

1.5 Least Common Multiple

Learning Target: Find the least common multiple of two numbers.

- Success Criteria:**
- I can explain the meaning of multiples of a number.
 - I can use lists of multiples to identify the least common multiple of numbers.
 - I can use prime factors to identify the least common multiple of numbers.

EXPLORATION 1

Identifying Common Multiples

Work with a partner. In parts (a)–(d), create a Venn diagram that represents the first several multiples of each number and identify any *common multiples*.

- 8 and 12
 - 4 and 14
 - 10 and 15
 - 20 and 35
- e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *least common multiple* of each pair of numbers. Then circle it in each diagram.

EXPLORATION 2

Using Prime Factors

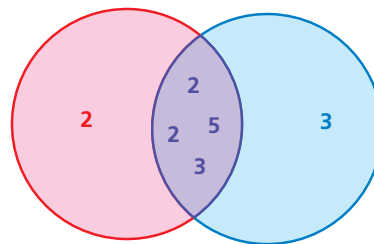
Work with a partner.

- Create a Venn diagram that represents the prime factorizations of 8 and 12.
- Repeat part (a) for the remaining number pairs in Exploration 1.
- MP STRUCTURE** Make a conjecture about the relationship between the least common multiples you found in Exploration 1 and the numbers in the Venn diagrams you just created.
- The Venn diagram shows the prime factors of two numbers.

Math Practice

Analyze Conjectures

How can you test your conjecture in part (c)?



Use the diagram to complete the following tasks.

- Identify the two numbers.
- Find the greatest common factor.
- Find the least common multiple.

1.5 Lesson

Key Vocabulary

common multiples,
p. 28

least common multiple,
p. 28

Multiples that are shared by two or more numbers are called **common multiples**. The least of the common multiples is called the **least common multiple (LCM)**. You can find the LCM of two or more numbers by listing multiples or using prime factors.

EXAMPLE 1 Finding the LCM Using Lists of Multiples

Find the LCM of 4 and 6.

List the multiples of each number.

Multiples of 4: 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, ... Circle the common multiples.

Multiples of 6: 6, **12**, 18, **24**, 30, **36**, ...

Some common multiples of 4 and 6 are 12, 24, and 36. The least of these common multiples is 12.

▶ So, the LCM of 4 and 6 is 12.

Try It Find the LCM of the numbers using lists of multiples.

1. 3, 8

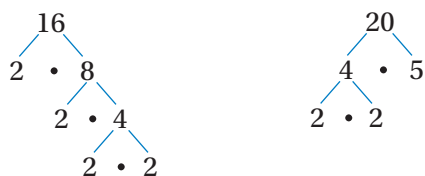
2. 9, 12

3. 6, 10

EXAMPLE 2 Finding the LCM Using Prime Factorizations

Find the LCM of 16 and 20.

Make a factor tree for each number.



Write the prime factorization of each number. Circle each different factor where it appears the greater number of times.

$$16 = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{2}$$

2 appears more often here, so circle all 2s.

$$20 = 2 \cdot 2 \cdot \textcircled{5}$$

5 appears once. Do not circle the 2s again.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 80$$

Find the product of the circled factors.

▶ So, the LCM of 16 and 20 is 80.

Try It Find the LCM of the numbers using prime factorizations.

4. 14, 18

5. 28, 36

6. 24, 90

Math Practice

Maintain Oversight

Explain why the method used in Example 2 works.

EXAMPLE 3**Finding the LCM of Three Numbers**

Find the LCM of 4, 15, and 18.

Write the prime factorization of each number. Circle each different factor where it appears the greatest number of times.

$$4 = 2 \cdot 2$$

2 appears most often here, so circle both 2s.

$$15 = 3 \cdot 5$$

5 appears here only, so circle 5.

$$18 = 2 \cdot 3 \cdot 3$$

3 appears most often here, so circle both 3s.

$$2 \cdot 2 \cdot 5 \cdot 3 \cdot 3 = 180$$

Find the product of the circled factors.

► So, the LCM of 4, 15, and 18 is 180.

Try It

Find the LCM of the numbers.

7. 2, 5, 8

8. 6, 10, 12

9. Write three numbers that have a least common multiple of 100.

**Self-Assessment for Concepts & Skills**

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FINDING THE LCM Find the LCM of the numbers.

10. 6, 9

11. 30, 40

12. 5, 11

13. **MP REASONING** Write two numbers such that 18 and 30 are multiples of the numbers. Justify your answer.

14. **MP REASONING** You need to find the LCM of 13 and 14. Would you rather list their multiples or use their prime factorizations? Explain.



15. **MP CHOOSE TOOLS** A student writes the prime factorizations of 8 and 12 in a table as shown. She claims she can use the table to find the greatest common factor and the least common multiple of 8 and 12. How is this possible?

8 =	2	2	2		
12 =	2	2		3	

16. **CRITICAL THINKING** How can you use least common multiples to add or subtract fractions with different denominators?

EXAMPLE 4

Modeling Real Life

One firefly flashes every 8 seconds. Another firefly flashes every 10 seconds. Both fireflies just flashed. After how many seconds will both fireflies flash at the same time again?

Understand the problem.

Make a plan.

Solve and check.

You are given the numbers of seconds between flashes for two different fireflies. You are asked when the fireflies will flash at the same time again.

The LCM of the numbers of seconds between flashes represents the number of seconds it will take for both fireflies to flash at the same time again. So, find the LCM of 8 and 10 by listing the multiples of each number.

Multiples of 8: 8, 16, 24, 32, **40**, . . .

Multiples of 10: 10, 20, 30, **40**, 50, . . .

The LCM of 8 and 10 is 40.

So, both fireflies will flash at the same time again after 40 seconds.

Another Method Find the LCM using prime factorizations.

$$8 = 2 \cdot 2 \cdot 2 \quad 10 = 2 \cdot 5$$

So, the LCM is $2 \cdot 2 \cdot 2 \cdot 5 = 40$. ✓



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



- A geyser erupts every fourth day. Another geyser erupts every sixth day. Today both geysers erupted. In how many days will both geysers erupt on the same day again?
- A water park has two large buckets that slowly fill with water. One bucket dumps water every 12 minutes. The other bucket dumps water every 10 minutes. Five minutes ago, both buckets dumped water. When will both buckets dump water at the same time again?
- DIG DEEPER!** You purchase disposable plates, cups, and forks for a cookout. Plates are sold in packages of 24 for \$3 per package, cups in packages of 32 for \$2 per package, and forks in packages of 48 for \$4 per package. You want to buy packages so that you have the same number of plates, cups, and forks. What is the least amount of money you need?

1.5 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Find the GCF of the numbers.

1. 18, 42

2. 72, 96

3. 38, 76, 114

Divide.

4. $900 \div 6$

5. $1944 \div 9$

6. $672 \div 12$

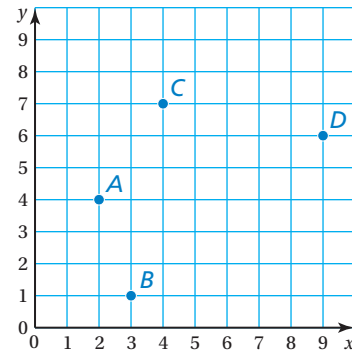
Write an ordered pair that corresponds to the point.

7. Point A

8. Point B

9. Point C

10. Point D



► Concepts, Skills, & Problem Solving

USING A VENN DIAGRAM Use a Venn diagram to find the least common multiple of the numbers. (See Exploration 1, p. 27.)

11. 3, 7

12. 6, 8

13. 4, 5

FINDING THE LCM Find the LCM of the numbers using lists of multiples.

14. 1, 5

15. 2, 6

16. 2, 3

17. 2, 9

18. 3, 4

19. 8, 9

20. 5, 8

21. 11, 12

22. 12, 18

FINDING THE LCM Find the LCM of the numbers using prime factorizations.

23. 7, 12

24. 5, 9

25. 4, 11

26. 9, 10

27. 12, 27

28. 18, 45

29. 22, 33

30. 36, 60

31. 35, 50

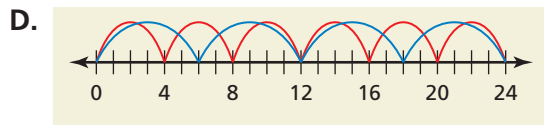
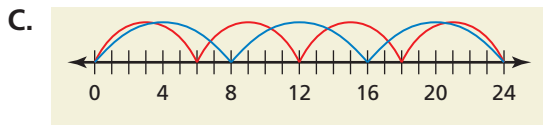
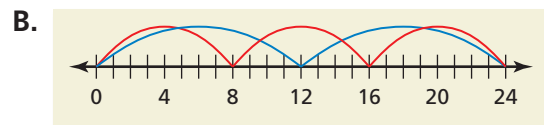
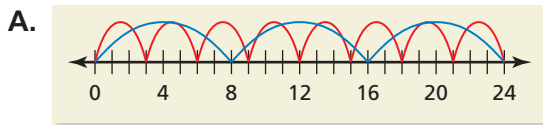
32. **MP YOU BE THE TEACHER** Your friend finds the LCM of 6 and 9. Is your friend correct? Explain your reasoning.

$6 \times 9 = 54$

The LCM of 6 and 9 is 54.

33. **MODELING REAL LIFE** You have diving lessons every fifth day and swimming lessons every third day. Today you have both lessons. In how many days will you have both lessons on the same day again?

34. **MP REASONING** Which model represents an LCM that is different from the other three? Explain your reasoning.



FINDING THE LCM Find the LCM of the numbers.

35. 2, 3, 7

36. 3, 5, 11

37. 4, 9, 12

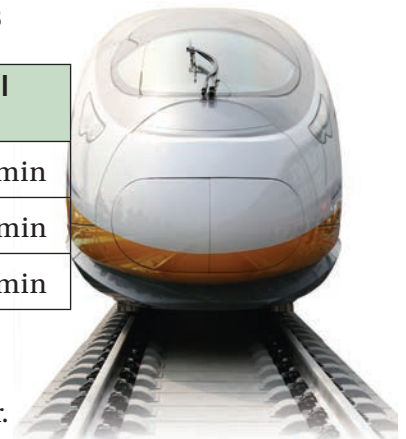
38. 6, 8, 15

39. 7, 18, 21

40. 9, 10, 28

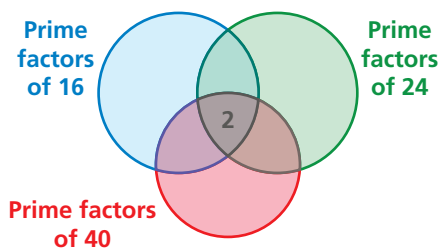
41. **MP PROBLEM SOLVING** At Union Station, you notice that three subway lines just arrived at the same time. How long must you wait until all three lines arrive at Union Station at the same time again?

Subway Line	Arrival Time
A	Every 10 min
B	Every 12 min
C	Every 15 min



42. **DIG DEEPER!** A radio station gives away \$15 to every 15th caller, \$25 to every 25th caller, and a free concert ticket to every 100th caller. When will the station first give away *all* three prizes to one caller? When this happens, how much money and how many tickets are given away?

43. **MP LOGIC** You and a friend are running on treadmills. You run 0.5 mile every 3 minutes, and your friend runs 2 miles every 14 minutes. You both start and stop running at the same time and run a whole number of miles. What are the least possible numbers of miles you and your friend can run?



44. **VENN DIAGRAM** Refer to the Venn diagram.

- Copy and complete the Venn diagram.
- What is the LCM of 16, 24, and 40?
- What is the LCM of 16 and 40? 24 and 40? 16 and 24? Explain how you found your answers.

CRITICAL THINKING Tell whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

- The LCM of two different prime numbers is their product.
- The LCM of a set of numbers is equal to one of the numbers in the set.
- The GCF of two different numbers is the LCM of the numbers.