10.5 Volumes of Pyramids

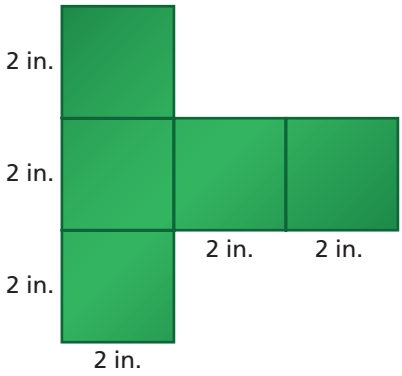
Learning Target: Find the volume of a pyramid.

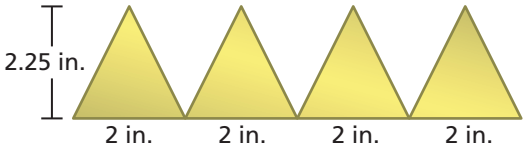
Success Criteria:
- I can use a formula to find the volume of a pyramid.
- I can use the volume of a pyramid to solve a real-life problem.

EXPLORATION 1

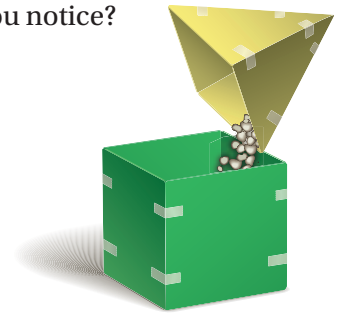
Finding a Formula for the Volume of a Pyramid

Work with a partner. Draw the two nets on cardboard and cut them out. Fold and tape the nets to form an open cube and an open square pyramid. Both figures should have the same size square base and the same height.



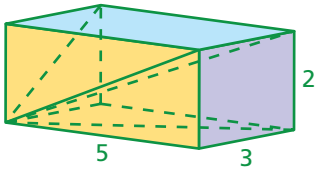


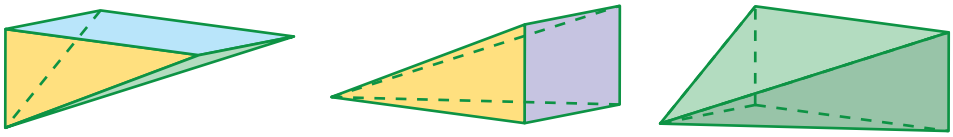
a. Compare the volumes of the figures. What do you notice?



b. Use your observations in part (a) to write a formula for the volume of a pyramid.

c. The rectangular prism below can be cut to form three pyramids. Use your formula in part (b) to show that the sum of the volumes of the three pyramids is equal to the volume of the prism.





Math Practice

Interpret a Solution

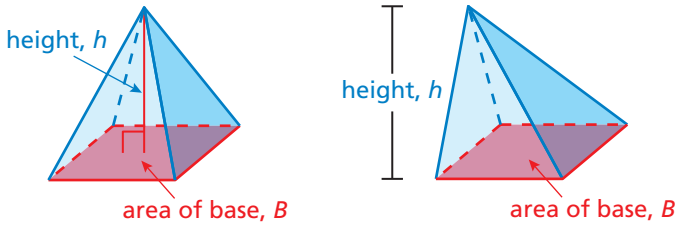
How do your calculations in part (c) help you verify that your formula is correct?

10.5 Lesson

Key Idea

Volume of a Pyramid

Words The volume V of a pyramid is one-third the product of the area of the base and the height of the pyramid.



Algebra $V = \frac{1}{3}Bh$

Area of base

Height of pyramid

Volumes of oblique pyramids are calculated the same way as volumes of right pyramids.

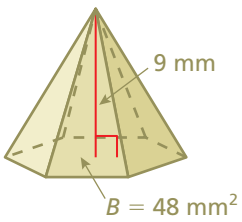
EXAMPLE 1 Finding the Volume of a Pyramid

Find the volume of the pyramid.

$V = \frac{1}{3}Bh$ Write the formula for volume.

$= \frac{1}{3}(48)(9)$ Substitute.

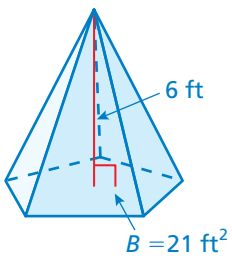
$= 144$ Multiply.



The volume is 144 cubic millimeters.

Try It Find the volume of the pyramid.

1.



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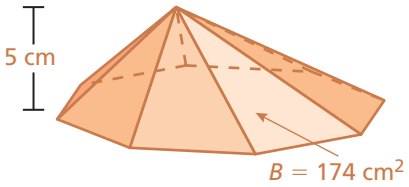
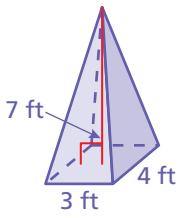


EXAMPLE 2 Finding the Volume of a Pyramid

Find the volume of the pyramid.

The area of the base of a rectangular pyramid is the product of the length ℓ and the width w. You can use $V = \frac{1}{3}\ell wh$ to find the volume of a rectangular pyramid.

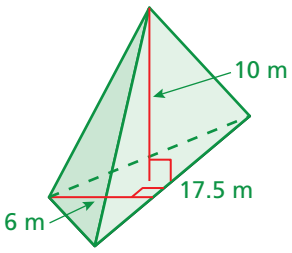
a.



$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(4)(3)(7)$$
$$= 28$$

The volume is 28 cubic feet.

b.

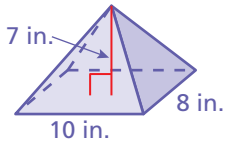


$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}\left(\frac{1}{2}\right)(17.5)(6)(10)$$
$$= 175$$

The volume is 175 cubic meters.

Try It **Find the volume of the pyramid.**

3.



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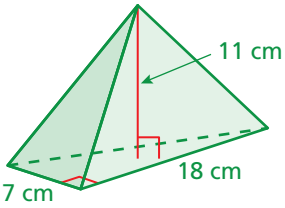


Self-Assessment for Concepts & Skills

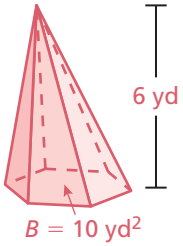
Solve each exercise. Then rate your understanding of the success criteria in your journal.

5. WRITING How is the formula for the volume of a pyramid different from the formula for the volume of a prism?

6. MP PROBLEM SOLVING How many different pyramids can you draw with the same height and volume? Explain.

FINDING THE VOLUME OF A PYRAMID **Find the volume of the pyramid.**

7.



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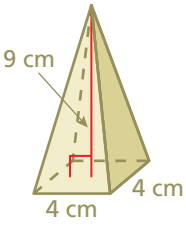
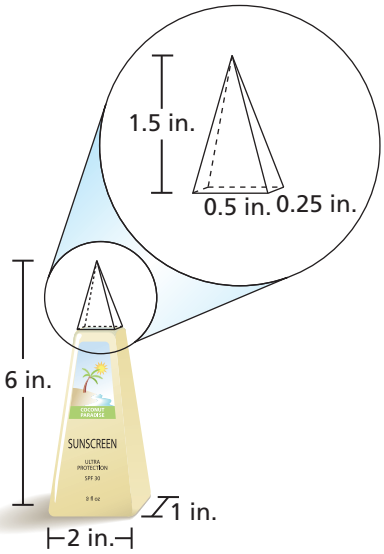


EXAMPLE 3 Modeling Real Life



Suncreen $9.96

The diagram shows the portion of a rectangular pyramid that is removed to make a sunscreen bottle. The portion that is removed is also a rectangular pyramid. Find the unit cost of the sunscreen.

Find the volume of the original pyramid and subtract the volume of the smaller pyramid.

Original Pyramid	*Smaller Pyramid*
$V = \frac{1}{3}Bh$	$V = \frac{1}{3}Bh$
$= \frac{1}{3}(2)(1)(6)$	$= \frac{1}{3}(0.5)(0.25)(1.5)$
$= 4 \text{ in.}^3$	$= 0.0625 \text{ in.}^3$

The volume of sunscreen in the bottle is $4 - 0.0625 = 3.9375$ cubic inches. The bottle of sunscreen costs $9.96. Find the unit rate.

$9.96 per 3.9375 cubic inches: $\frac{9.96}{3.9375} \approx \$2.53$ per cubic inch.

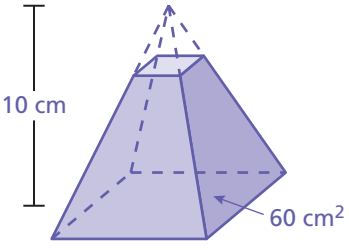
So, the unit cost of the sunscreen is about $2.53 per cubic inch.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. A resort features a square pyramid with a water slide. The length of the water slide is 90% of the height of the pyramid. The base of the pyramid has side lengths of 60 feet. The volume of the pyramid is 60,000 cubic feet. What is the length of the water slide?

10. DIG DEEPER! To make a candle, you use a mold to create the wax pyramid shown. You cut off the top 3 centimeters of the pyramid to make space for a wick. If the base area of the removed portion is 5.4 square centimeters, what percentage of the wax did you remove?



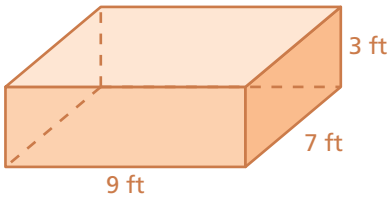
10.5 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the volume of the prism.

1.



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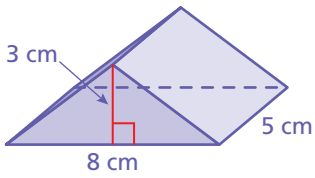


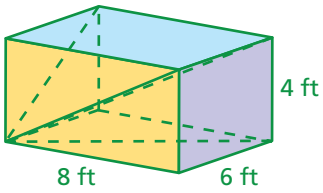
Solve the inequality. Graph the solution.

3. $r + 0.5 < -0.4$ **4.** $z - 2.4 \geq -0.6$ **5.** $h - 5 \leq -3.7$

Concepts, Skills, & Problem Solving

VOLUMES OF PYRAMIDS **The rectangular prism is cut to form three pyramids. Show that the sum of the volumes of the three pyramids is equal to the volume of the prism.** (See Exploration 1, p. 433.)

6.



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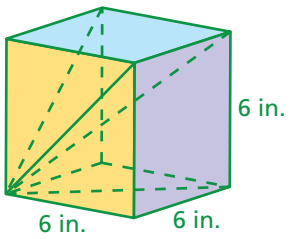
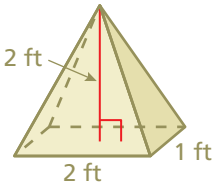


FINDING THE VOLUME OF A PYRAMID **Find the volume of the pyramid.**

8.



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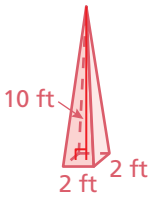

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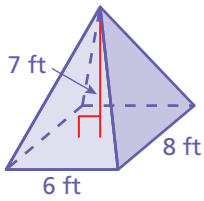

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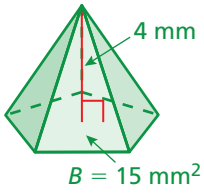

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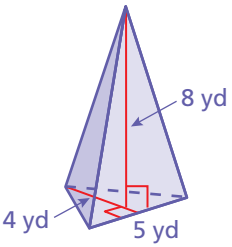

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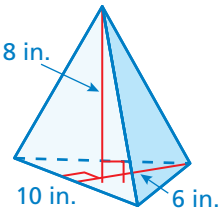
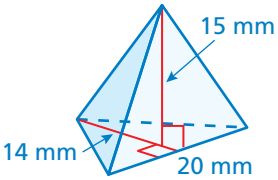


14.



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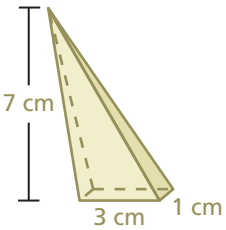

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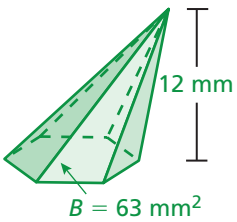
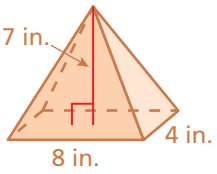
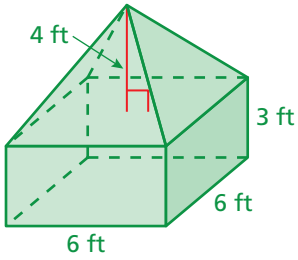


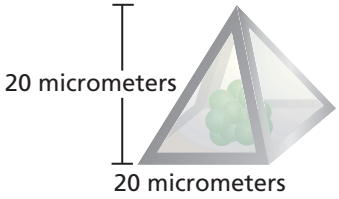
17. **YOU BE THE TEACHER** Your friend finds the volume of the pyramid. Is your friend correct? Explain your reasoning.



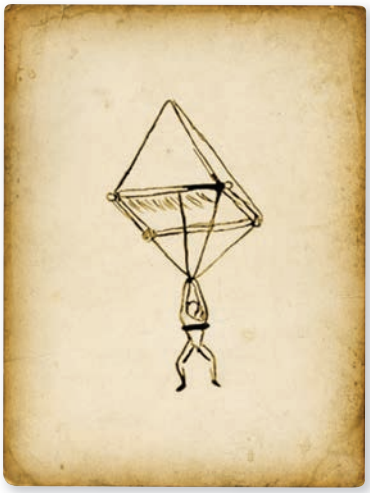
$V = Bh$
$= 8(4)(7)$
$= 224$ cubic inches

18. **MODELING REAL LIFE** A researcher develops a cage for a living cell in the shape of a square-based pyramid. A scale model of the cage is shown. What is the volume of the model?



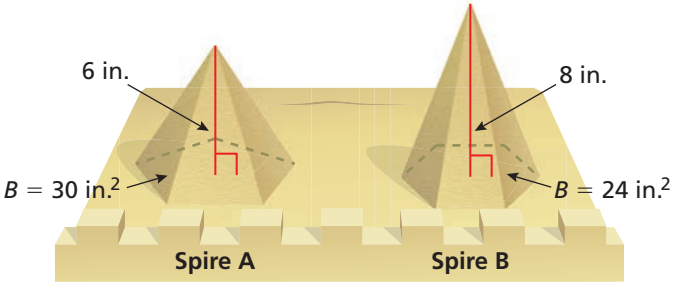


19. **FINDING VOLUME** Find the volume of the composite solid. Justify your answer.



Not drawn to scale

20. **MODELING REAL LIFE** In 1483, Leonardo da Vinci designed a parachute. It is believed that this was the first parachute ever designed. In a notebook, he wrote, "If a man is provided with a length of gummed linen cloth with a length of 12 yards on each side and 12 yards high, he can jump from any great height whatsoever without injury." Find the volume of air inside Leonardo's parachute.



21. **MODELING REAL LIFE** Which sandcastle spire has a greater volume? How much more sand do you need to make the spire with the greater volume?

22. **PROBLEM SOLVING** Use the photo of the tepee.

a. What is the shape of the base? How can you tell?

b. The tepee's height is about 10 feet. Estimate the volume of the tepee.

23. **OPEN-ENDED** A rectangular pyramid has a volume of 40 cubic feet and a height of 6 feet. Find one possible set of dimensions of the base.

24. **REASONING** Do the two solids have the same volume? Explain.