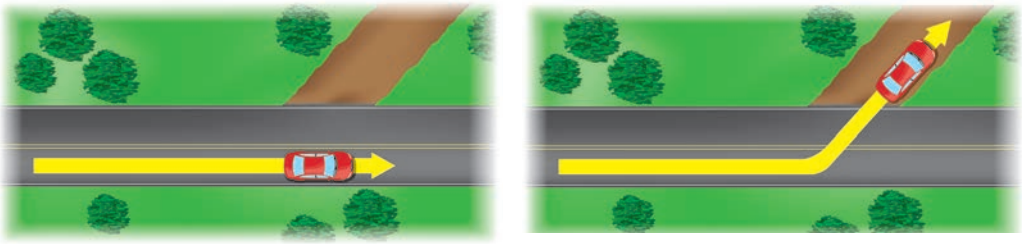
9.5 Mean Absolute Deviation

Learning Target: Find and interpret the mean absolute deviation of a data set.

Success Criteria:
- I can explain how the mean absolute deviation describes the variability of a data set with a single number.
- I can find the mean absolute deviation of a data set.
- I can compare data sets using the mean absolute deviation to draw conclusions.

The Meaning of a Word ▶ Deviate

When you **deviate** from something, you stray or depart from the normal course of action.



EXPLORATION 1 Finding Distances from the Mean

Work with a partner. The table shows the exam scores of 14 students in your class.

Exam Scores					
Ben	89	Omar	95	Dan	94
Emma	86	Hong	96	Lucy	89
Jeremy	80	Rob	92	Priya	84
Pete	80	Amy	90	Heather	85
Malik	96	Sue	76		

Math Practice

Use Operations
What operation can you use to find the distance from the mean? Explain.

a. Which exam score *deviates* the most from the mean? Which exam score *deviates* the least from the mean? Explain how you found your answers.

b. How far is each data value from the mean?

c. Divide the sum of the values in part (b) by the number of values. In your own words, what does this represent?

d. MP **REASONING** In a data set, what does it mean when the value you found in part (c) is close to 0? Explain.

9.5 Lesson

Key Vocabulary
mean absolute deviation, *p. 440*

Another measure of variation is the *mean absolute deviation*. The **mean absolute deviation** is an average of how much data values differ from the mean.

Key Idea

Finding the Mean Absolute Deviation (MAD)

Step 1: Find the mean of the data.

Step 2: Find the distance between each data value and the mean.

Step 3: Find the sum of the distances in Step 2.

Step 4: Divide the sum in Step 3 by the total number of data values.

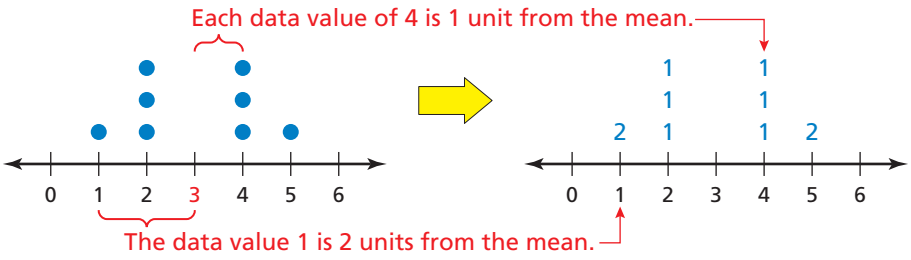
EXAMPLE 1 Finding the Mean Absolute Deviation

Find and interpret the mean absolute deviation of the data.

1, 2, 2, 2, 4, 4, 4, 5

Step 1: Mean $= \frac{1 + 2 + 2 + 2 + 4 + 4 + 4 + 5}{8} = \frac{24}{8} = 3$

Step 2: You can use a dot plot to organize the data. Replace each dot with its distance from the mean.



Step 3: The sum of the distances is $2 + 1 + 1 + 1 + 1 + 1 + 1 + 2 = 10$.

Step 4: The mean absolute deviation is $\frac{10}{8} = 1.25$.

So, the data values differ from the mean by an average of 1.25.

Math Practice

Maintain Oversight
When each data value in Example 1 increases by 5, do you need to repeat Steps 1–4 to find the mean absolute deviation? Explain your reasoning.

Try It

1. Find and interpret the mean absolute deviation of the data.

5, 8, 8, 10, 13, 14, 16, 22

EXAMPLE 2 Finding the Mean Absolute Deviation

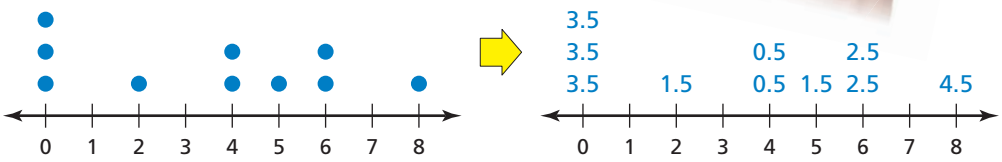
The smartphone shows the numbers of runs allowed by a pitcher in his last 10 starts. Find the mean, median, and mean absolute deviation of the data.

Order the runs allowed:

0, 0, 0, 2, 4, 4, 5, 6, 6, 8.

$\text{Mean} = \frac{35}{10} = 3.5$ $\quad$ $\text{Median} = \frac{4+4}{2} = 4$

Mean absolute deviation:



The mean absolute deviation is $\frac{24}{10} = 2.4$.

▶ The mean is 3.5, the median is 4, and the mean absolute deviation is 2.4.

Try It

2. **WHAT IF?** The pitcher allows 4 runs in the next game. How would you expect the mean absolute deviation to change? Explain.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

3. **WRITING** Explain why the variability of a data set can be described by the mean absolute deviation.

4. **FINDING THE MEAN ABSOLUTE DEVIATION** Find and interpret the mean absolute deviation of the data.

8, 12, 4, 3, 14, 1, 9, 13

5. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

range | interquartile range | mean | mean absolute deviation

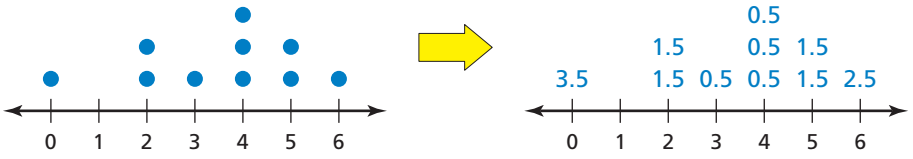
EXAMPLE 3 Modeling Real Life

Find the mean, median, and mean absolute deviation of the numbers of runs allowed by Pitcher B in his last 10 starts. Which measure can you use to distinguish these data from the data in Example 2? What can you conclude?

Order the runs allowed for Pitcher B: 0, 2, 2, 3, 4, 4, 4, 5, 5, 6.

$$\text{Mean} = \frac{35}{10} = 3.5 \qquad \text{Median} = \frac{4+4}{2} = 4$$

Mean absolute deviation:



The mean absolute deviation is $\frac{14}{10} = 1.4$.

You cannot use the measures of center to distinguish the data because they are the same for each data set. The measure of variation, MAD, is 2.4 for Pitcher A and 1.4 for Pitcher B. This indicates that the data for Pitcher B has less variation.

Using the MAD to distinguish the data, you can conclude that Pitcher B is more consistent than Pitcher A.

The greater the mean absolute deviation, the greater the variation of the data.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

Tiger Sharks	
Allison	3
Fito	6
Chuck	5
Sumila	4
Lauren	4
Antonio	2

Bear Cats	
Cherie	6
Carlos	1
Dominic	4
Jack	1
Gloria	8
Hannah	4

6. The tables show the numbers of questions answered correctly by members of two teams on a game show. Compare the mean, median, and mean absolute deviation of the numbers of correct answers for each team. What can you conclude?

7. The data set shows the numbers of books that students in your book club read last summer.

8, 6, 11, 12, 14, 12, 11, 6, 15, 9, 7, 10, 9, 13, 5, 8

A new student who read 18 books last summer joins the club. Is 18 an outlier? How does including this value in the data set affect the measures of center and variation? Explain.

9.5 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the range and interquartile range of the data.

1. 23, 45, 39, 34, 28, 41, 26, 33

2. 63, 53, 48, 61, 69, 63, 57, 72, 46

Graph the integer and its opposite.

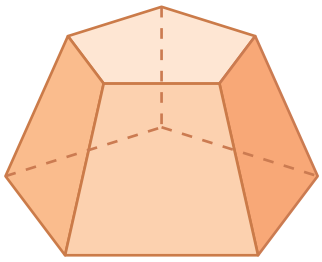
3. -15

4. 17

5. 16

6. -22

7. Find the numbers of faces, edges, and vertices of the solid.



Write the word sentence as an equation.

8. 17 plus a number q is 40.

9. The product of a number s and 14 is 49.

10. The difference of a number b and 9 is 32.

11. The quotient of 36 and a number g is 9.

Concepts, Skills, & Problem Solving

FINDING DISTANCES FROM THE MEAN **Find the average distance of each data value in the set from the mean.** (See Exploration 1, p. 439.)

12. Model years of used cars on a lot: 2014, 2006, 2009, 2011, 2005

13. Prices of kites at a shop: \$7, \$20, \$9, \$35, \$12, \$15, \$7, \$10, \$20, \$25

FINDING THE MEAN ABSOLUTE DEVIATION **Find and interpret the mean absolute deviation of the data.**

14. 69, 51, 71, 77, 71, 80, 75, 63, 73

15. 94, 86, 95, 99, 88, 90

16. 46, 54, 43, 57, 50, 62, 78, 42

17. 25, 28, 20, 22, 32, 28, 35, 34, 30, 36

18. 101, 115, 124, 125, 173, 165, 170

19. 1.1, 7.5, 4.9, 0.4, 2.2, 3.3, 5.1

20. $\frac{1}{4}, \frac{5}{8}, \frac{3}{8}, \frac{3}{4}, \frac{1}{2}$

21. 4.6, 8.5, 7.2, 6.6, 5.1, 6.2, 8.1, 10.3

22. MP **YOU BE THE TEACHER** Your friend finds and interprets the mean absolute deviation of the data set 35, 40, 38, 32, 42, and 41. Is your friend correct? Explain your reasoning.

$$\text{Mean} = \frac{35 + 40 + 38 + 32 + 42 + 41}{6} = 38$$

$$\text{MAD} = \frac{3 + 2 + 0 + 6 + 4 + 3}{6} = 3$$

So, the data values differ from the mean by an average of 3.

23. **MODELING REAL LIFE** The data set shows the admission prices at several glass-blowing workshops.

$20, $20, $16, $12, $15, $25, $11

Find and interpret the range, interquartile range, and mean absolute deviation of the data.

24. **MODELING REAL LIFE** The table shows the prices of the five most-expensive and least-expensive dishes on a menu. Find the MAD of each data set. Then compare their variations.

Five Most-Expensive Dishes					Five Least-Expensive Dishes				
$28	$30	$28	$39	$25	$7	$7	$10	$8	$12

25. **REASONING** The data sets show the years of the coins in two collections.

Your collection: 1950, 1952, 1908, 1902, 1955, 1954, 1901, 1910

Your friend's collection: 1929, 1935, 1928, 1930, 1925, 1932, 1933, 1920

Compare the measures of center and the measures of variation for each data set. What can you conclude?

Movies Watched			
7	5	14	5
6	9	10	12
15	4	5	8
11	10	9	2

26. **MODELING REAL LIFE** You survey students in your class about the numbers of movies they watched last month. A new student joins the class who watched 22 movies last month. Is 22 an outlier? How does including this value affect the measures of center and the measures of variation? Explain.

REASONING **Which data set would have the greater mean absolute deviation? Explain your reasoning.**

27. guesses for number of gumballs in a jar

guesses for number of baseballs in a jar

28. monthly rainfall amounts in a city

monthly amounts of water used in a home

29. **REASONING** Range, interquartile range, and mean absolute deviation are all measures of variation. Which measure of variation is most reliable? Explain your reasoning.

30. **DIG DEEPER!** Add and subtract the MAD from the mean in the original data set in Exercise 26.

a. What percent of the values are within one MAD of the mean? two MADs of the mean? Which values are more than twice the MAD from the mean?

b. What do you notice as you get more and more MADs away from the mean? Explain.