9.4 Constructing Polygons

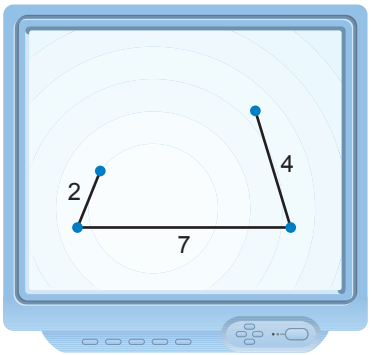
Learning Target: Construct a polygon with given measures.

Success Criteria:
- I can use technology to draw polygons.
- I can determine whether given measures result in one triangle, many triangles, or no triangle.
- I can draw polygons given angle measures or side lengths.

EXPLORATION 1 Using Technology to Draw Polygons

Work with a partner.

a. Use geometry software to draw each polygon with the given side lengths or angle measures, if possible. Complete the table.



Side Lengths or Angle Measures	How many figures are possible?
i. 4 cm, 6 cm, 7 cm	
ii. 2 cm, 6 cm, 7 cm	
iii. 2 cm, 4 cm, 7 cm	
iv. 2 cm, 4 cm, 6 cm	
v. 2 in., 3 in., 3 in., 5 in.	
vi. 1 in., 1 in., 3 in., 6 in.	
vii. 1 in., 1 in., 3 in., 4 in.	
viii. 90°, 60°, 30°	
ix. 100°, 40°, 20°	
x. 50°, 60°, 70°	
xi. 20°, 80°, 100°	
xii. 20°, 50°, 50°, 60°	
xiii. 30°, 80°, 120°, 130°	
xiv. 60°, 60°, 120°, 120°	

Math Practice

Use Technology to Explore

How does geometry software help you learn about characteristics of triangles and quadrilaterals?

b. Without constructing, how can you tell whether it is possible to draw a triangle given three angle measures? three side lengths? Explain your reasoning.

c. Without constructing, how can you tell whether it is possible to draw a quadrilateral given four angle measures? four side lengths? Explain your reasoning.

9.4 Lesson

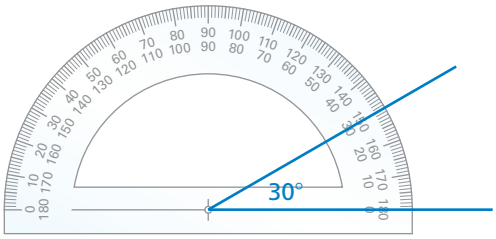
You can draw a triangle with three given angle measures when the sum of the angle measures is 180°.

EXAMPLE 1 Constructing Triangles Using Angle Measures

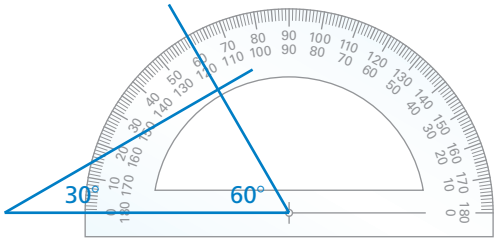
Draw a triangle with angle measures of 30°, 60°, and 90°, if possible.

Because 30° + 60° + 90° = 180°, you can draw a triangle with the given angle measures.

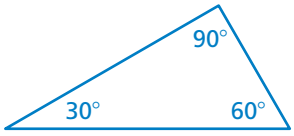
Step 1: Draw the 30° angle.



Step 2: Draw the 60° angle.



Step 3: Measure the remaining angle. The angle measure is 90°.



Try It **Draw a triangle with the given angle measures, if possible.**

1. 45°, 45°, 90°
2. 100°, 55°, 25°
3. 60°, 60°, 80°

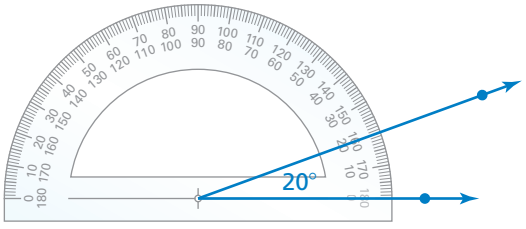
EXAMPLE 2 Constructing Triangles Using Angles and Sides

Draw a triangle with side lengths of 3 centimeters and 4 centimeters that meet at a 20° angle.

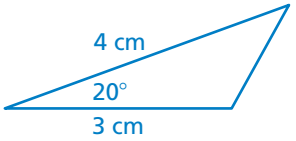
In Example 1, you can change the lengths of the sides to create many different triangles that meet the criteria. In Example 2, only one triangle is possible.

Step 1: Draw a 20° angle.

Step 2: Use a ruler to mark 3 centimeters on one ray and 4 centimeters on the other ray.



Step 3: Draw the third side to form the triangle.



Try It

4. Draw a triangle with side lengths of 1 inch and 2 inches that meet at a 60° angle.

You can draw a triangle with three given side lengths when the sum of the lengths of any two sides is greater than the length of the third side.

EXAMPLE 3 Constructing Triangles Using Side Lengths

Draw a triangle with the given side lengths, if possible.

a. 4 cm, 2 cm, 3 cm

The sum of the lengths of any two sides is greater than the length of the third side.

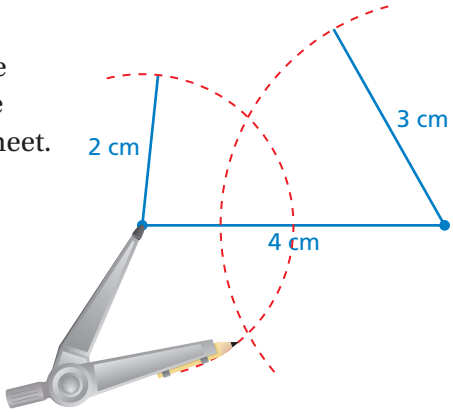
$4 \text{ cm} + 2 \text{ cm} > 3 \text{ cm}$ $\quad 4 \text{ cm} + 3 \text{ cm} > 2 \text{ cm}$ $\quad 2 \text{ cm} + 3 \text{ cm} > 4 \text{ cm}$

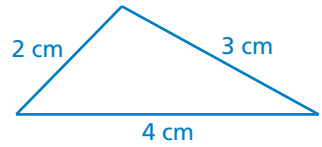
So, you can draw a triangle with the given side lengths.

Step 1: Draw a 4-centimeter side.

Step 2: Use a compass to determine where the 2-centimeter side and the 3-centimeter side meet.

Step 3: The third vertex can be at either intersection point. Draw the triangle.



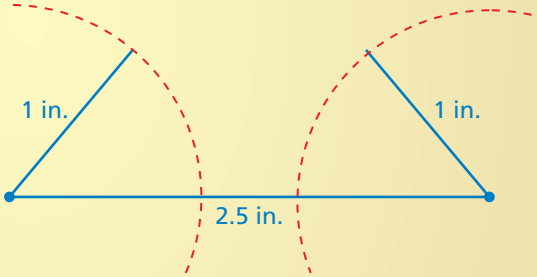


In Example 3, only one triangle is possible. You can start with a different side length in Step 1, but the resulting triangle will have the same size and shape.

b. 2.5 in., 1 in., 1 in.

Because 1 in. + 1 in. < 2.5 in., it is not possible to draw the triangle.

Check Try to draw the triangle. Draw a 2.5-inch side. Use a compass to show that the 1-inch sides cannot intersect.



So, it is not possible to draw the triangle. ✓

Math Practice

Look for Structure

How can you change one of the side lengths in part (b) so that they form a triangle? Compare answers with a classmate.

Try It **Draw a triangle with the given side lengths, if possible.**

5. 2 cm, 2 cm, 5 cm **6.** 4 cm, 3 cm, 3 cm **7.** 1 cm, 4 cm, 5 cm

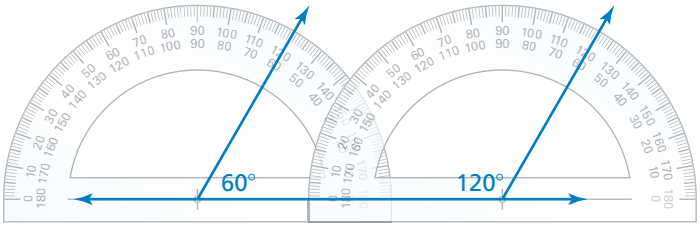
You can draw a quadrilateral with four given angle measures when the sum of the angle measures is 360°.

EXAMPLE 4 Constructing a Quadrilateral

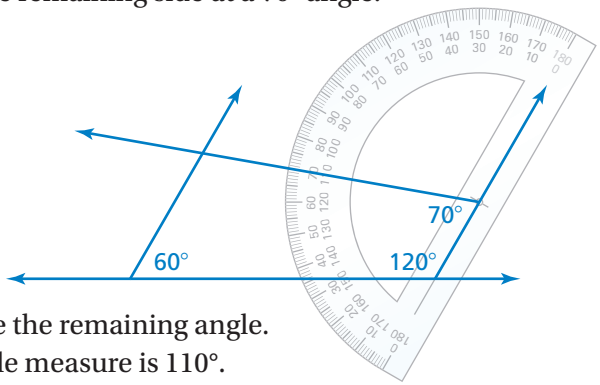
Draw a quadrilateral with angle measures of 60°, 120°, 70°, and 110°, if possible.

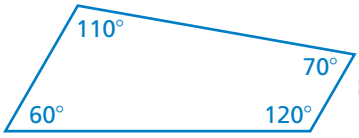
Because 60° + 120° + 70° + 110° = 360°, you can draw a quadrilateral with the given angle measures.

Step 1: Draw a 60° angle and a 120° angle that each have one side on a line.



Step 2: Draw the remaining side at a 70° angle.





Step 3: Measure the remaining angle. The angle measure is 110°.

Try It

Draw a quadrilateral with the given angle measures, if possible.

8. 100°, 90°, 65°, 105°

9. 100°, 40°, 20°, 20°

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

DRAWING POLYGONS **Draw a polygon with the given side lengths or angle measures, if possible.**

10. 25 mm, 36 mm, 38 mm

11. 10°, 15°, 155°

12. 20°, 45°, 50°, 65°

13. 50°, 90°, 110°, 110°

14. **USING SIDE LENGTHS** Can you construct *one*, *many*, or *no* triangle(s) with side lengths of 3 inches, 4 inches, and 8 inches? Explain.

EXAMPLE 5 Modeling Real Life

You enclose a flower bed using landscaping boards with lengths of 3 yards, 4 yards, and 5 yards. Estimate the area of the flower bed.

Understand the problem.

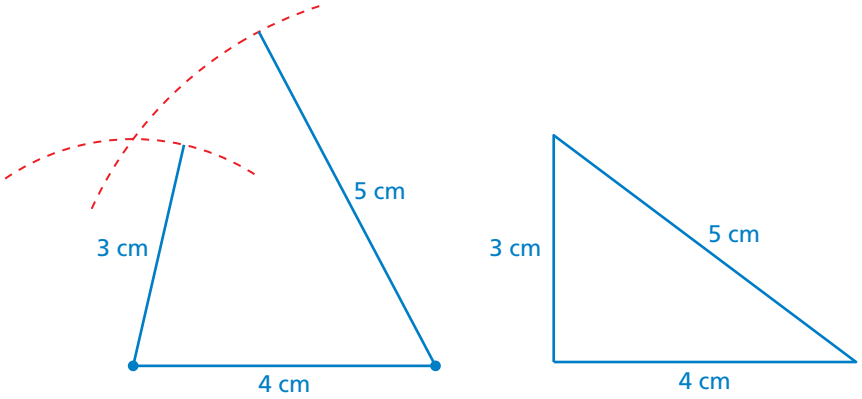
You know the lengths of boards used to enclose a triangular region. You are asked to estimate the area of the triangular region.

Make a plan.

Draw a triangle with side lengths of 3 yards, 4 yards, and 5 yards using a scale of 1 cm : 1 yd. Use the drawing to estimate the base and height of the flower bed. Then use the formula for the area of a triangle to estimate the area.

Solve and check.

Draw the triangle.



The shape of the flower bed appears to be a right triangle with a base length of 4 yards and a height of 3 yards.

So, the area of the flower bed is about $A = \frac{1}{2}(4)(3) = 6$ square yards.

Another Method

Using a ruler, the height from the largest angle to the 5-centimeter side is about 2.4 centimeters. So, the area is about $\frac{1}{2}(2.4)(5) = 6 \text{ yd}^2$. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

15. A triangular pen has fence lengths of 6 feet, 8 feet, and 10 feet. Create a scale drawing of the pen.

16. The front of a cabin is the shape of a triangle. The angles of the triangle are 40°, 70°, and 70°. Can you determine the height of the cabin? If not, what information do you need?

17. DIG DEEPER! Two rooftops have triangular patios. One patio has side lengths of 9 meters, 10 meters, and 11 meters. The other has side lengths of 6 meters, 10 meters, and 15 meters. Which patio has a greater area? Explain.

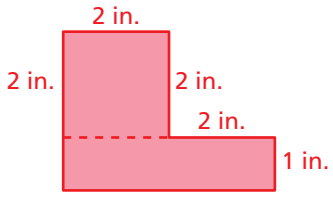
9.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

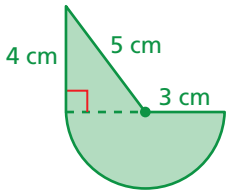
Review & Refresh

Find the perimeter and area of the figure.

1.



2.



Use a tree diagram to find the sample space and the total number of possible outcomes of the indicated event.

3. choosing a toothbrush

Toothbrush	
Type	Electric, Traditional
Strength	Extra soft, Soft, Medium

4. choosing a toy hoop

Toy Hoop	
Size	Small, Medium, Large
Color	Blue, Green, Orange, Pink, Purple, Yellow

Concepts, Skills, & Problem Solving

USING TECHNOLOGY TO DRAW POLYGONS **Use geometry software to draw the polygon with the given side lengths or angle measures, if possible.**
(See Exploration 1, p. 381.)

5. 30°, 65°, 85°

6. 2 in., 3 in., 5 in.

7. 80°, 90°,100°, 110°

8. 2 cm, 2 cm, 5 cm, 5 cm

CONSTRUCTING TRIANGLES USING ANGLE MEASURES **Draw a triangle with the given angle measures, if possible.**

9. 40°, 50°, 90°

10. 20°, 40°, 120°

11. 38°, 42°, 110°

12. 54°, 60°, 66°

13. MP **YOU BE THE TEACHER** Your friend determines whether he can draw a triangle with angle measures of 10°, 40°, and 130°. Is your friend correct? Explain your reasoning.

$10° + 40° < 130°$

Because the sum of the measure of two angles is not greater than the measure of the third angle, you cannot draw a triangle.

CONSTRUCTING TRIANGLES USING ANGLES AND SIDES **Draw a triangle with the given description.**

14. side lengths of 1 inch and 2 inches meet at a 50° angle

15. side lengths of 7 centimeters and 9 centimeters meet at a 120° angle

16. a 95° angle connects to a 15° angle by a side of length 2 inches

17. a 70° angle connects to a 70° angle by a side of length 4 centimeters

CONSTRUCTING TRIANGLES USING SIDE LENGTHS **Draw a triangle with the given side lengths, if possible.**

18. 4 in., 5 in., 10 in.

19. 10 mm, 30 mm, 50 mm

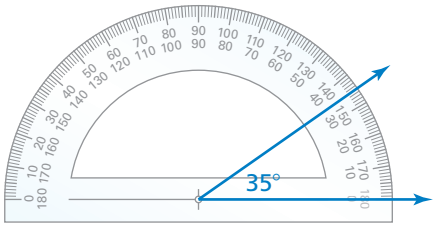
20. 5 cm, 5 cm, 8 cm

21. 8 mm, 12 mm, 13 mm

22. MP **MODELING REAL LIFE** Can you construct a triangular case using two pieces of wood that are 12 inches long and one piece of wood that is 25 inches long? Explain.

23. MP **MODELING REAL LIFE** Can you construct a warning triangle using three pieces of plastic that are each 6 inches long? Explain.

24. MP **LOGIC** You are constructing a triangle. You draw the first angle, as shown. Your friend says that you must be constructing an acute triangle. Is your friend correct? Explain your reasoning.



USING ANGLES AND SIDES **Determine whether you can construct *one, many,* or *no* triangle(s) with the given description. Explain your reasoning.**

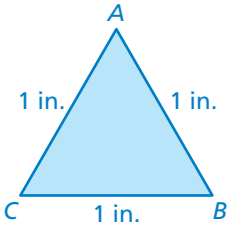
25. a triangle with one angle measure of 60° and one side length of 4 centimeters

26. a scalene triangle with side lengths of 3 centimeters and 7 centimeters

27. an isosceles triangle with two side lengths of 4 inches that meet at an 80° angle

28. a triangle with one angle measure of 60°, one angle measure of 70°, and a side length of 10 centimeters between the two angles

29. a triangle with one angle measure of 20°, one angle measure of 35°, and a side of length 3 inches that is between the two angles



30. MP **REASONING** A triangle is shown.

a. Construct a triangle with side lengths twice those of the triangle shown. Does the new triangle have the same angle measures?

b. How can you change the side lengths of the triangle so that the measure of $\angle A$ increases?

CONSTRUCTING QUADRILATERALS **Draw a quadrilateral with the given angle measures, if possible.**

31. 60°, 60°, 120°, 120°

32. 50°, 60°, 110°, 150°

33. 20°, 30°, 150°, 160°

34. 10°, 10°, 10°, 150°

CONSTRUCTING SPECIAL QUADRILATERALS **Construct a quadrilateral with the given description.**

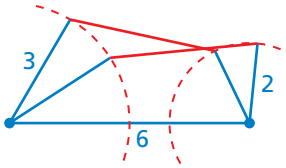
35. a rectangle with side lengths of 1 inch and 2 inches

36. a kite with side lengths of 4 centimeters and 7 centimeters

37. a trapezoid with base angles of 40°

38. a rhombus with side lengths of 10 millimeters

39. MP **REASONING** A quadrilateral has side lengths of 6 units, 2 units, and 3 units as shown. How many quadrilaterals can be formed given a fourth side with a fixed length? Explain.



40. MP **REASONING** What types of quadrilaterals can you form using four side lengths of 7 units? Use drawings to support your conclusion.

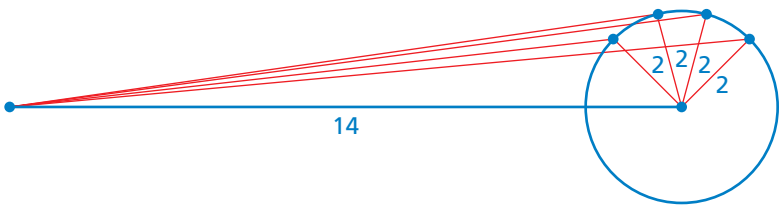
41. MP **MODELING REAL LIFE** A triangular section of a farm is enclosed by fences that are 2 meters, 6 meters, and 7 meters long. Estimate the area of the section.

42. MP **MODELING REAL LIFE** A chemical spill expert sets up a triangular caution zone using cones. Cones A and B are 14 meters apart. Cones B and C are 22 meters apart. Cones A and C are 34 meters apart. Estimate the area of the caution zone.

43. MP **MODELING REAL LIFE** A search region is in the shape of an equilateral triangle. The measure of one side of the region is 20 miles. Make a scale drawing of the search region. Estimate the area of the search region.

44. MP **REASONING** A triangle has fixed side lengths of 2 and 14.

a. How many triangles can you construct? Use the figure below to explain your reasoning.



b. Is the unknown side length of the triangle also fixed? Explain.