1.4 Perimeter and Area in the Coordinate Plane

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Learning Target Find perimeters and areas of polygons in the coordinate plane.

Success Criteria
- I can classify and describe polygons.
- I can find perimeters of polygons in the coordinate plane.
- I can find areas of polygons in the coordinate plane.

EXPLORE IT! Finding the Perimeter and Area of a Quadrilateral

TENNESSEE MATH STANDARDS
G.GPE.A.3, G.MG.A.1

Work with a partner.

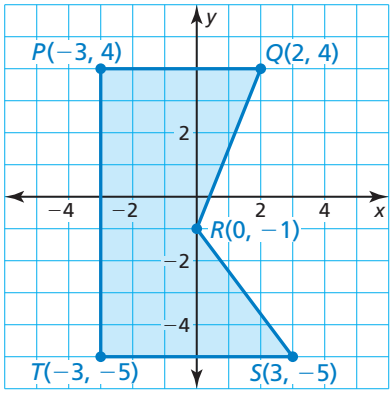
a. Use a piece of graph paper to draw a quadrilateral *ABCD* in a coordinate plane. At most two sides of your quadrilateral can be horizontal or vertical. Plot and label the vertices of *ABCD*.

b. Make several observations about quadrilateral *ABCD*. Can you use any other names to classify your quadrilateral? Explain.

c. Explain how you can find the perimeter of quadrilateral *ABCD*. Then find the perimeter. Compare your method with those of your classmates.

d. Explain how you can find the area of quadrilateral *ABCD*. Then find the area. Compare your method with those of your classmates.

e. Use the methods from parts (c) and (d) to find the perimeter and area of the polygon below. Explain your reasoning.



Math Practice

Look for Structure
In part (d), how might it be helpful to visualize a polygon as being composed of one or more smaller polygons?

Classifying Polygons

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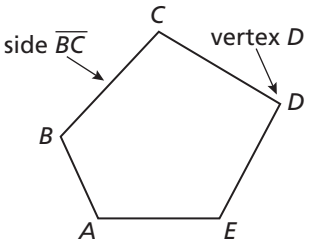
KEY IDEA

Polygons

In geometry, a figure that lies in a plane is called a plane figure. Recall that a *polygon* is a closed plane figure formed by three or more line segments called *sides*. Each side intersects exactly two sides, one at each *vertex*, so that no two sides with a common vertex are collinear.



polygon *ABCDE*

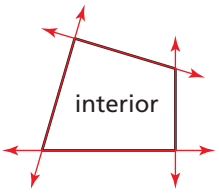
READING

You can name a polygon by listing the vertices in consecutive order.

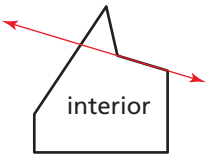
Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	*n*-gon

The number of sides determines the type of polygon, as shown in the table. You can also name a polygon using the term *n*-gon, where *n* is the number of sides. For instance, a 14-gon is a polygon with 14 sides.

A polygon is *convex* when no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is *concave*.



convex polygon



concave polygon

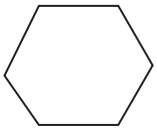
EXAMPLE 1 Classifying Polygons

WATCH

Classify each polygon by the number of sides. Tell whether it is *convex* or *concave*.

a.

b.



SOLUTION

a. The polygon has four sides. So, it is a quadrilateral. The polygon is concave.

b. The polygon has six sides. So, it is a hexagon. The polygon is convex.

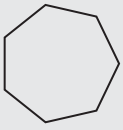
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

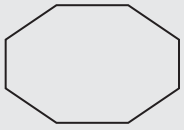
Classify the polygon by the number of sides. Tell whether it is *convex* or *concave*.

1.

2.



3.



4. MP **REASONING** Can you draw a concave triangle? If so, draw one. If not, explain why not.

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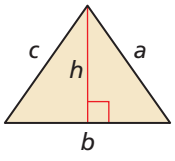
Finding Perimeter and Area in the Coordinate Plane

You can use the formulas below and the Distance Formula to find perimeters and areas of polygons in the coordinate plane.

Perimeter and Area

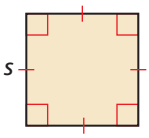
Triangle



$P = a + b + c$

$A = \frac{1}{2}bh$

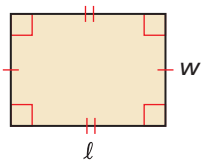
Square



$P = 4s$

$A = s^2$

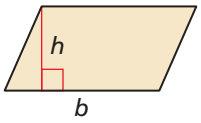
Rectangle



$P = 2\ell + 2w$

$A = \ell w$

Parallelogram



$A = bh$

READING

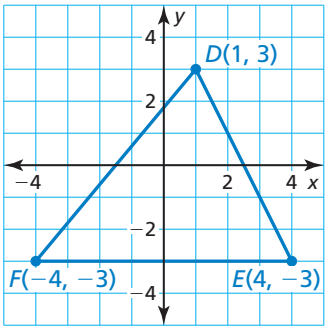
You can read the notation $\triangle DEF$ as "triangle *D E F*."

EXAMPLE 2 Finding Perimeter in the Coordinate Plane

WATCH

Find the perimeter of $\triangle DEF$ with vertices $D(1, 3)$, $E(4, -3)$, and $F(-4, -3)$.

SOLUTION



Step 1 Draw the triangle in a coordinate plane by plotting the vertices and connecting them.

Step 2 Find the length of each side.

$\overline{DE}$ Let $(x_1, y_1) = (1, 3)$ and $(x_2, y_2) = (4, -3)$.

$DE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(4 - 1)^2 + (-3 - 3)^2}$ Substitute.

$= \sqrt{3^2 + (-6)^2}$ Subtract.

$= \sqrt{45}$ Simplify.

$\overline{EF}$ $EF = |-4 - 4| = |-8| = 8$ Ruler Postulate

$\overline{FD}$ Let $(x_1, y_1) = (-4, -3)$ and $(x_2, y_2) = (1, 3)$.

$FD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[1 - (-4)]^2 + [3 - (-3)]^2}$ Substitute.

$= \sqrt{5^2 + 6^2}$ Subtract.

$= \sqrt{61}$ Simplify.

REMEMBER

Perimeter has linear units, such as feet or meters. Area has square units, such as square feet or square meters.

Step 3 Find the sum of the side lengths.

$DE + EF + FD = \sqrt{45} + 8 + \sqrt{61} \approx 22.52$ units

So, the perimeter of $\triangle DEF$ is about 22.52 units.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the perimeter of the polygon with the given vertices.

5. $G(-3, 2)$, $H(2, 2)$, $J(-1, -3)$

6. $Q(-4, -1)$, $R(1, 4)$, $S(4, 1)$, $T(-1, -4)$

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EXAMPLE 3 Finding Area in the Coordinate Plane

WATCH

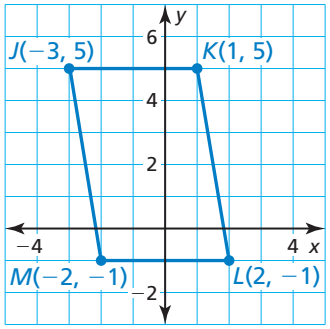
Find the area of $\square JKLM$ with vertices $J(-3, 5)$, $K(1, 5)$, $L(2, -1)$, and $M(-2, -1)$.

READING

You can read the notation $\square JKLM$ as "parallelogram J K L M."

SOLUTION

Step 1 Draw the parallelogram in a coordinate plane by plotting the vertices and connecting them.



Step 2 Find the length of the base and the height.

Base

Let $\overline{JK}$ be the base. Use the Ruler Postulate to find the length of $\overline{JK}$.

$JK = |1 - (-3)| = |4| = 4$ Ruler Postulate

So, the length of the base is 4 units.

Height

Let the height be the distance from point M to $\overline{JK}$. By counting grid lines, you can determine that the height is 6 units.

ANOTHER WAY

You can also find the area of $\square JKLM$ by decomposing the parallelogram into a rectangle and two triangles, then finding the sum of their areas.

Step 3 Substitute the values for the base and height into the formula for the area of a parallelogram.

$A = bh$ Write the formula for area of a parallelogram.

$= 4(6)$ Substitute.

$= 24$ Multiply.

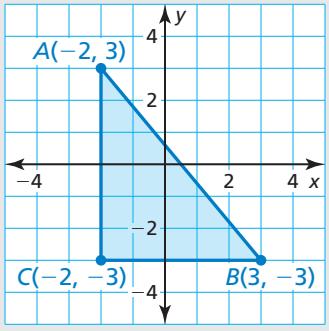
▶ So, the area of $\square JKLM$ is 24 square units.

SELF-ASSESSMENT

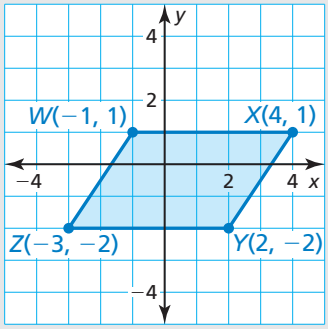
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the area of the polygon with the given vertices.

7.



8.



9. $N(-1, 1)$, $P(2, 1)$, $Q(2, -2)$, $R(-1, -2)$

10. $K(-3, 3)$, $L(3, 3)$, $M(3, -1)$, $N(-3, -1)$

EXAMPLE 4 Modeling Real Life

WATCH

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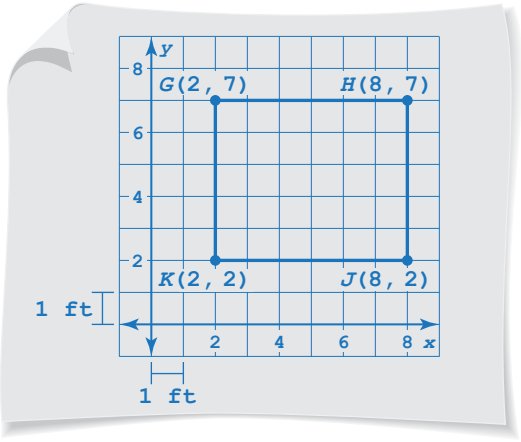
You are building a shed in your backyard. The diagram shows the four vertices of the shed floor. Each unit in the coordinate plane represents 1 foot. Find the perimeter and the area of the floor of the shed.



SOLUTION

1. **Understand the Problem**
   You are given the coordinates of the vertices of the shed floor. You need to find the perimeter and the area of the floor.

2. **Make a Plan** The floor of the shed is rectangular, so use the coordinates of the vertices to find the length and the width. Then use formulas to find the perimeter and area.

3. **Solve and Check**

   Step 1 Find the length and the width.

   Length $GH = |8 - 2| = 6$ Ruler Postulate

   Width $KG = |7 - 2| = 5$ Ruler Postulate

   The shed has a length of 6 feet and a width of 5 feet.

   Step 2 Substitute the values for the length and width into the formulas for the perimeter P and area A of a rectangle.

$P = 2\ell + 2w$	Write formulas.	$A = \ell w$
$= 2(6) + 2(5)$	Substitute.	$= 6(5)$
$= 22$	Evaluate.	$= 30$

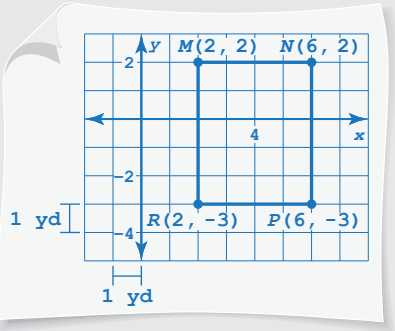
   ▶ The perimeter of the floor of the shed is 22 feet and the area is 30 square feet.

Check To check the perimeter, count the grid lines around the floor of the shed. There are 22 grid lines. To check the area, count the number of grid squares that make up the floor. There are 30 grid squares. ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. You are building a patio in your school's courtyard. The diagram shows the four vertices of the patio. Each unit in the coordinate plane represents 1 yard. Find the perimeter and the area of the patio.



1.4 Practice WITH CalcChat® AND CalcView®

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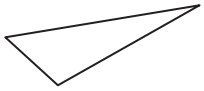
In Exercises 1–4, classify the polygon by the number of sides. Tell whether it is *convex* or *concave*.
Example 1

1.

2.



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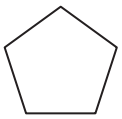
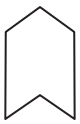


4.



In Exercises 5–10, find the perimeter of the polygon with the given vertices. *Example 2*

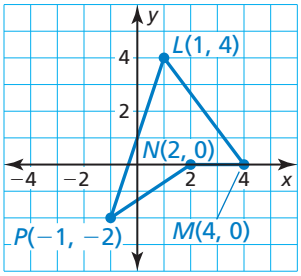
5. $G(2, 4), H(2, -3), J(-2, -3), K(-2, 4)$

6. $Q(-3, 2), R(1, 2), S(1, -2), T(-3, -2)$

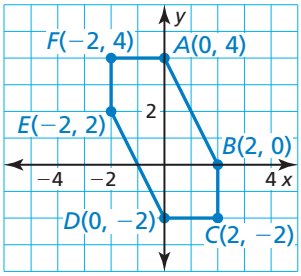
7. $U(-2, 4), V(3, 4), W(3, -4)$

8. $X(-1, 3), Y(3, 0), Z(-1, -2)$

9.



10.



In Exercises 11–14, find the area of the polygon with the given vertices. *Example 3*

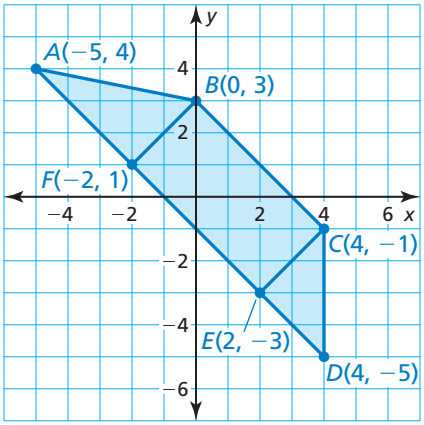
11. $E(3, 1), F(3, -2), G(-2, -2)$

12. $J(-3, 4), K(4, 4), L(3, -3)$

13. $W(0, 0), X(0, 3), Y(-3, 3), Z(-3, 0)$

14. $N(-4, 1), P(1, 1), Q(3, -1), R(-2, -1)$

In Exercises 15–18, use the diagram to find the perimeter and the area of the polygon.



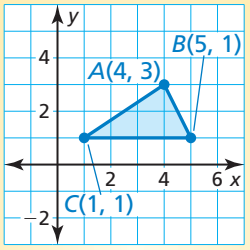
15. $\triangle CDE$

16. $\triangle ABF$

17. rectangle $BCEF$

18. quadrilateral $ABCD$

19. ERROR ANALYSIS Describe and correct the error in finding the area of the triangle.



$b = |5 - 1| = 4$

$h = \sqrt{(5 - 4)^2 + (1 - 3)^2} = \sqrt{5}$

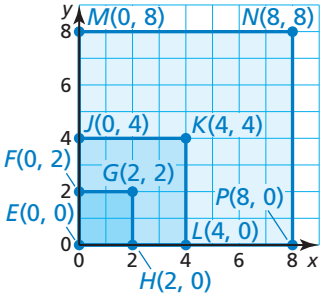
$A = \frac{1}{2}bh = \frac{1}{2}(4)(\sqrt{5}) = 2\sqrt{5}$

The area is $2\sqrt{5}$ square units.

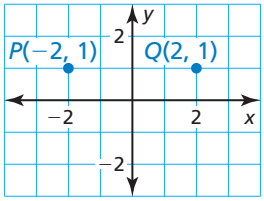
20. MP REPEATED REASONING Use the diagram.

a. Find the perimeter and area of each square.

b. What happens to the area of a square when its perimeter increases by a factor of n?



COLLEGE PREP **In Exercises 21 and 22, use the diagram.**



21. Determine which point is the remaining vertex of a triangle with an area of 4 square units.

Ⓐ $R(2, 0)$ Ⓑ $S(-2, -1)$

Ⓒ $T(-1, 0)$ Ⓓ $U(2, -2)$

22. Determine which points are the remaining vertices of a rectangle with a perimeter of 14 units.

Ⓐ $A(2, -1)$ and $B(-2, -1)$

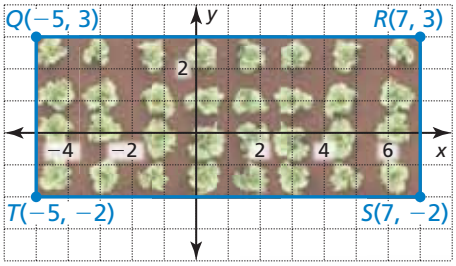
Ⓑ $C(-1, -2)$ and $D(1, -2)$

Ⓒ $E(-2, -2)$ and $F(2, -2)$

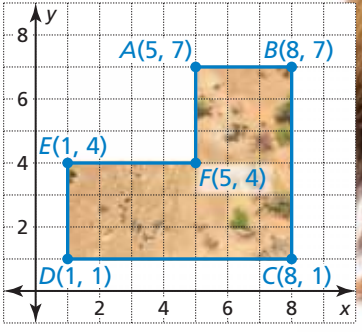
Ⓓ $G(2, 0)$ and $H(-2, 0)$

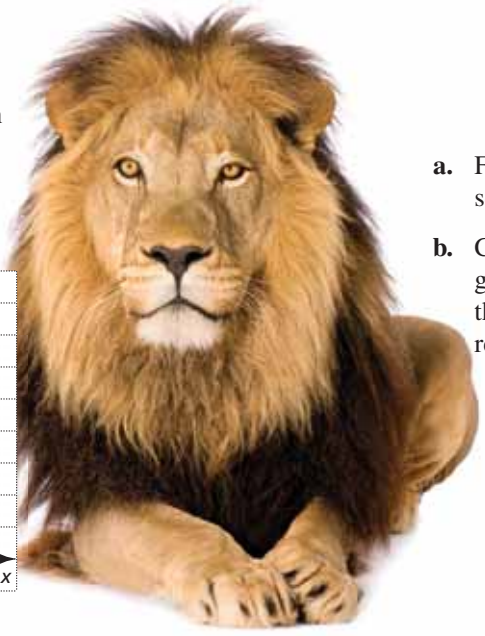
23. **MODELING REAL LIFE** You are building a school garden. The diagram shows the four vertices of the garden. Each unit in the coordinate plane represents 1 foot. Find the perimeter and the area of the garden.

Example 4



24. **MODELING REAL LIFE** The diagram shows the vertices of a lion sanctuary. Each unit in the coordinate plane represents 100 feet. Find the perimeter and the area of the sanctuary.



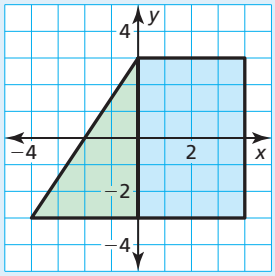


25. **MODELING REAL LIFE** You and your friend hike to a waterfall that is 4 miles east of where you left your bikes. You then hike to a lookout point that is 2 miles north of your bikes. From the lookout point, you return to your bikes.

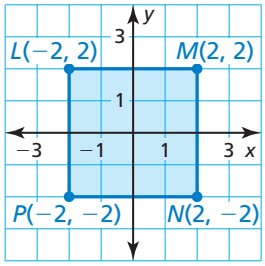
a. About how far do you hike? Assume you travel along straight paths.

b. From the waterfall, your friend hikes to a wishing well before going to the lookout point and returning to the bikes. The wishing well is 3 miles north and 2 miles west of the lookout point. About how far does your friend hike?

26. **HOW DO YOU SEE IT?**
Without performing any calculations, determine whether the triangle or the rectangle has a greater area. Which polygon has a greater perimeter? Explain your reasoning.



27. **ANALYZING RELATIONSHIPS** Use the diagram.



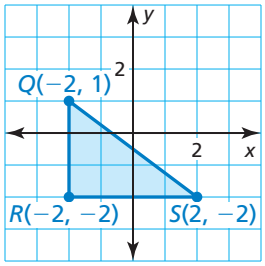
a. Find the perimeter and the area of the square.

b. Connect the midpoints of the sides of the given square to make a quadrilateral. Is this quadrilateral a square? Explain your reasoning.

c. Find the perimeter and the area of the quadrilateral you made in part (b). Compare this area to the area of the square you found in part (a).

28. CONNECTING CONCEPTS The lines $y_1 = 2x - 6$, $y_2 = -3x + 4$, and $y_3 = -\frac{1}{2}x + 4$ intersect to form the sides of a right triangle. Find the perimeter and the area of the triangle.

29. MAKING AN ARGUMENT Will a rectangle that has the same perimeter as $\triangle QRS$ have the same area as the triangle? Explain your reasoning.



30. THOUGHT PROVOKING
A café that has an area of 350 square feet is being expanded to occupy an adjacent space that has an area of 150 square feet. Draw a diagram of the remodeled café in a coordinate plane.

31. MP REASONING Triangle ABC has a perimeter of 12 units. The vertices of the triangle are $A(x, 2)$, $B(2, -2)$, and $C(-1, 2)$. Find the value of x.

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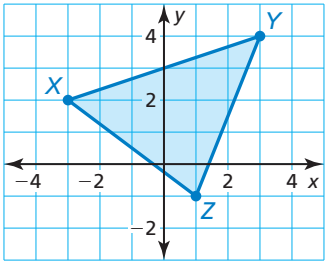
32. PERFORMANCE TASK As a graphic designer, your job is to create a company logo that includes at least two different polygons and has an area of at least 50 square units. Draw your logo in a coordinate plane and record its perimeter and area. Describe the company and create a proposal explaining how your logo relates to the company.

33. DIG DEEPER Find the area of $\triangle XYZ$. (*Hint:* Draw a rectangle whose sides contain points *X*, *Y*, and *Z*.)



REVIEW & REFRESH

WATCH

34. Does the table represent a *linear* or *nonlinear* function? Explain.

x	−1	0	1	2	3
y	−9	−7	−5	−3	−1

In Exercises 35–38, solve the equation.

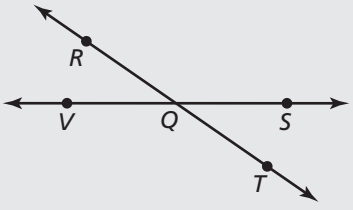
35. $3x - 7 = 2$

36. $4 = 9 + 5x$

37. $x + 4 = x - 12$

38. $\frac{x + 1}{2} = 4x - 3$

In Exercises 39 and 40, use the diagram.



39. Give another name for $\overline{RT}$.

40. Name two pairs of opposite rays.

In Exercises 41 and 42, the endpoints of a segment are given. Find the coordinates of the midpoint *M* and the length of the segment.

41. $J(4, 3)$ and $K(2, -3)$

42. $L(-4, 5)$ and $N(5, -3)$

43. Use a compass and straightedge to construct a copy of the line segment.

44. MODELING REAL LIFE You deposit $200 into a savings account that earns 5% annual interest compounded quarterly. Write a function that represents the balance y (in dollars) after t years.

In Exercises 45 and 46, graph the function. Then describe the transformations from the graph of $f(x) = |x|$ to the graph of the function.

45. $g(x) = |x - 4| + 5$

46. $h(x) = -3|x + 1|$

47. Find the perimeter and the area of $\square ABCD$ with vertices $A(3, 5)$, $B(6, 5)$, $C(4, -1)$, and $D(1, -1)$.