1.1 Points, Lines, and Planes

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Learning Target Use defined terms and undefined terms.

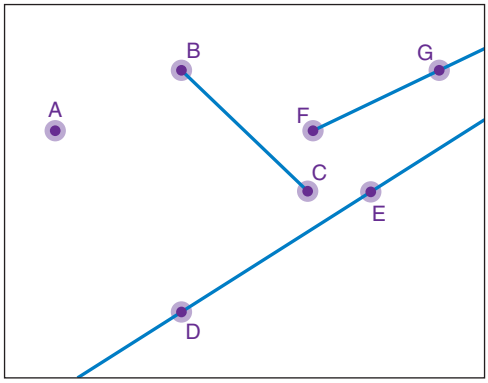
Success Criteria
- I can describe a point, a line, and a plane.
- I can define and name segments and rays.
- I can sketch intersections of lines and planes.

EXPLORE IT! Using Technology

TENNESSEE MATH STANDARDS
preparing for G.CO.A.3

Work with a partner.

a. Use technology to draw several points. Also, draw some lines, line segments, and rays.

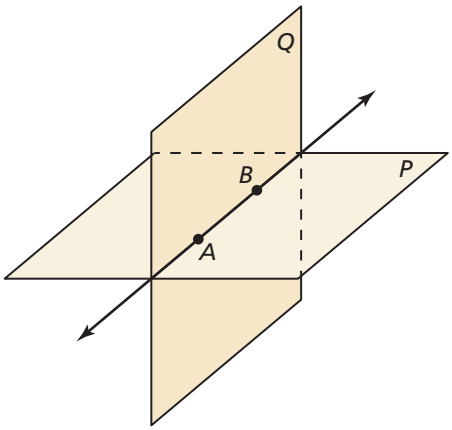


b. How would you describe a line? a point?

c. What is the difference between a line and a line segment? a line and a ray?

d. Write your own definitions for a line segment and a ray, based on how they relate to a line.

e. The diagram shows plane P and plane Q intersecting. How would you describe a plane?



f. MP **CHOOSE TOOLS** Describe the ways in which each of the following can intersect and not intersect. Provide a sketch or use real-life objects to model each type of intersection.

i. two lines

ii. a line and a plane

iii. two planes

Math Practice

Listen and Ask Questions
Ask a few classmates to read their answers to parts (b)–(d). Ask any questions you have about their answers.

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Vocabulary

VOCAB

undefined terms, *p. 4*
point, *p. 4*
line, *p. 4*
plane, *p. 4*
collinear points, *p. 4*
coplanar points, *p. 4*
defined terms, *p. 5*
line segment, or segment, *p. 5*
endpoints, *p. 5*
ray, *p. 5*
opposite rays, *p. 5*
intersection, *p. 6*

Using Undefined Terms

In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

KEY IDEA

Undefined Terms: Point, Line, and Plane

Point A **point** has no dimension. A dot represents a point.

point *A*

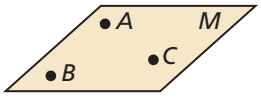
Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

Through any two points, there is exactly one line. You can use any two points on a line to name it.

line ℓ, line AB ($\overleftrightarrow{AB}$), or line BA ($\overleftrightarrow{BA}$)

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



plane *M*, or plane *ABC*

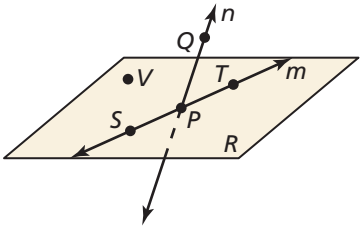
Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Naming Points, Lines, and Planes

WATCH

a. Give two other names for $\overleftrightarrow{PQ}$ and plane R.

b. Name three points that are collinear. Name four points that are coplanar.



SOLUTION

a. Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line n. Other names for plane R are plane SVT and plane PTV.

b. Points S, P, and T lie on the same line, so they are collinear. Points S, P, T, and V lie in the same plane, so they are coplanar.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Use the diagram in Example 1. Give two other names for $\overleftrightarrow{ST}$. Name a point that is *not* coplanar with points Q, S, and T.
2. **WRITING** Compare collinear points and coplanar points.

Using Defined Terms

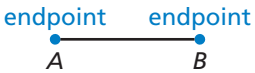
In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

KEY IDEA

Defined Terms: Segment and Ray

The diagrams below use the points A and B and parts of the line AB.

Segment A **line segment**, or **segment**, is a part of a line that consists of two **endpoints** and all points on the line between the endpoints.

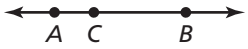


segment AB ($\overline{AB}$), or segment BA ($\overline{BA}$)

Ray A **ray** is a part of a line that consists of an endpoint and all points on the line on one side of the endpoint.

ray AB ($\overrightarrow{AB}$)

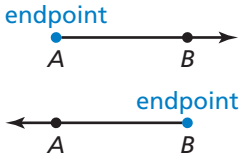
Opposite Rays Two rays that have the same endpoint and form a line are **opposite rays**.



$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.

STUDY TIP

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



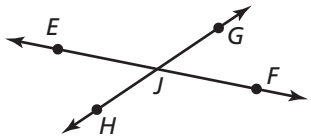
Segments and rays are collinear when they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar when they lie in the same plane.

EXAMPLE 2 Naming Segments, Rays, and Opposite Rays

WATCH

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint J. Which of these rays are opposite rays?



COMMON ERROR

In Example 2, $\overrightarrow{JG}$ and $\overrightarrow{JF}$ have a common endpoint, but they are not collinear. So, they are *not* opposite rays.

SOLUTION

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

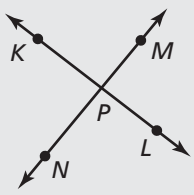
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram.

3. Give another name for $\overline{KL}$.

4. Are $\overrightarrow{KP}$ and $\overrightarrow{PK}$ the same ray? Are $\overrightarrow{NP}$ and $\overrightarrow{NM}$ the same ray? Explain.



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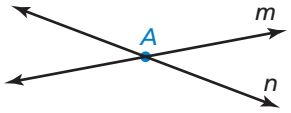
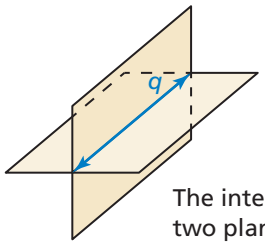
Sketching Intersections

Two or more geometric figures *intersect* when they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of these two lines is a point.



The intersection of these two planes is a line.

EXAMPLE 3 Sketching Intersections of Lines and Planes

WATCH

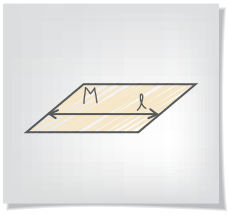
a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

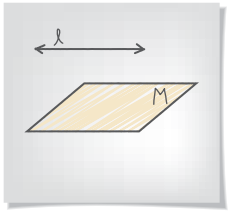
c. Sketch a plane and a line that intersects the plane at a point.

SOLUTION

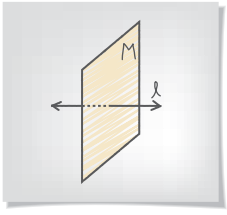
a.



b.



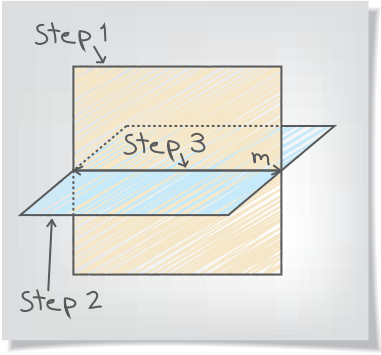
c.



EXAMPLE 4 Sketching an Intersection of Planes

WATCH

Sketch two planes that intersect in a line.



SOLUTION

Step 1 Draw a vertical plane. Shade the plane.

Step 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where planes are hidden.

Step 3 Draw the line of intersection.

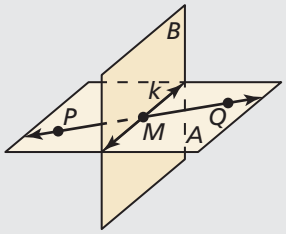
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. Sketch two different lines that intersect a plane at the same point.

6. Sketch two planes that do not intersect.

Use the diagram.



7. Name the intersection of $\overleftrightarrow{PQ}$ and line k.

8. Name the intersection of plane A and plane B.

9. Name the intersection of line k and plane A.

Electric utilities use sulfur hexafluoride as an insulator. Leaks in electrical equipment contribute to the release of sulfur hexafluoride into the atmosphere.

Solving Real-Life Problems

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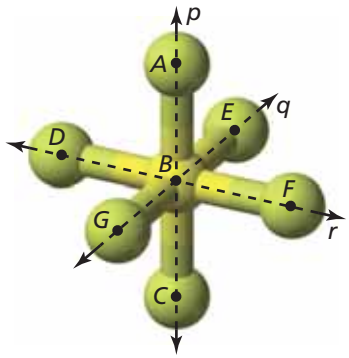
EXAMPLE 5 **Modeling Real Life** WATCH INFO

The diagram shows a model of a molecule of sulfur hexafluoride, the most potent greenhouse gas in the world. Name two different planes that contain line r.



SOLUTION

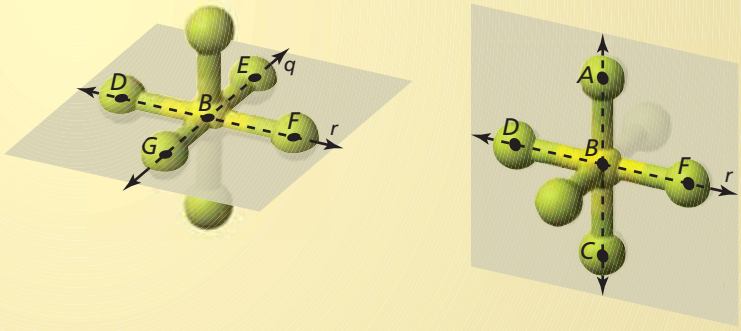
To name a plane that contains line r, use two points on line r and one point not on line r. Points D and F lie on line r. Points C and E do not lie on line r.

▶ So, plane DEF and plane CDF both contain line r.

COMMON ERROR

Because point B also lies on line r, you cannot use points D, B, and F to name a single plane. There are infinitely many planes that pass through these points.

Check The question asks for two *different* planes. Check whether plane DEF and plane CDF are two unique planes or the same plane named differently. Because point C does not lie in plane DEF, plane DEF and plane CDF are different planes.



SELF-ASSESSMENT

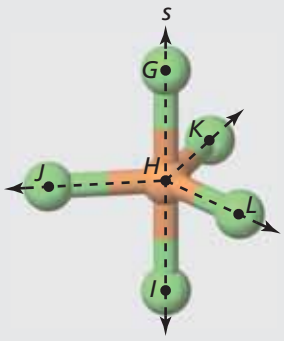
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram that shows a model of a molecule of phosphorus pentachloride.

10. Name two different planes that contain line s.

11. Name three different planes that contain point K.

12. Name two different planes that contain $\overleftrightarrow{HJ}$.



1.1 Practice WITH CalcChat® AND CalcView®

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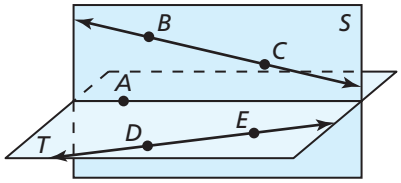
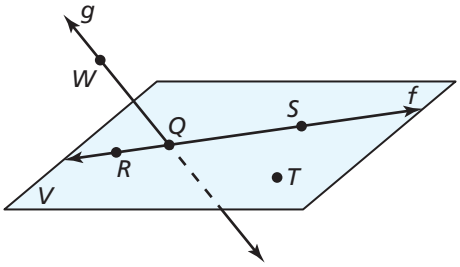
In Exercises 1–4, use the diagram.



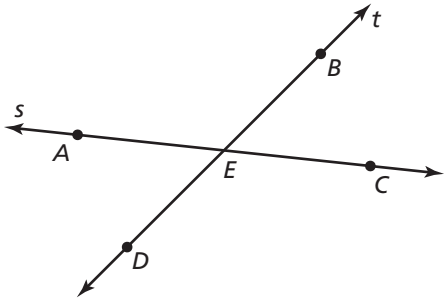
1. Name four points.
2. Name two lines.
3. Name the plane that contains points A, B, and C.
4. Name the plane that contains points A, D, and E.

In Exercises 5–8, use the diagram. *Example 1*



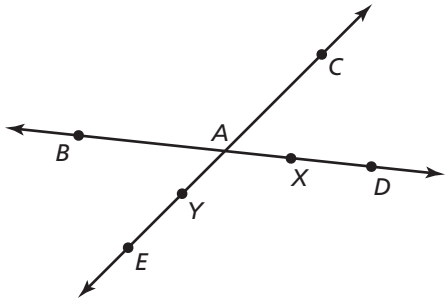
5. Give two other names for $\overleftrightarrow{WQ}$.
6. Give another name for plane V.
7. Name three points that are collinear. Then name a fourth point that is not collinear with these three points.
8. Name a point that is not coplanar with R, S, and T.

In Exercises 9–14, use the diagram. *Example 2*



9. What is another name for $\overline{BD}$?
10. What is another name for $\overline{AC}$?
11. What is another name for $\overrightarrow{AE}$?
12. Name all rays with endpoint E.
13. Name two pairs of opposite rays.
14. Name one pair of rays that are not opposite rays.

ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in naming opposite rays in the diagram.**



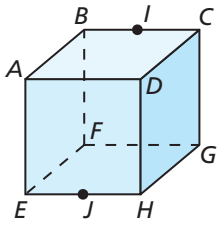
15. ✗ $\overrightarrow{AD}$ and $\overrightarrow{AC}$ are opposite rays.

16. ✗ $\overline{YC}$ and $\overline{YE}$ are opposite rays.

In Exercises 17–24, sketch the figure described. *Examples 3 and 4*

17. plane P and line ℓ intersecting at one point
18. plane K and line m intersecting at all points on line m
19. $\overrightarrow{AB}$ and $\overleftrightarrow{AC}$
20. $\overrightarrow{MN}$ and $\overrightarrow{NX}$
21. plane M and $\overrightarrow{NB}$ intersecting at point B
22. plane M and $\overrightarrow{NB}$ intersecting at point A
23. plane A and plane B not intersecting
24. plane C and plane D intersecting at $\overleftrightarrow{XY}$

In Exercises 25–32, use the diagram.



25. Name a point that is collinear with points E and H.

26. Name a point that is collinear with points B and I.

27. Name a point that is not collinear with points E and H.

28. Name a point that is not collinear with points B and I.

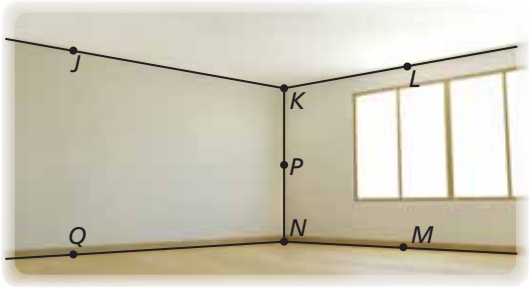
29. Name a point that is coplanar with points D, A, and B.

30. Name a point that is coplanar with points C, G, and F.

31. Name the intersection of plane AEH and plane FBE.

32. Name the intersection of plane BGF and plane HDG.

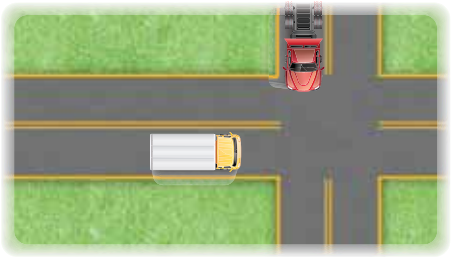
MODELING REAL LIFE **In Exercises 33 and 34, use the diagram.** *Example 5*



33. Name two points that are collinear with P.

34. Name two planes that contain J.

35. **MODELING REAL LIFE** When two trucks traveling in different directions approach an intersection at the same time, one of the trucks must change its speed or direction to avoid a collision. Two airplanes, however, can travel in different directions and cross paths without colliding. Explain how this is possible.



36. **CRITICAL THINKING** Given two points on a line and a third point not on the line, is it possible to draw a plane that includes the line and the third point? Explain your reasoning.

In Exercises 37–40, name the geometric term modeled by the part of the object indicated with an arrow.

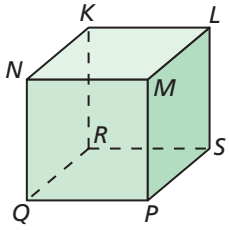
37.

38.

39.

40.

In Exercises 41–44, use the diagram to name all the points that are *not* coplanar with the given points.



41. N, K, and L

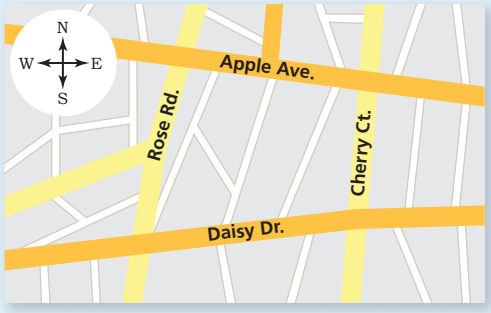
42. P, Q, and N

43. P, Q, and R

44. R, K, and N

45. **CRITICAL THINKING** Is it possible to draw two planes that intersect at one point? Explain your reasoning.

46. **HOW DO YOU SEE IT?**
You and your friend walk in opposite directions, forming opposite rays. You were originally on the corner of Apple Avenue and Cherry Court.



a. Name two possibilities of the road and direction you and your friend may have traveled.

b. Your friend claims he went north on Cherry Court, and you went east on Apple Avenue. Make an argument for why you know this could not have happened.

47. MP **REASONING** Explain why a four-legged chair may rock from side to side even if the floor is level. Would a three-legged chair on the same level floor rock from side to side? Why or why not?

48. **MODELING REAL LIFE** You are designing a living room. Counting the floor, walls, and ceiling, you want the design to contain at least eight different planes. Draw a diagram of your design. Label each plane in your design.

CONNECTING CONCEPTS **In Exercises 49 and 50, graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.**

49. $x \leq 3$

50. $-7 \leq x \leq 4$

CRITICAL THINKING **In Exercises 51–58, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

51. A line _____ has endpoints.

52. A line and a point _____ intersect.

53. A plane and a point _____ intersect.

54. Two planes _____ intersect in a line.

55. Two points _____ determine a line.

56. Any three points _____ determine a plane.

57. Any three points not on the same line _____ determine a plane.

58. Two lines that are not parallel _____ intersect.

59. MP **STRUCTURE** Two coplanar intersecting lines will always intersect at one point. What is the greatest number of intersection points that exist if you draw four coplanar lines? Explain.

60. **THOUGHT PROVOKING**
Is it possible for three planes to never intersect? to intersect in one line? to intersect in one point? Sketch the possible situations.

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REVIEW & REFRESH

WATCH

In Exercises 61 and 62, determine which of the lines, if any, are parallel or perpendicular. Explain.

61. Line a passes through $(1, 3)$ and $(-2, -3)$.
Line b passes through $(-1, -5)$ and $(0, -3)$.
Line c passes through $(3, 2)$ and $(1, 0)$.

62. Line a: $y + 4 = \frac{1}{2}x$
Line b: $2y = -4x + 6$
Line c: $y = 2x - 1$

In Exercises 63 and 64, solve the equation.

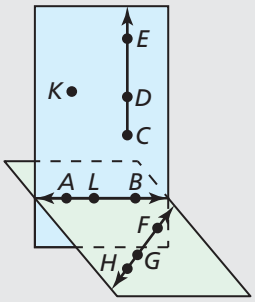
63. $18 + x = 43$

64. $x - 23 = 19$

65. **MODELING REAL LIFE** You bike at a constant speed of 10 miles per hour. You plan to bike 30 miles, plus or minus 5 miles. Write and solve an equation to find the minimum and maximum numbers of hours you bike.

66. Graph $f(x) = -\frac{1}{3}x + 5$ and $g(x) = f(x - 4)$. Describe the transformation from the graph of f to the graph of g.

In Exercises 67–73, use the diagram.



67. Name four points.

68. Name two lines.

69. Name three rays.

70. Name three collinear points.

71. Name three coplanar points.

72. Give two names for the plane shaded blue.

73. Name three line segments.

In Exercises 74–77, use zeros to graph the function.

74. $y = 2x(x - 5)(x + 8)$

75. $y = 4x^3 - 64x$

76. $y = 3x^3 + 3x^2 - 6x$

77. $y = -x(x + 1)(x - 7)$

In Exercises 78 and 79, make a box-and-whisker plot that represents the data.

78. Scores on a test: 76, 90, 84, 97, 82, 100, 92, 90, 88

79. Minutes spent at the gym: 60, 45, 50, 45, 65, 50, 55, 60, 60, 50