1.7 Piecewise Functions

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Learning Target Graph and write piecewise functions.

Success Criteria
- I can evaluate piecewise functions.
- I can graph piecewise functions.
- I can write piecewise functions.

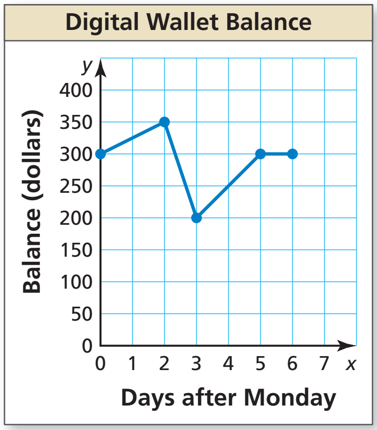
EXPLORE IT! Interpreting a Graph

TENNESSEE MATH STANDARDS
A2.F.IF.A.1,
A2.F.IF.B.4,
A2.F.IF.B.6,
A2.F.IF.B.6.b

Work with a partner. At the beginning of the day on Monday, your friend has $300 in her digital wallet. She sends and receives the following electronic payments through Sunday.

- $50 payment received on Wednesday
- $150 payment sent on Thursday
- $100 payment received on Saturday

She graphs the balance of her account during the week as shown.



a. Does the graph represent y as a function of x? Explain your reasoning.

b. According to the graph, what is your friend's balance when $x = 2$? when $x = 4$? Do these values match what you expect given the list of payments sent and received?

c. Does the graph accurately show how the account balance changes during the week? Explain your reasoning.

d. Create a graph that more accurately shows how the account balance changes during the week.

Math Practice

Simplify a Situation
How can finding the balance on each day of the week help you create a more accurate graph?

Evaluating Piecewise Functions

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Vocabulary VOCAB

piecewise function, *p. 46*
step function, *p. 48*

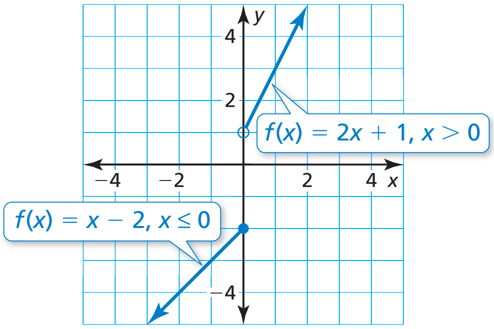
KEY IDEA

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

$$f(x) = \begin{cases} x - 2, & \text{if } x \leq 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0.
- The expression $2x + 1$ represents the value of f when x is greater than 0.



EXAMPLE 1 **Evaluating a Piecewise Function** WATCH

Evaluate $f(x) = \begin{cases} 3x + 1, & \text{if } x < 2 \\ x - 5, & \text{if } x \geq 2 \end{cases}$ when (a) $x = 2$ and (b) $x = -\frac{1}{3}$.

SOLUTION

a. Because $x = 2$ and $2 \geq 2$, use the second equation.

$f(x) = x - 5$ Write the second equation.

$f(2) = 2 - 5$ Substitute 2 for x.

$f(2) = -3$ Subtract.

▶ The value of f is -3 when $x = 2$.

b. Because $x = -\frac{1}{3}$ and $-\frac{1}{3} < 2$, use the first equation.

$f(x) = 3x + 1$ Write the first equation.

$f\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1$ Substitute $-\frac{1}{3}$ for x.

$f\left(-\frac{1}{3}\right) = 0$ Simplify.

▶ The value of f is 0 when $x = -\frac{1}{3}$.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Evaluate the function when $x = -6, -1, 0, \frac{5}{2}$, and 4.

1. $h(x) = \begin{cases} -3x, & \text{if } x < 0 \\ 2x + 7, & \text{if } x \geq 0 \end{cases}$

2. $f(x) = \begin{cases} 3, & \text{if } x < -2 \\ x + 2, & \text{if } -2 \leq x \leq 5 \\ 4x, & \text{if } x > 5 \end{cases}$

3. MP **REASONING** When evaluating a piecewise function, can two different inputs have the same output? Explain your reasoning.

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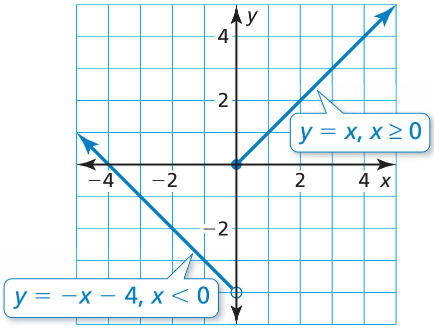
Graphing and Writing Piecewise Functions

EXAMPLE 2 Graphing a Piecewise Function

WATCH

Graph $y = \begin{cases} -x - 4, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$. Describe the domain, range, and end behavior of the function.



SOLUTION

Step 1 Graph $y = -x - 4$ for $x < 0$. Because 0 is not included in the domain for this equation, use an open circle at $(0, -4)$.

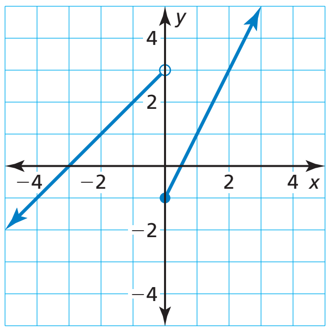
Step 2 Graph $y = x$ for $x \geq 0$. Because 0 is included in the domain for this equation, use a closed circle at $(0, 0)$.

▶ The domain is all real numbers, and the range is $y > -4$. The graph shows that $y \to +\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

EXAMPLE 3 Writing a Piecewise Function

WATCH

Write a piecewise function represented by the graph.



SOLUTION

Each "piece" of the function is linear.

Left Piece When $x < 0$, the graph is the line represented by $y = x + 3$.

Right Piece When $x \geq 0$, the graph is the line represented by $y = 2x - 1$.

▶ So, a piecewise function represented by the graph is

$$f(x) = \begin{cases} x + 3, & \text{if } x < 0 \\ 2x - 1, & \text{if } x \geq 0 \end{cases}.$$

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

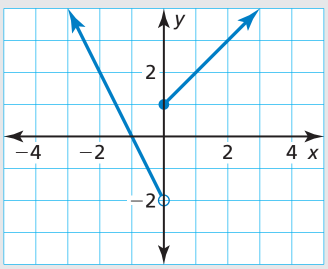
Graph the function. Describe the domain, range, and end behavior of the function.

4. $y = \begin{cases} x + 1, & \text{if } x \leq 0 \\ -x, & \text{if } x > 0 \end{cases}$

5. $y = \begin{cases} x - 2, & \text{if } x < -1 \\ 4x, & \text{if } x \geq -1 \end{cases}$

Write a piecewise function represented by the graph.

6.



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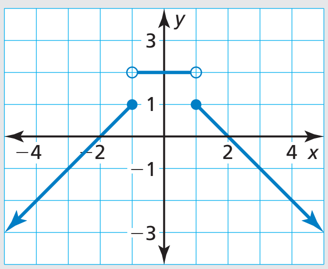

8. OPEN-ENDED Write a piecewise function that is positive over its entire domain.

Graphing and Writing Step Functions

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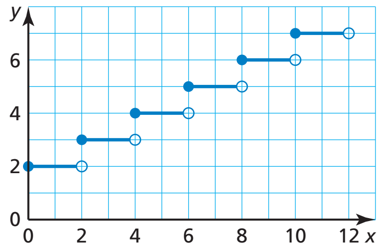
WORDS AND MATH

How does the graph of a step function relate to your understanding of the word *step*?

A **step function** is a piecewise function defined by a constant value over each part of its domain. The graph of a step function consists of a series of line segments.



$$f(x) = \begin{cases} 2, & \text{if } 0 \le x < 2 \\ 3, & \text{if } 2 \le x < 4 \\ 4, & \text{if } 4 \le x < 6 \\ 5, & \text{if } 6 \le x < 8 \\ 6, & \text{if } 8 \le x < 10 \\ 7, & \text{if } 10 \le x < 12 \end{cases}$$

EXAMPLE 4 Modeling Real Life

WATCH

You rent a karaoke machine for 5 days. The rental company charges \$50 for the first day and \$25 for each additional day or any portion of a day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the karaoke machine.

SOLUTION

Step 1 Use a table to organize the information.

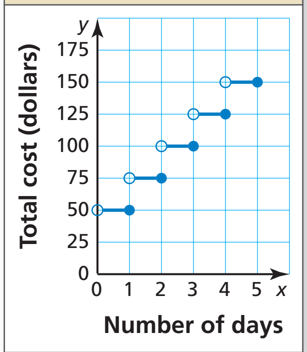
Number of days	Total cost (dollars)
$0 < x \le 1$	50
$1 < x \le 2$	75
$2 < x \le 3$	100
$3 < x \le 4$	125
$4 < x \le 5$	150

Step 2 Write the step function.

$$f(x) = \begin{cases} 50, & \text{if } 0 < x \le 1 \\ 75, & \text{if } 1 < x \le 2 \\ 100, & \text{if } 2 < x \le 3 \\ 125, & \text{if } 3 < x \le 4 \\ 150, & \text{if } 4 < x \le 5 \end{cases}$$

Step 3 Graph the step function.

Karaoke Machine Rental



SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

9. A landscaper rents a wood chipper for 4 days. The rental company charges \$150 for the first day and \$100 for each additional day or any portion of a day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the chipper.

10. **MP REASONING** Is it possible to perform a vertical translation on a step function f? a horizontal translation? If so, how would the equation that represents f change?

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Writing Absolute Value Functions

The absolute value function $f(x) = |x|$ can be written as a piecewise function.

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

REMEMBER

The vertex form of an absolute value function is $g(x) = a|x - h| + k$, where $a \neq 0$. The vertex of the graph of g is (h, k).

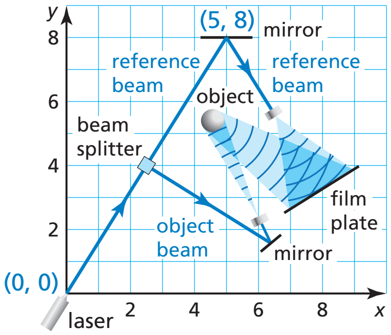
Similarly, the vertex form of an absolute value function $g(x) = a|x - h| + k$ can be written as a piecewise function.

$$g(x) = \begin{cases} a[-(x - h)] + k, & \text{if } x - h < 0 \\ a(x - h) + k, & \text{if } x - h \geq 0 \end{cases}$$

EXAMPLE 5 Modeling Real Life

WATCH

In holography, light from a laser beam is split into two beams, a reference beam and an object beam. Light from the object beam reflects off an object and is recombined with the reference beam to form images on film that can be used to create three-dimensional images.



a. Write an absolute value function that represents the path of the reference beam.

b. Write the function in part (a) as a piecewise function.

SOLUTION

a. The vertex of the path of the reference beam is (5, 8). So, the function has the form $g(x) = a|x - 5| + 8$. Substitute the coordinates of the point (0, 0) into the equation and solve for a.

$g(x) = a|x - 5| + 8$ — Vertex form of the function

$0 = a|0 - 5| + 8$ — Substitute 0 for x and 0 for $g(x)$.

$-1.6 = a$ — Solve for a.

▶ So, the function $g(x) = -1.6|x - 5| + 8$ represents the path of the reference beam.

STUDY TIP

Recall that the graph of an absolute value function is symmetric about the line $x = h$. So, it makes sense that the piecewise function "splits" at $x = 5$.

b. Write $g(x) = -1.6|x - 5| + 8$ as a piecewise function.

$$g(x) = \begin{cases} -1.6[-(x - 5)] + 8, & \text{if } x - 5 < 0 \\ -1.6(x - 5) + 8, & \text{if } x - 5 \geq 0 \end{cases}$$

Simplify each expression, and solve the inequalities.

▶ So, a piecewise function for $g(x) = -1.6|x - 5| + 8$ is

$$g(x) = \begin{cases} 1.6x, & \text{if } x < 5 \\ -1.6x + 16, & \text{if } x \geq 5 \end{cases}.$$

SELF-ASSESSMENT

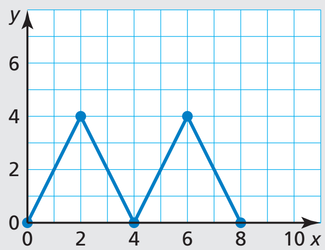
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. WHAT IF? In Example 5, the reference beam originates at (3, 0) and reflects off a mirror at (5, 4).

a. Write an absolute value function that represents the path of the reference beam.

b. Write the function in part (a) as a piecewise function.

12. WRITING Write a piecewise function represented by the graph. Then describe a real-life situation that can be modeled by the graph.



1.7 Practice WITH CalcChat® AND CalcView®

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In Exercises 1 and 2, evaluate the function when $x = -6, -2, 0, \frac{1}{2}$, and 2. *Example 1*

1. $f(x) = \begin{cases} 5x - 1, & \text{if } x \le -2 \\ x + 3, & \text{if } x > -2 \end{cases}$

2. $g(x) = \begin{cases} -x + 4, & \text{if } x < -1 \\ 3, & \text{if } -1 \le x < 2 \\ 2x - 5, & \text{if } x \ge 2 \end{cases}$

In Exercises 3–6, graph the function. Describe the domain, range, and end behavior of the function. *Example 2*

3. $y = \begin{cases} -3x - 2, & \text{if } x \le -1 \\ x + 2, & \text{if } x > -1 \end{cases}$

4. $y = \begin{cases} x + 8, & \text{if } x < 4 \\ 4x - 4, & \text{if } x \ge 4 \end{cases}$

5. $y = \begin{cases} 1, & \text{if } x < -3 \\ x - 1, & \text{if } -3 \le x \le 3 \\ -\frac{5}{3}x + 4, & \text{if } x > 3 \end{cases}$

6. $y = \begin{cases} 2x + 1, & \text{if } x \le -1 \\ -x + 2, & \text{if } -1 < x < 2 \\ -3.5, & \text{if } x \ge 2 \end{cases}$

7. MODELING REAL LIFE On a trip, the total distance (in miles) you travel in x hours is represented by the piecewise function

$$d(x) = \begin{cases} 55x, & \text{if } 0 \le x \le 2 \\ 65x - 20, & \text{if } 2 < x \le 5 \end{cases}.$$

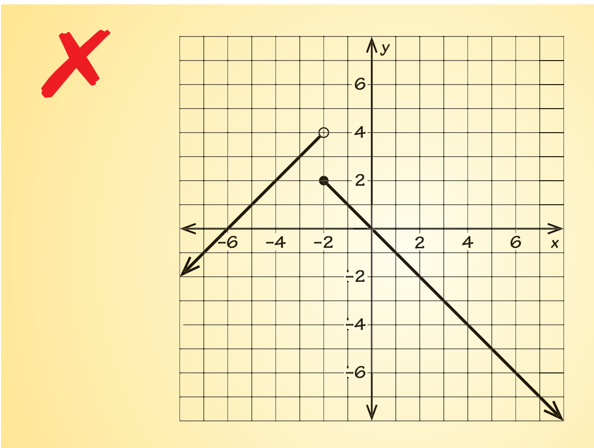
a. Interpret the domain and range of the function.

b. How far do you travel in 4 hours?

c. Compare the first 2 hours of the trip to the last 3 hours by calculating and interpreting the average rates of change.

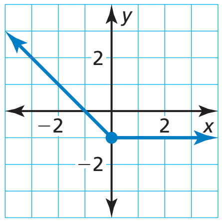
8. ERROR ANALYSIS Describe and correct the error in graphing

$$y = \begin{cases} x + 6, & \text{if } x \le -2 \\ -x, & \text{if } x > -2 \end{cases}.$$



In Exercises 9–14, write a piecewise function represented by the graph. *Example 3*

9.



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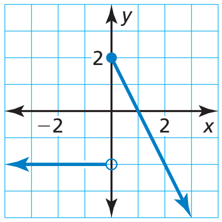

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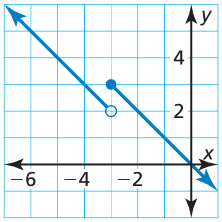

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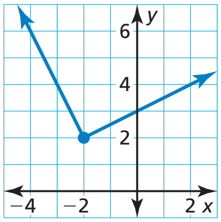

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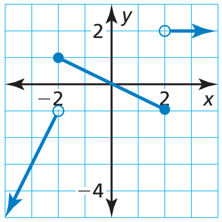

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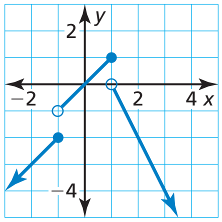


15. WRITING When you graph the solutions of a piecewise function and a linear function, how do the graphs differ?

16. MODELING REAL LIFE Write a piecewise function that represents the total cost y (in dollars) of ordering x custom shirts. Then interpret the domain and range of the function. Determine the total cost of ordering 26 shirts.

In Exercises 17 and 18, graph the step function. Find the domain and range.

17. $f(x) = \begin{cases} 2, & \text{if } 0 < x \le 2 \\ 3, & \text{if } 2 < x \le 4 \\ 4, & \text{if } 4 < x \le 6 \\ 5, & \text{if } 6 < x \le 8 \end{cases}$

18. $f(x) = \begin{cases} -\frac{3}{2}, & \text{if } -3 \le x < -2 \\ 0, & \text{if } -2 \le x < 0 \\ 2, & \text{if } 0 \le x < 1 \\ 4, & \text{if } 1 \le x < 3 \end{cases}$

19. **MODELING REAL LIFE** Miami-Dade County issues tickets for speeding as follows.

- no more than 9 miles per hour over: \$151
- more than 9 miles per hour to no more than 14 miles per hour over: \$226
- more than 14 miles per hour to no more than 19 miles per hour over: \$276
- more than 19 miles per hour to no more than 29 miles per hour over: \$301
- more than 30 miles per hour over: \$376

Write and graph a step function that represents the relationship between the number x of miles per hour over the speed limit and the cost y (in dollars) of the speeding ticket. *Example 4*

20. **MODELING REAL LIFE** A parking garage charges \$4 per hour or any portion of an hour, up to a daily maximum of \$15.

a. Write and graph a step function that represents the relationship between the number x of hours a car is parked in the garage and the total cost y (in dollars) of parking in the garage for up to 1 day. Interpret the domain, range, and any intercepts.

b. Is x a function of y? Explain your reasoning.

In Exercises 21–26, write the absolute value function as a piecewise function.

21. $y = |x| - 1$

22. $y = |x + 1|$

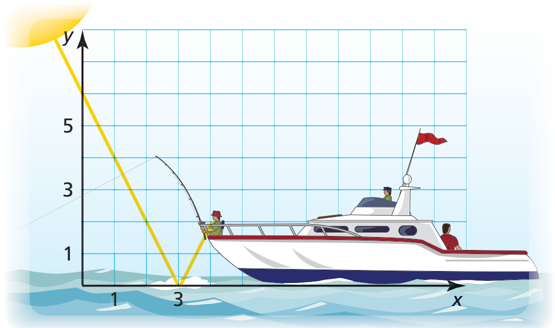
23. $y = 3|x - 1|$

24. $y = -2|x + 4|$

25. $y = -|x + 2| - 3$

26. $y = 2|x - 4| + 1$

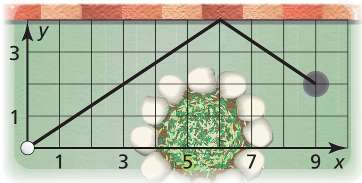
27. **MODELING REAL LIFE** You are sitting on a boat on a lake. You can get a sunburn from the sunlight that hits you directly and also from the sunlight that reflects off the water. *Example 5*



a. Write an absolute value function that represents the path of the sunlight that reflects off the water.

b. Write the function in part (a) as a piecewise function.

28. **MODELING REAL LIFE** You are trying to make a hole in one on the miniature golf green.



a. Write an absolute value function that represents the path of the golf ball.

b. Write the function in part (a) as a piecewise function.

29. **COLLEGE PREP** Which of the following are true about $f(x) = \begin{cases} -x - 3, & \text{if } x < -2 \\ \frac{1}{2}x, & \text{if } x \geq -2 \end{cases}$? Select all that apply.

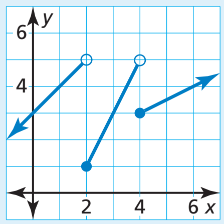
Ⓐ The y-intercept of the graph is -3.

Ⓑ The function is decreasing when $x < -2$ and increasing when $x > -2$.

Ⓒ The function is positive when $x < -3$, negative when $-3 < x < 0$, and positive when $x > 0$.

Ⓓ $f(-4) = f(2)$

30. **HOW DO YOU SEE IT?** The graph of a piecewise function f is shown. What is the value of $f(2)$? Which is greater, $f(3.9)$ or $f(4)$?



31. **CRITICAL THINKING** Describe how the graph of each piecewise function changes when $<$ is replaced with $\leq$ and $\geq$ is replaced with $>$. Do the domain and range change? Explain.

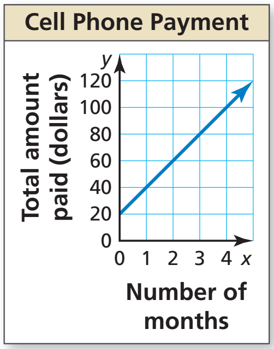
a. $f(x) = \begin{cases} x + 1, & \text{if } x < 3 \\ -x - 2, & \text{if } x \geq 3 \end{cases}$

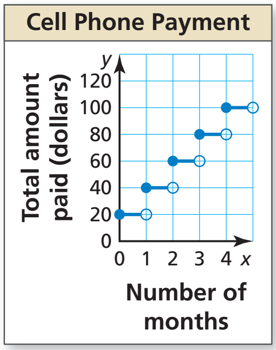
b. $f(x) = \begin{cases} \frac{3}{2}x + 2, & \text{if } x < 2 \\ -x + 7, & \text{if } x \geq 2 \end{cases}$

32. MP **STRUCTURE** Graph $y = \begin{cases} -x + 2, & \text{if } x \leq -2 \\ \big||x|\big|, & \text{if } x > -2 \end{cases}$.
Find the domain, range, and when the function is increasing or decreasing.

33. **MAKING AN ARGUMENT** Your friend says that $f(x) = \begin{cases} -3x - 4, & \text{if } x \leq -3 \\ 2, & \text{if } x \geq -3 \end{cases}$ represents a piecewise function. Is your friend correct? Explain.

34. MULTIPLE REPRESENTATIONS You purchase a new cell phone that costs \$250. You select a payment plan where you initially pay \$20, and then pay \$20 every month for the next 12 months. So far, you have paid \$120. Two graphs that model this situation are shown. Explain how each graph represents the situation. Then describe an advantage and disadvantage of each representation.





35. PERFORMANCE TASK You are the manager of a store. During a sale, you offer customers different discounts based on the total amounts they spend. Write and graph a step function that represents your discount policy. Then create an advertisement explaining your discount policy to customers.

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36. THOUGHT PROVOKING
The output y of the *greatest integer function* is the greatest integer less than or equal to the input value x. This function is written as $f(x) = [\![x]\!]$. Graph the function for $-4 \le x < 4$. Is it a piecewise function? a step function? Explain.

37. **DIG DEEPER** During an 8-hour snowstorm, it snows at a rate of 1 inch per hour for the first 2 hours, 2 inches per hour for the next 5 hours, and 1 inch per hour for the final hour. Write and graph a piecewise function that represents the snow accumulation during the storm. What is the total accumulation?

REVIEW & REFRESH

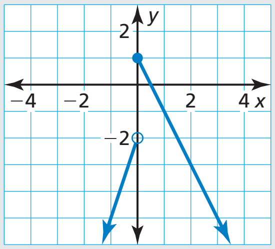
WATCH

In Exercises 38 and 39, graph the function. Describe the domain, range, and end behavior of the function.

38. $y = \begin{cases} -2x, & \text{if } x < 1 \\ x - 3, & \text{if } x \ge 1 \end{cases}$

39. $y = \begin{cases} 5x, & \text{if } x \le -4 \\ -5x, & \text{if } x > -4 \end{cases}$

40. Write a piecewise function represented by the graph.



In Exercises 41 and 42, graph *f*. Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

41. $f(x) = -|x - 9| - 2$ **42.** $f(x) = 4|x + 1| - 3$

43. Determine whether the data show a linear relationship. If so, write an equation of a line of fit. Then estimate y when $x = 10$ and explain its meaning in the context of the situation.

Minutes jogging, x	5	10	15	20	25
Distance (miles), y	0.75	1.5	2.25	3	3.75

44. MODELING REAL LIFE There must be 2 chaperones for every 25 students at a school dance. How many chaperones are needed for 200 students?

45. Graph

$$y = \begin{cases} -\frac{1}{4}x, & \text{if } x \le -4 \\ 2x + 3, & \text{if } x > -4 \end{cases}.$$

Describe the domain, range, and end behavior of the function.

In Exercises 46 and 47, solve the inequality. Graph the solution, if possible.

46. $|k - 8| < 11$ **47.** $-3|2p + 5| \le 18$

48. MODELING REAL LIFE The function $m(x) = |x - 35|$ represents the absolute deviation (in kilograms) of a male German shepherd's weight x (in kilograms) from the average weight of a full-grown male German shepherd. The function $f(x) = |x - 27|$ represents the absolute deviation (in kilograms) of a female German shepherd's weight x (in kilograms) from the average weight of a full-grown female German shepherd.

a. Interpret and compare the vertices of the graphs in this context.

b. Compare the absolute deviation for a male and a female German shepherd that both weigh 45 kilograms.