4.4 Graphing Linear Equations in Slope-Intercept Form

Learning Target: Graph linear equations in slope-intercept form.

Success Criteria:
- I can identify the slope and *y*-intercept of a line given an equation.
- I can rewrite a linear equation in slope-intercept form.
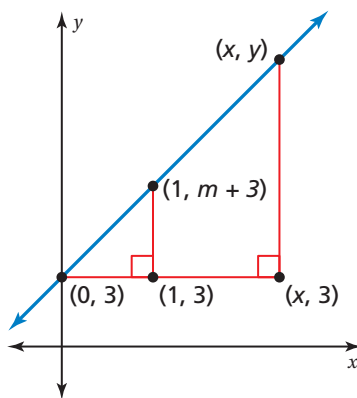- I can use the slope and *y*-intercept to graph linear equations.

EXPLORATION 1

Deriving an Equation

Work with a partner. In the previous section, you learned that the graph of a proportional relationship can be represented by the equation $y = mx$, where m is the constant of proportionality.
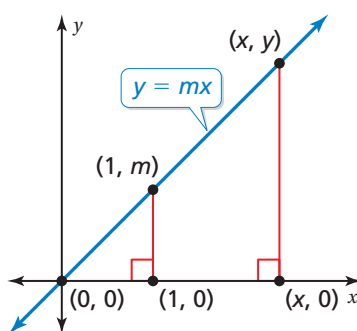
a. You translate the graph of a proportional relationship 3 units up as shown below. Let (x, y) represent any point on the graph. Make a conjecture about the equation of the line. Explain your reasoning.

Math Practice

Understand Quantities

How does the meaning of the equation $y = mx$ help you make a conjecture in part (a)?

b. Describe the relationship between the corresponding side lengths of the triangles. Explain your reasoning.

c. Use the relationship in part (b) to write an equation relating y, m, and x. Does your equation support your conjecture in part (a)? Explain.

d. You translate the graph of a proportional relationship b units up. Write an equation relating y, m, x, and b. Justify your answer.

Key Ideas

Intercepts

The **x-intercept** of a line is the *x*-coordinate of the point where the line crosses the *x*-axis. It occurs when $y = 0$.

The **y-intercept** of a line is the *y*-coordinate of the point where the line crosses the *y*-axis. It occurs when $x = 0$.

y-intercept = b
(0, b)
x-intercept = a
(a, 0)
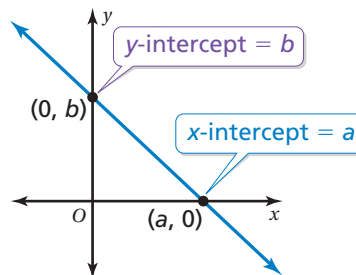
Slope-Intercept Form

Words   A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the *y*-intercept of the line is b.

Algebra $y = mx + b$

slope   y-intercept

> Linear equations can, but do not always, pass through the origin. So, proportional relationships are a special type of linear equation in which $b = 0$.

EXAMPLE 1 Identifying Slopes and *y*-Intercepts

Find the slope and the *y*-intercept of the graph of each linear equation.

a. $y = -4x - 2$

$$y = -4x + (-2)$$ Write in slope-intercept form.

▶ The slope is -4, and the *y*-intercept is -2.

b. $y - 5 = \dfrac{3}{2}x$

$$y = \dfrac{3}{2}x + 5$$ Add 5 to each side.

▶ The slope is $\dfrac{3}{2}$, and the *y*-intercept is 5.

Try It **Find the slope and the *y*-intercept of the graph of the linear equation.**

1. $y = 3x - 7$ **2.** $y - 1 = -\dfrac{2}{3}x$

EXAMPLE 2 **Graphing a Linear Equation in Slope-Intercept Form**

Graph $y = -3x + 3$. Identify the x-intercept.
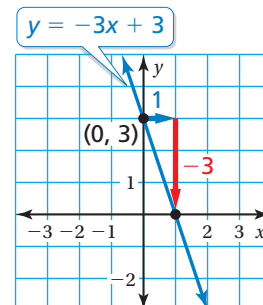
Step 1: Find the slope and the y-intercept.

$$y = -3x + 3$$

slope y-intercept

Step 2: The y-intercept is 3. So, plot $(0, 3)$.

Step 3: Use the slope to find another point and draw the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-3}{1}$$

Plot the point that is 1 unit right and 3 units down from $(0, 3)$. Draw a line through the two points.

▶ The line crosses the x-axis at $(1, 0)$. So, the x-intercept is 1.

Try It **Graph the linear equation. Identify the x-intercept.**

3. $y = x - 4$

4. $y = -\dfrac{1}{2}x + 1$

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

5. **IN YOUR OWN WORDS** Consider the graph of the equation $y = mx + b$.

   a. How does changing the value of m affect the graph of the equation?

   b. How does changing the value of b affect the graph of the equation?

IDENTIFYING SLOPE AND y-INTERCEPT **Find the slope and the y-intercept of the graph of the linear equation.**

6. $y = -x + 0.25$

7. $y - 2 = -\dfrac{3}{4}x$

GRAPHING A LINEAR EQUATION **Graph the linear equation. Identify the x-intercept.**

8. $y = x - 7$

9. $y = 2x + 8$

EXAMPLE 3   **Modeling Real Life**

The cost y (in dollars) of taking a taxi x miles is represented by the equation $y = 2.5x + 2$. Graph the equation. Interpret the y-intercept and the slope.

Understand the problem.

You are given an equation that represents the cost of taking a taxi. You are asked to graph the equation and interpret the y-intercept and the slope.
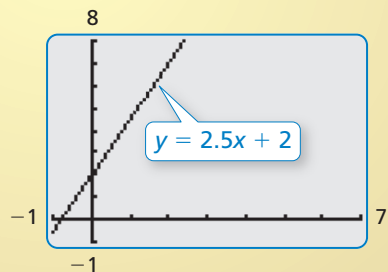
Make a plan.

Use the equation to identify the slope and the y-intercept. Then graph the equation and interpret the y-intercept and the slope.
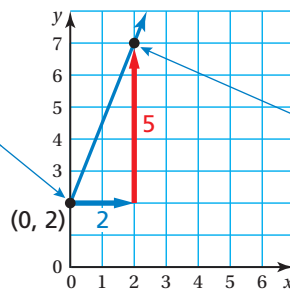
Solve and check.

The equation is already written in the form $y = mx + b$. So, the slope is $2.5 = \dfrac{5}{2}$ and the y-intercept is 2. Use the slope and the y-intercept to graph the equation.

Check

Use a graphing calculator to graph $y = 2.5x + 2$.

$y = 2.5x + 2$

The y-intercept is 2. So, plot (0, 2).

Use the slope to plot another point, (2, 7). Draw a line through the points.

(0, 2)

> The y-intercept is 2. So, there is an initial fee of $2 to take the taxi. The slope is 2.5. So, the cost per mile is $2.50.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. The height y (in feet) of a movable bridge after rising for x seconds is represented by the equation $y = 3x + 16$. Graph the equation. Interpret the y-intercept and slope. How many seconds does it take the bridge to reach a height of 76 feet? Justify your answer.

11. The number y of perfume bottles in storage after x months is represented by the equation $y = -20x + 460$. Graph the equation. Interpret the y-intercept and the slope. In how many months will there be no perfume bottles left in storage? Justify your answer.

▶ Review & Refresh

Tell whether x and y are in a proportional relationship. Explain your reasoning. If so, write an equation that represents the relationship.

1.

x	1	2	3	4
y	6	8	10	12

2.

x	−8	−4	4	8
y	4	2	−2	−4

Solve the equation for y.

3. $x = 4y - 2$

4. $3y = -6x + 1$

5. $1 + y = -\dfrac{4}{5}x - 2$

6. $2.5y = 5x - 5$

7. $1.3y + 5.2 = -3.9x$

8. $y - \dfrac{2}{3}x = -6$

▶▶ Concepts, Skills, & Problem Solving

GRAPHING A LINEAR EQUATION Graph the equation. (See Exploration 1, p. 161.)

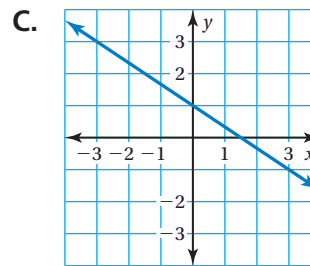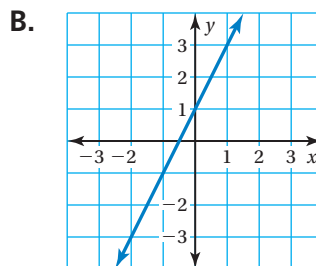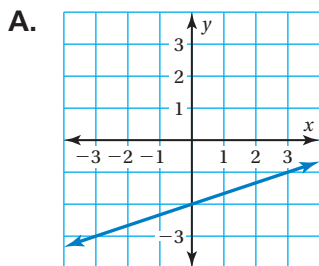9. The graph of $y = 3.5x$ is translated up 2 units.

10. The graph of $y = -5x$ is translated down 3 units.

MATCHING EQUATIONS AND GRAPHS Match the equation with its graph. Identify the slope and the y-intercept.

11. $y = 2x + 1$

12. $y = \dfrac{1}{3}x - 2$

13. $y = -\dfrac{2}{3}x + 1$

A.

B.

C.

IDENTIFYING SLOPES AND y-INTERCEPTS Find the slope and the y-intercept of the graph of the linear equation.

14. $y = 4x - 5$

15. $y = -7x + 12$

16. $y = -\dfrac{4}{5}x - 2$

17. $y = 2.25x + 3$

18. $y + 1 = \dfrac{4}{3}x$

19. $y - 6 = \dfrac{3}{8}x$

20. $y - 3.5 = -2x$

21. $y = -5 - \dfrac{1}{2}x$

22. $y = 11 + 1.5x$

23. **YOU BE THE TEACHER** Your friend finds the slope and y-intercept of the graph of the equation $y = 4x - 3$. Is your friend correct? Explain your reasoning.

> $y = 4x - 3$; The slope is 4 and the y-intercept is 3.

24. MODELING REAL LIFE The number y of seasonal allergy shots available at a facility x days after receiving a shipment is represented by $y = -15x + 375$.

    a. Graph the linear equation.

    b. Interpret the slope and the y-intercept.

GRAPHING AN EQUATION Graph the linear equation. Identify the x-intercept.

25. $y = x + 3$ **26.** $y = 4x - 8$ **27.** $y = -3x + 9$

28. $y = -5x - 5$ **29.** $y + 14 = -7x$ **30.** $y = 8 - 2x$

31. MP PRECISION You go to a harvest festival and pick apples.

    a. Which equation represents the cost (in dollars) of going to the festival and picking x pounds of apples? Explain.

         $y = 5x + 0.75$ $y = 0.75x + 5$

    b. Graph the equation you chose in part (a).

You Pick 'Em
Apples $0.75 per lb
Admission: $5

32. MP REASONING Without graphing, identify the equations of the lines that are parallel. Explain your reasoning.

$y = 2x + 4$ $y = \dfrac{1}{2}x + 1$ $y = 2x - 3$ $y = 2x + 1$ $y = \dfrac{1}{2}x + 2$

33. MP PROBLEM SOLVING A skydiver parachutes to the ground. The height y (in feet) of the skydiver after x seconds is $y = -10x + 3000$.

    a. Graph the linear equation.

    b. Interpret the slope, y-intercept, and x-intercept.

34. DIG DEEPER! Six friends create a website. The website earns money by selling banner ads. It costs $120 a month to operate the website.

    a. A banner ad earns $0.005 per click. Write a linear equation that represents the monthly profit after paying operating costs.

    b. Graph the equation in part (a). On the graph, label the number of clicks needed for the friends to start making a profit. Explain.