$.

$x \leq 10$

5 6 7 8 9 10 11 12 13 14 15

$x = 5$ is a solution. $x = 15$ is *not* a solution.

🔴 On Your Own
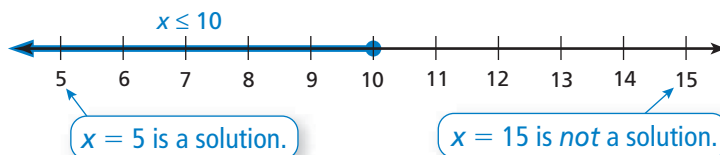
Now You're Ready
Exercises 6–9

Solve the inequality. Graph the solution.

1. $p \div 3 > 2$ **2.** $\dfrac{3}{5}q \leq 6$ **3.** $1 < \dfrac{s}{7}$

EXAMPLE **2** Solving an Inequality Using Division

Solve 4n > 32. Graph the solution.

4n > 32 — Write the inequality.

Undo the multiplication. → $\dfrac{4n}{4} > \dfrac{32}{4}$ — Division Property of Inequality

n > 8 — Simplify.

∴ The solution is n > 8.

$$n > 8$$

```
←——+——+——+——+——+——+——⊕——+——+——+——→
   −4   −2   0   2   4   6   8  10  12  14  16
```

EXAMPLE **3** Real-Life Application

A one-way bus ride costs $1.75. A 30-day bus pass costs $42.

a. Write and solve an inequality to find the least number of one-way rides you must take for the 30-day pass to be a better deal.

b. You ride the bus an average of 20 times each month. Is the pass a better deal? Explain.

a. **Words**   The price of a one-way ride   times   the number of one-way rides   is more than $42.

Variable Let r be the number of one-way rides.

Inequality   1.75   • r >   42

1.75r > 42 — Write the inequality.

$\dfrac{1.75r}{1.75} > \dfrac{42}{1.75}$ — Division Property of Inequality

r > 24 — Simplify.

∴ So, you need to take more than 24 one-way rides for the pass to be a better deal.

b. No. The cost of 20 one-way rides is less than $42. So, the pass is not a better deal.

On Your Own

Now You're Ready
Exercises 10–13

Solve the inequality. Graph the solution.

4. 11k ≤ 33   **5.** 5 • j > 20   **6.** 50 ≤ 2m

7. The sign shows the toll for driving on Alligator Alley. Write and solve an inequality to represent the number of times someone can drive on Alligator Alley with $15.

Passenger Cars
Toll $2.50

✔ Vocabulary and Concept Check

1. REASONING How is the graph of the solution of $2x \geq 10$ different from the graph of the solution of $2x = 10$?
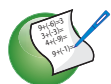
Name the property you should use to solve the inequality.

2. $3x \leq 27$ **3.** $7x > 49$ **4.** $\dfrac{x}{2} < 36$

5. OPEN-ENDED Write two inequalities that have the same solution set: one that you can solve using division and one that you can solve using multiplication.

📝 Practice and Problem Solving

Solve the inequality. Graph the solution.

① 6. $\dfrac{m}{8} < 4$ **7.** $n \div 6 > 2$ **8.** $\dfrac{t}{3} \geq 15$ **9.** $\dfrac{1}{11}c \geq 9$

② 10. $12x < 96$ **11.** $5x \geq 25$ **12.** $8 \cdot w \leq 72$ **13.** $7p \leq 42$

14. $\dfrac{3}{4}b > 15$ **15.** $6x < 90$ **16.** $3s \geq 36$ **17.** $\dfrac{5}{9}v \leq 45$

18. $4t > 72$ **19.** $\dfrac{3}{4}w \leq 24$ **20.** $12m < 132$ **21.** $\dfrac{5x}{8} \geq 30$

22. ERROR ANALYSIS Describe and correct the error in solving the inequality.

✗
$$\dfrac{x}{6} \leq 30$$
$$\dfrac{x}{6} \cdot 6 \leq \dfrac{30}{6}$$
$$x \leq 5$$

23. GEOMETRY The length of a rectangle is 8 feet, and its area is less than 168 square feet. Write and solve an inequality to represent the width of the rectangle.

24. PLAYGROUND Students at a playground are divided into 5 equal groups with at least 6 students in each group. Write and solve an inequality to represent the number of students at the playground.

Write the word sentence as an inequality. Then solve the inequality.

25. Eight times a number n is less than 72.

26. A number t divided by 32 is at most 4.25.

27. 225 is no less than 12 times a number w.

Graph the numbers that are solutions to both inequalities.

28. $x + 7 > 9$ and $8x \le 64$

29. $x - 3 \le 8$ and $6x < 72$

30. THRILL RIDE A thrill ride at an amusement park holds a maximum of 12 people per ride.

  a. Write and solve an inequality to find the least number of rides needed for 15,000 people.

  b. Do you think it is possible for 15,000 people to ride the thrill ride in 1 day? Explain.

31. FOOTBALL A winning football team more than doubled the offensive yards gained by its opponent. The opponent gained 272 offensive yards. The winning team had 80 offensive plays. Write and solve an inequality to find the possible number of yards per play for the winning team.

Park Hours
10:00 A.M.–10:00 P.M.

32. LOGIC Explain how you know that $7x < 7x$ has no solution.

33. OPEN-ENDED Give an example of a real-life situation in which you can list all the solutions of an inequality. Give an example of a real-life situation in which you cannot list all the solutions of an inequality.

School Fundraiser
Earn $5 for every $50 in sales! TAKE ONE.

34. FUNDRAISER You are selling items from a catalog for a school fundraiser. Write and solve two inequalities to find the range of sales that will earn you between $40 and $50.

Critical Thinking Let $a > b$ and $x > y$. Tell whether the statement is *always* true. Explain your reasoning.
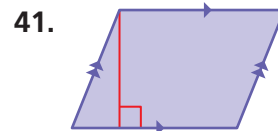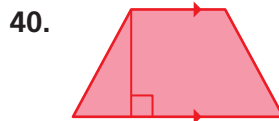
35. $a + x > b + y$

36. $a - x > b - y$

37. $ax > by$

38. $\dfrac{a}{x} > \dfrac{y}{b}$

Fair Game Review What you learned in previous grades & lessons

Classify the quadrilateral. *(Skills Review Handbook)*

39.

40.

41.
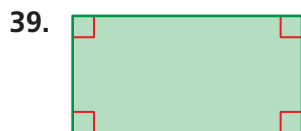
42. MULTIPLE CHOICE On a normal day, 12 airplanes arrive at an airport every 15 minutes. Which rate does not represent this situation? *(Section 5.3)*

  Ⓐ 24 airplanes every 30 minutes

  Ⓑ 4 airplanes every 5 minutes

  Ⓒ 6 airplanes every 5 minutes

  Ⓓ 48 airplanes each hour

Write the word sentence as an inequality. *(Section 7.5)*

1. A number x is greater than 0.

2. Twice a number c is at least -8.

Tell whether the given value is a solution of the inequality. *(Section 7.5)*

3. $2n > 16$; $n = 9$

4. $x - 1 \le 9$; $x = 10$

Graph the inequality on a number line. *(Section 7.5)*

5. $y > -4$

6. $m \le \dfrac{3}{5}$

Solve the inequality. Graph the solution. *(Section 7.6)*

7. $x + 4 \le 8$

8. $18 > 16 + g$

Write the word sentence as an inequality. Then solve the inequality. *(Section 7.6)*

9. Two less than a number is more than 15.

10. Seven more than a number is less than or equal to 27.

Solve the inequality. Graph the solution. *(Section 7.7)*

11. $\dfrac{3a}{2} < 24$

12. $121 \ge 11s$

Write the word sentence as an inequality. Then solve the inequality. *(Section 7.7)*

13. Three times a number x is more than 18.

14. 84 is no less than 7 times a number k.

15. WATER PARK Each visit to a water park costs $19.95. An annual pass to the park costs $89.95. Write an inequality to represent the number of times you would need to visit the park for the pass to be a better deal. *(Section 7.5)*

16. GARDEN You want to use a square section of your yard for a garden. You have at most 52 feet of fencing to surround the garden. Write and solve an inequality to represent the possible lengths of each side of the garden. *(Section 7.7)*

17. DELIVERY You were planning to spend $12 on a pizza. Write and solve an inequality to represent the additional amount you must spend to get free delivery. *(Section 7.6)*

PIZZA PALACE
FREE DELIVERY
for orders of $30.00 or more
516 E 31st   865-564

Review Key Vocabulary

equation, *p. 296*
solution, *p. 302*
inverse operations, *p. 303*
equation in two variables, *p. 316*

solution of an equation in
  two variables, *p. 316*
independent variable, *p. 316*
dependent variable, *p. 316*

inequality, *p. 326*
solution of an inequality, *p. 327*
solution set, *p. 327*
graph of an inequality, *p. 328*

Review Examples and Exercises

7.1 Writing Equations in One Variable *(pp. 294–299)*

Write the word sentence "The quotient of a number b and 6 is 9" as an equation.

The quotient of a number b and 6 is 9.

$$b \div 6 \qquad\qquad = 9 \qquad \textit{Quotient of means division.}$$

⠿ An equation is $b \div 6 = 9$.

Exercises

Write the word sentence as an equation.

1. The product of a number m and 2 is 8.

2. 6 less than a number t is 7.

3. A number m increased by 5 is 7.

4. 8 is the quotient of a number g and 3.

7.2 Solving Equations Using Addition or Subtraction *(pp. 300–307)*

Solve $z + 5 = 13$.

$z + 5 =\ 13$	Write the equation.	
$\underline{-5 \quad -5}$	Subtraction Property of Equality	
$z =\ 8$	Simplify.	

Undo the addition. →

Check
$$z + 5 = 13$$
$$8 + 5 \overset{?}{=} 13$$
$$13 = 13 ✓$$

⠿ The solution is $z = 8$.

Exercises

Solve the equation. Check your solution.

5. $x - 1 = 8$

6. $m + 7 = 11$

7. $21 = p - 12$

7.3 Solving Equations Using Multiplication or Division *(pp. 308–313)*

Solve $4c = 32$.

$4c = 32$	Write the equation.
$\dfrac{4c}{4} = \dfrac{32}{4}$	Division Property of Equality
$c = 8$	Simplify.

Undo the multiplication.

Check

$4c = 32$

$4(8) \overset{?}{=} 32$

$32 = 32$ ✔

Exercises

Solve the equation. Check your solution.

8. $7 \cdot q = 42$ **9.** $7k \div 3 = 21$ **10.** $\dfrac{5a}{7} = 25$

7.4 Writing Equations in Two Variables *(pp. 314–321)*

Tell whether (6, 16) is a solution of the equation $y = 3x - 4$.

$16 \overset{?}{=} 3(6) - 4$	Substitute.
$16 \neq 14$ ✗	Compare.

⋮ So, (6, 16) is *not* a solution.

Exercises

Tell whether the ordered pair is a solution of the equation.

11. $y = 3x + 1$; (2, 7) **12.** $y = 7x - 4$; (4, 22)

13. TAXI A taxi ride costs $3 plus $2.50 per mile. Write and graph an equation in two variables that represents the total cost of a taxi ride.

7.5 Writing and Graphing Inequalities *(pp. 324–331)*

Write the word sentence as an inequality.

a. A number x is more than -9.

A number x is more than -9.

$x \qquad > \qquad -9$

⋮ An inequality is $x > -9$.

b. A number r divided by 2 is at most 4.

A number r divided by 2 is at most 4.

$\dfrac{r}{2} \qquad \leq \qquad 4$

⋮ An inequality is $\dfrac{r}{2} \leq 4$.

Exercises

Write the word sentence as an inequality.

14. A number m is less than 5. **15.** A number h is at least -12.

Graph the inequality on a number line.

16. $x < 0$ **17.** $a \geq 3$ **18.** $n \leq -1$

Solving Inequalities Using Addition or Subtraction *(pp. 332–337)*
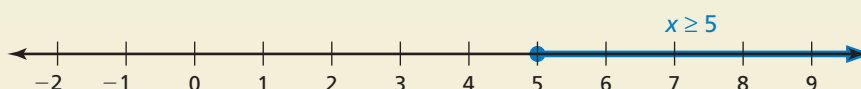
Solve $1 \leq x - 4$. Graph the solution.

$$1 \leq x - 4 \qquad \text{Write the inequality.}$$

Undo the subtraction. ⟶ $\underline{+ 4 \qquad + 4} \qquad$ Addition Property of Inequality

$$5 \leq x \qquad \text{Simplify.}$$

The inequality $5 \leq x$ is the same as $x \geq 5$.

∴ The solution is $x \geq 5$.

$x \geq 5$

Exercises

Solve the inequality. Graph the solution.

19. $x + 1 > 3$ **20.** $k - 7 \leq 0$ **21.** $y + 8 \geq 9$

22. $24 < 11 + x$ **23.** $4 \leq n - 4$ **24.** $x - 20 > 24$

25. $b + 12 \leq 26$ **26.** $s - 1.5 < 2.5$ **27.** $\dfrac{1}{4} + m \leq \dfrac{1}{2}$

7.7 **Solving Inequalities Using Multiplication or Division** *(pp. 338–343)*
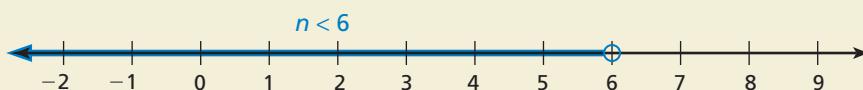
Solve $7n < 42$. Graph the solution.

$$7n < 42 \qquad \text{Write the inequality.}$$

Undo the multiplication. ⟶ $\dfrac{7n}{7} < \dfrac{42}{7} \qquad$ Division Property of Inequality

$$n < 6 \qquad \text{Simplify.}$$

∴ The solution is $n < 6$.

$n < 6$

Exercises

Solve the inequality. Graph the solution.

28. $x \div 2 < 4$ **29.** $9n \geq 63$ **30.** $\dfrac{5}{3}x \leq 10$

31. $9 \geq 3b$ **32.** $10p > 40$ **33.** $\dfrac{3}{11}k < 15$

34. TICKETS The cost of three tickets to a movie is at least $20. Write and solve an inequality that represents the situation.

Write the word sentence as an equation.

1. 7 times a number s is 84.

2. 13 is one-third of a number m.

Solve the equation. Check your solution.

3. $15 = 7 + b$

4. $v - 6 = 16$

5. $5x = 70$

6. $3b = 45$

7. $\dfrac{6m}{7} = 30$

8. $\dfrac{8k}{3} = 32$

Tell whether the ordered pair is a solution of the equation.

9. $y = 9x$; (3, 27)

10. $y = 4x + 2$; (8, 36)

Write an inequality for the situation.

11. An MP3 player holds up to 300 songs.

12. Riders must be at least 48 inches tall.

Graph the inequality on a number line.

13. $x \geq 5$

14. $m \leq -2$

Solve the inequality. Graph the solution.

15. $x - 3 < 7$

16. $12 \geq n + 6$

17. $\dfrac{4}{3}b \leq 12$

18. $72 > 12p$

19. **SCHOOL DANCE** Each ticket to a school dance is $4. The total amount collected in ticket sales is $332. Write and solve an equation to find the number of students attending the dance.

20. **T-SHIRTS** A soccer team will sell T-shirts for a fundraiser. The company that makes the T-shirts charges $10 per shirt plus a $20 shipping fee per order.

    a. Write and graph an equation in two variables that represents the total cost of ordering the shirts.

    b. Choose an ordered pair that lies on your graph in part (a). Interpret it in the context of the problem.

21. **HURRICANE** A hurricane has wind speeds that are greater than or equal to 74 miles per hour. Write an inequality to represent the possible wind speeds during a hurricane.

1. What is the area of the balcony shown below?

4.25 ft

4 ft

8.75 ft
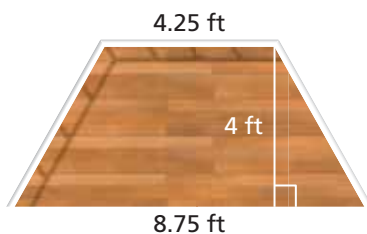
A. 9 ft^2

C. 26 ft^2

B. 18 ft^2

D. 52 ft^2

2. You are making identical fruit baskets using 16 apples, 24 pears, and 32 bananas. What is the greatest number of baskets you can make using all the fruit?

F. 2

H. 8

G. 4

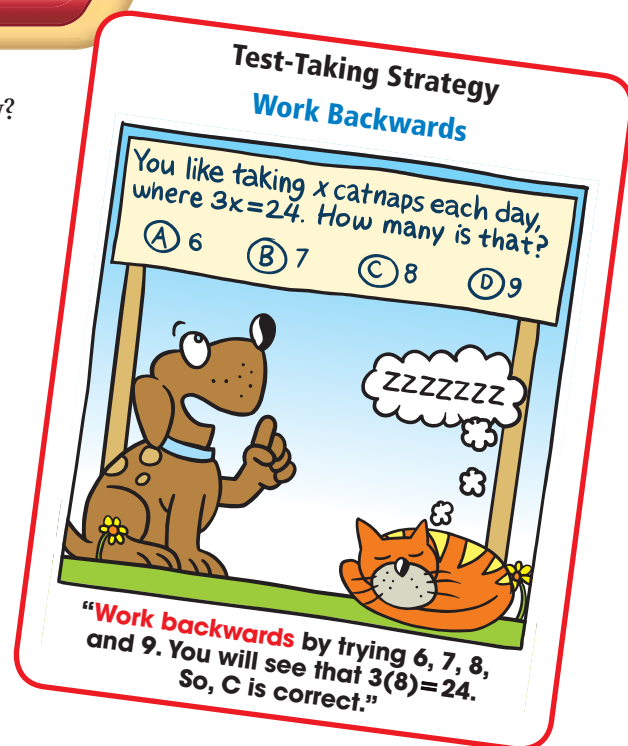I. 16

3. Which equation represents the word sentence below?

The sum of 18 and 5 is equal to 9 less than a number *y*.

A. $18 - 5 = 9 - y$

C. $18 + 5 = y - 9$

B. $18 + 5 = 9 - y$

D. $18 - 5 = y - 9$

4. Which number line is a graph of the solution of the inequality below?

$$x \geq 5$$

F.

1 2 3 4 5 6 7 8 9

G.

1 2 3 4 5 6 7 8 9

H.

1 2 3 4 5 6 7 8 9

I.

1 2 3 4 5 6 7 8 9

5. The steps your friend took to divide two mixed numbers are shown below.

$$3\frac{3}{5} \div 1\frac{1}{2} = \frac{18}{5} \times \frac{3}{2}$$

$$= \frac{27}{5}$$

$$= 5\frac{2}{5}$$

What should your friend change in order to divide the two mixed numbers correctly?

   A. Find a common denominator of 5 and 2.

   B. Multiply by the reciprocal of $\frac{18}{5}$.

   C. Multiply by the reciprocal of $\frac{3}{2}$.

   D. Rename $3\frac{3}{5}$ as $2\frac{8}{5}$.

6. An inequality is graphed on the number line below.
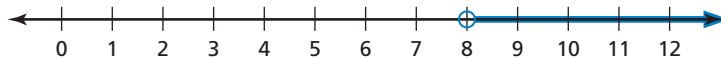
What is the least whole number value that is a solution of the inequality?

7. A company ordering parts receives a charge of $25 for shipping and handling plus $20 per part. Which equation represents the cost c of ordering p parts?

   F. $c = 25 + 20p$ **H.** $p = 25 + 20c$

   G. $c = 20 + 25p$ **I.** $p = 20 + 25c$

8. Which property is illustrated by the statement below?

$$5(3 + 6) = 5(3) + 5(6)$$

   A. Associative Property of Multiplication

   B. Commutative Property of Multiplication

   C. Commutative Property of Addition

   D. Distributive Property

9. What is the value of the expression below?

$$46.8 \div 0.156$$

10. In a fish tank, 75% of the fish are goldfish. How many fish are in the tank if there are 24 goldfish?
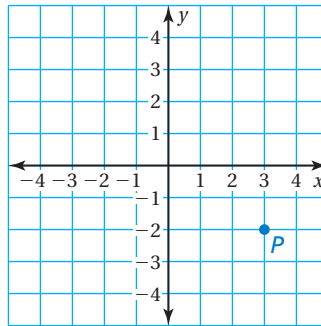
   F. 6   **H.** 32

   G. 18   **I.** 96

11. What are the coordinates of point P in the coordinate plane below?

   A. $(-3, -2)$ **C.** $(-2, -3)$

   B. $(3, -2)$ **D.** $(-2, 3)$

12. What is the first step in evaluating the expression below?
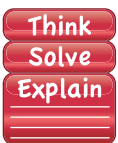
$$3 \cdot (5 + 2)^2 \div 7$$

   F. Multiply 3 and 5.   **H.** Evaluate 5^2.

   G. Add 5 and 2.   **I.** Evaluate 2^2.

13. Jeff wants to save $4000 to buy a used car. He has already saved $850. He plans to save an additional $150 each week.

Part A   Write and solve an equation to represent the number of weeks remaining until he can afford the car.

Jeff saves $150 per week by saving $\frac{3}{4}$ of what he earns at his job each week. He works 20 hours per week.

Part B   Write an equation to represent the amount per hour that Jeff must earn to save $150 per week. Explain your reasoning.

Part C   What is the amount per hour that Jeff must earn? Show your work and explain your reasoning.