3.2 Characteristics of Functions

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Learning Target Describe characteristics of functions.

Success Criteria
- I can estimate intercepts of a graph of a function.
- I can approximate when a function is positive, negative, increasing, or decreasing.
- I can sketch a graph of a function from a verbal description.

EXPLORE IT! Describing Characteristics of Functions

TENNESSEE MATH STANDARDS
A1.F.IF.B.4,
A1.F.IF.C.9.a,
A1.F.IF.C.9.b

Work with a partner. Consider the function $y = x^3 - 3x$.

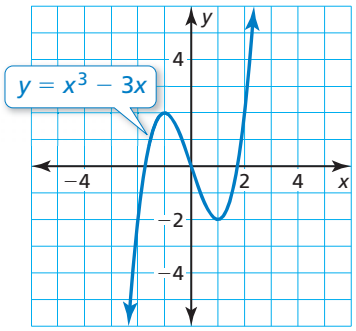
a. What do you think it means for a function to be *positive*? *negative*? *increasing*? *decreasing*?

b. Write a pair of numbers greater than 50 and a pair of numbers less than -50 to use as inputs for the function.

NUMBERS GREATER THAN 50	NUMBERS LESS THAN -50
First Number: _____	First Number: _____
Second Number: _____	Second Number: _____

Find the outputs for each pair of inputs. Do you think the function is increasing? decreasing? Explain your reasoning using your input-output pairs.

c. The graph of the function is shown below. Approximate when the function is positive, negative, increasing, or decreasing over its entire domain.



d. Explain whether it is possible for a graph to be decreasing over its entire domain but never negative. Justify your answer using a sketch.

Math Practice

Use Technology to Explore
How can you use technology to identify characteristics of a function?

Intercepts of Graphs of Functions

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Vocabulary

VOCAB

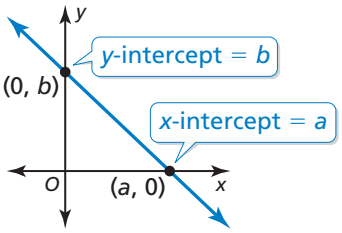
x-intercept, *p. 122*
y-intercept, *p. 122*
increasing, *p. 123*
decreasing, *p. 123*
end behavior, *p. 123*

KEY IDEA

Intercepts

An ***x*-intercept** of a graph is the x-coordinate of a point where the graph intersects the x-axis. It occurs when $y = 0$.

A ***y*-intercept** of a graph is the y-coordinate of a point where the graph intersects the y-axis. It occurs when $x = 0$.

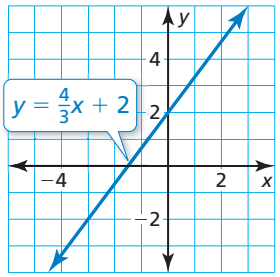


EXAMPLE 1 Estimating Intercepts

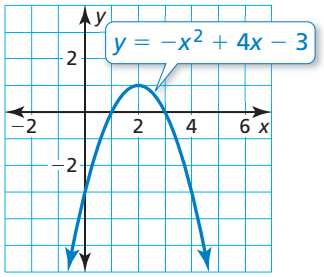
WATCH

Estimate the intercepts of the graph of each function.

a.



b.



STUDY TIP

You can use a graph to estimate intercepts, but your estimates may not be exact. Substitute your estimates into the equation to check whether they are exact.

SOLUTION

a. The graph appears to intersect the x-axis at $(-1.5, 0)$. It appears to intersect the y-axis at $(0, 2)$.

▶ So, the x-intercept is about -1.5, and the y-intercept is about 2.

b. The graph appears to intersect the x-axis at $(1, 0)$ and $(3, 0)$. It appears to intersect the y-axis at $(0, -3)$.

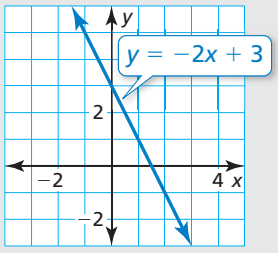
▶ So, the x-intercepts are about 1 and 3, and the y-intercept is about -3.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Estimate the intercepts of the graph of the function.

1.



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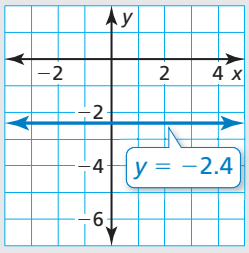
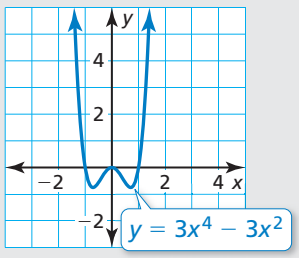


3.



4. MP **REASONING** Can the graph of a function have more than one y-intercept? Can the graph of a function have an infinite number of x-intercepts? Explain your reasoning.

Other Characteristics of Functions

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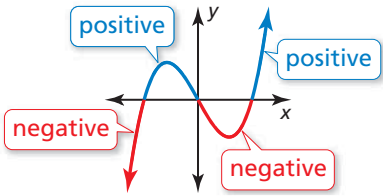
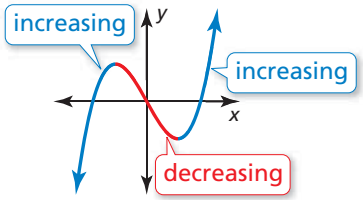
KEY IDEAS

Positive, Negative, Increasing, Decreasing, and End Behavior

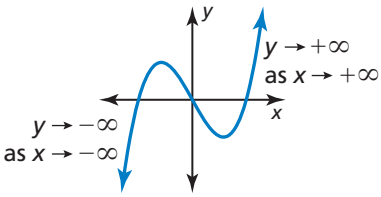
A function is *positive* when its graph lies above the x-axis. A function is *negative* when its graph lies below the x-axis.



A function is **increasing** when its graph moves up as x moves to the right. A function is **decreasing** when its graph moves down as x moves to the right.

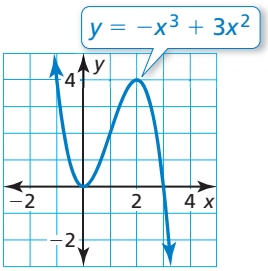


The **end behavior** of a function is the behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$).



EXAMPLE 2 Describing Characteristics

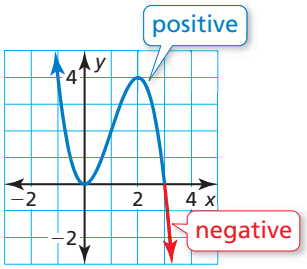
WATCH



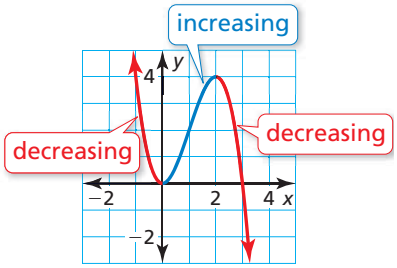
Approximate when the function $y = -x^3 + 3x^2$ is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

SOLUTION

Positive and Negative:
The function appears to be positive when $x < 0$, positive when $0 < x < 3$, and negative when $x > 3$.



Increasing and Decreasing:
The function appears to be decreasing when $x < 0$, increasing when $0 < x < 2$, and decreasing when $x > 2$.



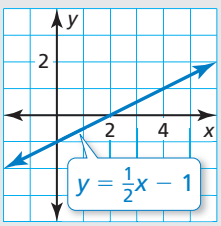
End behavior: The graph shows that the function values increase as x approaches negative infinity and the function values decrease as x approaches positive infinity. So, $y \to +\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

5.



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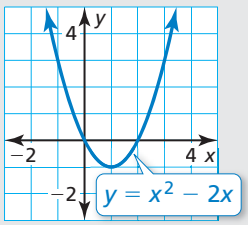

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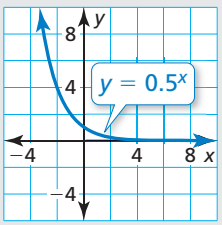

A Japanese mission used hopping robots to conduct experiments on an asteroid. Because of the asteroid's low gravity, a single hop can last 15 minutes or more.

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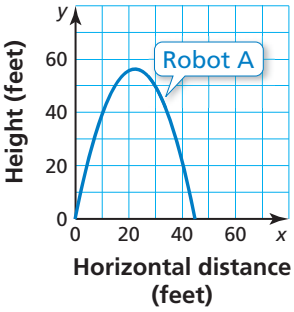
Solving Real-Life Problems

EXAMPLE 3 **Modeling Real Life** WATCH INFO

Researchers send two robots to explore an asteroid. The robots move by "hopping" between locations. The graph shows the path of Robot A's first hop. Robot B lands 60 feet from where it starts its first hop, reaching a maximum height of 48 feet after traveling a horizontal distance of 30 feet. Compare the two hops.

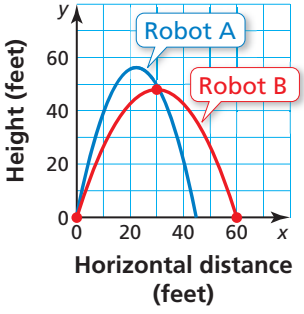


SOLUTION

Use the verbal description to sketch a graph that represents Robot B's hop. First identify several points on the graph.

- the start of the hop: (0, 0)
- the maximum height of the hop: (30, 48)
- the end of the hop: (60, 0)

So, the graph increases from (0, 0) to (30, 48) and then decreases to (60, 0). Sketch the graph using a curve similar to the graph that represents Robot A's hop.



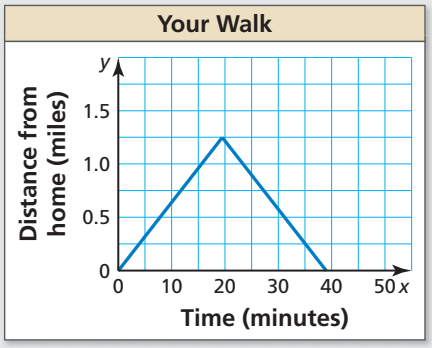
▶ **Intercepts:** Robot A's hop has x-intercepts of about 0 and 45. So, the robot lands about 45 feet from where it starts. This is about $60 - 45 = 15$ feet shorter than Robot B's hop.

Increasing and decreasing: Each hop reaches its maximum height when the height changes from increasing to decreasing. The graph shows that the maximum height of Robot A's hop is between 50 and 60, or about 55. So, Robot A's hop is about $55 - 48 = 7$ feet higher than Robot B's hop at its maximum height.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

8. The graph shows your distance from home while out on a walk. The next day, you jog the same route twice at a constant speed. The entire jog takes 1 hour. Compare your walk to your jog.



9. MP **REASONING** You throw a ball straight up into the air and notice that the speed of the ball decreases as it approaches its maximum height, then increases again on the way down. Sketch a graph that represents the relationship between time and height in this situation. Explain your reasoning.

3.2 Practice WITH CalcChat® AND CalcView®

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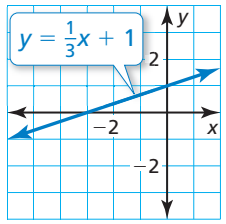
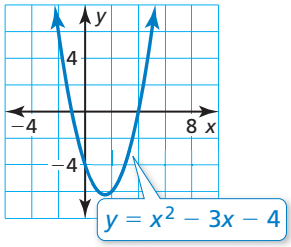
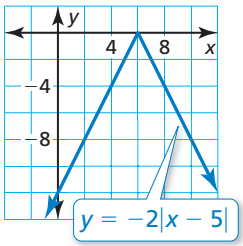
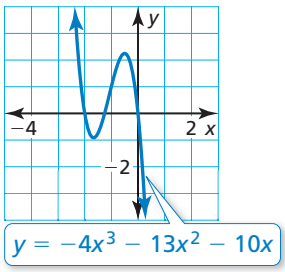
In Exercises 1–4, estimate the intercepts of the graph of the function. *Example 1*

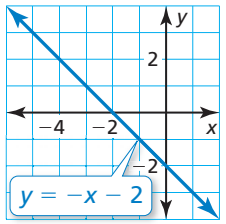
1. 

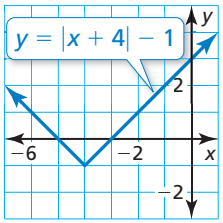
2. 

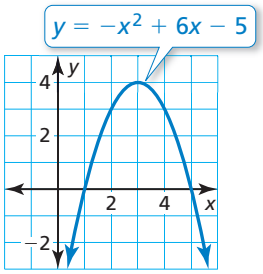
3. 

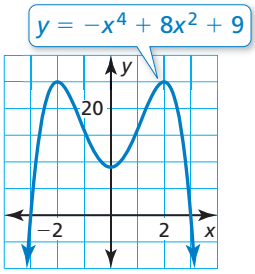
4. 

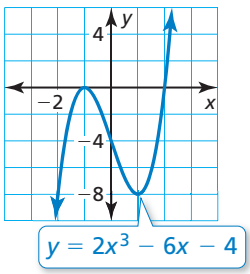
In Exercises 5–10, approximate when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function. *Example 2*

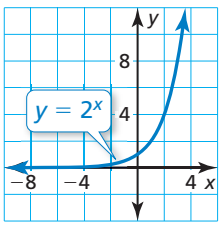
5. 

6. 

7. 

8. 

9. 

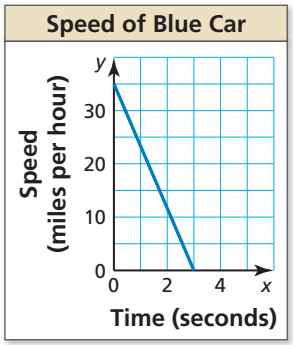
10. 

In Exercises 11 and 12, sketch a graph of a function with the given characteristics.

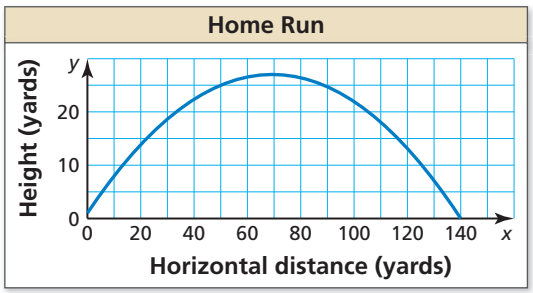
11.
- The function is increasing when $x < -6$ and decreasing when $x > -6$.
- The function is negative when $x < -8$, positive when $-8 < x < -4$, and negative when $x > -4$.

12.
- The x-intercepts are -0.5, 1, and 3.25.
- $y \to +\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to +\infty$.

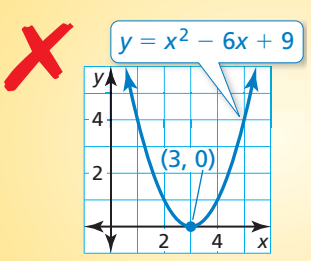
13. MODELING REAL LIFE The graph shows the speed of a blue car after the driver applies the brakes. The driver of a red car applies the brakes while traveling 30 miles per hour. The speed of the red car decreases at a constant rate until the car comes to a complete stop 4 seconds later. Compare the initial speeds and stopping times. *Example 3*



14. MODELING REAL LIFE The graph shows the path of a home run. During a second home run, a baseball player hits the baseball when it is 3 feet above home plate. The ball lands on the ground 402 feet from home plate, reaching a maximum height of 93 feet after traveling a horizontal distance of 199 feet. Compare the two home runs.

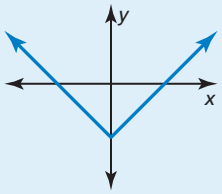


15. ERROR ANALYSIS Describe and correct the error in describing characteristics of the function.



The function is positive for all values of x. The function is decreasing when $x < 3$ and increasing when $x > 3$.

16. HOW DO YOU SEE IT?
The graph of a function is shown.



a. How many x-intercepts does the graph have? Is the y-intercept *positive* or *negative*?

b. Is the function *increasing* or *decreasing* when $x < 0$? $x > 0$?

17. COLLEGE PREP The graph of a function is a line that is decreasing for all values of x and has an x-intercept of 4. Which of the following are true? Select all that apply.

(A) The y-intercept is negative.

(B) The graph has only one x-intercept.

(C) The function is positive when $x < 4$ and negative when $x > 4$.

(D) $y \to -\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

18. MAKING AN ARGUMENT Consider the graph of a function that is negative over its entire domain. Can the graph have an x-intercept? Explain.

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19. DIG DEEPER You board a car at the bottom of a Ferris wheel. The Ferris wheel then makes several complete rotations before stopping again to let you off where you boarded. Sketch a graph that represents the relationship between time and your height above the ground, and describe the relationship.

20. THOUGHT PROVOKING
Sketch a graph of a function with the given characteristics.

- The function is decreasing for $x < 0$ and increasing for $x > 0$.
- As x approaches negative infinity, the function does *not* approach positive infinity or negative infinity.
- As x approaches positive infinity, the function does *not* approach positive infinity or negative infinity.
- The graph does *not* have a y-intercept.

REVIEW & REFRESH

WATCH

In Exercises 21 and 22, write the sentence as an inequality. Graph the inequality.

21. A number n is greater than or equal to -5 and less than -1.

22. A number k is no more than $-\frac{1}{2}$ or at least $2\frac{1}{2}$.

In Exercises 23–26, solve the equation.

23. $7 + b = -21$

24. $-3.2x = 16$

25. $-3(t - 5) = 14$

26. $30 = 2m - 8m + 12$

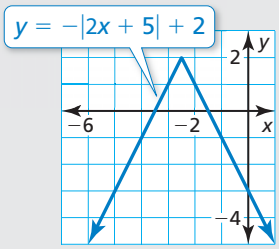
27. MODELING REAL LIFE You have a \$25 gift card to a coffee shop. You have already used \$19.85. You want to purchase one drink and one bakery item. Which pairs of items can you purchase with the amount left on the gift card?

Drink	Price
Coffee	\$2.09
Cappuccino	\$3.79

Bakery item	Price
Muffin	\$2.59
Bagel	\$1.49

28. Find the greatest common factor of 30 and 42.

29. Estimate the intercepts of the graph of the function.



In Exercises 30–33, solve the inequality. Graph the solution.

30. $5a > 20$

31. $\frac{r}{-2} + 6 \leq 11$

32. $1.5x + 7 - 5x > 11 - 3x$

33. $\left|\frac{1}{3}x + 6\right| + 2 > 3$

34. MP REASONING Complete the relation so that it is (a) a function and (b) *not* a function. Explain your reasoning.

$(-6, -1), (-4, 0), (-2, 1), (2, 2),$ ____