

5.3 Solving Systems of Linear Equations by Elimination

Learning Target: Understand how to solve systems of linear equations by elimination.

- Success Criteria:**
- I can add or subtract equations in a system.
 - I can use the Multiplication Property of Equality to produce equivalent equations.
 - I can solve a system of linear equations by elimination.

EXPLORATION 1

Solving a System Algebraically

Work with a partner. A student found the value of x in the system using substitution as shown.

$$3x + y = 1 \quad \text{Equation 1}$$

$$x - y = -5 \quad \text{Equation 2}$$

Step 1: $3x + y = 1$ Equation 1

$$x + 5 = y \quad \text{Revised Equation 2}$$

Step 2: $3x + x + 5 = 1$ Substitute $x + 5$ for y in Equation 1.

$$4x + 5 = 1 \quad \text{Combine like terms.}$$

$$4x = -4 \quad \text{Subtract 5 from each side.}$$

$$x = -1 \quad \text{Divide each side by 4.}$$

- Find another way to obtain the equation $4x = -4$ from the original system. Does your method produce an equation in one variable for any system? Explain.
- Can you use your method in part (a) to solve each system below? If so, solve the system. If not, replace one of the equations with an equivalent equation that allows you to use your method in part (a). Then solve the system.

Math Practice

Finding an Entry Point

What do you look for when deciding how to solve a system of equations?

System 1: $2x + 3y = -4$ Equation 1

$$2x - 3y = 8 \quad \text{Equation 2}$$

System 2: $x + 4y = -5$ Equation 1

$$3x - 2y = 13 \quad \text{Equation 2}$$

- Compare your solution methods in part (b) with other pairs of students.

5.3 Lesson

When the equations in a linear system have a pair of like terms with the same or opposite coefficients, you can add or subtract the equations to *eliminate* one of the variables. Then use the resulting equation to solve the system.

EXAMPLE 1 Solving a System of Linear Equations by Elimination

Solve the system by elimination.

$$x + 3y = -2 \quad \text{Equation 1}$$

$$x - 3y = 16 \quad \text{Equation 2}$$

Because the coefficients of x are the same, you can also subtract the equations in Step 1.

$$\begin{array}{r} x + 3y = -2 \\ x - 3y = 16 \\ \hline 6y = -18 \\ \text{So, } y = -3. \end{array}$$

Step 1: Notice that the coefficients of the y -terms are opposites. So, you can add the equations to obtain an equation in one variable, x .

$$x + 3y = -2 \quad \text{Equation 1}$$

$$x - 3y = 16 \quad \text{Equation 2}$$

$$\begin{array}{r} x + 3y = -2 \\ x - 3y = 16 \\ \hline 2x = 14 \end{array} \quad \text{Add the equations.}$$

Step 2: Solve for x .

$$2x = 14 \quad \text{Equation from Step 1}$$

$$x = 7 \quad \text{Divide each side by 2.}$$

Step 3: Substitute 7 for x in one of the original equations and solve for y .

$$x + 3y = -2 \quad \text{Equation 1}$$

$$7 + 3y = -2 \quad \text{Substitute 7 for } x.$$

$$3y = -9 \quad \text{Subtract 7 from each side.}$$

$$y = -3 \quad \text{Divide each side by 3.}$$

▶ The solution is $(7, -3)$.

Check

Equation 1

$$x + 3y = -2$$

$$7 + 3(-3) \stackrel{?}{=} -2$$

$$-2 = -2 \quad \checkmark$$

Equation 2

$$x - 3y = 16$$

$$7 - 3(-3) \stackrel{?}{=} 16$$

$$16 = 16 \quad \checkmark$$

Try It Solve the system by elimination. Check your solution.

1. $2x - y = 9$

$$4x + y = 21$$

2. $-5x + 2y = 13$

$$5x + y = -1$$

3. $3x + 4y = -6$

$$7x + 4y = -14$$

To solve a system by elimination, you may need to multiply one or both equations by a constant so a pair of like terms has the same or opposite coefficients.

EXAMPLE 2 Solving a System of Linear Equations by Elimination

Solve the system by elimination.

$$\begin{array}{r} -6x + 5y = 25 \quad \text{Equation 1} \\ -2x - 4y = 14 \quad \text{Equation 2} \end{array}$$

Step 1: Notice that no pairs of like terms have the same or opposite coefficients. One way to solve by elimination is to multiply Equation 2 by 3 so that the x -terms have a coefficient of -6 .

$$\begin{array}{r} -6x + 5y = 25 \quad \text{Equation 1} \\ -2x - 4y = 14 \quad \text{Multiply by 3.} \rightarrow -6x - 12y = 42 \quad \text{Revised Equation 2} \end{array}$$

Step 2: Subtract the equations to obtain an equation in one variable, y .

$$\begin{array}{r} -6x + 5y = 25 \quad \text{Equation 1} \\ -6x - 12y = 42 \quad \text{Revised Equation 2} \\ \hline 17y = -17 \quad \text{Subtract the equations.} \end{array}$$

Step 3: Solve for y .

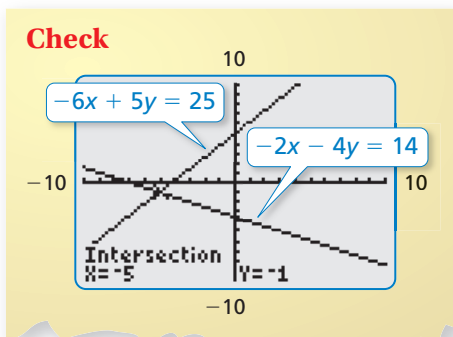
$$\begin{array}{r} 17y = -17 \quad \text{Equation from Step 2} \\ y = -1 \quad \text{Divide each side by 17.} \end{array}$$

Step 4: Substitute -1 for y in one of the original equations and solve for x .

$$\begin{array}{r} -2x - 4y = 14 \quad \text{Equation 2} \\ -2x - 4(-1) = 14 \quad \text{Substitute } -1 \text{ for } y. \\ -2x + 4 = 14 \quad \text{Multiply.} \\ -2x = 10 \quad \text{Subtract 4 from each side.} \\ x = -5 \quad \text{Divide each side by } -2. \end{array}$$

▶ The solution is $(-5, -1)$.

Notice that you can also multiply Equation 2 by -3 and then add the equations.



Try It Solve the system by elimination. Check your solution.

4. $3x + y = 11$
 $6x + 3y = 24$

5. $4x - 5y = -19$
 $-x - 2y = 8$

6. $5y = 15 - 5x$
 $y = -2x + 3$

EXAMPLE 3**Choosing a Solution Method**

Which are efficient approaches to solving the system?

$$x - 2y = 6 \quad \text{Equation 1}$$

$$-x + 4y = 6 \quad \text{Equation 2}$$

- A. Add the equations.
- B. Multiply Equation 1 by 2 and subtract the equations.
- C. Solve Equation 1 for x and substitute the result in Equation 2.
- D. Substitute $-x + 4y$ for 6 in Equation 1.

The methods in Choices A and C result in an equation in one variable, y . You can solve these equations and use the results to find the value of x .

The methods in Choices B and D will not result in an equation in one variable.

► So, Choices A and C are efficient approaches to solving the system.

Try It

- 7. Change one word in Choice B so that it represents an efficient approach to solving the system.

**Self-Assessment for Concepts & Skills**

Solve each exercise. Then rate your understanding of the success criteria in your journal.

SOLVING A SYSTEM OF LINEAR EQUATIONS Solve the system by elimination. Check your solution.

8. $2x + y = 4$
 $-2x + 2y = 5$

9. $-x + y = 1$
 $-3x + y = 7$

10. $y = -2x + 3$
 $4x - 5y = 13$

CHOOSING A SOLUTION METHOD Solve the system. Explain your choice of method.

11. $y = 6x - 1$
 $y = 3x - 4$

12. $3x = y + 2$
 $3x + 2y = 5$

13. $2x - y = 7$
 $x + y = 5$

14. **WHICH ONE DOESN'T BELONG?** Which system does *not* belong with the other three? Explain your reasoning.

$$\begin{array}{l} 3x + 3y = 3 \\ 2x - 3y = 7 \end{array}$$

$$\begin{array}{l} -2x + y = 6 \\ 2x - 3y = -10 \end{array}$$

$$\begin{array}{l} 2x + 3y = 11 \\ 3x - 2y = 10 \end{array}$$

$$\begin{array}{l} x + y = 5 \\ 3x - y = 3 \end{array}$$

EXAMPLE 4**Modeling Real Life**

You buy 8 hostas and 15 daylilies for \$193. Your friend buys 3 hostas and 12 daylilies for \$117. Find the cost of each daylily.



Use a verbal model to write a system of linear equations. Let x represent the cost of each hosta and let y represent the cost of each daylily.

Number of hostas	•	Cost of each hosta, x	+	Number of daylilies	•	Cost of each daylily, y	=	Total cost
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The system is: $8x + 15y = 193$ Equation 1 (You)

$3x + 12y = 117$ Equation 2 (Your friend)

Step 1: One way to find the cost of each daylily is to eliminate the x -terms and solve for y . Multiply Equation 1 by 3 and Equation 2 by 8.



$8x + 15y = 193$ **Multiply by 3.** $24x + 45y = 579$ Revised Equation 1

$3x + 12y = 117$ **Multiply by 8.** $24x + 96y = 936$ Revised Equation 2

Step 2: Subtract the revised equations.

$24x + 45y = 579$ Revised Equation 1

$24x + 96y = 936$ Revised Equation 2

$-51y = -357$ Subtract the equations.

Step 3: Solving the equation $-51y = -357$ gives $y = 7$.

▶ So, each daylily costs \$7.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

15. A fitness instructor purchases exercise bikes and treadmills for two gyms. For the first gym, 2 exercise bikes and 3 treadmills cost \$2200. For the second gym, 3 exercise bikes and 4 treadmills cost \$3000. How much does a treadmill cost?



16. **DIG DEEPER!** At your school, cooking club members raise \$5 per member for a charity and woodshop club members raise \$10 per member for a different charity. The cooking club has three times as many members as the woodshop club. The difference of the number of members in the two clubs is 12 members. How much does each club raise?

5.3 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Solve the system by substitution. Check your solution.

1. $x = 5 - y$
 $x - y = 3$

2. $x - 5y = 1$
 $-x + y = 7$

3. $x + 6y = -2$
 $-x = 3y - 10$

The vertices of a triangle are given. Draw the triangle and its image after a dilation with the given scale factor. Identify the type of dilation.

4. $A(-1, 1), B(1, 3), C(3, 1); k = 2$

5. $D(-8, -4), E(-4, 8), F(0, 0); k = 0.5$

► Concepts, Skills, & Problem Solving

SOLVING A SYSTEM ALGEBRAICALLY Explain how to obtain the equation

$3x = 6$ from the given system. (See Exploration 1, p. 211.)

6. $2x + y = 5$
 $x - y = 1$

7. $5x + 2y = 2$
 $x + y = -2$

8. $-x + y = -3$
 $6x - 3y = 15$

SOLVING A SYSTEM OF LINEAR EQUATIONS Solve the system by elimination.

Check your solution.

9. $x + 3y = 5$
 $-x - y = -3$

10. $x - 2y = -7$
 $3x + 2y = 3$

11. $4x + 3y = -5$
 $-x + 3y = -10$

12. $2x + 7y = 1$
 $2x - 4y = 12$

13. $2x + 5y = 16$
 $3x - 5y = -1$

14. $3x - 2y = 4$
 $6x - 2y = -2$

15. **YOU BE THE TEACHER** Your friend solves the system. Is your friend correct? Explain your reasoning.

$$5x + 2y = 9 \quad \text{Equation 1}$$

$$3x - 2y = -1 \quad \text{Equation 2}$$

$$2x = 10$$

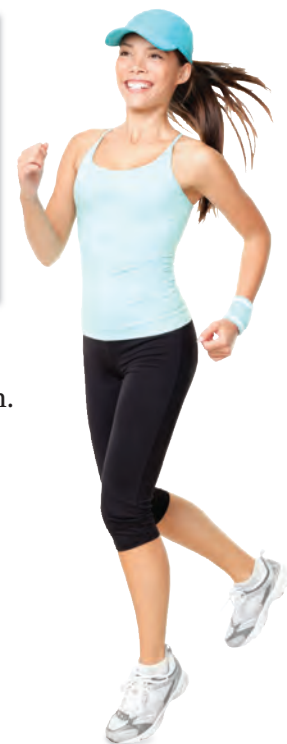
$$x = 5$$

The solution is $(5, -8)$.

16. **MODELING REAL LIFE** You and your friend are selling raffle tickets for a new laptop. You sell 14 more tickets than your friend sells. Together, you and your friend sell 58 tickets.

- Write a system of linear equations that represents this situation.
- How many tickets do each of you sell?

17. **MODELING REAL LIFE** You can jog around your block twice and the park once in 10 minutes. You can jog around your block twice and the park 3 times in 22 minutes. Write a system of linear equations that represents this situation. How long does it take you to jog around the park?



SOLVING A SYSTEM OF LINEAR EQUATIONS Solve the system by elimination.

Check your solution.

$$\begin{aligned} 18. \quad & 2x - y = 0 \\ & 3x - 2y = -3 \end{aligned}$$

$$\begin{aligned} 19. \quad & x + 4y = 1 \\ & 3x + 5y = 10 \end{aligned}$$

$$\begin{aligned} 20. \quad & -2x + 3y = 7 \\ & 5x + 8y = -2 \end{aligned}$$

$$\begin{aligned} 21. \quad & 3x + 3 = 3y \\ & 2x - 6y = 2 \end{aligned}$$

$$\begin{aligned} 22. \quad & 2x - 6 = 4y \\ & 7y = -3x + 9 \end{aligned}$$

$$\begin{aligned} 23. \quad & 5x = 4y + 8 \\ & 3y = 3x - 3 \end{aligned}$$

24. **YOU BE THE TEACHER** Your friend solves the system. Is your friend correct? Explain your reasoning.

$x + y = 1$ Equation 1
 $5x + 3y = -3$ Equation 2

Multiply by -5 .

$-5x + y = 1$
 $5x + 3y = -3$
 $4y = -2$
 $y = -0.5$

The solution is $(-0.3, -0.5)$.

CHOOSING A SOLUTION METHOD Solve the system. Explain your choice of method.

$$\begin{aligned} 25. \quad & x + y = 4 \\ & x - y = 4 \end{aligned}$$

$$\begin{aligned} 26. \quad & y = x - 3 \\ & y = -2x + 3 \end{aligned}$$

$$\begin{aligned} 27. \quad & x + 2y = 0 \\ & 2x - y = 4 \end{aligned}$$

$$\begin{aligned} 28. \quad & y + 5x = 1 \\ & 5y - x = 5 \end{aligned}$$

$$\begin{aligned} 29. \quad & 2 = x - 3y \\ & -2x + y = 4 \end{aligned}$$

$$\begin{aligned} 30. \quad & 8x + 5y = 6 \\ & 8x = 3 - 2y \end{aligned}$$

MP NUMBER SENSE For what value of a might you choose to solve the system by elimination? Explain.

$$\begin{aligned} 31. \quad & 4x - y = 3 \\ & ax + 10y = 6 \end{aligned}$$

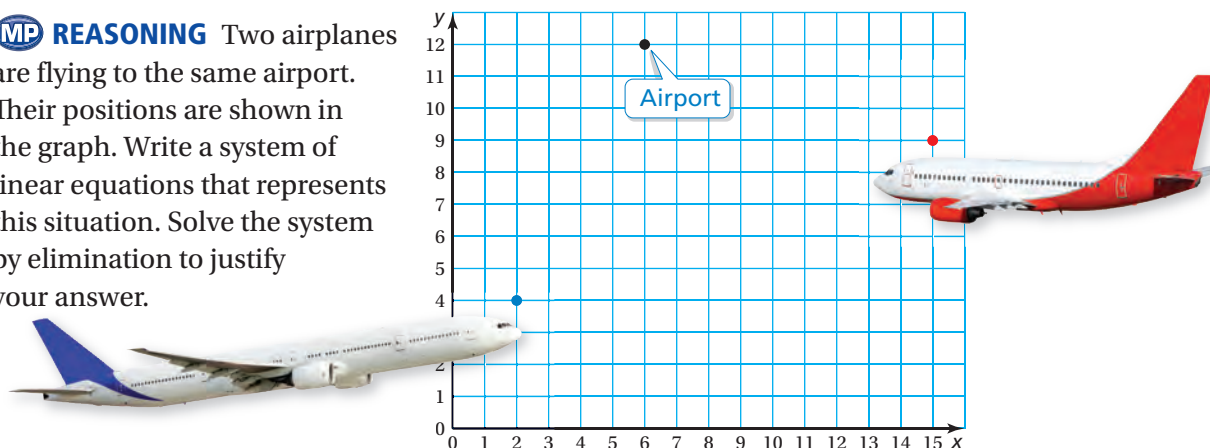
$$\begin{aligned} 32. \quad & x - 7y = 6 \\ & -6x + ay = 9 \end{aligned}$$

CRITICAL THINKING Determine whether the line through the first pair of points intersects the line through the second pair of points. Explain.

33. Line 1: $(-2, 1), (2, 7)$
Line 2: $(-4, -1), (0, 5)$

34. Line 1: $(-2, 8), (0, 2)$
Line 2: $(3, -2), (6, 4)$

35. **MP REASONING** Two airplanes are flying to the same airport. Their positions are shown in the graph. Write a system of linear equations that represents this situation. Solve the system by elimination to justify your answer.



36. **MODELING REAL LIFE** A laboratory uses liquid nitrogen tanks of two different sizes. The combined volume of 3 large tanks and 2 small tanks is 24 liters. The combined volume of 2 large tanks and 3 small tanks is 21 liters. What is the volume of each size of tank? Justify your answer.
37. **MP PROBLEM SOLVING** The table shows the numbers of correct answers on a practice standardized test. You score 86 points on the test and your friend scores 76 points. How many points is each type of question worth?

	You	Your Friend
Multiple Choice	23	28
Short Response	10	5



38. **MP LOGIC** You solve a system of equations in which x represents the number of adult memberships sold and y represents the number of student memberships sold. Can $(-6, 24)$ be the solution of the system? Explain your reasoning.
39. **MP PROBLEM SOLVING** The table shows the activities of two tourists at a vacation resort. You want to go parasailing for 1 hour and horseback riding for 2 hours. How much do you expect to pay?

	Parasailing	Horseback Riding	Total Cost
Tourist 1	2 hours	5 hours	\$205
Tourist 2	3 hours	3 hours	\$240

40. **MP REASONING** Write a system of linear equations containing $2x + y = 0$ and that has the solution $(2, -4)$.

41. **MP REASONING** A metal alloy is a mixture of two or more metals. A jeweler wants to make 8 grams of 18-karat gold, which is 75% gold. The jeweler has an alloy that is 90% gold and an alloy that is 50% gold. How much of each alloy should the jeweler use?



42. **MP PROBLEM SOLVING** It takes a powerboat traveling with the current 30 minutes to go 10 miles. The return trip takes 50 minutes traveling against the current. What is the speed of the current?
43. **DIG DEEPER!** Solve the system of equations by elimination.

$$2x - y + 3z = -1$$

$$x + 2y - 4z = -1$$

$$y - 2z = 0$$