## Expressions and Equations

### 3.1 Algebraic Expressions

3.2 Adding and Subtracting Linear Expressions
3.3 Solving Equations Using Addition or subtraction

### 3.4 Solving Equations Using Multiplication orpDivision

## 3.5 solving Jwo-Step Equations

chers lhope the
"Now I will be able $2 x+(-2)=2$."
"I can't find my algebra tiles biscuits."

$$
2(00+1)=8
$$


"Descartes, if you solve for

in the equation, what do you get?"

## What You Learned Before

- Evaluating Expressions


## Example 1 Evaluate $6 x+2 y$ when

 $x=-3$ and $y=5$.$6 x+2 y=6(-3)+2(5)$
$=-18+10$
$=-8$

Substitute -3 for $x$ and 5 for $y$.
Using order of operations, multiply 6 and -3 , and 2 and 5 . Add -18 and 10 .

Example 2 Evaluate $6 x^{2}-3(y+2)+8$ when $x=-2$ and $y=4$.
$6 x^{2}-3(y+2)+8=6(-2)^{2}-3(4+2)+8$ Substitute -2 for $x$ and 4 for $y$.
$=6(-2)^{2}-3(6)+8 \quad$ Using order of operations, evaluate within the parentheses.
$=6(4)-3(6)+8 \quad$ Using order of operations, evaluate the exponent.
$=24-18+8 \quad$ Using order of operations, multiply 6 and 4 , and 3 and 6 .
$=14 \quad$ Subtract 18 from 24. Add the result to 8 .

## Try It Yourself

Evaluate the expression when $x=-\frac{1}{4}$ and $y=3$.

1. $2 x y$
2. $12 x-3 y$
3. $-4 x-y+4$
4. $8 x-y^{2}-3$

## - Writing Algebraic Expressions

Example 3 Write the phrase as an algebraic expression.
a. the sum of twice a number $m$ and four

$$
2 m+4
$$

b. eight less than three times a number $x$

$$
3 x-8
$$

## Try It Yourself

Write the phrase as an algebraic expression.
5. five more than three times a number $q$
6. nine less than a number $n$
7. the product of a number $p$ and six
8. the quotient of eight and a number $h$
9. four more than three times a number $t$
10. two less than seven times a number $c$

### 3.1 Algebraic Expressions

## Essential Question How can you simplify an algebraic expression?

## 1 ACIIVITY: Simplifying Algebraic Expressions

## Work with a partner.

a. Evaluate each algebraic expression when $x=0$ and when $x=1$. Use the results to match each expression in the left table with its equivalent expression in the right table.

b. Compare each expression in the left table with its equivalent expression in the right table. In general, how do you think you obtain the equivalent expression in the right column?

## 2 ACJIVIJY: Writing a Math Lesson

## Math <br> Practice

## Communicate

 PreciselyWhat can you do to make sure that you are communicating exactly what is needed in the Key Idea?

## Work with a partner. Use your results from Activity 1 to write a lesson on simplifying an algebraic expression.

| Simplifying an Algebraic Expression |
| :--- | :--- | :--- | :--- |

## What is Your Answer?

3. IN YOUR OWN WORDS How can you simplify an algebraic expression? Give an example that demonstrates your procedure.
4. REASONING Why would you want to simplify an algebraic expression? Discuss several reasons. complete Exercises 12-14 on page 84.

## Key Vocabulary 4

like terms, p. 82
simplest form, p. 82

Parts of an algebraic expression are called terms. Like terms are terms that have the same variables raised to the same exponents. Constant terms are also like terms. To identify terms and like terms in an expression, first write the expression as a sum of its terms.

## EXAMPLE (1) Identifying Terms and Like Terms

Identify the terms and like terms in each expression.
a. $9 x-2+7-x$
b. $z^{2}+5 z-3 z^{2}+z$

Rewrite as a sum of terms.
Terms: $\underbrace{9 x}_{9 x,}+\underbrace{(-2)}_{-2,}+\underbrace{7}_{7,}+\underbrace{(-x)}_{-x}$
Like terms: $9 x$ and $-x,-2$ and 7

Rewrite as a sum of terms.

$$
z^{2}+5 z+\left(-3 z^{2}\right)+z
$$

Terms: $z^{2}, 5 z,-3 z^{2}, \quad z$
Like terms: $z^{2}$ and $-3 z^{2}, 5 z$ and $z$

An algebraic expression is in simplest form when it has no like terms and no parentheses. To combine like terms that have variables, use the Distributive Property to add or subtract the coefficients.

EXAMPLE

## Study Tip

To subtract a variable term, add the term with the opposite coefficient.

## 2 Simplifying an Algebraic Expression

$$
\begin{aligned}
& \text { Simplify } \frac{3}{4} y+12-\frac{1}{2} y-6 . \\
& \frac{3}{4} y+12-\frac{1}{2} y-6=\frac{3}{4} y+12+\left(-\frac{1}{2} y\right)+(-6) \quad \text { Rewrite as a sum. } \\
& =\frac{3}{4} y+\left(-\frac{1}{2} y\right)+12+(-6) \quad \begin{array}{l}
\text { Commutative Property } \\
\text { of Addition }
\end{array} \\
& =\left[\frac{3}{4}+\left(-\frac{1}{2}\right)\right] y+12+(-6) \quad \text { Distributive Property } \\
& =\frac{1}{4} y+6 \quad \text { Combine like terms. }
\end{aligned}
$$

## On Your Own

Identify the terms and like terms in the expression.

Now You're Ready
Exercises 5-10
and 12-17

1. $y+10-\frac{3}{2} y$
2. $2 r^{2}+7 r-r^{2}-9$
3. $7+4 p-5+p+2 q$

Simplify the expression.
4. $14-3 z+8+z$
5. $2.5 x+4.3 x-5$
6. $\frac{3}{8} b-\frac{3}{4} b$

$$
\text { Simplify }-\frac{1}{2}(6 n+4)+2 n
$$

$$
\begin{aligned}
-\frac{1}{2}(6 n+4)+2 n & =-\frac{1}{2}(6 n)+\left(-\frac{1}{2}\right)(4)+2 n & & \text { Distributive Property } \\
& =-3 n+(-2)+2 n & & \text { Multiply. }
\end{aligned}
$$

$$
=-3 n+2 n+(-2) \quad \text { Commutative Property }
$$ of Addition

$$
=(-3+2) n+(-2) \quad \text { Distributive Property }
$$

$$
\begin{equation*}
=-n-2 \tag{Simplify.}
\end{equation*}
$$

## On Your Own

Now You're Ready
Exercises 18-20

Simplify the expression.
7. $3(q+1)-4$
8. $-2(g+4)+7 g$
9. $7-4\left(\frac{3}{4} x-\frac{1}{4}\right)$

## EXAMPLE



Each person in a group buys a ticket, a medium drink, and a large popcorn. Write an expression in simplest form that represents the amount of money the group spends at the movies. Interpret the expression.

Words Each is each medium is and each large is $\$ 4$. ticket \$7.50, drink \$2.75, popcorn

Variable The same number of each item is purchased. So, $x$ can represent the number of tickets, the number of medium drinks, and the number of large popcorns.
Expression

$$
7.50 x
$$

$$
+
$$

$$
2.75 x
$$

$$
+
$$$4 x$

$$
\begin{aligned}
7.50 x+2.75 x+4 x & =(7.50+2.75+4) x & & \text { Distributive Property } \\
& =14.25 x & & \text { Add coefficients. }
\end{aligned}
$$

$\therefore$ The expression $14.25 x$ indicates that the total cost per person is $\$ 14.25$.

## On Your Own

10. WHAT IF? Each person buys a ticket, a large drink, and a small popcorn. How does the expression change? Explain.

### 3.1 Exergises

## Vocabulary and Concept Check

1. WRITING Explain how to identify the terms of $3 y-4-5 y$.
2. WRITING Describe how to combine like terms in the expression $3 n+4 n-2$.
3. VOCABULARY Is the expression $3 x+2 x-4$ in simplest form? Explain.
4. REASONING Which algebraic expression is in simplest form? Explain.

| $5 x-4+6 y$ |
| :---: |
| $3(7+y)$ |
| $12 n-n$ |

## Practice and Problem Solving

Identify the terms and like terms in the expression.
5. $t+8+3 t$
6. $3 z+4+2+4 z$
7. $2 n-n-4+7 n$
8. $-x-9 x^{2}+12 x^{2}+7$
9. $1.4 y+5-4.2-5 y^{2}+z$
10. $\frac{1}{2} s-4+\frac{3}{4} s+\frac{1}{8}-s^{3}$
11. ERROR ANALYSIS Describe and correct the error in identifying the like terms in the expression.

$$
\begin{aligned}
& 3 x-5+2 x^{2}+9 x=3 x+2 x^{2}+9 x-5 \\
& \text { Like Terms: } 3 x, 2 x^{2} \text {, and } 9 x
\end{aligned}
$$

## Simplify the expression.

(2) 12. $12 g+9 g$
15. $4.2 v-5-6.5 v$
18. $4(b-6)+19$
13. $11 x+9-7$
14. $8 s-11 s+6$
16. $8+4 a+6.2-9 a$
19. $4 p-5(p+6)$

21. HIKING On a hike, each hiker carries the items shown. Write an expression in simplest form that represents the weight carried by $x$ hikers. Interpret the expression.
20. $-\frac{2}{3}(12 c-9)+14 c$
22. STRUCTURE Evaluate the expression $-8 x+5-2 x-4+5 x$ when $x=2$ before and after simplifying. Which method do you prefer? Explain.
23. REASONING Are the expressions $8 x^{2}+3\left(x^{2}+y\right)$ and $7 x^{2}+7 y+4 x^{2}-4 y$ equivalent? Explain your reasoning.
24. CRITICAL THINKING Which solution shows a correct way of simplifying $6-4(2-5 x)$ ? Explain the errors made in the other solutions.
(A) $6-4(2-5 x)=6-4(-3 x)=6+12 x$
(B) $6-4(2-5 x)=6-8+20 x=-2+20 x$
(C) 6-4(2-5x)=2(2-5x)=4-10x
(D) $6-4(2-5 x)=6-8-20 x=-2-20 x$
25. BANNER Write an expression in simplest form that represents the area of the banner.


|  | Car | Truck |
| :--- | :---: | :---: |
| Wash | $\$ 8$ | $\$ 10$ |
| Wax | $\$ 12$ | $\$ 15$ |

26. CAR WASH Write an expression in simplest form 3 ft We're \#1 $\frac{\text { W) }}{(3+x) \mathrm{ft}}$ that represents the earnings for washing and waxing $x$ cars and $y$ trucks.

MODELING Draw a diagram that shows how the expression can represent the area of a figure. Then simplify the expression.
27. $5(2+x+3)$
28. $(4+1)(x+2 x)$
29. Trificeal You apply gold foil to a piece of red poster board to make the design shown.
a. Write an expression in simplest form that represents the area of the gold foil.
b. Find the area of the gold foil when $x=3$.
c. The pattern at the right is called "St. George's Cross." Find a country that uses this pattern as its flag.


## (A) Fair Game Review what you learned in previous grades \& lessons

Order the lengths from least to greatest. (Skills Review Handbook)
30. 15 in., 14.8 in., 15.8 in., 14.5 in., 15.3 in. 31. $0.65 \mathrm{~m}, 0.6 \mathrm{~m}, 0.52 \mathrm{~m}, 0.55 \mathrm{~m}, 0.545 \mathrm{~m}$
32. MULTIPLE CHOICE A bird's nest is 12 feet above the ground. A mole's den is 12 inches below the ground. What is the difference in height of these two positions? (Section 1.3)
(A) 24 in .
(B) 11 ft
(C) 13 ft
(D) 24 ft

## 3.2

## Addfing and Subtracting

Lhear Expressions

Essential Question How can you use algebra tiles to add or subtract algebraic expressions?

Key: + = variable $\quad-$ = variable $+\square-\square$ = zero pair $\pm=1 \quad \Xi=-1 \quad \pm \Theta$ = zero pair

## ACTIVITY: Writing Algebraic Expressions

Work with a partner. Write an algebraic expression shown by the algebra tiles.
a.

b.

c.

d.


## 2 ACIIVIIT: Adding Algebraic Expressions

Work with a partner. Write the sum of two algebraic expressions modeled by the algebra tiles. Then use the algebra tiles to simplify the expression.
a. $(++ \pm \pm)+(\square+ \pm \pm \pm)$
b. $(+\square-\Theta-)+(+\square)$

Linear Expressions
In this lesson, you will

- apply properties of operations to add and subtract linear expressions.
- solve real-life problems.
c. $(\square+ \pm+ \pm \pm)+\left(\begin{array}{c}+ \\ +\square \\ +\end{array}\right)$



## 3 ACJIV/IY: Subtracting Algebraic Expressions

## Math

Practice
Use Expressions
What do the tiles represent? How does this help you write an expression?

Work with a partner. Write the difference of two algebraic expressions modeled by the algebra tiles. Then use the algebra tiles to simplify the expression.
a. $(\square \pm \pm \pm)-(+ \pm)$
b. $(+\square-\Xi-(+)-(+\square-\Theta)$
c. $\left(\begin{array}{c}+ \\ + \\ +\end{array} \pm \pm \pm \pm\right)-\left(\begin{array}{l}+\square)\end{array}\right.$


## ACTIVITY: Adding and Subtracting Algebraic Expressions

Work with a partner. Use algebra tiles to model the sum or difference. Then use the algebra tiles to simplify the expression.
a. $(2 x+1)+(x-1)$
b. $(2 x-6)+(3 x+2)$
c. $(2 x+4)-(x+2)$
d. $(4 x+3)-(2 x-1)$

## What Is Your Answer?

5. IN YOUR OWN WORDS How can you use algebra tiles to add or subtract algebraic expressions?
6. Write the difference of two algebraic expressions modeled by the algebra tiles. Then use the algebra tiles to simplify the expression.
$(- \pm \pm \pm)-\left(\begin{array}{l}- \\ - \\ -\end{array}\right)$

## Practice

Use what you learned about adding and subtracting algebraic expressions to complete Exercises 6 and 7 on page 90.

## Key Vocabulary

linear expression, p. 88

A linear expression is an algebraic expression in which the exponent of the variable is 1 .

| Linear Expressions | $-4 x$ | $3 x+5$ | $5-\frac{1}{6} x$ |
| :--- | :---: | :---: | :---: |
| Nonlinear Expressions | $x^{2}$ | $-7 x^{3}+x$ | $x^{5}+1$ |

You can use a vertical or a horizontal method to add linear expressions.

## EXAMPLE (1) Adding Linear Expressions

## Find each sum.

a. $(x-2)+(3 x+8)$

Vertical method: Align

$$
x-2
$$

like terms vertically and add. $\frac{+3 x+8}{4 x+6}$
b. $(-4 y+3)+(11 y-5)$

Horizontal method: Use properties of operations to group like terms and simplify.

$$
\begin{aligned}
(-4 y+3)+(11 y-5) & =-4 y+3+11 y-5 & & \text { Rewrite the sum. } \\
& =-4 y+11 y+3-5 & & \text { Commutative Property } \\
& =(-4 y+11 y)+(3-5) & & \text { Group like terms. } \\
& =7 y-2 & & \text { Combine like terms. }
\end{aligned}
$$

## EXAMPLE 2 Adding Linear Expressions

Find $2(-7.5 z+3)+(5 z-2)$.

$$
\begin{array}{rlrl}
2(-7.5 z+3)+(5 z-2) & =-15 z+6+5 z-2 & & \text { Distributive Property } \\
& =-15 z+5 z+6-2 & & \text { Commutative Property } \\
& =-10 z+4 & & \text { of Addition } \\
& & \text { Combine like terms. }
\end{array}
$$

## On Your Own

Now You're Ready
Exercises 8-16

## Find the sum.

1. $(x+3)+(2 x-1)$
2. $(-8 z+4)+(8 z-7)$
3. $(4-n)+2(-5 n+3)$
4. $\frac{1}{2}(w-6)+\frac{1}{4}(w+12)$

To subtract one linear expression from another, add the opposite of each term in the expression. You can use a vertical or a horizontal method.

## EXAMPLE

## Study Tip

To find the opposite of a linear expression, you can multiply the expression by -1 .

## 3 Subtracting Linear Expressions

Find each difference.
a. $(5 x+6)-(-x+6)$
b. $(7 y+5)-2(4 y-3)$
a. Vertical method: Align like terms vertically and subtract.

$$
\begin{array}{r}
(5 x+6) \\
-(-x+6)
\end{array} \text { Add the opposite. } \quad \begin{array}{r}
5 x+6 \\
+\quad x-6 \\
\hline 6 x
\end{array}
$$

b. Horizontal method: Use properties of operations to group like terms and simplify.

$$
\begin{array}{rlrl}
(7 y+5)-2(4 y-3) & =7 y+5-8 y+6 & & \text { Distributive Property } \\
& =7 y-8 y+5+6 & & \text { Commutative Property } \\
& =(7 y-8 y)+(5+6) & & \text { of Addition } \\
& =-y+11 & & \text { Group like terms. } \\
\text { Combine like terms. }
\end{array}
$$

## EXAMPLE <br> 4 Real-Life Application

The original price of a cowboy hat is $d$ dollars. You use a coupon and buy the hat for ( $d-2$ ) dollars. You decorate the hat and sell it for $(2 d-4)$ dollars. Write an expression that represents your earnings from buying and selling the hat. Interpret the expression.

$$
\begin{aligned}
\text { earnings } & =\text { selling price }- \text { purchase price } & & \text { Use a model. } \\
& =(2 d-4)-(d-2) & & \text { Write the difference. } \\
& =(2 d-4)+(-d+2) & & \text { Add the opposite. } \\
& =2 d-d-4+2 & & \text { Group like terms. } \\
& =d-2 & & \text { Combine like terms. }
\end{aligned}
$$

$\therefore \quad$ You earn $(d-2)$ dollars. You also paid $(d-2)$ dollars, so you doubled your money by selling the hat for twice as much as you paid for it.

## On Your Own

Now You're Ready
Exercises 19-24

Find the difference.
5. $(m-3)-(-m+12)$
6. $-2(c+2.5)-5(1.2 c+4)$
7. WHAT IF? In Example 4, you sell the hat for $(d+2)$ dollars. How much do you earn from buying and selling the hat?

## Vocabulary and Concept Check

VOCABULARY Determine whether the algebraic expression is a linear expression. Explain.

1. $x^{2}+x+1$
2. $-2 x-8$
3. $x-x^{4}$
4. WRITING Describe two methods for adding or subtracting linear expressions.
5. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Subtract $x$ from $3 x-1$.

What is $x$ more than $3 x-1$ ?

Find $3 x-1$ decreased by $x$.

What is the difference of $3 x-1$ and $x$ ?

## Practice and Problem Solving

Write the sum or difference of two algebraic expressions modeled by the algebra tiles. Then use the algebra tiles to simplify the expression.
6. $\left(\begin{array}{ccc}+ & - & - \\ + & - & -\end{array}\right)+\left(\begin{array}{c}+ \\ +\end{array}+ \pm \pm\right)$
7. $\left(\begin{array}{c}+ \\ + \pm \pm \pm \pm \pm \\ +\end{array}+\left(\begin{array}{c}+ \\ + \\ +\end{array}\right)\right.$

## Find the sum.

8. $(n+8)+(n-12)$
9. $(7-b)+(3 b+2)$
10. $(2 w-9)+(-4 w-5)$
11. $(2 x-6)+4(x-3)$
12. $5(-3.4 k-7)+(3 k+21)$
13. $3(2-0.9 h)+(-1.3 h-4)$
14. $\frac{1}{3}(9-6 m)+\frac{1}{4}(12 m-8)$
15. $(1-5 q)+2(2.5 q+8)$
16. $-\frac{1}{2}(7 z+4)+\frac{1}{5}(5 z-15)$
17. BANKING You start a new job. After $w$ weeks, you have $(10 w+120)$ dollars in your savings account and $(45 w+25)$ dollars in your checking account. Write an expression that represents the total in both accounts.
18. FIREFLIES While catching fireflies, you and a friend decide to have a competition. After $m$ minutes, you have $(3 m+13)$ fireflies and your friend has $(4 m+6)$ fireflies.
a. Write an expression that represents the number of fireflies you and your friend caught together.
b. The competition ends after 5 minutes. Who has more fireflies?

## Find the difference.

(3)
19. $(-2 g+7)-(g+11)$
20. $(6 d+5)-(2-3 d)$
21. $(4-5 y)-2(3.5 y-8)$
22. $(2 n-9)-5(-2.4 n+4)$
23. $\frac{1}{8}(-8 c+16)-\frac{1}{3}(6+3 c)$
24. $\frac{3}{4}(3 x+6)-\frac{1}{4}(5 x-24)$
25. ERROR ANALYSIS Describe and correct the error in finding the difference.

$$
\begin{aligned}
(4 m+9)-3(2 m-5) & =4 m+9-6 m-15 \\
& =4 m-6 m+9-15 \\
& =-2 m-6
\end{aligned}
$$

26. STRUCTURE Refer to the expressions in Exercise 18.
a. How many fireflies are caught each minute during the competition?
b. How many fireflies are caught before the competition starts?
27. LOGIC Your friend says the sum of two linear expressions is always a linear expression. Is your friend correct? Explain.
28. GEOMETRY The expression $17 n+11$ represents the perimeter (in feet) of the triangle. Write an expression that represents the measure of the third side.

29. TAXI Taxi Express charges $\$ 2.60$ plus $\$ 3.65$ per mile, and Cab Cruiser charges $\$ 2.75$ plus $\$ 3.90$ per mile. Write an expression that represents how much more Cab Cruiser charges than Taxi Express.

30. MODELING A rectangular room is 10 feet longer than it is wide. One-foot-by-one-foot tiles cover the entire floor. Write an expression that represents the number of tiles along the outside of the room.
31. SReasoning . Write an expression in simplest form that represents the vertical distance between the two lines shown. What is the distance when $x=3$ ? when $x=-3$ ?

## (4) Fair Game Review what you learned in previous grades \& lessons

Evaluate the expression when $x=-\frac{4}{5}$ and $y=\frac{1}{3}$. (Section 2.2)
32. $x+y$
33. $2 x+6 y$
34. $-x+4 y$
35. MULTIPLE CHOICE What is the surface area of a cube that has a side length of 5 feet? (Skills Review Handbook)
(A) $25 \mathrm{ft}^{2}$
(B) $75 \mathrm{ft}^{2}$
(C) $125 \mathrm{ft}^{2}$
(D) $150 \mathrm{ft}^{2}$

## Key Vocabulary

factoring an
expression, p. 92

When factoring an expression, you write the expression as a product of factors. You can use the Distributive Property to factor expressions.

## EXAMPLE (1) Factoring Out the GCF

## Factor $24 x$ - 18 using the GCF.

Find the GCF of $24 x$ and 18 by writing their prime factorizations.

$$
\begin{aligned}
24 x & =2 \cdot 2 \cdot 2,3 \cdot x \quad \text { Circle the common prime factors. } \\
18 & =2 \cdot 3 \cdot 3
\end{aligned}
$$

So, the GCF of $24 x$ and 18 is $2 \cdot 3=6$. Use the GCF to factor the expression.

$$
\begin{aligned}
24 x-18 & =6(4 x)-6(3) & & \text { Rewrite using GCF. } \\
& =6(4 x-3) & & \text { Distributive Property }
\end{aligned}
$$

$$
\because \quad \text { So, } 24 x-18=6(4 x-3)
$$

You can also use the Distributive Property to factor out any rational number from an expression.

## EXAMPLE 2 Factoring Out a Fraction

Factor $\frac{1}{2}$ out of $\frac{1}{2} x+\frac{3}{2}$.
Write each term as a product of $\frac{1}{2}$ and another factor.

$$
\begin{aligned}
\frac{1}{2} x & =\frac{1}{2} \cdot x \\
\frac{3}{2} & =\frac{1}{2} \cdot 3
\end{aligned}
$$

$$
\text { Think: } \frac{1}{2} x \text { is } \frac{1}{2} \text { times what? }
$$

$$
\text { Think: } \frac{3}{2} \text { is } \frac{1}{2} \text { times what? }
$$

## Linear Expressions

In this extension, you will

- factor linear expressions.

Use the Distributive Property to factor out $\frac{1}{2}$.

$$
\begin{array}{rlrl}
\frac{1}{2} x+\frac{3}{2} & =\frac{1}{2} \cdot x+\frac{1}{2} \cdot 3 \\
& =\frac{1}{2}(x+3) & \text { Rewrite the expression. } \\
\therefore \quad \text { So, } \frac{1}{2} x+\frac{3}{2} & =\frac{1}{2}(x+3) . & \text { Distributive Property }
\end{array}
$$

## Factor $\mathbf{- 2}$ out of $\mathbf{- 4 p + 1 0}$.

## Wath Practice

## View as Components

How does rewriting each term as a product help you see the common factor?

Write each term as a product of -2 and another factor.

$$
\begin{array}{rlrl}
-4 p & =-2 \cdot 2 p & \text { Think: }-4 p \text { is }-2 \text { times what? } \\
10 & =-2 \cdot(-5) & & \text { Think: } 10 \text { is }-2 \text { times what? }
\end{array}
$$

Use the Distributive Property to factor out -2 .

$$
\begin{aligned}
-4 p+10 & =-2 \cdot 2 p+(-2) \cdot(-5) & & \text { Rewrite the expression. } \\
& =-2[2 p+(-5)] & & \text { Distributive Property } \\
& =-2(2 p-5) & & \text { Simplify. } \\
\therefore \quad \text { So, }-4 p+10 & =-2(2 p-5) . & &
\end{aligned}
$$

## Practice

Factor the expression using the GCF.

1. $9+21$
2. $32-48$
3. $8 x+2$
4. $3 y-24$
5. $20 z-8$
6. $15 w+65$
7. $36 a+16 b$
8. $21 m-49 n$

Factor out the coefficient of the variable.
9. $\frac{1}{3} b-\frac{1}{3}$
10. $\frac{3}{8} d+\frac{3}{4}$
11. $2.2 x+4.4$
12. $4 h-3$
13. Factor $-\frac{1}{2}$ out of $-\frac{1}{2} x+6$.
14. Factor $-\frac{1}{4}$ out of $-\frac{1}{2} x-\frac{5}{4} y$.
15. WRESTLING A square wrestling mat has a perimeter of $(12 x-32)$ feet. Write an expression that represents the side length of the mat (in feet).

16. MAKING A DIAGRAM A table is 6 feet long and 3 feet wide. You extend the table by inserting two identical table leaves. The longest side length of each rectangular leaf is 3 feet. The extended table is rectangular with an area of $(18+6 x)$ square feet.
a. Make a diagram of the table and leaves.
b. Write an expression that represents the length of the extended table. What does $x$ represent?
17. STRUCTURE The area of the trapezoid is $\left(\frac{3}{4} x-\frac{1}{4}\right)$ square centimeters. Write two different pairs of expressions that represent possible lengths of the bases.


You can use a four square to organize information about a topic. Each of the four squares can be a category, such as definition, vocabulary, example, non-example, words, algebra, table, numbers, visual, graph, or equation. Here is an example of a four square for like terms.

| Definition <br> Terms that have the same <br> variables raised to the <br> same exponents | Examples <br> 2 and $-3,3 x$ and $-7 x$, <br> $x^{2}$ and $6 x^{2}$ |
| :--- | :--- |

## On Your Own

Make four squares to help you study these topics.

1. simplest form
2. linear expression
3. factoring expressions

After you complete this chapter, make four squares for the following topics.
4. equivalent equations
5. solving equations using addition or subtraction
6. solving equations using multiplication or division


> "My four square shows that my new red skateboard is faster than my old blue skateboard."
7. solving two-step equations

Identify the terms and like terms in the expression. (Section 3.1)

1. $11 x+2 x$
2. $9 x-5 x$
3. $21 x+6-x-5$
4. $8 x+14-3 x+1$

Simplify the expression. (Section 3.1)
5. $2(3 x+x)$
6. $-7+3 x+4 x$
7. $2 x+4-3 x+2+3 x$
8. $7 x+6+3 x-2-5 x$

Find the sum or difference. (Section 3.2)
9. $(s+12)+(3 s-8)$
10. $(9 t+5)+(3 t-6)$
11. $(2-k)+3(-4 k+2)$
12. $\frac{1}{4}(q-12)+\frac{1}{3}(q+9)$
13. $(n-8)-(-2 n+2)$
14. $-3(h-4)-2(-6 h+5)$

Factor out the coefficient of the variable. (Section 3.2)
15. $5 c-15$
16. $\frac{2}{9} j+\frac{2}{3}$
17. $2.4 n+9.6$
18. $-6 z+12$

19. PAINTING You buy the same number of brushes, rollers, and paint cans. Write an expression in simplest form that represents the total amount of money you spend for painting supplies.
(Section 3.1)
20. APPLES A basket holds $n$ apples. You pick $2 n-3$ apples, and your friend picks $n+4$ apples. Write an expression that represents the number of apples you and your friend picked. Interpret the expression. (Section 3.2)
21. EXERCISE Write an expression in simplest form for the perimeter of the exercise mat.
(Section 3.1)


# Solving Equations Using Addfition or Subtraction 

## Essential Question How can you use algebra ties to solve

 addition or subtraction equations?(1) ACIIVIJY: Solving Equations

Work with a partner. Use algebra tiles to model and solve the equation.
a. $x-3=-4$

Model the equation $x-3=-4$.

To get the variable tile by itself, remove
the tiles on the left side by adding

tiles to each side.
How many zero pairs can you remove from each side?


Circle them.
The remaining tile shows the value of $x$.

$\therefore$ So, $x=$ $\qquad$
b. $z-6=2$
c. $p-7=-3$
d. $-15=t-5$

## 2 ACIIVITY: Solving Equations

Solving Equations In this lesson, you will

- write simple equations.
- solve equations using addition or subtraction. - solve real-life problems.

Work with a partner. Use algebra tiles to model and solve the equation.
a. $-5=n+2$

Model the equation $-5=n+2$.


How many zero pairs can you remove from the right side?
Circle them.

$\therefore$ So, $n=$ $\square$
b. $y+10=-5$
c. $7+b=-1$
d. $8=12+z$

## Interpret

Results
How can you add tiles to make zero pairs? Explain how this helps you solve the equation.

## 3 ACJIV/JY: Writing and Solving Equations

Work with a partner. Write an equation shown by the algebra tiles. Then solve.
a.

b.

c.

d.

(4) ACTIV/JY: Using a Different Method to Find a Solution


Work with a partner. The melting point of a solid is the temperature at which the solid melts to become a liquid. The melting point of the element bromine is about $19^{\circ} \mathrm{F}$. This is about $57^{\circ} \mathrm{F}$ more than the melting point of mercury.
a. Which of the following equations can you use to find the melting point of mercury? What is the melting point of mercury?

$$
\begin{array}{ll|l|l}
x+57=19 & x-57=19 & x+19=57 & x+19=-57
\end{array}
$$

b. CHOOSE TOOLS How can you solve this problem without using an equation? Explain. How are these two methods related?

## What Is Your Answer?

5. IN YOUR OWN WORDS How can you use algebra tiles to solve addition or subtraction equations? Give an example of each.
6. STRUCTURE Explain how you could use inverse operations to solve addition or subtraction equations without using algebra tiles.
7. What makes the cartoon funny?
8. The word variable comes from the word vary. For example, the temperature in Maine varies a lot from winter to summer.

"Dear Sir: Yesterday you said $\boldsymbol{x}=\mathbf{2}$. Today you are saying $x=3$. Please make up your mind."

Write two other English sentences that use the word vary.

## Practice

Use what you learned about solving addition or subtraction equations to complete Exercises 5-8 on page 100.

## Key Vocabulary 4

 equivalent equations, p. 98Two equations are equivalent equations if they have the same solutions. The Addition and Subtraction Properties of Equality can be used to write equivalent equations.

## GO Key Ideas

## Addition Property of Equality

Words Adding the same number to each side of an equation produces an equivalent equation.
Algebra If $a=b$, then $a+c=b+c$.

## Subtraction Property of Equality

Words Subtracting the same number from each side of an equation produces an equivalent equation.
Algebra If $a=b$, then $a-c=b-c$.

Addition and subtraction are inverse operations.

## EXAMPLE (1) Solving Equations

a. Solve $x-5=-1$.

$$
x-5=-1 \quad \text { Write the equation. }
$$

Undo the subtraction. $\longrightarrow+5+5$

$$
\underline{5}
$$

$$
x=4
$$

Addition Property of Equality
Simplify.
$\therefore \quad$ The solution is $x=4$.

## Check

$$
\begin{array}{r}
x-5=-1 \\
4-5 \stackrel{?}{=}-1 \\
-1=-1
\end{array}
$$

b. Solve $z+\frac{3}{2}=\frac{1}{2}$.
$z+\frac{3}{2}=\frac{1}{2} \quad$ Write the equation.


Subtraction Property of Equality

Simplify.
$\therefore$ The solution is $z=-1$.

Check

$$
\begin{array}{r}
z+\frac{3}{2}=\frac{1}{2} \\
-1+\frac{3}{2} \stackrel{?}{=} \frac{1}{2} \\
\frac{1}{2}=\frac{1}{2}
\end{array}
$$

## On Your Own

Solve the equation. Check your solution.

1. $p-5=-2$
2. $w+13.2=10.4$
3. $x-\frac{5}{6}=-\frac{1}{6}$

## EXAMPLE 2 Writing an Equation

A company has a profit of $\mathbf{\$ 7 5 0}$ this week. This profit is $\mathbf{\$ 9 0 0}$ more than the profit $P$ last week. Which equation can be used to find $P$ ?
(A) $750=900-P$
(B) $750=P+900$
(C) $900=P-750$
(D) $900=P+750$

Words The profit this week is $\$ 900$ more than the profit last week.
Equation $750 \quad=P \quad+\quad 900$
$\therefore$ The equation is $750=P+900$. The correct answer is (B).

## On Your Own

4. A company has a profit of $\$ 120.50$ today. This profit is $\$ 145.25$ less than the profit $P$ yesterday. Write an equation that can be used to find $P$.

## EXAMPLE

3 Real-Life Application
The line graph shows the scoring while you and your friend played a video game. Write and solve an equation to find your score after Level 4.

You can determine the following from the graph.
Words Your friend's score is 33 points less than your score. Variable Let $s$ be your score after Level 4 .

| Equation | -8 | $=$ | $s$ | - | 33 |
| :--- | :--- | :--- | :--- | :--- | :--- |

$$
\begin{array}{rll}
-8=s-33 & & \text { Write equation. } \\
\underline{+33} & \underline{+33} & \\
\text { Addition Property of Equality } \\
25 & =s & \\
\text { Simplify. }
\end{array}
$$

$\therefore$ Your score after Level 4 is 25 points.
Reasonable? From the graph, your score after Level 4 is between 20 points and 30 points. So, 25 points is a reasonable answer.

## On Your Own

5. WHAT IF? You have -12 points after Level 1 . Your score is 27 points less than your friend's score. What is your friend's score?

## Vocabulary and Concepł Check

1. VOCABULARY What property would you use to solve $m+6=-4$ ?
2. VOCABULARY Name two inverse operations.
3. WRITING Are the equations $m+3=-5$ and $m=-2$ equivalent? Explain.
4. WHICH ONE DOESN'T BELONG? Which equation does not belong with the other three? Explain your reasoning.

$$
\begin{array}{lll}
x+3=-1 & x+1=-5 & x-2=-6
\end{array} \quad x-9=-13
$$

## Practice and Problem Solving

Solve the equation. Check your solution.
5. $a-6=13$
6. $-3=z-8$
7. $-14=k+6$
8. $x+4=-14$
9. $c-7.6=-4$
10. $-10.1=w+5.3$
11. $\frac{1}{2}=q+\frac{2}{3}$
12. $p-3 \frac{1}{6}=-2 \frac{1}{2}$
13. $g-9=-19$
14. $-9.3=d-3.4$
15. $4.58+y=2.5$
16. $x-5.2=-18.73$
17. $q+\frac{5}{9}=\frac{1}{6}$
18. $-2 \frac{1}{4}=r-\frac{4}{5}$
19. $w+3 \frac{3}{8}=1 \frac{5}{6}$
20. $4 \frac{2}{5}+k=-3 \frac{2}{11}$
21. ERROR ANALYSIS Describe and correct the error in finding the solution.

$$
\geqslant \begin{aligned}
x+8 & =10 \\
\frac{+8}{x} & =\frac{+8}{18}
\end{aligned}
$$

Write the word sentence as an equation. Then solve.
22. 4 less than a number $n$ is -15 .
23. 10 more than a number $c$ is 3 .
24. The sum of a number $y$ and -3 is -8 .
25. The difference between a number $p$ and 6 is -14 .

## In Exercises 26-28, write an equation. Then solve.

26. DRY ICE The temperature of dry ice is $-109.3^{\circ} \mathrm{F}$. This is $184.9^{\circ} \mathrm{F}$ less than the outside temperature. What is the outside temperature?
27. PROFIT A company makes a profit of $\$ 1.38$ million. This is $\$ 2.54$ million more than last year. What was the profit last year?
28. HELICOPTER The difference in elevation of a helicopter and a submarine is $18 \frac{1}{2}$ meters. The elevation of the submarine is $-7 \frac{3}{4}$ meters. What is the elevation of the helicopter?

GEOMETRY Write and solve an equation to find the unknown side length.
29. Perimeter $=12 \mathrm{~cm}$

30. Perimeter $=24.2$ in.

31. Perimeter $=34.6 \mathrm{ft}$


In Exercises 32-36, write an equation. Then solve.

32. STATUE OF LIBERTY The total height of the Statue of Liberty and its pedestal is 153 feet more than the height of the statue. What is the height of the statue?
33. BUNGEE JUMPING Your first jump is $50 \frac{1}{6}$ feet higher than your second jump. Your first jump reaches $-200 \frac{2}{5}$ feet. What is the height of your second jump?
34. TRAVEL Boatesville is $65 \frac{3}{5}$ kilometers from Stanton. A bus traveling from Stanton is $24 \frac{1}{3}$ kilometers from Boatesville. How far has the bus traveled?
35. GEOMETRY The sum of the measures of the angles of a triangle equals $180^{\circ}$. What is the measure of the missing angle?

36. SKATEBOARDING The table shows your scores in a skateboarding competition. The leader has 311.62 points. What score

| Round | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Points | 63.43 | 87.15 | 81.96 | $?$ | do you need in the fourth round to win?

37. CRITICAL THINKING Find the value of $2 x-1$ when $x+6=2$.


Find the values of $x$.
38. $|x|=2$
39. $|x|-2=4$
40. $|x|+5=18$

## Fair Game Review what you learned in previous grades \& lessons

Multiply or divide. (Section 1.4 and Section 1.5)
41. $-7 \times 8$
42. $6 \times(-12)$
43. $18 \div(-2)$
44. $-26 \div 4$
45. MULTIPLE CHOICE A class of 144 students voted for a class president. Three-fourths of the students voted for you. Of the students who voted for you, $\frac{5}{9}$ are female. How many female students voted for you? (Section 2.4)
(A) 50
(B) 60
(C) 80
(D) 108

## 3.4

## Solving Equations Using Multiplication or Division

## Essential Question How can you use mutiplication or division

 to solve equations?
## (1) ACIIVIJY: Using Division to Solve Equations

Work with a partner. Use algebra tiles to model and solve the equation.
a. $3 x=-12$

Model the equation $3 x=-12$.


Keep one of the groups. This shows the value of $x$.

$\therefore$ So, $x=\square$.
b. $2 k=-8$
c. $-15=3 t$
d. $-20=5 m$
e. $4 h=-16$

## 2 ACTIVIJY: Writing and Solving Equations

Solving Equations In this lesson, you will

- solve equations using multiplication or division.
- solve real-life problems.

Work with a partner. Write an equation shown by the algebra tiles. Then solve.
a.

b.

c.

d.


## 3 ACTIVITY: Using a Different Method to Find a Solution

## Math

Practice
Analyze Givens
How can you use the given information to decide which equation represents the situation?

Work with a partner. Choose the equation you can use to solve each problem.
Solve the equation. Then explain how to solve the problem without using an equation. How are the two methods related?
a. For the final part of a race, a handcyclist travels 32 feet each second across a distance of 400 feet. How many seconds does it take for the handcyclist to travel the last 400 feet of the race?

b. The melting point of the element radon is about $-96^{\circ} \mathrm{F}$. The melting point of nitrogen is about 3.6 times the melting point of radon. What is the melting point of nitrogen?

$$
\begin{array}{c|c}
3.6 x=-96 & x+96=3.6 \\
\frac{x}{3.6}=-96 & -96 x=3.6
\end{array}
$$

c. This year, a hardware store has a profit of $-\$ 6.0$ million. This profit is $\frac{3}{4}$ of last year's profit. What is last year's profit?

$$
\begin{aligned}
\frac{x}{-6}=\frac{3}{4} & -6 x=\frac{3}{4} \\
\frac{3}{4}+x=-6 & \frac{3}{4} x=-6
\end{aligned}
$$



## What Is Your Answer?

4. IN YOUR OWN WORDS How can you use multiplication or division to solve equations? Give an example of each.

## Practice

Use what you learned about solving equations to complete Exercises 7-10 on page 106.

## GO Key Ideas

## Multiplication Property of Equality

Words Multiplying each side of an equation by the same number produces an equivalent equation.
Algebra If $a=b$, then $a \cdot c=b \cdot c$.

## Remember

Multiplication and division are inverse operations.

## Division Property of Equality

Words Dividing each side of an equation by the same number produces an equivalent equation.
Algebra If $a=b$, then $a \div c=b \div c, c \neq 0$.

## EXAMPLE (7) Solving Equations

a. Solve $\frac{x}{3}=-6$.

$$
\frac{x}{3}=-6 \quad \text { Write the equation. }
$$

Undo the division. $\longrightarrow 3 \cdot \frac{x}{3}=3 \cdot(-6) \quad$ Multiplication Property of Equality

$$
x=-18 \quad \text { Simplify. }
$$

$\therefore$ The solution is $x=-18$.

Check

$$
\begin{aligned}
\frac{x}{3} & =-6 \\
\frac{-18}{3} & \stackrel{?}{=}-6 \\
-6 & =-6
\end{aligned}
$$

b. Solve $18=-4 y$.

$$
18=-4 y \quad \text { Write the equation. } \quad \text { Check }
$$

| Undo the multiplication. $\longrightarrow$ | $\frac{18}{-4}=\frac{-4 y}{-4}$ |
| ---: | :--- |
| -4.5 | $=y$ |

Division Property of Equality

$$
\begin{aligned}
& 18=-4 y \\
& 18 \stackrel{?}{=}-4(-4.5) \\
& 18=18
\end{aligned}
$$

$\because$ The solution is $y=-4.5$.

## On Your Own

Now You're Ready
Exercises 7-18

Solve the equation. Check your solution.

1. $\frac{x}{5}=-2$
2. $-a=-24$
3. $3=-1.5 n$

Solve $-\frac{4}{5} x=-8$

$$
-\frac{4}{5} x=-8 \quad \text { Write the equation. }
$$

$$
\begin{array}{ll}
\begin{array}{l}
\text { Multiply each side by }-\frac{5}{4^{\prime}} \\
\text { the reciprocal of }-\frac{4}{5}
\end{array}
\end{array} \longrightarrow-\frac{5}{4} \cdot\left(-\frac{4}{5} x\right)=-\frac{5}{4} \cdot(-8) \quad \text { Multiplicative Inverse Property }
$$

$\therefore$ The solution is $x=10$.

## On Your Own

Now You're Ready
Exercises 19-22

Solve the equation. Check your solution.
4. $-14=\frac{2}{3} x$
5. $-\frac{8}{5} b=5$
6. $\frac{3}{8} h=-9$

EXAMPLE


Record low temperature in Arizona

The record low temperature in Arizona is 1.6 times the record low temperature in Rhode Island. What is the record low temperature in Rhode Island?

Words The record low in Arizona is 1.6 times the record low in Rhode Island.

Variable Let $t$ be the record low in Rhode Island.
Equation $\quad-40 \quad=1.6 \times \quad t$

$$
\begin{aligned}
-40 & =1.6 t & & \text { Write equation. } \\
-\frac{40}{1.6} & =\frac{1.6 t}{1.6} & & \text { Division Property of Equality } \\
-25 & =t & & \text { Simplify. }
\end{aligned}
$$

$\therefore$ The record low temperature in Rhode Island is $-25^{\circ} \mathrm{F}$.

## On Your Own

Now You're Ready
Exercises 24-27
7. The record low temperature in Hawaii is -0.15 times the record low temperature in Alaska. The record low temperature in Hawaii is $12^{\circ} \mathrm{F}$. What is the record low temperature in Alaska?

## Vocabulary and Concept Check

1. WRITING Explain why you can use multiplication to solve equations involving division.
2. OPEN-ENDED Turning a light on and then turning the light off are considered to be inverse operations. Describe two other real-life situations that can be thought of as inverse operations.

Describe the inverse operation that will undo the given operation.
3. multiplying by 5
4. subtracting 12
5. dividing by -8
6. adding -6

## Practice and Problem Solving

Solve the equation. Check your solution.
7. $3 h=15$
8. $-5 t=-45$
9. $\frac{n}{2}=-7$
10. $\frac{k}{-3}=9$
11. $5 m=-10$
12. $8 t=-32$
13. $-0.2 x=1.6$
14. $-10=-\frac{b}{4}$
15. $-6 p=48$
16. $-72=8 d$
17. $\frac{n}{1.6}=5$
18. $-14.4=-0.6 p$
19. $\frac{3}{4} g=-12$
20. $8=-\frac{2}{5} c$
21. $-\frac{4}{9} f=-3$
22. $26=-\frac{8}{5} y$
(2)
23. ERROR ANALYSIS Describe and correct the error in finding the solution.

$$
\text { N } \begin{aligned}
-4.2 x & =21 \\
\frac{-4.2 x}{4.2} & =\frac{21}{4.2} \\
x & =5
\end{aligned}
$$

Write the word sentence as an equation. Then solve.
24. A number divided by -9 is -16 .
25. A number multiplied by $\frac{2}{5}$ is $\frac{3}{20}$.
26. The product of 15 and a number is -75 .
27. The quotient of a number and -1.5 is 21 .

In Exercises 28 and 29, write an equation. Then solve.
28. NEWSPAPERS You make $\$ 0.75$ for every newspaper you sell. How many newspapers do you have to sell to buy the soccer cleats?

29. ROCK CLIMBING A rock climber averages $12 \frac{3}{5}$ feet per minute. How many feet does the rock climber climb in 30 minutes?


OPEN-ENDED (a) Write a multiplication equation that has the given solution.
(b) Write a division equation that has the same solution.
30. -3
31. -2.2
32. $-\frac{1}{2}$
33. $-1 \frac{1}{4}$
34. REASONING Which of the methods can you use to solve $-\frac{2}{3} c=16$ ?

Multiply each side by $-\frac{2}{3}$.

Divide each side by $-\frac{2}{3}$.

Multiply each side by $-\frac{3}{2}$.
Multiply each side by 3 , then divide each side by -2 .
35. STOCK A stock has a return of $-\$ 1.26$ per day. Write and solve an equation to find the number of days until the total return is $-\$ 10.08$.
36. ELECTION In a school election, $\frac{3}{4}$ of the students vote. There are 1464 ballots. Write and solve an equation to find the number of students.
37. OCEANOGRAPHY Aquarius is an underwater ocean laboratory located in the Florida Keys National Marine Sanctuary. Solve the equation $\frac{31}{25} x=-62$ to find the value of $x$.
38. SHOPPING The price of a bike at Store A is $\frac{5}{6}$ the price at Store B. The price at Store A is $\$ 150.60$. Write and solve an equation to find how much you save by buying the bike at Store A.
39. CRITICAL THINKING Solve $-2|m|=-10$.
40.

Nomber In four days, your family drives $\frac{5}{7}$ of a trip.


Your rate of travel is the same throughout the trip. The total trip is 1250 miles. In how many more days will you reach your destination?

## Fair Game Review what you learned in previous grades \& lessons

Subtract. (Section 1.3)
41. $5-12$
42. $-7-2$
43. $4-(-8)$
44. $-14-(-5)$
45. MULTIPLE CHOICE Of the 120 apartments in a building, 75 have been scheduled to receive new carpet. What fraction of the apartments have not been scheduled to receive new carpet? (Skills Review Handbook)
(A) $\frac{1}{4}$
(B) $\frac{3}{8}$
(C) $\frac{5}{8}$
(D) $\frac{3}{4}$

### 3.5 Solving Two-Step Equations

## Essential Question How can you use algebratilest osove a

 two-step equation?
## (1) ACIIVIJY: Solving a Two-Step Equation

Work with a partner. Use algebra tiles to model and solve $2 x-3=-5$.
Model the equation $2 x-3=-5$.

Remove the red tiles on the left side by adding yellow tiles to each side.

How many zero pairs can you remove from each side?


Circle them.
Because there are $\square$ green tiles, divide the red tiles into equal groups. Circle the groups.

Keep one of the groups. This shows the value of $x$.

$\because$ So, $x=$ $\square$

## 2 ACIIVIIY: The Math behind the Tiles

Work with a partner. Solve $2 x-3=-5$ without using algebra tiles. Complete each step. Then answer the questions.

Use the steps in Activity 1 as a guide.

## Solving Equations

In this lesson, you will

- solve two-step equations.
- solve real-life problems.

| $2 x-3$ | $=-5$ |  | Write the equation. |
| ---: | :--- | ---: | :--- |
| $2 x-3+\square$ | $=-5+\square$ |  | Add to each side. |
| $2 x$ | $=\square$ |  | Simplify. |
| $2 x$ | $=\square$ |  | Divide each side by $\square$. |
| $x$ | $=\square$ |  | Simplify. |

$\therefore$ So, $x=\square$.
a. Which step is first, adding 3 to each side or dividing each side by 2 ?
b. How are the above steps related to the steps in Activity 1?

## 3 ACTIVIJY: Solving Equations Using Algebra Tiles

Work with a partner.

- Write an equation shown by the algebra tiles.
- Use algebra tiles to model and solve the equation.
- Check your answer by solving the equation without using algebra tiles.
a.

b.



## 4 ACTIVITY: Working Backwards

## Math Practice

## Maintain

 OversightHow does working backwards help you decide which operation to do first? Explain.

## Work with a partner.

a. Sample: Your friend pauses a video game to get a drink. You continue the game. You double the score by saving a princess. Then you lose 75 points because you do not collect the treasure. You finish the game with -25 points. How many points did you have when you started? One way to solve the problem is to work backwards. To do this, start with the end result and retrace the events.
You have -25 points at the end of the game. $\mathbf{- 2 5}$
You lost 75 points for not collecting the treasure, $\quad \mathbf{- 2 5 + 7 5}=\mathbf{5 0}$ so add 75 to -25 .

You doubled your score for saving the princess, $\quad \mathbf{5 0} \div \mathbf{2}=\mathbf{2 5}$ so find half of 50 .
$\therefore$ So, you started the game with 25 points.
b. You triple your account balance by making a deposit. Then you withdraw $\$ 127.32$ to buy groceries. Your account is now overdrawn by $\$ 10.56$. By working backwards, find your account balance before you made the deposit.

## What is Your Answer?

5. IN YOUR OWN WORDS How can you use algebra tiles to solve a two-step equation?
6. When solving the equation $4 x+1=-11$, what is the first step?
7. REPEATED REASONING Solve the equation $2 x-75=-25$. How do your steps compare with the strategy of working backwards in Activity 4?

## Practice

Use what you learned about solving two-step equations to complete Exercises 6-11 on page 112.

Solve $-3 x+5=2$. Check your solution.

$$
-3 x+5=2 \quad \text { Write the equation. }
$$



$$
-3 x=-3 \quad \text { Simplify. }
$$

Undo the multiplication. $\longrightarrow \frac{-3 x}{-3}=\frac{-3}{-3}$ Division Property of Equality $x=1 \quad$ Simplify.

Check

$$
\begin{array}{r}
-3 x+5=2 \\
-3(1)+5 \stackrel{?}{=} 2 \\
-3+5 \stackrel{?}{=} 2 \\
2=2
\end{array}
$$

$\therefore$ The solution is $x=1$.

## On Your Own

Solve the equation. Check your solution.

1. $2 x+12=4$
2. $-5 c+9=-16$
3. $3(x-4)=9$

EXAMPLE

## Study Tip

You can simplify the equation in Example 2 before solving. Multiply each side by the LCD of the fractions, 8 .

$$
\begin{aligned}
\frac{x}{8}-\frac{1}{2} & =-\frac{7}{2} \\
x-4 & =-28 \\
x & =-24
\end{aligned}
$$

## 2 Solving a Two-Step Equation

Solve $\frac{x}{8}-\frac{1}{2}=-\frac{7}{2}$. Check your solution.

$$
\begin{aligned}
& \frac{x}{8}-\frac{1}{2}=-\frac{7}{2} \quad \text { Write the equation. } \\
& +\frac{1}{2}+\frac{1}{2} \quad \text { Addition Property of Equality } \\
& \frac{x}{8}=-3 \quad \text { Simplify. } \\
& 8 \cdot \frac{x}{8}=8 \cdot(-3) \quad \begin{array}{l}
\text { Multiplication Property of } \\
\text { Equality }
\end{array} \\
& x=-24 \quad \text { Simplify. } \\
& \text { Check } \\
& \frac{x}{8}-\frac{1}{2}=-\frac{7}{2} \\
& \frac{-24}{8}-\frac{1}{2} \stackrel{?}{=}-\frac{7}{2} \\
& -3-\frac{1}{2} \stackrel{?}{=}-\frac{7}{2} \\
& -\frac{7}{2}=-\frac{7}{2}
\end{aligned}
$$

$\therefore$ The solution is $x=-24$.

## On Your Own

Now You're Ready
Exercises 20-25

Solve the equation. Check your solution.
4. $\frac{m}{2}+6=10$
5. $-\frac{z}{3}+5=9$
6. $\frac{2}{5}+4 a=-\frac{6}{5}$

## EXAMPLE

3 Combining Like Terms Before Solving
Solve $3 y-8 y=25$.

| $3 y-8 y$ | $=25$ |  | Write the equation. |
| ---: | :--- | ---: | :--- |
| $-5 y$ | $=25$ |  | Combine like terms. |
| $y$ | $=-5$ |  | Divide each side by -5. |

$\therefore$ The solution is $y=-5$.

## EXAMPLE (4) Real-Life Application



The height at the top of a roller coaster hill is 10 times the height $h$ of the starting point. The height decreases 100 feet from the top to the bottom of the hill. The height at the bottom of the hill is $\mathbf{- 1 0}$ feet. Find $h$.

| Location | Verbal Description | Expression |
| :--- | :--- | :---: |
| Start | The height at the start is $h$. | $h$ |
| Top of hill | The height at the top of the hill is <br> 10 times the starting height $h$. | $10 h$ |
| Bottom of hill | The height decreases by 100 feet. <br> So, subtract 100. | $10 h-100$ |

The height at the bottom of the hill is -10 feet. Solve $10 h-100=-10$ to find $h$.

$$
\begin{aligned}
10 h-100 & =-10 & & \text { Write equation. } \\
10 h & =90 & & \text { Add } 100 \text { to each side. } \\
h & =9 & & \text { Divide each side by } 10 .
\end{aligned}
$$

$\therefore$ So, the height at the start is 9 feet.

## On Your Own

Exercises 29-34

Solve the equation. Check your solution.
7. $4-2 y+3=-9$
8. $7 x-10 x=15$
9. $-8=1.3 m-2.1 m$
10. WHAT IF? In Example 4, the height at the bottom of the hill is -5 feet. Find the height $h$.

## Vocabulary and Concept Check

1. WRITING How do you solve two-step equations?

Match the equation with the first step to solve it.
2. $4+4 n=-12$
3. $4 n=-12$
4. $\frac{n}{4}=-12$
5. $\frac{n}{4}-4=-12$
A. Add 4.
B. Subtract 4 .
C. Multiply by 4 .
D. Divide by 4 .

## Practice and Problem Solving

Solve the equation. Check your solution.
(1)
6. $2 v+7=3$
7. $4 b+3=-9$
8. $17=5 k-2$
9. $-6 t-7=17$
10. $8 n+16.2=1.6$
11. $-5 g+2.3=-18.8$
12. $2 t-5=-10$
13. $-4 p+9=-5$
14. $11=-5 x-2$
15. $4+2.2 h=-3.7$
16. $-4.8 f+6.4=-8.48$
17. $7.3 y-5.18=-51.9$

ERROR ANALYSIS Describe and correct the error in finding the solution.
18.

$$
\begin{aligned}
-6+2 x & =-10 \\
-6+\frac{2 x}{2} & =-\frac{10}{2} \\
-6+x & =-5 \\
x & =1
\end{aligned}
$$

19. 

$$
1 \begin{aligned}
-3 x+2 & =-7 \\
-3 x & =-9 \\
-\frac{3 x}{3} & =\frac{-9}{3} \\
x & =-3
\end{aligned}
$$

Solve the equation. Check your solution.
(2)
20. $\frac{3}{5} g-\frac{1}{3}=-\frac{10}{3}$
21. $\frac{a}{4}-\frac{5}{6}=-\frac{1}{2}$
22. $-\frac{1}{3}+2 z=-\frac{5}{6}$
23. $2-\frac{b}{3}=-\frac{5}{2}$
24. $-\frac{2}{3} x+\frac{3}{7}=\frac{1}{2}$
25. $-\frac{9}{4} v+\frac{4}{5}=\frac{7}{8}$

In Exercises 26-28, write an equation. Then solve.
26. WEATHER Starting at 1:00 P.m., the temperature changes -4 degrees per hour. How long will it take to reach $-1^{\circ}$ ?
27. BOWLING It costs $\$ 2.50$ to rent bowling shoes. Each game costs $\$ 2.25$. You have $\$ 9.25$. How many games can you bowl?
28. CELL PHONES A cell phone company charges a monthly fee plus $\$ 0.25$ for each text message. The monthly fee is $\$ 30.00$ and you owe $\$ 59.50$. How many text messages did you have?


Solve the equation. Check your solution.
(3)
29. $3 v-9 v=30$
30. $12 t-8 t=-52$
31. $-8 d-5 d+7 d=72$
32. $6(x-2)=-18$
33. $-4(m+3)=24$
34. $-8(y+9)=-40$
35. WRITING Write a real-world problem that can be modeled by $\frac{1}{2} x-2=8$. Then solve the equation.
36. GEOMETRY The perimeter of the parallelogram is 102 feet. Find $m$.


REASONING Exercises 37 and 38 are missing information. Tell what information you need to solve the problem.
37. TAXI A taxi service charges an initial fee plus $\$ 1.80$ per mile. How far can you travel for \$12?
38. EARTH The coldest surface temperature on the Moon is 57 degrees colder than twice the coldest surface temperature on Earth. What is the coldest surface temperature on Earth?
39. PROBLEM SOLVING On Saturday, you catch insects for your science class. Five of the insects escape. The remaining insects are divided into three groups to share in class. Each group has nine insects. How many insects did you catch on Saturday?
a. Solve the problem by working backwards.
b. Solve the equation $\frac{x-5}{3}=9$. How does the answer compare with the
answer to part (a)? answer to part (a)?

40. UNDERWATER HOTEL You must scuba dive to the entrance of your room at Jules' Undersea Lodge in Key Largo, Florida. The diver is 1 foot deeper than $\frac{2}{3}$ of the elevation of the entrance. What is the elevation of the entrance?
41. Geometry How much should you change the length of the rectangle so that the perimeter is 54 centimeters? Write an equation that shows how you found your answer.


## Fair Game Review what you learned in previous grades \& lessons

Multiply or divide. (Section 2.4)
42. $-6.2 \times 5.6$
43. $\frac{8}{3} \times\left(-2 \frac{1}{2}\right)$
44. $\frac{5}{2} \div\left(-\frac{4}{5}\right)$
45. $-18.6 \div(-3)$
46. MULTIPLE CHOICE Which fraction is not equivalent to 0.75 ? (Skills Review Handbook)
(A) $\frac{15}{20}$
(B) $\frac{9}{12}$
(C) $\frac{6}{9}$
(D) $\frac{3}{4}$

Solve the equation. Check your solution. (Section 3.3, Section 3.4, and Section 3.5)

1. $-6.5+x=-4.12$
2. $4 \frac{1}{2}+p=-5 \frac{3}{4}$
3. $-\frac{b}{7}=4$
4. $-2 w+3.7=-0.5$

Write the word sentence as an equation. Then solve. (Section 3.3 and Section 3.4)
5. The difference between a number $b$ and 7.4 is -6.8 .
6. $5 \frac{2}{5}$ more than a number $a$ is $7 \frac{1}{2}$.
7. A number $x$ multiplied by $\frac{3}{8}$ is $-\frac{15}{32}$.
8. The quotient of two times a number $k$ and -2.6 is 12 .

Write and solve an equation to find the value of $\boldsymbol{x}$. (Section 3.3 and Section 3.5)
9. Perimeter $=26$

10. Perimeter $=23.59$

11. Perimeter $=33$

12. BANKING You withdraw $\$ 29.79$ from your bank account. Now your balance is $-\$ 20.51$. Write and solve an equation to find the amount of money in your bank account before you withdrew the money. (Section 3.3)
13. WATER LEVEL During a drought, the water level of a lake changes $-3 \frac{1}{5}$ feet per day. Write and solve an equation to find how long it takes for the water level to change -16 feet. (Section 3.4)
14. BASKETBALL A basketball game has four quarters. The length of a game is 32 minutes. You play the entire game except for $4 \frac{1}{2}$ minutes. Write and solve an equation to find the mean time you play per quarter. (Section 3.5)
15. SCRAPBOOKING The mat needs to be cut to have a 0.5 -inch border on all four sides. (Section 3.5)
a. How much should you cut from the left and right sides?
b. How much should you cut from the top and bottom?


## Review Key Vocabulary

like terms, p. 82 simplest form, p. 82
linear expression, p. 88
factoring an expression, p. 92
equivalent equations, p. 98

## Review Examples and Exercises

### 3.1 Algebraic Expressions (pp. 80-85)

a. Identify the terms and like terms in the expression $6 y+9+3 y-7$.

Rewrite as a sum of terms.

$$
\text { Terms: } 6 y, 9, \quad \underbrace{6 y}_{3 y,}+\underbrace{9+3 y}_{-7}+\underbrace{(-7)}_{-7}
$$

Like terms: $6 y$ and $3 y, 9$ and -7
b. Simplify $\frac{2}{3} y+14-\frac{1}{6} y-8$.

$$
\begin{aligned}
\frac{2}{3} y+14-\frac{1}{6} y-8 & =\frac{2}{3} y+14+\left(-\frac{1}{6} y\right)+(-8) & & \text { Rewrite as a sum. } \\
& =\frac{2}{3} y+\left(-\frac{1}{6} y\right)+14+(-8) & & \text { Commutative Property of Addition } \\
& =\left[\frac{2}{3}+\left(-\frac{1}{6}\right)\right] y+14+(-8) & & \text { Distributive Property } \\
& =\frac{1}{2} y+6 & & \text { Combine like terms. }
\end{aligned}
$$

## Exercises

## Identify the terms and like terms in the expression.

1. $z+8-4 z$
2. $3 n+7-n-3$
3. $10 x^{2}-y+12-3 x^{2}$

Simplify the expression.
4. $4 h-8 h$
5. $6.4 r-7-2.9 r$
6. $\frac{3}{5} x+19-\frac{3}{20} x-7$
7. $3(2+q)+15$
8. $\frac{1}{8}(16 m-8)-17$
9. $-1.5(4-n)+2.8$

### 3.2 Adding and Subtracting Linear Expressions (pp. 86-93)

a. Find $(5 z+4)+(3 z-6)$.

$$
\begin{array}{r}
5 z+4 \\
+3 z-6 \\
\hline 8 z-2
\end{array} \quad \text { Align like terms vertically and add. }
$$

b. Factor $\frac{1}{4}$ out of $\frac{1}{4} x-\frac{3}{4}$.

Write each term as a product of $\frac{1}{4}$ and another factor.

$$
\frac{1}{4} x=\frac{1}{4} \cdot x \quad-\frac{3}{4}=\frac{1}{4} \cdot(-3)
$$

Use the Distributive Property to factor out $\frac{1}{4}$.

$$
\begin{aligned}
& \frac{1}{4} x-\frac{3}{4}=\frac{1}{4} \cdot x+\frac{1}{4} \cdot(-3)=\frac{1}{4}(x-3) \\
\therefore \quad & \text { So, }, \frac{1}{4} x-\frac{3}{4}=\frac{1}{4}(x-3) .
\end{aligned}
$$

## Exercises

## Find the sum or difference.

10. $(c-4)+(3 c+9)$
11. $\frac{2}{5}(d-10)-\frac{2}{3}(d+6)$

Factor out the coefficient of the variable.
12. $2 b+8$
13. $\frac{1}{4} y+\frac{3}{8}$
14. $1.7 j-3.4$
15. $-5 p+20$

## З„3 Solving Equations Using Addition or Subtraction (pp. 96-101)

Solve $x-9=-6$.

$$
x-9=-6
$$

Undo the subtraction.

$+9+9$
$x=3$

Write the equation.
Addition Property of Equality
Simplify.

Check

$$
\begin{array}{r}
x-9=-6 \\
3-9 \stackrel{?}{=}-6 \\
-6=-6
\end{array}
$$

## Exercises

Solve the equation. Check your solution.
16. $p-3=-4$
17. $6+q=1$
18. $-2+j=-22$
19. $b-19=-11$
20. $n+\frac{3}{4}=\frac{1}{4}$
21. $v-\frac{5}{6}=-\frac{7}{8}$
22. $t-3.7=1.2$
23. $\ell+15.2=-4.5$

## 3. $\sqrt[4]{ }$ Solving Equations Using Multiplication or Division (pp. 102-107)

Solve $\frac{x}{5}=-7$.

$$
\frac{x}{5}=-7
$$

Write the equation.

Multiplication Property of Equality
Check

$$
\begin{aligned}
\frac{x}{5} & =-7 \\
\frac{-35}{5} & \stackrel{?}{=}-7 \\
-7 & =-7
\end{aligned}
$$

$$
x=-35 \quad \text { Simplify. }
$$

## Exercises

Solve the equation. Check your solution.
24. $\frac{x}{3}=-8$
25. $-7=\frac{y}{7}$
26. $-\frac{z}{4}=-\frac{3}{4}$
27. $-\frac{w}{20}=-2.5$
28. $4 x=-8$
29. $-10=2 y$
30. $-5.4 z=-32.4$
31. $-6.8 w=3.4$
32. TEMPERATURE The mean temperature change is $-3.2^{\circ} \mathrm{F}$ per day for 5 days. What is the total change over the 5-day period?

### 3.5 Solving Two-Step Equations (pp. 108-113)

Solve $-6 y+7=-5$. Check your solution.

$$
\begin{array}{rlrl}
-6 y+7 & =-5 & & \text { Write the equation. } \\
\frac{-7}{-6 y} & =-12 & & \text { Subtraction Property of Equality } \\
\frac{-6 y}{-6} & =\frac{-12}{-6} & & \text { Simplify. } \\
y & =2 & & \text { Division Property of Equality } \\
& & \text { Simplify. }
\end{array}
$$

Check

$$
\begin{array}{r}
-6 y+7=-5 \\
-6(2)+7 \stackrel{?}{=}-5 \\
-12+7 \stackrel{?}{=}-5 \\
-5=-5
\end{array}
$$

$\therefore$ - The solution is $y=2$.

## Exercises

Solve the equation. Check your solution.
33. $-2 c+6=-8$
34. $3(3 w-4)=-20$
35. $\frac{w}{6}+\frac{5}{8}=-1 \frac{3}{8}$
36. $-3 x-4.6=5.9$
37. EROSION The floor of a canyon has an elevation of -14.5 feet. Erosion causes the elevation to change by -1.5 feet per year. How many years will it take for the canyon floor to have an elevation of -31 feet?

## Simplify the expression.

1. $8 x-5+2 x$
2. $2.5 w-3 y+4 w$
3. $3(5-2 n)+9 n$
4. $\frac{5}{7} x+15-\frac{9}{14} x-9$

## Find the sum or difference.

5. $(3 j+11)+(8 j-7)$
6. $\frac{3}{4}(8 p+12)+\frac{3}{8}(16 p-8)$
7. $(2 r-13)-(-6 r+4)$
8. $-2.5(2 s-5)-3(4.5 s-5.2)$

Factor out the coefficient of the variable.
9. $3 n-24$
10. $\frac{1}{2} q+\frac{5}{2}$

Solve the equation. Check your solution.
11. $7 x=-3$
12. $2(x+1)=-2$
13. $\frac{2}{9} g=-8$
14. $z+14.5=5.4$
15. $-14=6 c$
16. $\frac{2}{7} k-\frac{3}{8}=-\frac{19}{8}$
17. HAIR SALON Write an expression in simplest form that represents the income from $w$ women and $m$ men getting a haircut and a shampoo.

|  | Women | Men |
| :--- | :---: | :---: |
| Haircut | $\$ 45$ | $\$ 15$ |
| Shampoo | $\$ 12$ | $\$ 7$ |

18. RECORD A runner is compared with the world record holder during a race. A negative number means the runner is ahead of the time of the world record holder. A positive number means that the runner is behind the time of the world record holder. The table shows the time difference between the runner and the world record holder for each lap. What time

| Lap | Time Difference |
| :---: | :---: |
| 1 | -1.23 |
| 2 | 0.45 |
| 3 | 0.18 |
| 4 | $?$ | difference does the runner need for the fourth lap to match the world record?

19. GYMNASTICS You lose 0.3 point for stepping out of bounds during a floor routine. Your final score is 9.124 . Write and solve an equation to find your score before the penalty.
20. PERIMETER The perimeter of the triangle is 45 . Find the value of $x$.

21. Which equation represents the word sentence shown below?

The quotient of a number $b$ and 0.3 equals negative 10 .
A. $0.3 b=10$
B. $\frac{b}{0.3}=-10$
C. $\frac{0.3}{b}=-10$
D. $\frac{b}{0.3}=10$
2. What is the value of the expression below when $c=0$ and $d=-6$ ?

$$
\frac{c d-d^{2}}{4}
$$

Test-Taking Strategy

3. What is the value of the expression below?

$$
-38-(-14)
$$

F. -52
G. -24
H. 24
I. 52
4. The daily low temperatures last week are shown below.


What is the mean low temperature of last week?
A. $-2^{\circ} \mathrm{F}$
B. $6^{\circ} \mathrm{F}$
C. $8^{\circ} \mathrm{F}$
D. $10^{\circ} \mathrm{F}$
5. Which equation is equivalent to the equation shown below?

$$
-\frac{3}{4} x+\frac{1}{8}=-\frac{3}{8}
$$

F. $-\frac{3}{4} x=-\frac{3}{8}-\frac{1}{8}$
G. $-\frac{3}{4} x=-\frac{3}{8}+\frac{1}{8}$
H. $x+\frac{1}{8}=-\frac{3}{8} \cdot\left(-\frac{4}{3}\right)$
I. $x+\frac{1}{8}=-\frac{3}{8} \cdot\left(-\frac{3}{4}\right)$
6. What is the value of the expression below?

$$
-0.28 \div(-0.07)
$$

7. Karina was solving the equation in the box below.

$$
\begin{aligned}
-96 & =-6(x-15) \\
-96 & =-6 x-90 \\
-96+90 & =-6 x-90+90 \\
-6 & =-6 x \\
\frac{-6}{-6} & =\frac{-6 x}{-6} \\
1 & =x
\end{aligned}
$$

What should Karina do to correct the error that she made?
A. First add 6 to both sides of the equation.
B. First subtract $x$ from both sides of the equation.
C. Distribute the -6 to get $6 x-90$.
D. Distribute the -6 to get $-6 x+90$.
8. The perimeter of the rectangle is 400 inches. What is the value of $j$ ? (All measurements are in inches.)

F. 35
H. 140
G. 85
I. 200
9. Jacob was evaluating the expression below when $x=-2$ and $y=4$.

$$
3+x^{2} \div y
$$

His work is in the box below.

$$
\begin{aligned}
3+x^{2} \div y & =3+\left(-2^{2}\right) \div 4 \\
& =3-4 \div 4 \\
& =3-1 \\
& =3
\end{aligned}
$$

What should Jacob do to correct the error that he made?
A. Divide 3 by 4 before subtracting.
B. Square -2 , then divide.
C. Square then divide.
D. Subtract 4 from 3 before dividing.
10. Which number is equivalent to the expression shown below?

$$
-2 \frac{1}{4}-\left(-8 \frac{3}{8}\right)
$$

F. $-10 \frac{5}{8}$
G. $-10 \frac{1}{3}$
H. $6 \frac{1}{8}$
I. $6 \frac{1}{2}$
11. You want to buy the bicycle. You already have $\$ 43.50$ saved and plan to save an additional $\$ 7.25$ every week.

Part A Write and solve an equation to find the number of weeks you need to save before you can purchase the bicycle.

Part B How much sooner could you purchase the bicycle if you had a coupon for $\$ 20$ off and saved $\$ 8.75$ every week? Explain your reasoning.


