4.3 Graphing Proportional Relationships

Learning Target: Graph proportional relationships.

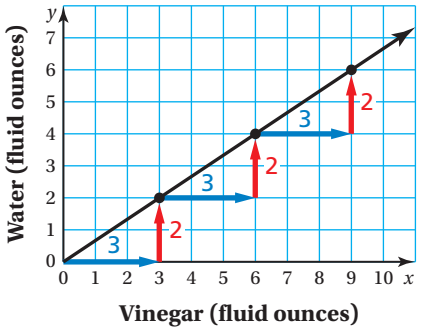
Success Criteria:
- I can graph an equation that represents a proportional relationship.
- I can write an equation that represents a proportional relationship.
- I can use graphs to compare proportional relationships.

EXPLORATION 1 Using a Ratio Table to Find Slope

Work with a partner. The graph shows amounts of vinegar and water that can be used to make a cleaning product.

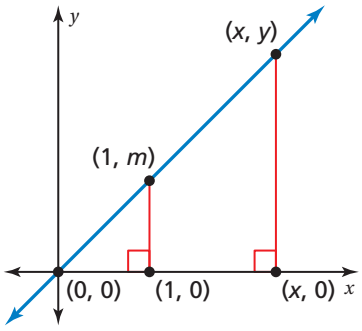
a. Use the graph to make a ratio table relating the quantities. Explain how the slope of the line is represented in the table.

b. Make a ratio table that represents a different ratio of vinegar to water. Use the table to describe the slope of the graph of the new relationship.



EXPLORATION 2 Deriving an Equation

Work with a partner. Let (x, y) represent any point on the graph of a proportional relationship.



Math Practice

Use a Graph

How can you find the side lengths of the triangles in the graph?

a. Describe the relationship between the corresponding side lengths of the triangles shown in the graph. Explain your reasoning.

b. Use the relationship in part (a) to write an equation relating y, m, and x. Then solve the equation for y.

c. What does your equation in part (b) describe? What does m represent? Explain your reasoning.

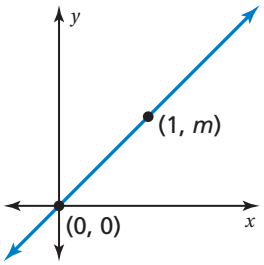
4.3 Lesson

Key Idea

Proportional Relationships

Words When two quantities x and y are proportional, the relationship can be represented by the equation $y = mx$, where m is the constant of proportionality.

Graph The graph of $y = mx$ is a line with a slope of m that passes through the origin.



In the equation $y = mx$, m represents the constant of proportionality, the slope, and the unit rate.

EXAMPLE 1 Graphing a Proportional Relationship

The cost y (in dollars) for x ounces of frozen yogurt is represented by $y = 0.5x$. Graph the equation and interpret the slope.

Method 1: Make a table of values.

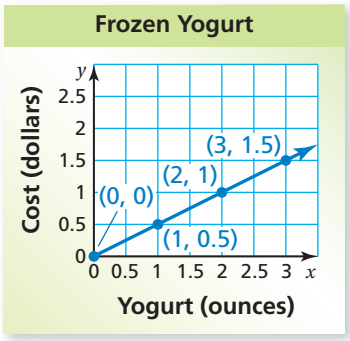
x	$y = 0.5x$	y	(x, y)
0	$y = 0.5(0)$	0	(0, 0)
1	$y = 0.5(1)$	0.5	(1, 0.5)
2	$y = 0.5(2)$	1	(2, 1)
3	$y = 0.5(3)$	1.5	(3, 1.5)

Method 2: Use the slope.

The equation shows that the slope m is 0.5. So, the graph passes through (0, 0) and (1, 0.5).

Plot the ordered pairs and draw a line through the points. Because negative values of x do not make sense in this context, graph in the first quadrant only.

The slope indicates that the unit cost is \$0.50 per ounce.



Try It

1. **WHAT IF?** The cost of frozen yogurt is represented by $y = 0.75x$. Graph the equation and interpret the slope.

EXAMPLE 2 Writing and Using an Equation

The weight y of an object on Titan, one of Saturn's moons, is proportional to the weight x of the object on Earth. An object that weighs 105 pounds on Earth would weigh 15 pounds on Titan.

The slope indicates that the weight of an object on Titan is one-seventh its weight on Earth.

a. Write an equation that represents the situation.

Use the point (105, 15) to find the slope of the line.

$y = mx$ Equation of a proportional relationship

$15 = m(105)$ Substitute 15 for y and 105 for x.

$\frac{1}{7} = m$ Simplify.

So, an equation that represents the situation is $y = \frac{1}{7}x$.

b. How much would a chunk of ice that weighs 3.5 pounds on Titan weigh on Earth?

$3.5 = \frac{1}{7}x$ Substitute 3.5 for y.

$24.5 = x$ Multiply each side by 7.

So, the chunk of ice would weigh 24.5 pounds on Earth.

Try It

2. How much would a spacecraft that weighs 3500 kilograms on Earth weigh on Titan?

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

GRAPHING A PROPORTIONAL RELATIONSHIP **Graph the equation.**

3. $y = 4x$ **4.** $y = -3x$ **5.** $y = 8x$

6. WRITING AND USING AN EQUATION The number y of objects a machine produces is proportional to the time x (in minutes) that the machine runs. The machine produces five objects in four minutes.

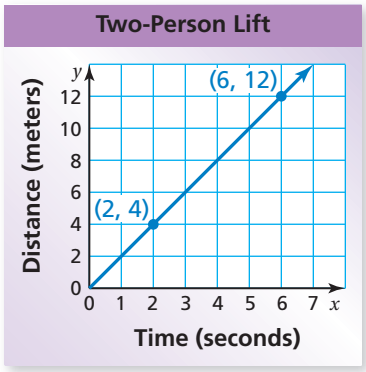
a. Write an equation that represents the situation.

b. Graph the equation in part (a) and interpret the slope.

c. How many objects does the machine produce in one hour?

EXAMPLE 3 Modeling Real Life

The distance y (in meters) that a four-person ski lift travels in x seconds is represented by the equation $y = 2.5x$. The graph shows the distance that a two-person ski lift travels.



a. Which ski lift is faster?

Identify the slope of the graph for each lift. Then interpret each slope as a unit rate.

Four-Person Lift

$y = 2.5x$

The slope is 2.5.

The four-person lift travels 2.5 meters per second.

Two-Person Lift

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

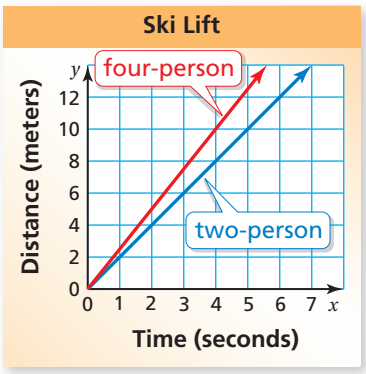
$$= \frac{8}{4} = 2$$

The two-person lift travels 2 meters per second.

So, the four-person lift is faster than the two-person lift.

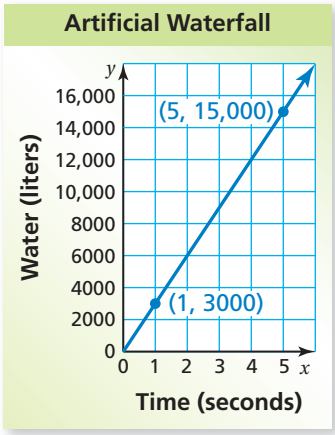
b. Graph the equation that represents the four-person lift in the same coordinate plane as the two-person lift. Compare and interpret the steepness of each graph.

The graph that represents the four-person lift is steeper than the graph that represents the two-person lift. So, the four-person lift is faster.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



7. The amount y (in liters) of water that flows over a natural waterfall in x seconds is represented by the equation $y = 500x$. The graph shows the number of liters of water that flow over an artificial waterfall. Which waterfall has a greater flow? Justify your answer.

8. The speed of sound in air is 343 meters per second. You see lightning and hear thunder 12 seconds later.

a. Is there a proportional relationship between the amount of time that passes and your distance from a lightning strike? Explain.

b. Estimate your distance from the lightning strike.

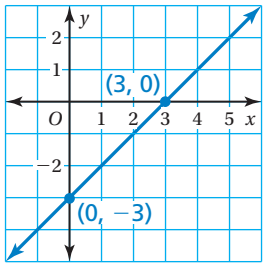
4.3 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the slope of the line.

1.



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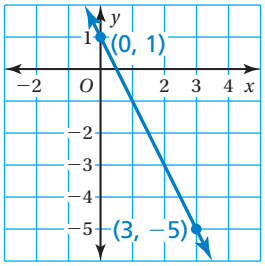

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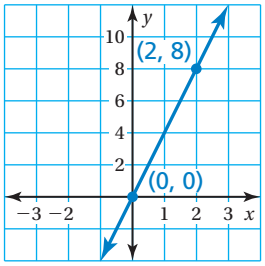


Solve the equation. Check your solution.

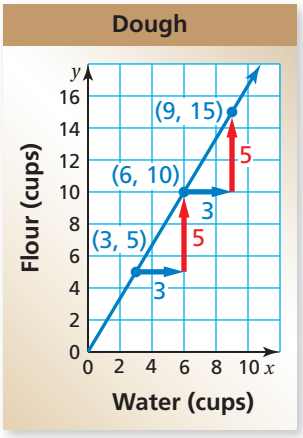
4. $2x + 3x = 10$

5. $x + \frac{1}{6} = 4 - 2x$

6. $2(1 - x) = 11$

Concepts, Skills, & Problem Solving

USING EQUIVALENT RATIOS **The graph shows amounts of water and flour that can be used to make dough.** (See Exploration 1, p. 155.)



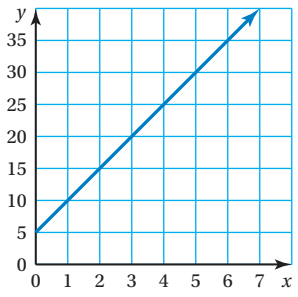
7. Use the graph to make a ratio table relating the quantities. Explain how the slope of the line is represented in the table.

8. Make a ratio table that represents a different ratio of flour to water. Use the table to describe the slope of the graph of the new relationship.

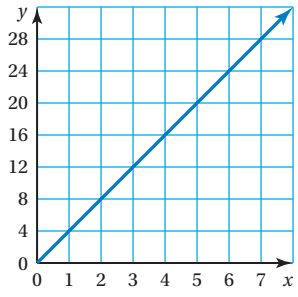
9. **GRAPHING AN EQUATION** The amount y (in dollars) that you raise by selling x fundraiser tickets is represented by the equation $y = 5x$. Graph the equation and interpret the slope.

IDENTIFYING PROPORTIONAL RELATIONSHIPS **Tell whether x and y are in a proportional relationship. Explain your reasoning. If so, write an equation that represents the relationship.**

10.



11.



12.

x	3	6	9	12
y	1	2	3	4

13.

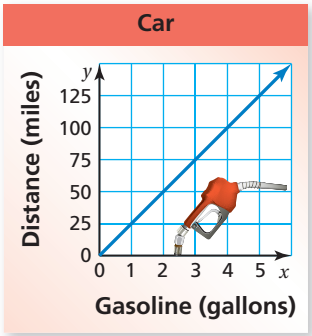
x	2	5	8	10
y	4	8	13	23

14. MODELING REAL LIFE The cost y (in dollars) to rent a kayak is proportional to the number x of hours that you rent the kayak. It costs \$27 to rent the kayak for 3 hours.

a. Write an equation that represents the situation.

b. Interpret the slope of the graph of the equation.

c. How much does it cost to rent the kayak for 5 hours? Justify your answer.



15. MODELING REAL LIFE The distance y (in miles) that a truck travels on x gallons of gasoline is represented by the equation $y = 18x$. The graph shows the distance that a car travels.

a. Which vehicle gets better gas mileage? Explain how you found your answer.

b. How much farther can the vehicle you chose in part (a) travel on 8 gallons of gasoline?

16. MP PROBLEM SOLVING Toenails grow about 13 millimeters per year. The table shows fingernail growth.

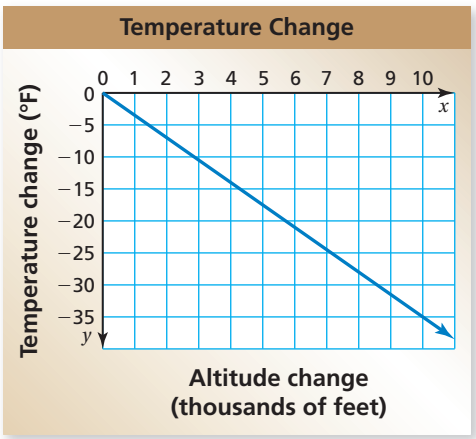
Weeks	1	2	3	4
Fingernail Growth (millimeters)	0.7	1.4	2.1	2.8

a. Do fingernails or toenails grow faster? Explain.

b. In the same coordinate plane, graph equations that represent the growth rates of toenails and fingernails. Compare and interpret the steepness of each graph.

17. MP REASONING The quantities x and y are in a proportional relationship. What do you know about the ratio of y to x for any point (x, y) on the graph of x and y?

18. DIG DEEPER! The graph relates the temperature change y (in degrees Fahrenheit) to the altitude change x (in thousands of feet).



a. Is the relationship proportional? Explain.

b. Write an equation of the line. Interpret the slope.

c. You are at the bottom of a mountain where the temperature is 74°F. The top of the mountain is 5500 feet above you. What is the temperature at the top of the mountain? Justify your answer.

19. CRITICAL THINKING Consider the distance equation $d = rt$, where d is the distance (in feet), r is the rate (in feet per second), and t is the time (in seconds). You run for 50 seconds. Are the distance you run and the rate you run at proportional? Use a graph to justify your answer.