8 Exponents and Scientific Notation

Chapter Learning Target:
Understand exponents and scientific notation.

Chapter Success Criteria:
- I can write products using exponents.
- I can describe the value of powers.
- I can evaluate expressions.
- I can compare quantities using scientific notation.

STEAM Video

Carbon Atoms

Carbon is one of the four main elements of life. The number of carbon atoms in a compound can be represented using exponents. In what other real-life situations are exponents used?

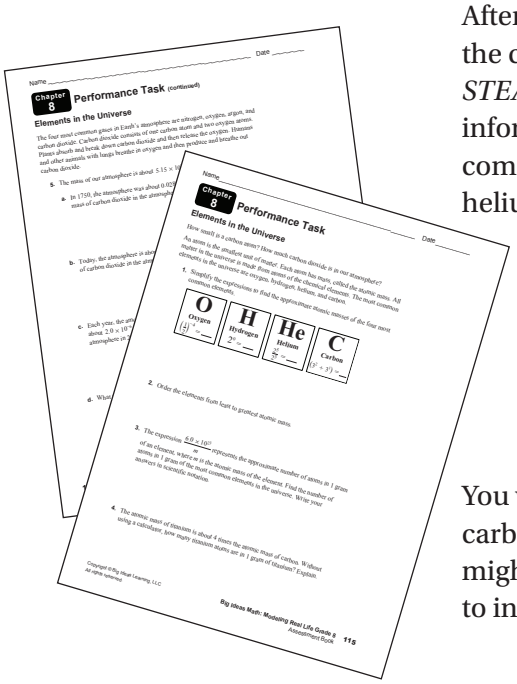
Watch the STEAM Video "Carbon Atoms." Then answer the following questions.

1. The table shows the percents carbon by weight for humans and plants. How many pounds of carbon are in a 130-pound person? a 25-pound plant?

	Percent Carbon by Weight
Human	18%
Plant	45%

2. Steven says 5×10^{22}, or 50,000,000,000,000,000,000,000, carbon atoms are in 1 gram of carbon. How many carbon atoms are in 3 grams of carbon?

Performance Task


Chapter 8 Performance Task (continued)
Elements in the Universe
Chapter 8 Performance Task
Elements in the Universe
O Oxygen
H Hydrogen
He Helium
C Carbon

Elements in the Universe

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Video Performance Task*. You will be given information about the *atomic masses* of the four most common elements in the universe: oxygen, hydrogen, helium, and carbon.

O	H	He	C
Oxygen	**Hydrogen**	**Helium**	**Carbon**
$\left(\frac{1}{2}\right)^{-4} =$ ___	$2^0 =$ ___	$\frac{2^5}{2^3} =$ ___	$(3^2 + 3^1) =$ ___

You will be asked to solve problems about the amounts of carbon dioxide in Earth's atmosphere for several years. What might cause the amount of carbon dioxide in the atmosphere to increase over time?

Getting Ready for Chapter 8

Chapter Exploration

1. Work with a partner. Write each distance as a whole number. Which numbers do you know how to write in words? For instance, in words, 10^2 is equal to *one hundred*.

a. 10^{27} meters: diameter of the observable universe

b. 10^{21} meters: diameter of the Milky Way galaxy

c. 10^{16} meters: diameter of the solar system

d. 10^{7} meters: diameter of Earth

e. 10^{4} meters: diameter of Halley's Comet

f. 10^{3} meters: diameter of a meteor crater

2. Work with a partner. Write the numbers of wives, sacks, cats, and kits as powers.

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives?

Nursery Rhyme, 1730

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

power

base of a power

exponent of a power

scientific notation

8.1 Exponents

Learning Target: Use exponents to write and evaluate expressions.

Success Criteria:
- I can write products using exponents.
- I can evaluate expressions involving powers.
- I can use exponents to solve real-life problems.

The expression 3^5 is called a *power*. The *base* is 3. The *exponent* is 5.

EXPLORATION 1 Using Exponent Notation

Work with a partner.

a. Copy and complete the table.

Power	Repeated Multiplication Form	Value
$(-3)^1$	-3	-3
$(-3)^2$	$(-3) \cdot (-3)$	9
$(-3)^3$		
$(-3)^4$		
$(-3)^5$		
$(-3)^6$		
$(-3)^7$		

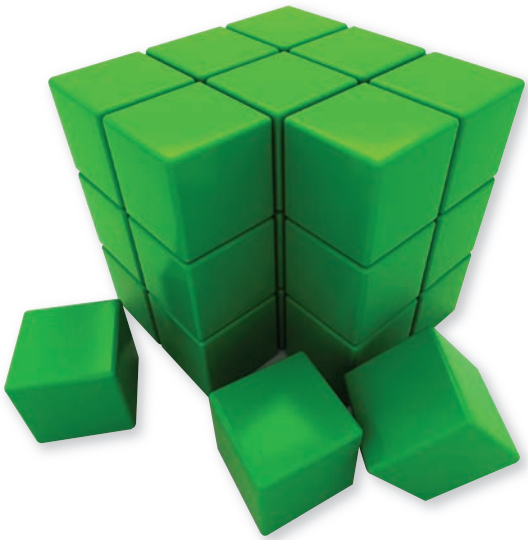
Math Practice

Build Arguments
When is the value of $(-3)^n$ positive? negative?

b. Describe what is meant by the expression $(-3)^n$. How can you find the value of $(-3)^n$?

EXPLORATION 2 Using Exponent Notation

Work with a partner. On a game show, each small cube is worth \$3. The small cubes are arranged to form a large cube. Show how you can use a power to find the total value of the large cube. Then write an explanation to convince a friend that your answer is correct.



8.1 Lesson

Key Vocabulary
power, *p. 320*
base, *p. 320*
exponent, *p. 320*

A **power** is a product of repeated factors. The **base** of a power is the repeated factor. The **exponent** of a power indicates the number of times the base is used as a factor.

base → $\left(\frac{1}{2}\right)^5$ ← exponent $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

power

$\frac{1}{2}$ is used as a factor 5 times.

EXAMPLE 1 Writing Expressions Using Exponents

Write each product using exponents.

a. $(-7) \cdot (-7) \cdot (-7)$

Because -7 is used as a factor 3 times, its exponent is 3.

▶ So, $(-7) \cdot (-7) \cdot (-7) = (-7)^3$.

b. $\pi \cdot \pi \cdot r \cdot r \cdot r$

Because π is used as a factor 2 times, its exponent is 2. Because r is used as a factor 3 times, its exponent is 3.

▶ So, $\pi \cdot \pi \cdot r \cdot r \cdot r = \pi^2 r^3$.

Math Practice

Communicate Precisely
Explain why you need to use parentheses to write powers when the base is negative.

Try It **Write the product using exponents.**

1. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

2. $0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot x \cdot x$

EXAMPLE 2 Evaluating Expressions

Evaluate each expression.

a. $(-2)^4$

$(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2)$ Write as repeated multiplication.

The base is -2.

$= 16$ Simplify.

b. -2^4

$-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2)$ Write as repeated multiplication.

The base is 2.

$= -16$ Simplify.

Try It **Evaluate the expression.**

3. 12^2 **4.** $(-2)^6$ **5.** -5^4 **6.** $\left(-\frac{1}{6}\right)^3$

EXAMPLE 3 Using Order of Operations

Evaluate each expression.

a. $3 + 2 \cdot 3^4 = 3 + 2 \cdot 81$ Evaluate the power.

$= 3 + 162$ Multiply.

$= 165$ Add.

b. $3^3 - 8^2 \div 2 = 27 - 64 \div 2$ Evaluate the powers.

$= 27 - 32$ Divide.

$= -5$ Subtract.

c. $-3 \cdot (-10^2 + 70) = -3 \cdot (-100 + 70)$ Evaluate the power.

$= -3 \cdot (-30)$ Perform operation in parentheses.

$= 90$ Multiply.

Math Practice

Look for Structure

Can you use the Distributive Property to evaluate the expression in part (c)? Explain.

Try It **Evaluate the expression.**

7. $9 - 2^5 \cdot 0.5$ **8.** $\left| -3^3 \div 27 \right|$ **9.** $(7 \cdot 4 - 4^3) \div 6$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

WRITING EXPRESSIONS USING EXPONENTS **Write the product using exponents.**

10. $(-0.9) \cdot (-0.9) \cdot (-0.9)$ **11.** $\frac{1}{8} \cdot \frac{1}{8} \cdot y \cdot y \cdot y$

EVALUATING EXPRESSIONS **Evaluate the expression.**

12. 11^2 **13.** -6^3 **14.** $(-0.3)^4$

USING ORDER OF OPERATIONS **Evaluate the expression.**

15. $\left| -24 \div 2^2 \right|$ **16.** $(3^3 - 6 \cdot 8) \div 7$

17. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$(-2)^6$ -8^2

8^2 2^6

EXAMPLE 4 Modeling Real Life

The annual profit P (in thousands of dollars) earned by a technology company x years after opening is represented by the equation $P = 0.1x^3 + 3$. How much more profit is earned in year 5 than in year 4?

Use the equation to find the profits earned in year 4 and year 5. Then subtract the profit in year 4 from the profit in year 5 to determine how much more profit is earned in year 5.

Year 4		*Year 5*
$P = 0.1x^3 + 3$	Write the equation.	$P = 0.1x^3 + 3$
$= 0.1(4)^3 + 3$	Substitute.	$= 0.1(5)^3 + 3$
$= 0.1(64) + 3$	Evaluate the power.	$= 0.1(125) + 3$
$= 9.4$	Simplify.	$= 15.5$

So, the company earns $15.5 - 9.4 = 6.1$, or \$6100 more profit in year 5 than in year 4.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

18. DIG DEEPER! Consider the diameters of three planets.

Planet A: 10^9 m **Planet B:** 10^7 m **Planet C:** 10^8 m

a. Write each diameter as a whole number.

b. A dwarf planet is discovered with a radius that is $\frac{1}{100}$ the radius of Planet C. Write the diameter of the dwarf planet as a power.

19. A fish jumps out of the water at a speed of 12 feet per second. The height y (in feet) of the fish above the surface of the water is represented by the equation $y = -16x^2 + 12x$, where x is the time (in seconds) since the jump began. The fish reaches its highest point above the surface of the water after 0.375 second. How far above the surface is the fish at this time?

8.1 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Sketch a graph that represents the situation.

1. A trading card becomes more valuable over time. The value increases at a constant rate, and then at a faster and faster rate.
2. The water level of a river remains constant, and then decreases at a constant rate.

The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.

3. $A(0, -4), B(0, -1), C(2, -1)$
   $90°$ clockwise about the origin
4. $E(1, 2), F(1, 3), G(4, 3), H(4, 2)$
   $180°$ about the origin

Concepts, Skills, & Problem Solving

USING EXPONENT NOTATION **Write the power in repeated multiplication form. Then find the value of the power.** (See Exploration 1, p. 319.)

5. 4^4
6. $(-8)^2$
7. $(-2)^3$

WRITING EXPRESSIONS USING EXPONENTS **Write the product using exponents.**

8. $3 \cdot 3 \cdot 3 \cdot 3$
9. $(-6) \cdot (-6)$
10. $\left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right)$
11. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$
12. $\pi \cdot \pi \cdot \pi \cdot x \cdot x \cdot x \cdot x$
13. $(-4) \cdot (-4) \cdot (-4) \cdot y \cdot y$
14. $6.4 \cdot 6.4 \cdot 6.4 \cdot 6.4 \cdot b \cdot b \cdot b$
15. $(-t) \cdot (-t) \cdot (-t) \cdot (-t) \cdot (-t)$
16. $-(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7)$
17. $-\left(\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right)$

EVALUATING EXPRESSIONS **Evaluate the expression.**

18. 5^2
19. -11^3
20. $(-1)^6$
21. $\left(\frac{1}{2}\right)^6$
22. $\left(-\frac{1}{12}\right)^2$
23. $-\left(\frac{1}{9}\right)^3$

24. **MP YOU BE THE TEACHER** Your friend evaluates the power -6^2. Is your friend correct? Explain your reasoning.

$-6^2 = (-6) \cdot (-6) = 36$

STRUCTURE **Write the prime factorization of the number using exponents.**

25. 675 **26.** 280 **27.** 363

28. **PATTERNS** The largest doll is 12 inches tall. The height of each of the other dolls is $\frac{7}{10}$ the height of the next larger doll. Write an expression involving a power that represents the height of the smallest doll. What is the height of the smallest doll?

USING ORDER OF OPERATIONS **Evaluate the expression.**

29. $5 + 3 \cdot 2^3$ **30.** $2 + 7 \cdot (-3)^2$ **31.** $(13^2 - 12^2) \div 5$

32. $\frac{1}{2}(4^3 - 6 \cdot 3^2)$ **33.** $\left|\frac{1}{2}(7 + 5^3)\right|$ **34.** $\left|\left(-\frac{1}{2}\right)^3 \div \left(\frac{1}{4}\right)^2\right|$

35. $(9^2 - 15 \cdot 2) \div 17$ **36.** $-6 \cdot (-5^2 + 20)$ **37.** $(-4 + 12 - 6^2) \div 7$

38. **STRUCTURE** Copy and complete the table. Compare the values of $2^h - 1$ with the values of 2^{h-1}. When are the values the same?

h	1	2	3	4	5
$2^h - 1$					
2^{h-1}					

39. **MODELING REAL LIFE** Scientists use carbon-14 dating to determine the age of a sample of organic material.

a. The amount C (in grams) of carbon-14 remaining after t years of a sample of organic material is represented by the equation $C = 100(0.99988)^t$. Find the amount of carbon-14 remaining after 4 years.

b. What percent of the carbon-14 remains after 4 years?

40. **DIG DEEPER!** The frequency (in vibrations per second) of a note on a piano is represented by the equation $F = 440(1.0595)^n$, where n is the number of notes above A440. Each black or white key represents one note. Use the frequencies of A and A440 to make a conjecture about frequencies of notes on a piano. Explain your reasoning.

8.2 Product of Powers Property

Learning Target: Generate equivalent expressions involving products of powers.

Success Criteria:
- I can find products of powers that have the same base.
- I can find powers of powers.
- I can find powers of products.

EXPLORATION 1

Finding Products of Powers

Work with a partner.

a. Copy and complete the table. Use your results to write a *general rule* for finding $a^m \cdot a^n$, a product of two powers with the same base.

Product	Repeated Multiplication Form	Power
$2^2 \cdot 2^4$		
$(-3)^2 \cdot (-3)^4$		
$7^3 \cdot 7^2$		
$5.1^1 \cdot 5.1^6$		
$(-4)^2 \cdot (-4)^2$		
$10^3 \cdot 10^5$		
$\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5$		

Math Practice

Consider Similar Problems

How are the expressions in part (b) similar to the expressions in part (a)?

b. Show how to use your rule in part (a) to write each expression below as a single power. Then write a *general rule* for finding $(a^m)^n$, a power of a power.

$(7^3)^2$ $(6^2)^2$ $(3^2)^3$ $(2^2)^4$ $\left(\left(\frac{1}{2}\right)^2\right)^5$

EXPLORATION 2

Finding Powers of Products

Work with a partner. Copy and complete the table. Use your results to write a *general rule* for finding $(ab)^m$, a power of a product.

Power	Repeated Multiplication Form	Product of Powers
$(2 \cdot 3)^3$		
$(2 \cdot 5)^2$		
$(5 \cdot 4)^3$		
$(-2 \cdot 4)^2$		
$(-3 \cdot 2)^4$		

8.2 Lesson

Key Ideas

Common Error

When multiplying powers, do not multiply the bases.

$4^2 \cdot 4^3 = 4^5$, not 16^5.

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $4^2 \cdot 4^3 = 4^{2+3} = 4^5$

Algebra $a^m \cdot a^n = a^{m+n}$

Power of a Power Property

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$

Algebra $(a^m)^n = a^{mn}$

Power of a Product Property

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$

Algebra $(ab)^m = a^m b^m$

EXAMPLE 1 Multiplying Powers with the Same Base

a. $2^4 \cdot 2^5 = 2^{4+5}$ Product of Powers Property

$= 2^9$ Simplify.

When a number is written without an exponent, its exponent is 1.

b. $-5 \cdot (-5)^6 = (-5)^1 \cdot (-5)^6$ Rewrite -5 as $(-5)^1$.

$= (-5)^{1+6}$ Product of Powers Property

$= (-5)^7$ Simplify.

c. $x^3 \cdot x^7 = x^{3+7}$ Product of Powers Property

$= x^{10}$ Simplify.

Try It **Simplify the expression. Write your answer as a power.**

1. $6^2 \cdot 6^4$
2. $\left(-\frac{1}{2}\right)^3 \cdot \left(-\frac{1}{2}\right)^6$
3. $z \cdot z^{12}$

EXAMPLE 2 Finding a Power of a Power

a. $(3^4)^3 = 3^{4 \cdot 3}$ Power of a Power Property

$= 3^{12}$ Simplify.

b. $(w^5)^4 = w^{5 \cdot 4}$ Power of a Power Property

$= w^{20}$ Simplify.

Try It **Simplify the expression. Write your answer as a power.**

4. $(4^3)^5$ **5.** $(y^2)^4$ **6.** $\left((-4)^3\right)^2$

EXAMPLE 3 Finding a Power of a Product

a. $(2x)^3 = 2^3 \cdot x^3$ Power of a Product Property

$= 8x^3$ Simplify.

b. $(3xy)^2 = 3^2 \cdot x^2 \cdot y^2$ Power of a Product Property

$= 9x^2y^2$ Simplify.

Try It **Simplify the expression.**

7. $(5y)^4$ **8.** $(ab)^5$ **9.** $(0.5mn)^2$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FINDING POWERS **Simplify the expression. Write your answer as a power.**

10. $4^7 \cdot 4^4$ **11.** $(g^6)^3$ **12.** $\left(-\frac{1}{3}\right)^5 \cdot \left(-\frac{1}{3}\right)^7$

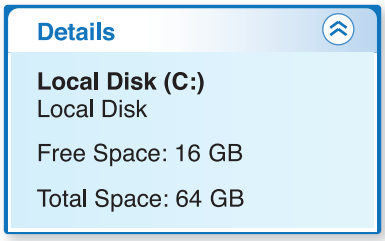
FINDING A POWER OF A PRODUCT **Simplify the expression.**

13. $(8t)^4$ **14.** $(yz)^6$ **15.** $\left(\frac{1}{4}gh\right)^3$

16. **CRITICAL THINKING** Can you use the Product of Powers Property to simplify $5^2 \cdot 6^4$? Explain.

17. **OPEN-ENDED** Write an expression that simplifies to x^{12} using the Product of Powers Property.

EXAMPLE 4 Modeling Real Life



One gigabyte (GB) of computer storage space is 2^{30} bytes. The storage details of a computer are shown. How many bytes of total storage space does the computer have?

The computer has 64 gigabytes of total storage space. Notice that you can write 64 as a power, 2^6.

Use a verbal model to solve the problem.

$$\text{Total number of bytes} = \text{Number of bytes in a gigabyte} \cdot \text{Number of gigabytes}$$

$= 2^{30} \cdot 2^6$ Substitute.

$= 2^{30+6}$ Product of Powers Property

$= 2^{36}$ Simplify.

So, the computer has 2^{36} bytes of total storage space.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

18. A newborn blue whale weighs 3^7 kilograms. An adult blue whale weighs 81 times the weight of the newborn. How many kilograms does the adult blue whale weigh?

19. One megabyte of cell phone storage space is 2^{20} bytes. An app uses 4^4 megabytes of storage space. How many bytes of storage space does the app use?

20. **DIG DEEPER!** The diagram shows the area of a small circular rug. The radius of a large circular rug is 3 times the radius of the small rug. Write an expression for the area of the large rug in terms of x. Justify your answer.

$A = \frac{1}{4}\pi x^2$

8.2 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Write the product using exponents.

1. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

2. $(-6) \cdot (-6) \cdot (-6) \cdot z \cdot z$

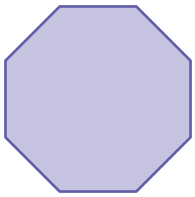
Find the value of y for the given value of x.

3. $y = -4x;\ x = 7$

4. $y = 5x + 6;\ x = -2$

5. $y = 10 - 3x;\ x = 3$

6. What is the measure of each interior angle of the regular polygon?



A. 45°

B. 135°

C. 1080°

D. 1440°

Concepts, Skills, & Problem Solving

FINDING PRODUCTS OF POWERS **Write the expression in repeated multiplication form. Then write the expression as a power.** (See Exploration 1, p. 325.)

7. $5^6 \cdot 5^3$

8. $(6^4)^2$

9. $(-8)^3 \cdot (-8)^4$

FINDING POWERS **Simplify the expression. Write your answer as a power.**

10. $3^2 \cdot 3^2$

11. $8^{10} \cdot 8^4$

12. $(5^4)^3$

13. $\left((-3)^2\right)^4$

14. $(-4)^5 \cdot (-4)^7$

15. $h^6 \cdot h$

16. $(b^{12})^3$

17. $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^6$

18. $(3.8^3)^4$

19. $(n^3)^5$

20. $\left(\left(-\frac{3}{4}\right)^5\right)^2$

21. $\left(-\frac{5}{7}\right)^8 \cdot \left(-\frac{5}{7}\right)^9$

MP YOU BE THE TEACHER **Your friend simplifies the expression. Is your friend correct? Explain your reasoning.**

22.

$$5^2 \cdot 5^9 = (5 \cdot 5)^{2+9}$$
$$= 25^{11}$$

23.

$$(r^6)^4 = r^{6+4}$$
$$= r^{10}$$

FINDING A POWER OF A PRODUCT **Simplify the expression.**

24. $(6g)^3$

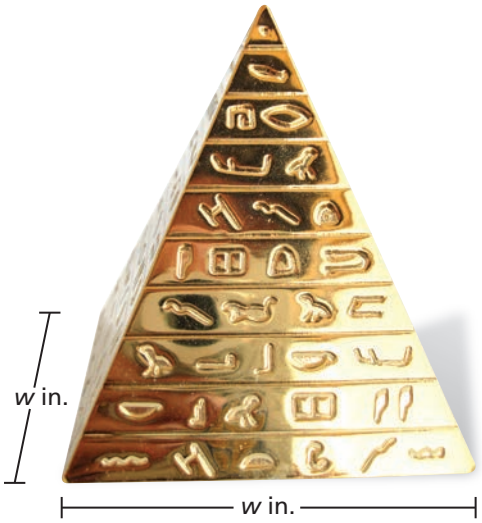
25. $(-3v)^5$

26. $\left(\frac{1}{5}k\right)^2$

27. $(1.2m)^4$

28. $(rt)^{12}$

29. $\left(-\frac{3}{4}p\right)^3$



30. MP **PRECISION** Is $3^2 + 3^3$ equal to 3^5? Explain.

31. MP **PROBLEM SOLVING** A display case for the artifact shown is in the shape of a cube. Each side of the display case is three times longer than the width w of the artifact.

a. Write a power that represents the volume of the case.

b. Simplify your expression in part (a).

32. MP **LOGIC** Show that $(3 \cdot 8 \cdot x)^7 = 6^7 \cdot 4^7 \cdot x^7$.

33. MP **MODELING REAL LIFE** The lowest altitude of an altocumulus cloud is about 3^8 feet. The highest altitude of an altocumulus cloud is about 3 times the lowest altitude. What is the highest altitude of an altocumulus cloud? Write your answer as a power.

34. **GEOMETRY** A square pyramid has a height h and a base with side length s. The side lengths of the base increase by 50%. Write a formula for the volume of the new pyramid in terms of s and h.

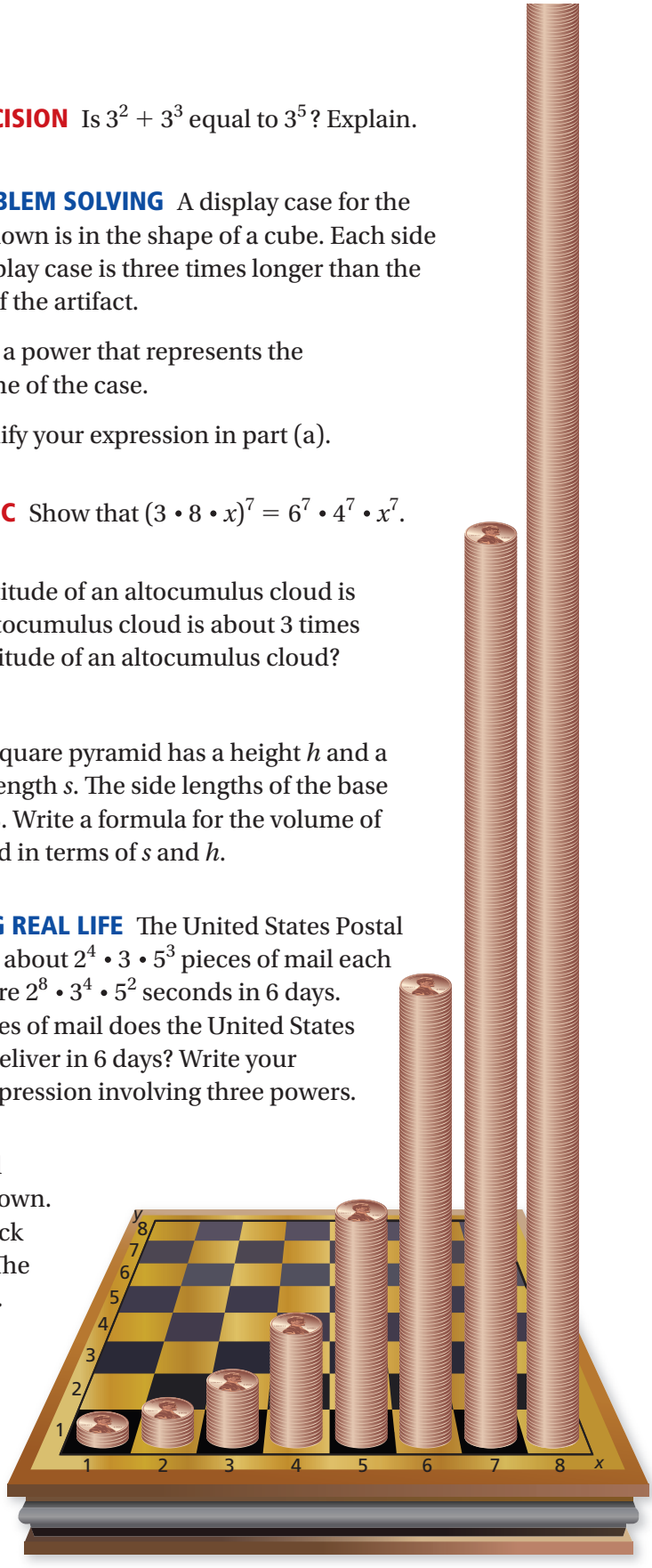
35. MP **MODELING REAL LIFE** The United States Postal Service delivers about $2^4 \cdot 3 \cdot 5^3$ pieces of mail each second. There are $2^8 \cdot 3^4 \cdot 5^2$ seconds in 6 days. How many pieces of mail does the United States Postal Service deliver in 6 days? Write your answer as an expression involving three powers.

36. MP **REASONING** The row numbers y and column numbers x of a chessboard are shown. Each position on the chessboard has a stack of pennies. (Only the first row is shown.) The number of pennies in each stack is $2^x \cdot 2^y$.

a. Which locations have 32 pennies in their stacks?

b. How much money (in dollars) is in the location with the tallest stack?

c. A penny is about 0.06 inch thick. About how tall is the tallest stack?



37. **CRITICAL THINKING** Find the value of x in the equation without evaluating the power.

a. $2^5 \cdot 2^x = 256$

b. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^x = \frac{1}{729}$

8.3 Quotient of Powers Property

Learning Target: Generate equivalent expressions involving quotients of powers.

Success Criteria:
- I can find quotients of powers that have the same base.
- I can simplify expressions using the Quotient of Powers Property.
- I can solve real-life problems involving quotients of powers.

EXPLORATION 1 Finding Quotients of Powers

Work with a partner.

a. Copy and complete the table. Use your results to write a *general rule* for finding $\frac{a^m}{a^n}$, a quotient of two powers with the same base.

Math Practice

Find General Methods

How does writing the expanded form of each expression help you find a general rule?

Quotient	Repeated Multiplication Form	Power
$\frac{2^4}{2^2}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2}$	
$\frac{(-4)^5}{(-4)^2}$		
$\frac{7^7}{7^3}$		
$\frac{8.5^9}{8.5^6}$		
$\frac{10^8}{10^5}$		
$\frac{3^{12}}{3^4}$		
$\frac{(-5)^7}{(-5)^5}$		
$\frac{11^4}{11^1}$		
$\frac{x^6}{x^2}$		

b. Use your rule in part (a) to simplify the quotients in the first column of the table above. Does your rule give the results in the third column?

8.3 Lesson

Key Idea

Quotient of Powers Property

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^5}{4^2} = 4^{5-2} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

EXAMPLE 1 Dividing Powers with the Same Base

a. $\frac{2^6}{2^4} = 2^{6-4}$ Quotient of Powers Property

$= 2^2$ Simplify.

b. $\frac{(-7)^9}{(-7)^3} = (-7)^{9-3}$ Quotient of Powers Property

$= (-7)^6$ Simplify.

c. $\frac{h^7}{h^6} = h^{7-6}$ Quotient of Powers Property

$= h^1 = h$ Simplify.

Common Error

When dividing powers, do not divide the bases.

$\frac{2^6}{2^4} = 2^2$, not 1^2.

Try It **Simplify the expression. Write your answer as a power.**

1. $\frac{9^7}{9^4}$
2. $\frac{4.2^6}{4.2^5}$
3. $\frac{(-8)^8}{(-8)^4}$
4. $\frac{x^8}{x^3}$

EXAMPLE 2 Simplifying an Expression

Simplify $\frac{3^4 \cdot 3^2}{3^3}$. Write your answer as a power.

The numerator is a product of powers. Add the exponents in the numerator.

$\frac{3^4 \cdot 3^2}{3^3} = \frac{3^{4+2}}{3^3}$ Product of Powers Property

$= \frac{3^6}{3^3}$ Simplify.

$= 3^{6-3}$ Quotient of Powers Property

$= 3^3$ Simplify.

Try It **Simplify the expression. Write your answer as a power.**

5. $\frac{6^7 \cdot 6^3}{6^5}$
6. $\frac{2^{15}}{2^3 \cdot 2^5}$
7. $\frac{m^8 \cdot m^6}{m^5}$

EXAMPLE 3 Simplifying Expressions

a. $\frac{(-4)^9}{(-4)^5} \cdot \frac{(-4)^8}{(-4)^2} = (-4)^{9-5} \cdot (-4)^{8-2}$ Quotient of Powers Property

$= (-4)^4 \cdot (-4)^6$ Simplify.

$= (-4)^{4+6}$ Product of Powers Property

$= (-4)^{10}$ Simplify.

b. $\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} = a^{10-6} \cdot a^{7-4}$ Quotient of Powers Property

$= a^4 \cdot a^3$ Simplify.

$= a^{4+3}$ Product of Powers Property

$= a^7$ Simplify.

Math Practice

Look for Structure

Show how you can simplify the expression in part (b) by first multiplying the numerators and then multiplying the denominators.

Try It **Simplify the expression. Write your answer as a power.**

8. $\frac{(-5)^7 \cdot (-5)^6}{(-5)^5 \cdot (-5)^2}$ **9.** $\frac{d^5}{d} \cdot \frac{d^9}{d^8}$ **10.** $\frac{p^3 \cdot p^6}{p^2} \cdot \frac{p^4}{p}$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

SIMPLIFYING EXPRESSIONS **Simplify the expression. Write your answer as a power.**

11. $\frac{(-3)^9}{(-3)^2}$ **12.** $\frac{8^6 \cdot 8^2}{8^5}$ **13.** $\frac{x^{11}}{x^4 \cdot x^6}$

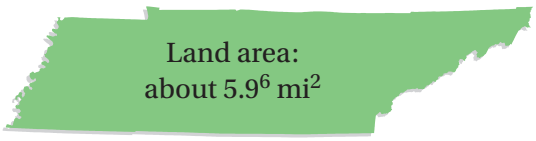
14. $\frac{5^6}{5} \cdot \frac{5^3}{5^2}$ **15.** $\frac{(-2)^9 \cdot (-2)^4}{(-2)^4 \cdot (-2)^4}$ **16.** $\frac{b^{10} \cdot b^3}{b^2} \cdot \frac{b^5}{b^3}$

17. **WHICH ONE DOESN'T BELONG?** Which quotient does *not* belong with the other three? Explain your reasoning.

$\frac{(-10)^7}{(-10)^2}$ $\frac{6^3}{6^2}$

$\frac{(-4)^8}{(-3)^4}$ $\frac{5^6}{5^3}$

EXAMPLE 4 Modeling Real Life



The projected population of Tennessee in 2030 is about $5 \cdot 5.9^8$. Predict the average number of people per square mile in Tennessee in 2030.

You can find the average number of people per square mile in 2030 by dividing the projected population of Tennessee in 2030 by the land area.

$$\text{People per square mile} = \frac{\text{Population in 2030}}{\text{Land area}}$$

$$= \frac{5 \cdot 5.9^8}{5.9^6} \quad \text{Substitute.}$$

$$= 5 \cdot \frac{5.9^8}{5.9^6} \quad \text{Rewrite.}$$

$$= 5 \cdot 5.9^2 \quad \text{Quotient of Powers Property}$$

$$= 174.05 \quad \text{Evaluate.}$$

So, you can predict that there will be about 174 people per square mile in Tennessee in 2030.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

18. You want to purchase a cat tracker. Tracker A detects your cat within a radius of $4 \cdot 10^2$ feet of your home. Tracker B detects your cat within a radius of 10^4 feet of your home. Which tracker has a greater radius? How many times greater?

19. DIG DEEPER! An earthquake of magnitude 3.0 is 10^2 times stronger than an earthquake of magnitude 1.0. An earthquake of magnitude 8.0 is 10^7 times stronger than an earthquake of magnitude 1.0. How many times stronger is an earthquake of magnitude 8.0 than an earthquake of magnitude 3.0?

20. The edge length of a cube-shaped crate is the square of the edge length of a cube-shaped box. Write an expression for the number of boxes that can fit in the crate. Justify your answer.

8.3 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

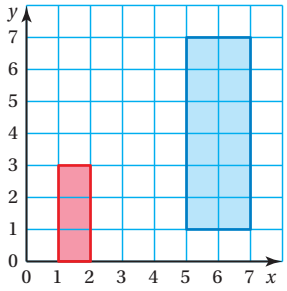
Review & Refresh

Simplify the expression. Write your answer as a power.

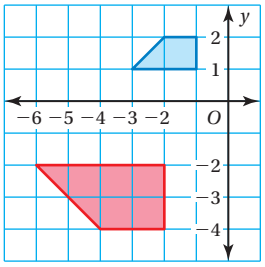
1. $4^2 \cdot 4^3$ **2.** $(a^5)^5$ **3.** $(xy)^7$

The red figure is similar to the blue figure. Describe a similarity transformation between the figures.

4.



5.



Concepts, Skills, & Problem Solving

FINDING QUOTIENTS OF POWERS **Write the quotient as repeated multiplication. Then write the quotient as a power.** (See Exploration 1, p. 331.)

6. $\frac{7^9}{7^6}$ **7.** $\frac{(-4.5)^6}{(-4.5)^2}$ **8.** $\frac{m^{10}}{m^5}$

DIVIDING POWERS WITH THE SAME BASE **Simplify the expression. Write your answer as a power.**

9. $\frac{6^{10}}{6^4}$ **10.** $\frac{8^9}{8^7}$ **11.** $\frac{(-3)^4}{(-3)^1}$ **12.** $\frac{4.5^5}{4.5^3}$

13. $\frac{64^4}{64^3}$ **14.** $\frac{(-17)^5}{(-17)^2}$ **15.** $\frac{(-6.4)^8}{(-6.4)^6}$ **16.** $\frac{\pi^{11}}{\pi^7}$

17. MP **YOU BE THE TEACHER** Your friend simplifies the quotient. Is your friend correct? Explain your reasoning.

$$\frac{6^{15}}{6^5} = 6^{15/5} = 6^3$$

SIMPLIFYING AN EXPRESSION **Simplify the expression. Write your answer as a power.**

18. $\frac{7^5 \cdot 7^3}{7^2}$ **19.** $\frac{6^{13}}{6^4 \cdot 6^2}$ **20.** $\frac{(-6.1)^{11}}{(-6.1)^7 \cdot (-6.1)^2}$

21. $\frac{\pi^{30}}{\pi^{18} \cdot \pi^4}$ **22.** $\frac{c^{22}}{c^8 \cdot c^9}$ **23.** $\frac{z^8 \cdot z^6}{z^8}$

24. **MODELING REAL LIFE** The sound intensity of a normal conversation is 10^6 times greater than the quietest noise a person can hear. The sound intensity of a jet at takeoff is 10^{14} times greater than the quietest noise a person can hear. How many times more intense is the sound of a jet at takeoff than the sound of a normal conversation?

SIMPLIFYING AN EXPRESSION **Simplify the expression. Write your answer as a power.**

25. $\dfrac{(-4)^8 \cdot (-4)^3}{(-4)^4 \cdot (-4)^2}$

26. $\dfrac{6^2}{6} \cdot \dfrac{6^{12}}{6^8}$

27. $\dfrac{3^2 \cdot 3^6}{3^2} \cdot \dfrac{3^5}{3}$

28. $\dfrac{z^7 \cdot z^6}{z \cdot z^2}$

29. $\dfrac{x^5}{x^4} \cdot \dfrac{x^{13}}{x^8}$

30. $\dfrac{y^8 \cdot y^2}{y^7} \cdot \dfrac{y^4}{y} \cdot \dfrac{y^7}{y^2}$

Device	Storage (GB)	Price
A	2^5	\$30
B	2^6	\$50
C	2^7	\$70
D	2^8	\$90
E	2^9	\$110

31. **REASONING** The storage capacities and prices of five devices are shown in the table.

a. How many times more storage does Device D have than Device B?

b. Do storage and price have a linear relationship? Explain.

32. **DIG DEEPER!** Consider the equation $\dfrac{9^m}{9^n} = 9^{2n}$.

a. Find two numbers m and n that satisfy the equation.

b. Describe the number of solutions that satisfy the equation. Explain your reasoning.

Milky Way galaxy: $10 \cdot 10^{10}$ stars

33. **MODELING REAL LIFE** A scientist estimates that there are about 10^{24} stars in the universe and that each galaxy has, on average, approximately the same number of stars as the Milky Way galaxy. About how many galaxies are in the universe?

34. **NUMBER SENSE** Find the value of x that makes $\dfrac{8^{3x}}{8^{2x+1}} = 8^9$ true. Explain how you found your answer.

8.4 Zero and Negative Exponents

Learning Target: Understand the concepts of zero and negative exponents.

Success Criteria:
- I can explain the meanings of zero and negative exponents.
- I can evaluate numerical expressions involving zero and negative exponents.
- I can simplify algebraic expressions involving zero and negative exponents.

EXPLORATION 1 Understanding Zero Exponents

Work with a partner.

a. Copy and complete the table.

Quotient	Quotient of Powers Property	Power
$\frac{5^3}{5^3}$		
$\frac{6^2}{6^2}$		
$\frac{(-3)^4}{(-3)^4}$		
$\frac{(-4)^5}{(-4)^5}$		

Math Practice

Find Entry Points

How can you use what you know about division to evaluate the expressions in the table?

b. Evaluate each expression in the first column of the table in part (a). How can you use these results to define a^0, where $a \neq 0$?

EXPLORATION 2 Understanding Negative Exponents

Work with a partner.

a. Copy and complete the table.

Product	Product of Powers Property	Power	Value
$5^{-3} \cdot 5^3$			
$6^2 \cdot 6^{-2}$			
$(-3)^4 \cdot (-3)^{-4}$			
$(-4)^{-5} \cdot (-4)^5$			

b. How can you use the Multiplicative Inverse Property to rewrite the powers containing negative exponents in the first column of the table?

c. Use your results in parts (a) and (b) to define a^{-n}, where $a \neq 0$ and n is an integer.

8.4 Lesson

Key Ideas

Zero Exponents

Words For any nonzero number a, $a^0 = 1$. The power 0^0 is *undefined*.

Numbers $4^0 = 1$ **Algebra** $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any nonzero number a, a^{-n} is the reciprocal of a^n.

Numbers $4^{-2} = \frac{1}{4^2}$ **Algebra** $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

EXAMPLE 1 Evaluating Expressions

a. $3^{-4} = \frac{1}{3^4}$ Definition of a negative exponent

$= \frac{1}{81}$ Evaluate the power.

b. $(-8.5)^{-4} \cdot (-8.5)^4 = (-8.5)^{-4+4}$ Product of Powers Property

$= (-8.5)^0$ Simplify.

$= 1$ Definition of a zero exponent

c. $\frac{2^6}{2^8} = 2^{6-8}$ Quotient of Powers Property

$= 2^{-2}$ Simplify.

$= \frac{1}{2^2}$ Definition of a negative exponent

$= \frac{1}{4}$ Evaluate the power.

Try It

Evaluate the expression.

1. 4^{-2}
2. $(-2)^{-5}$
3. $6^{-8} \cdot 6^8$
4. $\frac{(-3)^5}{(-3)^6}$
5. $\frac{1}{5^7} \cdot \frac{1}{5^{-4}}$
6. $\frac{4^5 \cdot 4^{-3}}{4^2}$

EXAMPLE 2 Simplifying Expressions

a. $-5x^0 = -5(1)$ Definition of a zero exponent

$= -5$ Multiply.

b. $\frac{9y^{-3}}{y^5} = 9y^{-3-5}$ Quotient of Powers Property

$= 9y^{-8}$ Simplify.

$= \frac{9}{y^8}$ Definition of a negative exponent

c. $\frac{n^4 \cdot n^{-7}}{6} = \frac{n^{4+(-7)}}{6}$ Product of Powers Property

$= \frac{n^{-3}}{6}$ Simplify.

$= \frac{1}{6n^3}$ Definition of a negative exponent

Try It **Simplify. Write the expression using only positive exponents.**

7. $8x^{-2}$ **8.** $b^0 \cdot b^{-10}$ **9.** $\frac{z^6}{15z^9}$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

EVALUATING EXPRESSIONS **Evaluate the expression.**

10. 7^{-2} **11.** $4^{-3} \cdot 4^0$ **12.** $\frac{(-9)^5}{(-9)^7}$

SIMPLIFYING EXPRESSIONS **Simplify. Write the expression using only positive exponents.**

13. $10t^{-5}$ **14.** $w^3 \cdot w^{-9}$ **15.** $\frac{r^8 \cdot r^{-8}}{4}$

16. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Write $\frac{1}{3 \cdot 3 \cdot 3}$ using a negative exponent.

Write 3 to the negative third.

Write $\frac{1}{3}$ cubed as a power with an integer base.

Write $(-3) \cdot (-3) \cdot (-3)$ as a power with an integer base.

EXAMPLE 3 Modeling Real Life

Drop of water: 50^{-2} liter

One drop of water leaks from a faucet every second. How many liters of water leak from the faucet in 1 hour?

Because you know how much water leaks per second, convert 1 hour to seconds.

$$1 \cancel{h} \times \frac{60 \cancel{\text{ min}}}{1 \cancel{h}} \times \frac{60 \text{ sec}}{1 \cancel{\text{ min}}} = 3600 \text{ sec}$$

Multiply the rate that water leaks by 3600 seconds.

$$3600 \cancel{\text{ sec}} \cdot \frac{50^{-2} \text{ L}}{1 \cancel{\text{ sec}}} = 3600 \cdot \frac{1}{50^2} \text{ L} \qquad \text{Definition of a negative exponent}$$

$$= 3600 \cdot \frac{1}{2500} \text{ L} \qquad \text{Evaluate the power.}$$

$$= \frac{3600}{2500} \text{ L} \qquad \text{Multiply.}$$

$$= 1\frac{11}{25}, \text{ or } 1.44 \text{ L} \qquad \text{Simplify.}$$

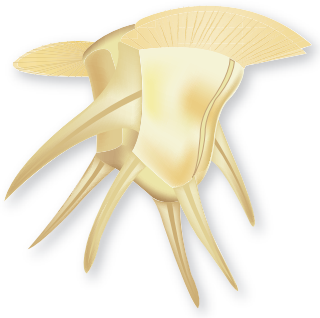
So, 1.44 liters of water leak from the faucet in 1 hour.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. The mass of a grain of sand is about 10^{-3} gram. About how many grains of sand are in a 10-kilogram bag of sand?

18. A one-celled, aquatic organism called a *dinoflagellate* is 1000 micrometers long. A microscope magnifies the dinoflagellate 100 times. What is the magnified length of the dinoflagellate in meters? (1 micrometer is 10^{-6} meter.)



Speed: 5^{-2} foot per second

19. DIG DEEPER! A garden is 12 yards long. Assuming the snail moves at a constant speed, how many minutes does it take the snail to travel the length of the garden? Justify your answer.

8.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

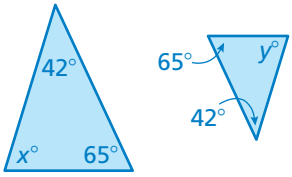
Review & Refresh

Simplify the expression. Write your answer as a power.

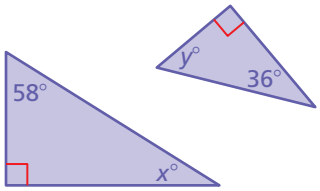
1. $\frac{10^8}{10^4}$ **2.** $\frac{y^9}{y^7}$ **3.** $\frac{(-3)^8 \cdot (-3)^3}{(-3)^2}$

Tell whether the triangles are similar. Explain.

4.



5.



6. Which data display best orders numerical data and shows how they are distributed?

A. bar graph **B.** line graph

C. scatter plot **D.** stem-and-leaf plot

Concepts, Skills, & Problem Solving

UNDERSTANDING NEGATIVE EXPONENTS **Copy and complete the table.**
(See Exploration 2, p. 337.)

	Product	Product of Powers Property	Power	Value
7.	$7^{-4} \cdot 7^4$			
8.	$(-2)^5 \cdot (-2)^{-5}$			

EVALUATING EXPRESSIONS **Evaluate the expression.**

9. $\frac{8^7}{8^7}$ **10.** $5^0 \cdot 5^3$ **11.** $(-2)^{-8} \cdot (-2)^8$ **12.** $9^4 \cdot 9^{-4}$

13. 6^{-2} **14.** 158^0 **15.** $\frac{4^3}{4^5}$ **16.** $\frac{-3}{(-3)^2}$

17. $2^2 \cdot 2^{-4}$ **18.** $3^{-3} \cdot 3^{-2}$ **19.** $\frac{1}{5^{-3}} \cdot \frac{1}{5^6}$ **20.** $\frac{(1.5)^2}{(1.5)^{-2} \cdot (1.5)^4}$

21. MP **YOU BE THE TEACHER** Your friend evaluates 4^{-3}. Is your friend correct? Explain your reasoning.

$4^{-3} = (-4)(-4)(-4)$
$= -64$

22. **CRITICAL THINKING** How can you write the number 1 as a power with base 2? a power with base 10?

23. MP **NUMBER SENSE** Without evaluating, order 5^0, 5^4, and 5^{-5} from least to greatest. Explain your reasoning.

SIMPLIFYING EXPRESSIONS **Simplify. Write the expression using only positive exponents.**

24. $6y^{-4}$

25. $8^{-2} \cdot a^7$

26. $\frac{9c^3}{c^{-4}}$

27. $\frac{5b^{-2}}{b^{-3}}$

28. $\frac{8x^3}{2x^9}$

29. $3d^{-4} \cdot 4d^4$

30. $m^{-2} \cdot n^3$

31. $\frac{3^{-2} \cdot k^0 \cdot w^0}{w^{-6}}$

32. **OPEN-ENDED** Write two different powers with negative exponents that have the same value. Justify your answer.

MP **REASONING** **In Exercises 33–36, use the table.**

33. How many millimeters are in a decimeter?

34. How many micrometers are in a centimeter?

35. How many nanometers are in a millimeter?

36. How many micrometers are in a meter?

Unit of Length	Length (meter)
Decimeter	10^{-1}
Centimeter	10^{-2}
Millimeter	10^{-3}
Micrometer	10^{-6}
Nanometer	10^{-9}

37. MP **MODELING REAL LIFE** A bacterium is 100 micrometers long. A virus is 1000 times smaller than the bacterium.

a. Using the table above, find the length of the virus in meters.

b. Is the answer to part (a) *less than*, *greater than*, or *equal to* 1 micrometer?

38. **DIG DEEPER!** Every 2 seconds, someone in the United States needs blood. A sample blood donation is shown.

a. One cubic millimeter of blood contains about 10^4 white blood cells. How many white blood cells are in the donation? ($1 \text{ mm}^3 = 10^{-3} \text{ mL}$)

b. One cubic millimeter of blood contains about 5×10^6 red blood cells. How many red blood cells are in the donation?

c. Compare your answers for parts (a) and (b).

39. MP **PRECISION** Describe how to rewrite a power with a positive exponent as a fraction with a power in the denominator. Use the definition of negative exponents to justify your reasoning.

40. MP **REASONING** The definition of a negative exponent states that $a^{-n} = \frac{1}{a^n}$. Explain why this rule does not apply when $a = 0$.

8.5 Estimating Quantities

Learning Target: Round numbers and write the results as the product of a single digit and a power of 10.

Success Criteria:
- I can round very large and very small numbers.
- I can write a multiple of 10 as a power.
- I can compare very large or very small quantities.

EXPLORATION 1

Using Powers of 10

Work with a partner. Match each picture with the most appropriate distance. Explain your reasoning.

6×10^3 m 1×10^1 m 2×10^{-1} m 6×10^{-2} m

a.

b.

c.

d.

EXPLORATION 2

Approximating Numbers

Work with a partner. Match each number in List 1 with its closest approximation in List 2. Explain your method.

Math Practice

Look for Patterns

How can you use the number of zeros to determine the value of the exponent for each number in List 1?

	List 1		*List 2*
a.	180,000,000,000,000	**A.**	3×10^{11}
b.	0.0000000011	**B.**	1×10^{-5}
c.	302,000,000,000	**C.**	2×10^{14}
d.	0.00000028	**D.**	3×10^{13}
e.	0.0000097	**E.**	3×10^{-7}
f.	330,000,000,000,000	**F.**	1×10^{-9}
g.	26,000,000,000,000	**G.**	2×10^{-5}
h.	0.000023	**H.**	3×10^{14}

8.5 Lesson

Round the number so that it contains exactly one nonzero digit.

One way to approximate a very large or a very small number is to round the number and write the result as the product of a single digit and a power of 10.

EXAMPLE 1 Approximating a Large Number

Earth contains about 332,500,000 cubic miles of water. The blue sphere represents all of the water on Earth, relative to the size of the planet.

Round the volume of water on Earth. Write the result as the product of a single digit and a power of 10.

$$\begin{aligned} 332{,}500{,}000 &\approx 300{,}000{,}000 && \text{Round to the nearest 100,000,000.} \\ &= 3 \times 100{,}000{,}000 && \text{Factor out 3.} \\ &= 3 \times 10^8 && \text{Write 100,000,000 as a power of 10.} \end{aligned}$$

▶ Earth contains about 3×10^8 cubic miles of water.

Try It **Round the number. Write the result as the product of a single digit and a power of 10.**

1. 8,031,426,100

2. 98,247,836,218

EXAMPLE 2 Approximating a Small Number

A blood vessel has a diameter of 0.0000924 meter. Round the diameter of the blood vessel. Write the result as the product of a single digit and a power of 10.

$$\begin{aligned} 0.0000924 &\approx 0.00009 && \text{Round to the nearest 0.00001.} \\ &= 9 \times 0.00001 && \text{Factor out 9.} \\ &= 9 \times 10^{-5} && \text{Write 0.00001 as a power of 10.} \end{aligned}$$

▶ The diameter of the blood vessel is about 9×10^{-5} meter.

Try It **Round the number. Write the result as the product of a single digit and a power of 10.**

3. 0.000384509

4. 0.00000726

EXAMPLE 3 Approximating a Quantity

The distance from Saturn to Neptune is about 1,911,674,960 miles. The distance from Mercury to Neptune is about 1.5 times this distance. What is the approximate distance from Mercury to Neptune?

A. 2×10^9 miles **B.** 3×10^9 miles

C. 2×10^{10} miles **D.** 3×10^{10} miles

Round the distance from Saturn to Neptune. Write the result as the product of a single digit and a power of 10.

$1{,}911{,}674{,}960 \approx 2{,}000{,}000{,}000$ Round to the nearest 1,000,000,000.

$= 2 \times 1{,}000{,}000{,}000$ Factor out 2.

$= 2 \times 10^9$ Write 1,000,000,000 as a power of 10.

Math Practice

Justify Conclusions

Explain to a classmate why you do not use the Distributive Property to multiply 1.5 and (2×10^9).

The distance from Mercury to Neptune is about 1.5 times the distance from Saturn to Neptune. So, the distance from Mercury to Neptune is about $1.5(2 \times 10^9)$, or 3×10^9, miles.

The correct answer is **B**.

Try It

5. The distance from Mercury to Mars is about 105,651,744 miles. The distance from Saturn to Jupiter is about 4 times this distance. What is the approximate distance from Saturn to Jupiter?

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

APPROXIMATING A NUMBER **Round the number. Write the result as the product of a single digit and a power of 10.**

6. 899,032,878,300 **7.** 62,322,118,987

8. 0.00000278101 **9.** 0.000013094

10. APPROXIMATING A QUANTITY Lake A has a volume of 21,150,427,000 cubic meters. Lake B has a volume that is 2.5 times the volume of Lake A. What is the approximate volume of Lake B?

EXAMPLE 4 Modeling Real Life

The population of the Philippines is about 104,260,000 and the population of India is about 1,282,000,000. Approximately how many times greater is the population of India than the population of the Philippines?

Understand the problem.

You are given the populations of the Philippines and India. You are asked to approximate the number of times greater the population of India is than the population of the Philippines.

Make a plan.

Round each number. Write each result as the product of a single digit and a power of 10. Then divide the population of India by the population of the Philippines.

Solve and check.

Philippines	*India*
$104{,}260{,}000 \approx 100{,}000{,}000$	$1{,}282{,}000{,}000 \approx 1{,}000{,}000{,}000$
$= 1 \times 100{,}000{,}000$	$= 1 \times 1{,}000{,}000{,}000$
$= 1 \times 10^8$	$= 1 \times 10^9$

Divide the population of India by the population of the Philippines.

$$\frac{1 \times 10^9}{1 \times 10^8} = \frac{10^9}{10^8} \qquad \text{Multiplication Property of One}$$

$$= 10^{9-8} \qquad \text{Quotient of Powers Property}$$

$$= 10^1 \qquad \text{Simplify.}$$

So, the population of India is about 10 times greater than the population of the Philippines.

Check Use a calculator to divide the numbers.

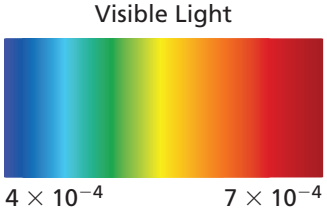
$$\frac{1{,}282{,}000{,}000}{104{,}260{,}000} \approx 12.3 \approx 10 \checkmark$$

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

11. On average, a small dog's heart beats about 530,000,000 times during its lifetime, and a large dog's heart beats about 1.4 times this amount. What is the approximate number of heartbeats in the lifetime of a large dog?

12. DIG DEEPER! A physicist observes a gamma ray with a wavelength of 0.00000000135 millimeter and an X-ray with a wavelength of 0.00000012 millimeter. (a) About how many times shorter is the wavelength of the gamma ray than the wavelength of the X-ray? (b) The diagram shows wavelengths of visible light. Which ray has a wavelength closer to the wavelength of dark blue light?



8.5 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Simplify. Write the expression using only positive exponents.

1. $3x^{-5}$ **2.** $d^0 \cdot d^{-4}$ **3.** $\frac{a^6}{2a^{11}}$

Write an equation in point-slope form of the line that passes through the given point and has the given slope.

4. $(-1, 2);\ m = -\frac{1}{3}$ **5.** $(3, 4);\ m = \frac{3}{4}$ **6.** $(1, -4);\ m = -2$

Concepts, Skills, & Problem Solving

APPROXIMATING NUMBERS **Match the number with its closest approximation.** (See Exploration 2, p. 343.)

7. 0.000618 **8.** 7,257,993,201 **9.** 0.0006781004 **10.** 782,309,441

A. 8×10^8 **B.** 6×10^{-4} **C.** 7×10^{-4} **D.** 7×10^9

APPROXIMATING A LARGE NUMBER **Round the number. Write the result as a product of a single digit and a power of 10.**

11. 414,148,636,008 **12.** 231,210 **13.** 28,007,806,203

14. 38,108,996,999 **15.** 1,003,111,391,008 **16.** 627,638,538

17. **APPROXIMATING A LARGE NUMBER** A company earns \$518,204,500. Round the number. Write the result as a product of a single digit and a power of 10.

APPROXIMATING A SMALL NUMBER **Round the number. Write the result as a product of a single digit and a power of 10.**

18. 0.00000124 **19.** 0.00003946 **20.** 0.00001726

21. 0.00063718 **22.** 0.00000000305 **23.** 0.000000000994

24. MP **YOU BE THE TEACHER** Your friend rounds 0.000468 to the nearest ten thousandth and writes the result as a product of a single digit and a power of 10. Is your friend correct? Explain your reasoning.

$$0.000468 \approx 0.0005$$
$$= 5 \times 0.0001$$
$$= 5 \times 10^{-4}$$

25. APPROXIMATING A QUANTITY A series of mystery books contains 2,029,242 words. A series of science fiction books contains about 3.5 times the number of words as the mystery book series. What is the approximate number of words in the science fiction book series?

26. APPROXIMATING A QUANTITY A volcanic eruption ejects about 43,600,000,000 cubic feet of volcanic rock. A smaller volcanic eruption ejects about 75% of this amount. What is the approximate amount of volcanic rock that the smaller volcanic eruption ejects?

27. MP STRUCTURE Find a number that is approximately 1.5 times 61,040,000,100. Write the result as the product of a single digit and a power of 10.

Mitochondrion

28. APPROXIMATING A QUANTITY A mitochondrion has a diameter of about 0.00000031 meter. The diameter of a chloroplast is about 3 times that of the mitochondrion. What is the approximate diameter of the chloroplast?

29. MP MODELING REAL LIFE A photo taken with a smartphone has 1,227,104 pixels. A photo taken with a camera has 11,943,936 pixels. Approximately how many times more pixels are in the photo taken with the camera?

30. MP MODELING REAL LIFE A star has a core temperature of about 115,000,000°F. The temperature of a lightning strike is about 10,300°F. Approximately how many times hotter is the core temperature of the star than the temperature of the lightning strike?

31. MP REASONING The table shows the diameters of five types of animal hairs.

Animal	Buffalo	Rat	Camel	Cow	Donkey
Diameter (meter)	0.00011	0.00004	0.00008	0.00016	0.00005

a. Order the hair types from greatest to least diameter.

b. What unit should be used to represent these data? Explain your reasoning.

32. MP PROBLEM SOLVING The distance between New York City and Princeton is about 68,500 meters. The distance between New York City and San Antonio is about 40 times this distance. What is the approximate distance between New York City and San Antonio? Write the result as the product of a single digit and a power of 10.

33. MP REASONING Is 5×10^6 a better approximation of 5,447,040 or 5,305,004? Explain.

34. MP NUMBER SENSE A proton weighs 0.00000000000167 nanogram. About how much do 8 protons weigh? Write the result as the product of a single digit and a power of 10. Is your answer an overestimate or an underestimate?

8.6 Scientific Notation

Learning Target: Understand the concept of scientific notation.

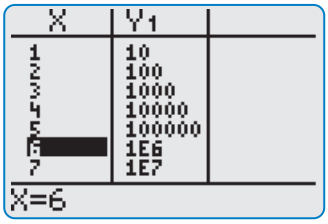
Success Criteria:
- I can convert between scientific notation and standard form.
- I can choose appropriate units to represent quantities.
- I can use scientific notation to solve real-life problems.

EXPLORATION 1 Using a Graphing Calculator

Work with a partner. Use a graphing calculator.

a. Experiment with multiplying very large numbers until your calculator displays an answer that is *not* in standard form. What do you think the answer means?

b. Enter the function $y = 10^x$ into your graphing calculator. Use the *table* feature to evaluate the function for positive integer values of x until the calculator displays a y-value that is not in standard form. Do the results support your answer in part (a)? Explain.

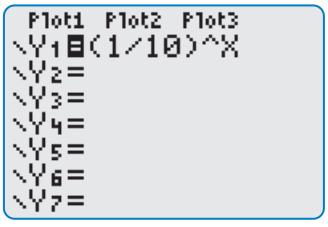


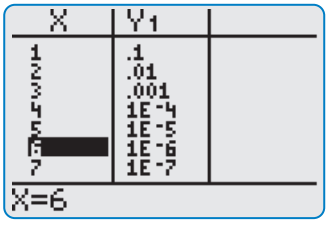
c. Repeat part (a) with very small numbers.

Math Practice

Make Sense of Quantities
How can writing $\frac{1}{10}$ as a power of 10 help you understand the calculator display?

d. Enter the function $y = \left(\frac{1}{10}\right)^x$ into your graphing calculator. Use the *table* feature to evaluate the function for positive integer values of x until the calculator displays a y-value that is not in standard form. Do the results support your answer in part (c)? Explain.





8.6 Lesson

Key Vocabulary
scientific notation, *p. 350*

A number is written in **scientific notation** when it is represented as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

The factor is greater than or equal to 1 and less than 10. → 8.3×10^{-7} ← The power of 10 has an integer exponent.

Key Idea

Writing Numbers in Scientific Notation

Move the decimal point so it is located to the right of the leading nonzero digit. The number of places you moved the decimal point indicates the exponent of the power of 10, as shown below.

If the number is greater than or equal to 10, then the exponent is positive. If the number is between 0 and 1, then the exponent is negative.

Number Greater Than or Equal to 10

Use a positive exponent when you move the decimal point to the left.

$$8600 = 8.6 \times 10^{3}$$
(3)

Number Between 0 and 1

Use a negative exponent when you move the decimal point to the right.

$$0.0024 = 2.4 \times 10^{-3}$$
(3)

EXAMPLE 1 Writing Numbers in Scientific Notation

a. Write 173,000,000 in scientific notation.

Move the decimal point 8 places to the left. → $173{,}000{,}000 = 1.73 \times 10^{8}$ (8) ← The number is greater than 10. So, the exponent is positive.

b. Write 0.0000032 in scientific notation.

Move the decimal point 6 places to the right. → $0.0000032 = 3.2 \times 10^{-6}$ (6) ← The number is between 0 and 1. So, the exponent is negative.

Try It

Write the number in scientific notation.

1. 50,000
2. 25,000,000
3. 683
4. 0.005
5. 0.00000033
6. 0.000506

Key Idea

Writing Numbers in Standard Form

The absolute value of the exponent indicates how many places to move the decimal point.

- If the exponent is negative, move the decimal point to the left.
- If the exponent is positive, move the decimal point to the right.

EXAMPLE 2 Writing Numbers in Standard Form

a. Write 3.22×10^{-4} in standard form.

$3.22 \times 10^{-4} = 0.000322$ Move the decimal point $|-4| = 4$ places to the left.

4

b. Write 7.9×10^{5} in standard form.

$7.9 \times 10^{5} = 790{,}000$ Move the decimal point $|5| = 5$ places to the right.

5

Try It **Write the number in standard form.**

7. 6×10^{7} **8.** 9.9×10^{-5} **9.** 1.285×10^{4}

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

WRITING NUMBERS IN SCIENTIFIC NOTATION **Write the number in scientific notation.**

10. 675,000,000 **11.** 0.000000084 **12.** 0.000012001

WRITING NUMBERS IN STANDARD FORM **Write the number in standard form.**

13. 8×10^{-7} **14.** 3.876×10^{7} **15.** 1.11×10^{-5}

16. WHICH ONE DOESN'T BELONG? Which number does *not* belong with the other three? Explain.

2.8×10^{15} 4.3×10^{-30} 1.05×10^{28} 10×9.2^{-13}

EXAMPLE 3 Modeling Real Life

A female flea consumes about 1.4×10^{-5} liter of blood each day.

A dog has 100 female fleas. What is the total amount of blood consumed by the fleas each day? Express your answer using more-appropriate units.

Write 1.4×10^{-5} in standard form. Then multiply the number by 100 to determine the amount of blood that 100 female fleas consume each day.

$$1.4 \times 10^{-5} = 0.000014$$

5

Move the decimal point $|-5| = 5$ places to the left.

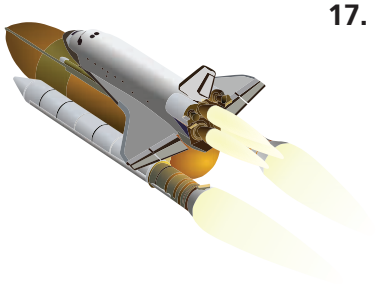
So, 100 female fleas consume about $100(0.000014) = 0.0014$ liter of blood per day. You can use milliliters to express this quantity using more-appropriate units.

$$0.0014 \text{ L} = 0.0014 \text{ L} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 1.4 \text{ mL}$$

The fleas consume about 0.0014 liter, or 1.4 milliliters, of blood each day.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



Total Power = 1.174×10^{10} watts

17. A series of movies is about 3.285×10^4 seconds long. How long does it take to watch the series twice? Express your answer using more-appropriate units.

18. The total power of a space shuttle during launch is the sum of the power from its solid rocket boosters and the power from its main engines. The power from the solid rocket boosters is 9,750,000,000 watts. What is the power from the main engines?

19. The area of a trampoline is about 1.8×10^4 square inches. Write this number in standard form. Then represent the area of the trampoline using more-appropriate units.

20. DIG DEEPER! The *epidermis, dermis,* and *hypodermis* are layers of your skin. The dermis is about 3.5 millimeters thick. The epidermis is about 1.25×10^{-3} meter thick. The hypodermis is about 0.15 centimeter thick. What is the difference in thickness of the thickest layer and the thinnest layer? Justify your answer.

8.6 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Round the number. Write the result as the product of a single digit and a power of 10.

1. 0.00000129

2. 4,241,933,200

3. 0.0000001801

4. 879,679,466

Write the product using exponents.

5. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

6. $3 \cdot 3 \cdot 3 \cdot y \cdot y \cdot y$

7. $(-2) \cdot (-2) \cdot (-2)$

Concepts, Skills, & Problem Solving

MP USING TOOLS **Use a graphing calculator to evaluate the function when $x = 10$. Write the number in standard form.**

(See Exploration 1, p. 349.)

8. $y = \left(\frac{1}{10}\right)^x$

9. $y = 20^x$

WRITING NUMBERS IN SCIENTIFIC NOTATION **Write the number in scientific notation.**

10. 0.0021

11. 5,430,000

12. 321,000,000

13. 0.00000625

14. 0.00004

15. 10,700,000

16. 45,600,000,000

17. 0.000000000009256

18. 840,000

WRITING NUMBERS IN STANDARD FORM **Write the number in standard form.**

19. 7×10^7

20. 8×10^{-3}

21. 5×10^2

22. 2.7×10^{-4}

23. 4.4×10^{-5}

24. 2.1×10^3

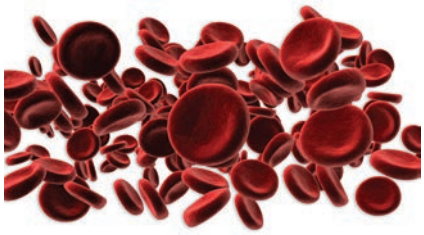
25. 1.66×10^9

26. 3.85×10^{-8}

27. 9.725×10^6

28. **MP MODELING REAL LIFE** The U.S. Brig *Niagara*, a warship from the Battle of Lake Erie in 1813, uses about 28,300 feet of rope to operate its sails and spars. Write this number in scientific notation.

29. **MP MODELING REAL LIFE** The radius of a fishing line is 2.5×10^{-4} feet. Write this number in standard form. Then write your answer using inches.



Blood: 2.7×10^8 platelets per milliliter

30. MP **MODELING REAL LIFE** Platelets are cell-like particles in the blood that help form blood clots.

a. How many platelets are in 3 milliliters of blood? Write your answer in standard form.

b. An adult human body contains about 5 liters of blood. How many platelets are in an adult human body?

CHOOSING APPROPRIATE UNITS **Match each value with the most appropriate unit of measurement.**

31. height of a skyscraper: 2.6×10^2

32. distance between two asteroids: 2.5×10^5

33. depth of a bathtub: 1.6×10^1

34. length of memory chip: 7.8×10^0

A. inches

B. millimeters

C. miles

D. meters

35. MP **NUMBER SENSE** Describe how the value of a number written in scientific notation changes when you increase the exponent by 1.

36. MP **PROBLEM SOLVING** The area of the Florida Keys National Marine Sanctuary is about 9600 square kilometers. The area of the Florida Reef Tract is about 16.2% of the area of the sanctuary. What is the area of the Florida Reef Tract? Write your answer in scientific notation.

37. MP **REASONING** A gigameter is 1.0×10^6 kilometers. How many square kilometers are in 5 square gigameters?

38. MP **PROBLEM SOLVING** There are about 1.4×10^9 cubic kilometers of water on Earth. About 2.5% of the water is freshwater. How much freshwater is on Earth?

39. **CRITICAL THINKING** The table shows the speed of light through each of five media. Determine in which media light travels the fastest and the slowest.

Medium	Speed
Air	6.7×10^8 mi/h
Glass	6.6×10^8 ft/sec
Ice	2.3×10^5 km/sec
Vacuum	3.0×10^8 m/sec
Water	2.3×10^{10} cm/sec

Equivalent to 1 Atomic Mass Unit
8.3×10^{-24} carat
1.66×10^{-21} milligram

40. MP **STRUCTURE** The mass of an atom or molecule is measured in atomic mass units. Which is greater, a *carat* or a *milligram*? Explain.

8.7 Operations in Scientific Notation

Learning Target: Perform operations with numbers written in scientific notation.

Success Criteria:
- I can explain how to add and subtract numbers in scientific notation.
- I can explain how to multiply and divide numbers in scientific notation.
- I can use operations in scientific notation to solve real-life problems.

EXPLORATION 1 Adding and Subtracting in Scientific Notation

Work with a partner.

a. Complete the table by finding the sum and the difference of Expression 1 and Expression 2. Write your answers in scientific notation. Explain your method.

Expression 1	Expression 2	Sum	Difference
3×10^4	1×10^4		
4×10^{-3}	2×10^{-3}		
4.1×10^{-7}	1.5×10^{-7}		
8.3×10^6	1.5×10^6		

Math Practice

Look for Structure

How might you find the sum or difference of two expressions in scientific notation that contain different powers of 10?

b. Use your results in part (a) to explain how to find $(a \times 10^n) + (b \times 10^n)$ and $(a \times 10^n) - (b \times 10^n)$.

EXPLORATION 2 Multiplying and Dividing in Scientific Notation

Work with a partner.

a. Complete the table by finding the product and the quotient of Expression 1 and Expression 2. Write your answers in scientific notation. Explain your method.

Expression 1	Expression 2	Product	Quotient
3×10^4	1×10^4		
4×10^3	2×10^2		
7.7×10^{-2}	1.1×10^{-3}		
4.5×10^5	3×10^{-1}		

b. Use your results in part (a) to explain how to find $(a \times 10^n) \times (b \times 10^m)$ and $(a \times 10^n) \div (b \times 10^m)$. Describe any properties that you use.

8.7 Lesson

To add or subtract numbers written in scientific notation with the same power of 10, add or subtract the factors. When the numbers have different powers of 10, first rewrite the numbers so they have the same power of 10.

EXAMPLE 1 Adding and Subtracting in Scientific Notation

Find the sum or difference.

a. $(4.6 \times 10^3) + (8.72 \times 10^3)$

$= (4.6 + 8.72) \times 10^3$ Distributive Property

$= 13.32 \times 10^3$ Add.

$= (1.332 \times 10^1) \times 10^3$ Write 13.32 in scientific notation.

$= 1.332 \times 10^4$ Product of Powers Property

b. $(3.5 \times 10^{-2}) - (6.6 \times 10^{-3})$

Rewrite 6.6×10^{-3} so that it has the same power of 10 as 3.5×10^{-2}.

$6.6 \times 10^{-3} = 6.6 \times 10^{-1} \times 10^{-2}$ Rewrite 10^{-3} as $10^{-1} \times 10^{-2}$.

$= 0.66 \times 10^{-2}$ Rewrite 6.6×10^{-1} as 0.66.

Subtract the factors.

$(3.5 \times 10^{-2}) - (0.66 \times 10^{-2})$

$= (3.5 - 0.66) \times 10^{-2}$ Distributive Property

$= 2.84 \times 10^{-2}$ Subtract.

Math Practice

Look for Structure

Solve Example 1(b) by rewriting 3.5×10^{-2} as 35×10^{-3}.

Try It **Find the sum or difference.**

1. $(8.2 \times 10^2) + (3.41 \times 10^{-1})$
2. $(7.8 \times 10^{-5}) - (4.5 \times 10^{-5})$

To multiply or divide numbers written in scientific notation, multiply or divide the factors and powers of 10 separately.

EXAMPLE 2 Multiplying in Scientific Notation

Find $(3 \times 10^{-5}) \times (5 \times 10^{-2})$.

$(3 \times 10^{-5}) \times (5 \times 10^{-2})$

$= 3 \times 5 \times 10^{-5} \times 10^{-2}$ Commutative Property of Multiplication

$= (3 \times 5) \times (10^{-5} \times 10^{-2})$ Associative Property of Multiplication

$= 15 \times 10^{-7}$ Simplify.

$= (1.5 \times 10^1) \times 10^{-7}$ Write 15 in scientific notation.

$= 1.5 \times 10^{-6}$ Product of Powers Property

Check

Use standard form to check your answer.

(3×10^{-5})
$\times (5 \times 10^{-2})$
$= 0.00003 \times 0.05$
$= 0.0000015$
$= 1.5 \times 10^{-6}$ ✓

Try It **Find the product.**

3. $6 \times (8 \times 10^{-5})$

4. $(7 \times 10^{2}) \times (3 \times 10^{5})$

5. $(2 \times 10^{4}) \times (6 \times 10^{-7})$

6. $(3 \times 10^{8}) \times (9 \times 10^{3})$

EXAMPLE 3 Dividing in Scientific Notation

Find $\dfrac{1.5 \times 10^{-8}}{6 \times 10^{7}}$.

$$\frac{1.5 \times 10^{-8}}{6 \times 10^{7}} = \frac{1.5}{6} \times \frac{10^{-8}}{10^{7}}$$ Rewrite as a product of fractions.

$$= 0.25 \times \frac{10^{-8}}{10^{7}}$$ Divide 1.5 by 6.

$$= 0.25 \times 10^{-15}$$ Quotient of Powers Property

$$= (2.5 \times 10^{-1}) \times 10^{-15}$$ Write 0.25 in scientific notation.

$$= 2.5 \times 10^{-16}$$ Product of Powers Property

Try It **Find the quotient.**

7. $(9.2 \times 10^{12}) \div 4.6$

8. $(1.5 \times 10^{-3}) \div (7.5 \times 10^{2})$

9. $(3.75 \times 10^{-8}) \div (1.25 \times 10^{-7})$

10. $(9.2 \times 10^{6}) \div (2.3 \times 10^{12})$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

11. **WRITING** Describe how to add or subtract two numbers written in scientific notation with the same power of 10.

12. MP **NUMBER SENSE** Two numbers written in scientific notation have different powers of 10. Do you have to rewrite the numbers so they have the same power of 10 before multiplying or dividing? Explain.

OPERATIONS IN SCIENTIFIC NOTATION **Evaluate the expression. Write your answer in scientific notation.**

13. $(7.26 \times 10^{4}) + (3.4 \times 10^{4})$

14. $(2.8 \times 10^{-5}) - (1.6 \times 10^{-6})$

15. $(2.4 \times 10^{4}) \times (3.8 \times 10^{-6})$

16. $(5.2 \times 10^{-3}) \div (1.3 \times 10^{-12})$

EXAMPLE 4 Modeling Real Life

Diameter ≈ 1,400,000,000 m

An aluminum ion has a diameter of about 5×10^{-11} meter. How many times greater is the diameter of the Sun than the diameter of the ion?

Write the diameter of the Sun in scientific notation.

$$1{,}400{,}000{,}000 = 1.4 \times 10^{9}$$

9

Divide the diameter of the Sun by the diameter of the aluminum ion.

$$\frac{1.4 \times 10^{9}}{5 \times 10^{-11}} = \frac{1.4}{5} \times \frac{10^{9}}{10^{-11}}$$ Rewrite as a product of fractions.

$$= 0.28 \times \frac{10^{9}}{10^{-11}}$$ Divide 1.4 by 5.

$$= 0.28 \times 10^{20}$$ Quotient of Powers Property

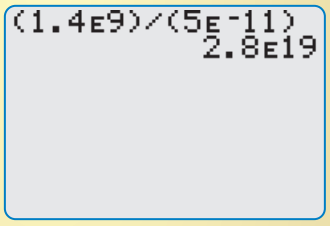
$$= (2.8 \times 10^{-1}) \times 10^{20}$$ Write 0.28 in scientific notation.

$$= 2.8 \times 10^{19}$$ Product of Powers Property

The diameter of the Sun is about 2.8×10^{19} times greater than the diameter of the aluminum ion.

Check

Use a calculator to check your answer.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. It takes the Sun about 2.3×10^{8} years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^{2} years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the Milky Way?

18. A person typically breathes about 8.64×10^{3} liters of air per day. The life expectancy of a person in the United States at birth is about 29,200 days. Estimate the total amount of air a person born in the United States breathes over a lifetime.

19. DIG DEEPER! In one week, about 4100 movie theaters each sold an average of 2200 tickets for Movie A. About 3.6×10^{7} total tickets were sold at the theaters during the week. An article claims that about 25% of all tickets sold during the week were for Movie A. Is this claim accurate? Justify your answer.

8.7 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

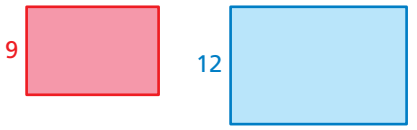
Review & Refresh

Write the number in scientific notation.

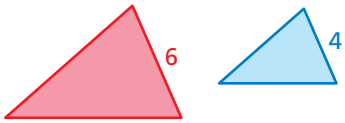
1. 0.0038 **2.** 74,000,000 **3.** 0.0000475

Find the values of the ratios (red to blue) of the perimeters and areas of the similar figures.

4.



5.



Concepts, Skills, & Problem Solving

OPERATIONS IN SCIENTIFIC NOTATION **Find the sum, difference, product, and quotient of Expression 1 and Expression 2. Write your answers in scientific notation.**
(See Explorations 1 and 2, p. 355.)

6. 3×10^3 Expression 1
2×10^3 Expression 2

7. 6×10^{-4} Expression 1
1.5×10^{-4} Expression 2

ADDING AND SUBTRACTING IN SCIENTIFIC NOTATION **Find the sum or difference. Write your answer in scientific notation.**

8. $(2 \times 10^5) + (3.8 \times 10^5)$

9. $(6.33 \times 10^{-9}) - (4.5 \times 10^{-9})$

10. $(9.2 \times 10^8) - (4 \times 10^8)$

11. $(7.2 \times 10^{-6}) + (5.44 \times 10^{-6})$

12. $(7.8 \times 10^7) - (2.45 \times 10^6)$

13. $(5 \times 10^{-5}) + (2.46 \times 10^{-3})$

14. $(9.7 \times 10^6) + (6.7 \times 10^5)$

15. $(2.4 \times 10^{-1}) - (5.5 \times 10^{-2})$

16. **MP YOU BE THE TEACHER**
Your friend adds 2.5×10^9 and 5.3×10^8. Is your friend correct? Explain your reasoning.

$$(2.5 \times 10^9) + (5.3 \times 10^8) = (2.5 \times 10^9) + (0.53 \times 10^9)$$
$$= (2.5 + 0.53) \times 10^9$$
$$= 3.03 \times 10^9$$

MULTIPLYING AND DIVIDING IN SCIENTIFIC NOTATION **Find the product or quotient. Write your answer in scientific notation.**

17. $5 \times (7 \times 10^7)$

18. $(5.8 \times 10^{-6}) \div (2 \times 10^{-3})$

19. $(1.2 \times 10^{-5}) \div 4$

20. $(5 \times 10^{-7}) \times (3 \times 10^6)$

21. $(3.6 \times 10^7) \div (7.2 \times 10^7)$

22. $(7.2 \times 10^{-1}) \times (4 \times 10^{-7})$

23. $(6.5 \times 10^8) \times (1.4 \times 10^{-5})$

24. $(2.8 \times 10^4) \div (2.5 \times 10^6)$

MATCHING **You use technology to find four sums. Match the sum with its standard form.**

25. 4.3E8 **26.** 4.3E−8 **27.** 4.3E10 **28.** 4.3E−10

A. 0.00000000043 **B.** 0.000000043 **C.** 430,000,000 **D.** 43,000,000,000

29. MP **MODELING REAL LIFE** How many times greater is the thickness of a dime than the thickness of a dollar bill?

Thickness = 0.135 cm

Thickness = 1.0922×10^{-2} cm

30. **MULTIPLE CHOICE** On a social media website, Celebrity A has about 8.6×10^7 followers and Celebrity B has about 4.1×10^6 followers. Determine which of the following is the best estimate for the number of followers for Celebrity A compared to the number of followers for Celebrity B.

A. more than 2 times greater **B.** less than 2 times greater

C. more than 20 times greater **D.** less than 20 times greater

MP **REASONING** **Evaluate the expression. Write your answer in scientific notation.**

31. $5{,}200{,}000 \times (8.3 \times 10^2) - (3.1 \times 10^8)$

32. $(9 \times 10^{-3}) + (2.4 \times 10^{-5}) \div 0.0012$

33. **GEOMETRY** Find the perimeter of the rectangle at the right.

34. DIG DEEPER! A human heart pumps about 7×10^{-2} liter of blood per heartbeat. The average human heart beats about 72 times per minute. How many liters of blood does a heart pump in 1 year? 70 years?

35. MP **MODELING REAL LIFE** Use the Internet or another reference to find the populations and areas (in square miles) of India, China, Argentina, the United States, and Egypt. Round each population to the nearest million and each area to the nearest thousand square miles.

a. Write each population and area in scientific notation.

b. Use your answers to part (a) to find and order the population densities (people per square mile) of each country from least to greatest.

8 Connecting Concepts

Using the Problem-Solving Plan

1. Atoms are made of protons, neutrons, and electrons. The table shows the numbers of protons and the masses of several atoms. Use a line of best fit to estimate the mass (in grams) of an atom that has 29 protons.

Protons, x	Mass (gram), y
1	1.67×10^{-24}
5	1.79×10^{-23}
53	2.11×10^{-22}
20	6.65×10^{-23}
14	4.66×10^{-23}
3	1.15×10^{-23}
40	1.51×10^{-22}
16	5.32×10^{-23}

Understand the problem.

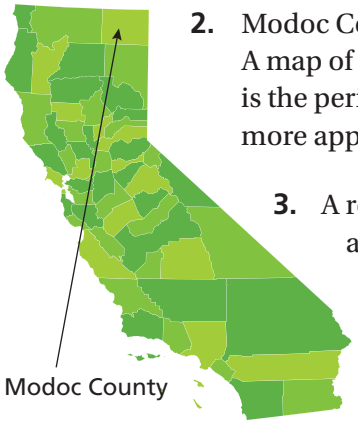
You know the numbers of protons and the masses of several atoms. You are asked to use the line of best fit to estimate the mass of an atom that has 29 protons.

Make a plan.

Use a graphing calculator to find an equation of the line of best fit. Then evaluate the equation when $x = 29$.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. Modoc County, California, is 74.9 miles long and 56.2 miles wide. A map of the county is drawn using a scale factor of 2.11×10^{-6}. What is the perimeter of the county on the map? Express your answer using more appropriate units.

3. A research company estimates that in the United States, about 8.37×10^7 adult males and 6.59×10^7 adult females watch NFL football, while 3.13×10^7 adult males and 5.41×10^7 adult females do *not* watch NFL football. Organize the results in a two-way table. Include the marginal frequencies.

Performance Task

Elements in the Universe

At the beginning of this chapter, you watched a STEAM Video called "Carbon Atoms." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

8 Chapter Review

Go to *BigIdeasMath.com* to download blank graphic organizers.

Review Vocabulary

Write the definition and give an example of each vocabulary term.

power, *p. 320*
base, *p. 320*
exponent, *p. 320*
scientific notation, *p. 350*

Graphic Organizers

You can use a **Definition and Example Chart** to organize information about a concept. Here is an example of a Definition and Example Chart for the vocabulary term ***power***.

Power: product of repeated factors

Example

$x \cdot x \cdot x = x^3$

Example

$\frac{1}{4} \cdot \frac{1}{4} = \left(\frac{1}{4}\right)^2$

Example

$(-2)^5$

Choose and complete a graphic organizer to help you study the concept.

1. Product of Powers Property
2. Power of a Power Property
3. Power of a Product Property
4. Quotient of Powers Property
5. negative exponents
6. scientific notation
7. adding and subtracting numbers in scientific notation
8. multiplying and dividing numbers in scientific notation

"Here is my Definition and Example Chart. I'm going to take a selfie from the top of the pyramid. Do you want to hold the camera?"

Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

1	2	3	4
I do not understand.	I can do it with help.	I can do it on my own.	I can teach someone else.

8.1 Exponents *(pp. 319–324)*

Learning Target: Use exponents to write and evaluate expressions.

Write the product using exponents.

1. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$

2. $2 \cdot 2 \cdot 2 \cdot n \cdot n$

Evaluate the expression.

3. 11^3

4. $-\left(\frac{1}{2}\right)^4$

5. $\left|\frac{1}{2}(16 - 6^3)\right|$

6. The profit P (in dollars) earned by a local merchant selling x items is represented by the equation $P = 0.2x^3 - 10$. How much more profit does he earn selling 15 items than 5 items?

8.2 Product of Powers Property *(pp. 325–330)*

Learning Target: Generate equivalent expressions involving products of powers.

Simplify the expression. Write your answer as a power.

7. $p^5 \cdot p^2$

8. $(n^{11})^2$

9. $\left(-\frac{2}{5}\right)^3 \cdot \left(-\frac{2}{5}\right)^2$

10. Simplify $(-2k)^4$.

11. Write an expression that simplifies to x^{24} using the Power of a Power Property.

12. You send an email with a file size of 4 kilobytes. One kilobyte is 2^{10} bytes. What is the file size of your email in bytes?

13. Explain how to use properties of exponents to simplify the expression $27 \cdot 3^2$.

8.3 Quotient of Powers Property *(pp. 331–336)*

Learning Target: Generate equivalent expressions involving quotients of powers.

Simplify the expression. Write your answer as a power.

14. $\frac{8^8}{8^3}$

15. $\frac{5^2 \cdot 5^9}{5}$

16. $\frac{w^8}{w^7} \cdot \frac{w^5}{w^2}$

17. $\frac{m^8}{m^6} \cdot \frac{m^{10} \cdot m^2}{m^9}$

18. Write an expression that simplifies to x^3 using the Quotient of Powers Property.

19. At the end of a fiscal year, a company has made 1.62×7^7 dollars in profit. The company employs 7^3 people. How much will each person receive if the company divides the profit equally among its employees?

8.4 Zero and Negative Exponents *(pp. 337–342)*

Learning Target: Understand the concepts of zero and negative exponents.

Evaluate the expression.

20. 2^{-4}

21. 95^0

22. $\frac{8^2}{8^4}$

23. $(-12)^{-7} \cdot (-12)^7$

24. $\frac{1}{7^9} \cdot \frac{1}{7^{-6}}$

25. $\frac{9^4 \cdot 9^{-2}}{9^2}$

Simplify. Write the expression using only positive exponents.

26. $x^{-2} \cdot x^0$

27. $y^{-8}y^3$

28. $\frac{3^{-1} \cdot z^5}{z^{-2}}$

29. Write an expression that simplifies to x^{-4}.

30. Water flows from a showerhead at a rate of 24^{-1} gallon per second. How many gallons do you use when taking a 15-minute shower? a 20-minute shower?

31. Explain two different methods for simplifying $w^{-2} \cdot w^5$.

8.5 Estimating Quantities *(pp. 343–348)*

Learning Target: Round numbers and write the results as the product of a single digit and a power of 10.

Round the number. Write the result as a product of a single digit and a power of 10.

32. 29,197,543

33. 0.000000647

34. The speed of light is 299,792,458 meters per second. About how far can a light beam travel in 3 seconds? Write your answer as a product of a single digit and a power of 10.

35. The population of Albany, New York is about 98,989 and the population of Moscow, Russia is about 12,235,448. Approximately how many times greater is the population of Moscow than the population of Albany?

8.6 Scientific Notation *(pp. 349–354)*

Learning Target: Understand the concept of scientific notation.

Write the number in scientific notation.

36. 0.00036

37. 800,000

38. 79,200,000

Write the number in standard form.

39. 2×10^{7}

40. 4.8×10^{-3}

41. 6.25×10^{5}

42. The mass of a single dust particle is 7.52×10^{-10} kilogram. What is the mass of a dust ball made of 100 dust particles? Express your answer using more-appropriate units.

8.7 Operations in Scientific Notation *(pp. 355–360)*

Learning Target: Perform operations with numbers written in scientific notation.

Evaluate the expression. Write your answer in scientific notation.

43. $(4.2 \times 10^{8}) + (5.9 \times 10^{9})$

44. $(5.9 \times 10^{-4}) - (1.8 \times 10^{-4})$

45. $(7.7 \times 10^{8}) \times (4.9 \times 10^{-5})$

46. $(3.6 \times 10^{5}) \div (1.8 \times 10^{9})$

Diameter $\approx 8 \times 10^{-6}$ m

47. A white blood cell has a diameter of about 0.000012 meter. How many times greater is the diameter of a white blood cell than the diameter of a red blood cell?

8 Practice Test

Write the product using exponents.

1. $(-15) \cdot (-15) \cdot (-15)$

2. $4 \cdot 4 \cdot x \cdot x \cdot x$

Evaluate the expression.

3. $10 + 3^3 \div 9$

4. $\dfrac{-2 \cdot (-2)^{-4}}{(-2)^{-2}}$

Simplify the expression. Write your answer as a power.

5. $9^{10} \cdot 9$

6. $(6^6)^5$

7. $\dfrac{(-3.5)^{13} \cdot (-3.5)^2}{(-3.5)^9}$

8. Simplify $(2y)^7$.

Round the number. Write the result as a product of a single digit and a power of 10.

9. 4,610,428,970

10. 0.00000572

Write the number in standard form.

11. 3×10^7

12. 9.05×10^{-3}

Evaluate the expression. Write your answer in scientific notation.

13. $(7.8 \times 10^7) + (9.9 \times 10^7)$

14. $(6.4 \times 10^5) - (5.4 \times 10^4)$

15. $(3.1 \times 10^6) \times (2.7 \times 10^{-2})$

16. $(9.6 \times 10^7) \div (1.2 \times 10^{-4})$

17. Is $(xy^2)^3$ the same as $(xy^3)^2$? Explain.

18. One scoop of rice weighs about 3^9 milligrams.

a. Write a linear function that relates the weight of rice to the number of scoops. What is the weight of 5 scoops of rice?

b. A grain of rice weighs about 3^3 milligrams. About how many grains of rice are in 1 scoop?

19. There are about 10,000 taste buds on a human tongue. Write this number in scientific notation.

20. From 1978 to 2008, the amount of lead allowed in the air in the United States was 1.5×10^{-6} gram per cubic meter. In 2008, the amount allowed was reduced by 90%. What is the new amount of lead allowed in the air?

8 Cumulative Practice

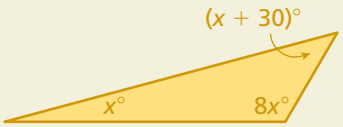
1. Mercury's distance from the Sun is approximately 5.79×10^7 kilometers. What is this distance in standard form?

A. 5,790,000 km **B.** 57,900,000 km

C. 579,000,000 km **D.** 5,790,000,000 km

2. Your friend solves the problem. What should your friend change to correctly answer the question?

How many *degrees* are in the largest angle in the triangle below?



$x + 8x + x + 30 = 180$

$10x = 150$

$x = 15$

F. The left side of the equation should equal 360° instead of 180°.

G. The sum of the acute angles should equal 90°.

H. Evaluate the smallest angle when $x = 15$.

I. Evaluate the largest angle when $x = 15$.

Test-Taking Strategy

Use Intelligent Guessing

"It can't be 40 or 5000 because they aren't divisible by 3. So, you can intelligently guess between 3000 and 3072."

3. Which expression is equivalent to the expression $2^4 2^3$?

A. 2^{12} **B.** 4^7

C. 48 **D.** 128

4. You randomly survey students in your school about whether they have a pet. You display your results in the two-way table. How many female students took the survey?

		Pet	
		Yes	No
Gender	Male	33	8
	Female	35	11

5. A bank account pays interest so that the amount in the account doubles every 10 years. The account started with $5,000 in 1940. Which expression represents the amount (in dollars) in the account n decades later?

F. $2^n \cdot 5000$

G. $5000(n + 1)$

H. 5000^n

I. $2^n + 5000$

6. The formula for the volume V of a pyramid is $V = \frac{1}{3}Bh$.

Which equation represents a formula for the height h of the pyramid?

A. $h = \frac{1}{3}VB$

B. $h = \frac{3V}{B}$

C. $h = \frac{V}{3B}$

D. $h = V - \frac{1}{3}B$

7. Think Solve Explain

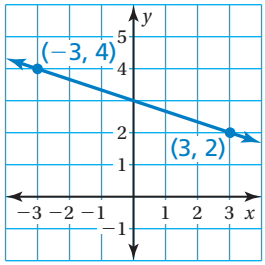
The gross domestic product (GDP) is a way to measure how much a country produces economically in a year. The table below shows the approximate population and GDP for the United States.

United States, 2016	
Population	324,000,000
GDP	$18,600,000,000,000

Part A Write the population and the GDP using scientific notation.

Part B Find the GDP per person for the United States using your answers from *Part A*. Write your answer in scientific notation. Show your work and explain your reasoning.

8. What is the equation of the line shown in the graph?



F. $y = -\frac{1}{3}x + 3$

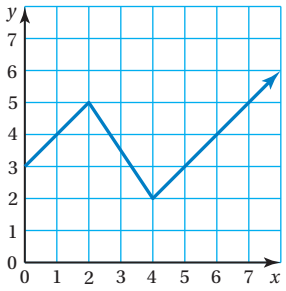
G. $y = \frac{1}{3}x + 1$

H. $y = -3x + 3$

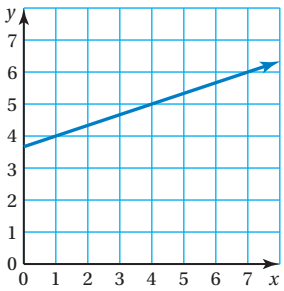
I. $y = 3x - \frac{1}{3}$

9. Which graph represents a linear function?

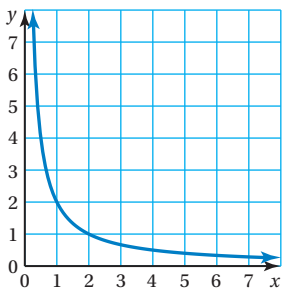
A.



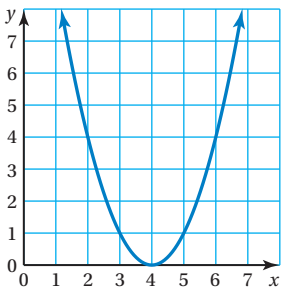
B.



C.



D.



10. Find $(-2.5)^{-2}$.

11. Two lines have the same y-intercept. The slope of one line is 1, and the slope of the other line is -1. What can you conclude?

F. The lines are parallel.

G. The lines meet at exactly one point.

H. The lines meet at more than one point.

I. The situation described is impossible.

12. Which list of ordered pairs represents the mapping diagram?

A. $(1, 2), (2, 0), (3, -2)$

B. $(1, 0), (2, 2), (3, -2)$

C. $(1, 0), (2, 2), (2, -2), (3, -2)$

D. $(0, 1), (2, 2), (-2, 2), (-2, 3)$