

3.4 Factoring Expressions

Learning Target: Factor algebraic expressions.

- Success Criteria:**
- I can identify the greatest common factor of terms, including variable terms.
 - I can use the Distributive Property to factor algebraic expressions.
 - I can write a term as a product involving a given factor.

EXPLORATION 1

Finding Dimensions

Work with a partner.

- a. The models show the areas (in square units) of parts of rectangles. Use the models to find the missing values that complete the expressions. Explain your reasoning.

	?	?	
?	$\frac{4}{5}$	$\frac{8}{5}$	$\frac{4}{5} + \frac{8}{5} = ?(? + ?)$

	?	?	
?	$\frac{3}{2}x$	$\frac{1}{2}$	$\frac{3}{2}x + \frac{1}{2} = ?(? + ?)$

	?	?	
?	$2.5x$	$3.75y$	$2.5x + 3.75y = ?(? + ?)$

Math Practice

View as Components

How does viewing each rectangle as two distinct parts help you complete the expressions?

- b. Are the expressions you wrote in part (a) equivalent to the original expressions? Explain your reasoning.
- c. Explain how you can use the Distributive Property to find rational number factors of an expression.

3.4 Lesson

Key Vocabulary

factoring an expression, p. 110

When **factoring an expression**, you write the expression as a product of factors. You can use the Distributive Property to factor any rational number from an expression.

EXAMPLE 1 Factoring Out the GCF

Factor $24x - 18$ using the GCF.

Find the GCF of $24x$ and 18 .

$$\begin{aligned}24x &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \\18 &= 2 \cdot 3 \cdot 3\end{aligned}$$

Circle the common prime factors.

So, the GCF of $24x$ and 18 is $2 \cdot 3 = 6$. Use the GCF to factor the expression.

$$\begin{aligned}24x - 18 &= 6(4x) - 6(3) \\&= 6(4x - 3)\end{aligned}$$

Rewrite using GCF.

Distributive Property

Try It Factor the expression using the GCF.

1. $15x + 25$

2. $4y - 20$

3. $36c + 24d$

EXAMPLE 2 Factoring Out a Rational Number

Factor $\frac{1}{2}$ out of $\frac{1}{2}x + \frac{3}{2}$.

Write each term as a product of $\frac{1}{2}$ and another factor.

$$\frac{1}{2}x = \frac{1}{2} \cdot x$$

Think: $\frac{1}{2}x$ is $\frac{1}{2}$ times what?

$$\frac{3}{2} = \frac{1}{2} \cdot 3$$

Think: $\frac{3}{2}$ is $\frac{1}{2}$ times what?

Use the Distributive Property to factor out $\frac{1}{2}$.

$$\begin{aligned}\frac{1}{2}x + \frac{3}{2} &= \frac{1}{2} \cdot x + \frac{1}{2} \cdot 3 \\&= \frac{1}{2}(x + 3)\end{aligned}$$

Rewrite the expression.

Distributive Property

Try It Factor out the coefficient of the variable term.

4. $\frac{1}{2}n - \frac{1}{2}$

5. $\frac{3}{4}p - \frac{3}{2}$

6. $5 + 2.5q$

EXAMPLE 3 Factoring Out a Negative Number

Factor -2 out of $-4p + 10$.

Write each term as a product of -2 and another factor.

$$-4p = -2 \cdot 2p \quad \text{Think: } -4p \text{ is } -2 \text{ times what?}$$

$$10 = -2 \cdot (-5) \quad \text{Think: } 10 \text{ is } -2 \text{ times what?}$$

Use the Distributive Property to factor out -2 .

$$-4p + 10 = -2 \cdot 2p + (-2) \cdot (-5) \quad \text{Rewrite the expression.}$$

$$= -2[2p + (-5)] \quad \text{Distributive Property}$$

$$= -2(2p - 5) \quad \text{Simplify.}$$

▶ So, $-4p + 10 = -2(2p - 5)$.

Try It

- Factor -5 out of $-5d + 30$.
- Factor -4 out of $-8k - 12$.



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FACTORIZING OUT THE GCF Factor the expression using the GCF.

9. $16n - 24$

10. $42a + 14b$

FACTORIZING OUT A RATIONAL NUMBER Factor out the coefficient of the variable term.

11. $\frac{1}{10}k - \frac{7}{10}$

12. $42 + 3.5h$

FACTORIZING OUT A NEGATIVE NUMBER Factor out the indicated number.

13. Factor -8 out of $-32d + 56$.

14. Factor -12 out of $-24k + 120$.

15. **WRITING** Describe the relationship between using the Distributive Property to simplify an expression and to factor an expression. Give an example to justify your answer.

EXAMPLE 4

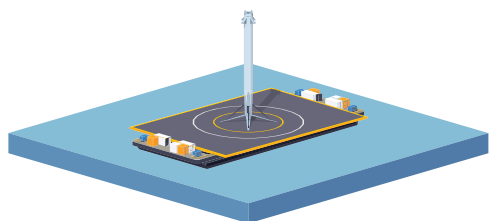
Modeling Real Life

A rectangular landing platform for a rocket is 60 yards wide and has an area of $(60x + 3600)$ square yards. Write an expression that represents the perimeter (in yards) of the platform.

Factor the width of 60 yards out of the given area expression to find an expression that represents the length (in yards) of the platform.

$$\begin{aligned} 60x + 3600 &= 60 \cdot x + 60 \cdot 60 && \text{Rewrite the expression.} \\ &= 60(x + 60) && \text{Distributive Property} \end{aligned}$$

So, the length (in yards) of the platform can be represented by $x + 60$. Use the perimeter formula to write an expression that represents the perimeter of the platform.



$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter of a rectangle} \\ &= 2(x + 60) + 2(60) && \text{Substitute for } \ell \text{ and } w. \\ &= 2x + 120 + 120 && \text{Multiply.} \\ &= 2x + 240 && \text{Add.} \end{aligned}$$

So, an expression that represents the perimeter (in yards) of the platform is $2x + 240$.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



16. An organization drills 3 wells to provide access to clean drinking water. The cost (in dollars) to drill and maintain the wells for n years is represented by $34,500 + 540n$. Write and interpret an expression that represents the cost to drill and maintain one well for n years.

17. A photograph is 16 inches long and has an area of $(16x + 96)$ square inches. A custom-made frame is 2 inches wide and costs \$0.50 per square inch. Write an expression that represents the cost of the frame.



3.4 Practice



Go to BigIdeasMath.com to get HELP with solving the exercises.

► Review & Refresh

Simplify the expression.

1. $8(k - 5)$

2. $-4.5(-6 + 2d)$

3. $-\frac{1}{4}(3g - 6 - 5g)$

Find the difference. Write fractions in simplest form.

4. $\frac{2}{3} - \left(-\frac{5}{3}\right)$

5. $-4.7 - 5.6$

6. $-4\frac{3}{8} - \left(-2\frac{1}{4}\right)$

Evaluate the expression when $x = 4$, $y = -6$, and $z = -3$.

7. $y \div z$

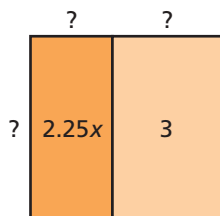
8. $\frac{4y}{2x}$

9. $\frac{3x - 2y}{z}$

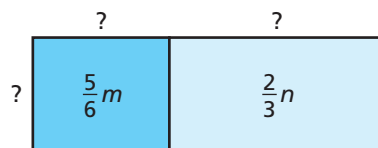
► Concepts, Skills, & Problem Solving

FINDING DIMENSIONS The model shows the area (in square units) of each part of a rectangle. Use the model to find the missing values that complete the expression. Explain your reasoning. (See Exploration 1, p. 109.)

10. $2.25x + 3 = \square (\square + \square)$



11. $\frac{5}{6}m + \frac{2}{3}n = \square (\square + \square)$



FACTORIZING OUT THE GCF Factor the expression using the GCF.

12. $9b + 21$

13. $32z - 48$

14. $8x + 2$

15. $3y - 24$

16. $14p - 28$

17. $6 + 16k$

18. $21 - 14d$

19. $20z - 8$

20. $15w + 65$

21. $36a + 16b$

22. $21m - 49n$

23. $12 + 9g - 30h$

FACTORIZING OUT A RATIONAL NUMBER Factor out the coefficient of the variable term.

24. $\frac{1}{7}a + \frac{1}{7}$

25. $\frac{1}{3}b - \frac{1}{3}$

26. $\frac{3}{8}d + \frac{3}{4}$

27. $2.2x + 4.4$

28. $1.5y - 6$

29. $0.8w + 3.6$

30. $\frac{15}{4} + \frac{3}{8}x$

31. $4h - 3$

32. $0.15c - 0.072$

33. $\frac{3}{8}z + 1$

34. $6s - \frac{3}{4}$

35. $\frac{5}{2}k - 2$

YOU BE THE TEACHER Your friend factors the expression. Is your friend correct? Explain your reasoning.

36.

$$16p - 28 = 4(4p - 28)$$

37.

$$\begin{aligned} \frac{2}{3}y - \frac{14}{3} &= \frac{2}{3} \cdot y - \frac{2}{3} \cdot 7 \\ &= \frac{2}{3}(y - 7) \end{aligned}$$

FACTORING OUT A NEGATIVE NUMBER Factor out the indicated number.

38. Factor -4 out of $-8d + 20$.

39. Factor -6 out of $18z - 15$.

40. Factor -0.25 out of $7g + 3.5$.

41. Factor $-\frac{1}{2}$ out of $-\frac{1}{2}x + 6$.

42. Factor -1.75 out of $-14m - 5.25n$.

43. Factor $-\frac{1}{4}$ out of $-\frac{1}{2}x - \frac{5}{4}y$.

44. **MP STRUCTURE** A rectangle has an area of $(4x + 12)$ square units. Write three multiplication expressions that can represent the product of the length and the width of the rectangle.

45. **MODELING REAL LIFE** A square wrestling mat has a perimeter of $(12x - 32)$ feet. Explain how to use the expression to find the length (in feet) of the mat. Justify your answer.



46. **MODELING REAL LIFE** A table is 6 feet long and 3 feet wide. You extend the length of the table by inserting two identical table *leaves*. The extended table is rectangular with an area of $(18 + 6x)$ square feet. Write and interpret an expression that represents the length (in feet) of the extended table.

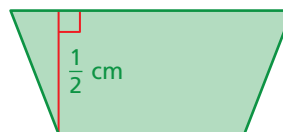


47. **DIG DEEPER!** A three-dimensional printing pen uses heated plastic to create three-dimensional objects. A kit comes with one 3D-printing pen and p packages of plastic. An art club purchases 6 identical kits for $(180 + 58.5p)$ dollars. Write and interpret an expression that represents the cost of one kit.

48. **MP STRUCTURE** The area of the trapezoid is

$$\left(\frac{3}{4}x - \frac{1}{4}\right) \text{ square centimeters. Write two different}$$

pairs of expressions that represent the possible base lengths (in centimeters). Justify your answers.



3

Connecting Concepts

► Using the Problem-Solving Plan

- The runway shown has an area of $(0.05x + 0.125)$ square miles. Write an expression that represents the perimeter (in feet) of the runway.



Understand the problem.

You know the area of the rectangular runway in square miles and the width of the runway in miles. You want to know the perimeter of the runway in feet.

Make a plan.

Factor the width of 0.05 mile out of the expression that represents the area to find an expression that represents the length of the runway. Then write an expression that represents the perimeter (in miles) of the runway. Finally, use a measurement conversion to write the expression in terms of feet.

Solve and check.

Use the plan to solve the problem. Then check your solution.

- The populations of two towns after t years can be modeled by $-300t + 7000$ and $-200t + 5500$. What is the combined population of the two towns after t years? The combined population of the towns in Year 10 is what percent of the combined population in Year 0?

FREEDOM	
POP	7000
ELEV	5900

Performance Task



Chlorophyll in Plants

At the beginning of this chapter, you watched a STEAM Video called "Tropic Status." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



3

Chapter Review



Go to BigIdeasMath.com to download blank graphic organizers.

▶ Review Vocabulary

Write the definition and give an example of each vocabulary term.

like terms, p. 92

linear expression, p. 98

factoring an expression,

simplest form, p. 92

p. 110

▶ Graphic Organizers

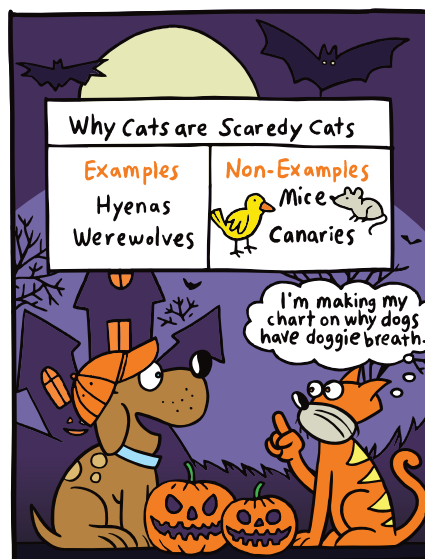
You can use an **Example and Non-Example Chart** to list examples and non-examples of a concept. Here is an Example and Non-Example Chart for *like terms*.

Like Terms

Examples	Non-Examples
2 and -3	y and 4
$3x$ and $-7x$	$3x$ and $3y$
x^2 and $6x^2$	$4x$ and $-2x^2$
y and $5y$	$2y$ and 5

Choose and complete a graphic organizer to help you study the concept.

1. simplest form
2. equivalent expressions
3. linear expression
4. Distributive Property
5. factoring an expression



"Here is my **Example and Non-Example Chart** about things that scare cats."

▶ Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.



3.1 Algebraic Expressions (pp. 91–96)

Learning Target: Simplify algebraic expressions.

Identify the terms and like terms in the expression.

1. $z + 8 - 4z$

2. $3n + 7 - n - 3$

3. $10x^2 - y + 12 - 3x^2$

Simplify the expression.

4. $4h - 8h$

5. $6.4r - 7 - 2.9r$

6. $2m - m - 7m$

7. $6y + 9 + 3y - 7$

8. $\frac{3}{5}x + 19 - \frac{3}{20}x - 7$

9. $\frac{2}{3}y + 14 - \frac{1}{6}y - 8$

10. Write an expression with 4 different terms that is equivalent to $5x^2 - 8$. Justify your answer.

11. Find the earnings for selling the same number of each type of sandwich. Justify your answer.

	Turkey	Ham
Pretzel Roll	2.25	1.55
Bagel	2.00	1.30

12. You buy the same number of brushes, rollers, and paint cans.

a. Write and interpret an expression in simplest form that represents the total amount of money you spend on painting supplies.

b. How much do you spend when you buy one set of supplies for each of 3 painters?





3.2 Adding and Subtracting Linear Expressions (pp. 97–102)

Learning Target: Find sums and differences of linear expressions.

Find the sum.

13. $(c - 4) + (3c + 9)$

14. $(5z + 4) + (3z - 6)$

15. $(-2.1m - 5) + (3m - 7)$

16. $\left(\frac{5}{4}q + 1\right) + (q - 4) + \left(-\frac{1}{4}q + 2\right)$

Find the difference.

17. $(x - 1) - (3x + 2)$

18. $(4y + 3) - (2y - 9)$

19. $\left(\frac{1}{2}h + 7\right) - \left(\frac{3}{2}h + 9\right)$

20. $(4 - 3.7b) - (-5.4b - 4) - (1.2b + 1)$

21. A basket holds n apples. You pick $(2n - 3)$ apples, and your friend picks $(n + 4)$ apples. How many apples do you and your friend pick together? How many baskets do you need to carry all the apples? Justify your answer.
22. Greenland has a population of x people. Barbados has a population of about 4500 more than 5 times the population of Greenland. Find and interpret the difference in the populations of these two countries.



3.3 The Distributive Property (pp. 103–108)

Learning Target: Apply the Distributive Property to generate equivalent expressions.

Simplify the expression.

23. $2(a - 3)$

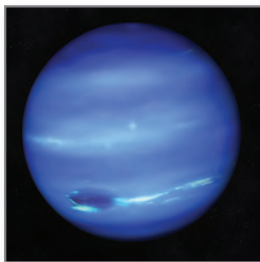
24. $-3(4x - 10)$

25. $-2.5(8 - b)$

26. $-7(1 - 3d - 5)$

27. $9(-3w - 6.2 + 2w)$

28. $\frac{3}{4}\left(8g - \frac{1}{4} - \frac{2}{3}g\right)$



29. Mars has m moons. The number of moons of Pluto is one more than twice the number of moons of Mars. The number of moons of Neptune is one less than 3 times the number of moons of Pluto. Write and interpret a simplified expression that represents the number of moons of Neptune.

Simplify the expression.

30. $3(2 + q) + 15$

31. $\frac{1}{8}(16m - 8) - 17$

32. $-1.5(4 - n) + 2.8$

33. $\frac{2}{5}(d - 10) - \frac{2}{3}(d + 6)$

34. The expression for degrees Fahrenheit is $\frac{9}{5}C + 32$, where C represents degrees Celsius. The temperature today is 5 degrees Celsius more than yesterday. Write and simplify an expression for the difference in degrees Fahrenheit for these two days.



3.4 Factoring Expressions (pp. 109–114)

Learning Target: Factor algebraic expressions.

Factor the expression using GCF.

35. $18a - 12$

36. $2b + 8$

37. $9 - 15x$

Factor out the coefficient of the variable term.

38. $\frac{1}{4}y + \frac{3}{8}$

39. $1.7j - 3.4$

40. $-5p + 20$

41. Factor $-\frac{3}{4}$ out of $\frac{3}{2}x - \frac{9}{4}y$.

42. You and 4 friends are buying tickets for a concert. The cost to buy one ticket is c dollars. If you buy all the tickets together, there is a discount and the cost is $(5c - 12.5)$ dollars. How much do you save per ticket when you buy the tickets together?

43. The rectangular pupil of an octopus is estimated to be 20 millimeters long with an area of $(20x - 200)$ square millimeters. Write an expression that represents the perimeter (in millimeters) of the octopus pupil.



44. A building block has a square base that has a perimeter of $(12x - 9)$ inches. Explain how to use the expression to find the length (in inches) of the wall shown.

3

Practice Test

1. Identify the terms and like terms in $4x + 9x^2 - 2x + 2$.

Simplify the expression.

2. $8x - 5 + 2x$

3. $2.5w - 3y + 4w$

4. $\frac{5}{7}x + 15 - \frac{9}{14}x - 9$

5. $(3j + 11) + (8j - 7)$

6. $(2r - 13) - (-6r + 4)$

7. $-2(4 - 3n)$

8. $3(5 - 2n) + 9n$

9. $\frac{1}{3}(6x + 9) - 2$

10. $\frac{3}{4}(8p + 12) + \frac{3}{8}(16p - 8)$

11. $-2.5(2s - 5) - 3(4.5s - 5.2)$

Factor out the coefficient of the variable term.

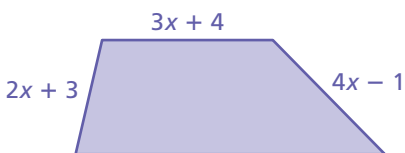
12. $6n - 24$

13. $\frac{1}{2}q + \frac{5}{2}$

14. $-4x + 36$

15. Find the earnings for giving a haircut and a shampoo to m men and w women. Justify your answer.

	Women	Men
Haircut	\$45	\$15
Shampoo	\$12	\$7



16. The expression $15x + 11$ represents the perimeter of the trapezoid. What is the length of the fourth side? Explain your reasoning.

17. The maximum number of charms that will fit on a bracelet is $3\left(d - \frac{2}{3}\right)$, where d is the diameter (in centimeters) of the bracelet.

- Write and interpret a simplified expression that represents the maximum number of charms on a bracelet.
- What is the maximum number of charms that fit on a bracelet that has a diameter of 6 centimeters?

18. You expand a rectangular garden so the perimeter is now twice the perimeter of the old garden. The expression $12w + 16$ represents the perimeter of the new garden, where w represents the width of the old garden.

- Write an expression that represents the perimeter of the old garden. Justify your answer.
- Write an expression that represents the area of the old garden.

3

Cumulative Practice

1. What is the simplified form of the expression?

$$3.7x - 5 - 2.3x$$

- A. $-3.6x$
- B. $6x - 5$
- C. $1.4x - 5$
- D. $3.7x - 7.3$

2. What is the value of the expression when $c = 0$ and $d = -6$?



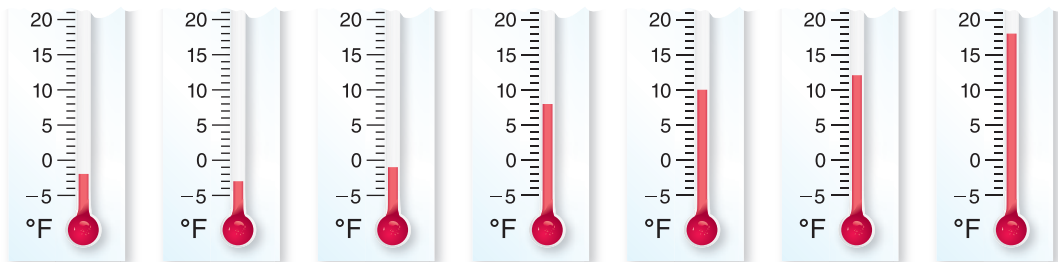
$$\frac{cd - d^2}{4}$$

3. What is the value of the expression?

$$-38 - (-14)$$

- F. -52
- G. -24
- H. 24
- I. 52

4. The daily low temperatures for a week are shown.



What is the mean low temperature of the week?

- A. -2°F
- B. 6°F
- C. 8°F
- D. 10°F

Test-Taking Strategy
Answer Easy Questions First

x mummies are added to $(x + 2)$ mummies.
 How many mummies are there?
 (A) $2x + 2$ (B) 1 million
 (C) 11 (D) 0

Tests about mummies get me all wound up.

"Scan the test and answer the easy questions first. Because x must be in the expression, A is correct."

5. You and a friend collect seashells on a beach. After h minutes, you have collected $(11 + 2h)$ seashells and your friend has collected $(5h - 2)$ seashells. How many total seashells have you and your friend collected?

F. $7h + 9$

G. $3h - 13$

H. $16h$

I. $7h + 13$

6. What is the value of the expression?



$$-0.28 \div (-0.07)$$

7. Which list is ordered from least to greatest?

A. $-\left|\frac{3}{4}\right|, -\frac{1}{2}, \left|\frac{3}{8}\right|, -\frac{1}{4}, -\frac{7}{8}$

B. $-\frac{1}{2}, -\frac{1}{4}, \left|\frac{3}{8}\right|, -\left|\frac{3}{4}\right|, -\frac{7}{8}$

C. $\left|-\frac{7}{8}\right|, \left|\frac{3}{8}\right|, -\frac{1}{4}, -\frac{1}{2}, -\left|\frac{3}{4}\right|$

D. $-\left|\frac{3}{4}\right|, -\frac{1}{2}, -\frac{1}{4}, \left|\frac{3}{8}\right|, -\frac{7}{8}$

8. Which number is equivalent to the expression shown?

$$-2\frac{1}{4} - \left(-8\frac{3}{8}\right)$$

F. $-10\frac{5}{8}$

G. $-10\frac{1}{3}$

H. $6\frac{1}{8}$

I. $6\frac{1}{2}$

9. What is the simplified form of the expression?

$$7x - 2(3x + 6)$$

A. $15x + 30$

B. $x - 12$

C. $13x + 12$

D. $-11x$

10. Which expression is *not* equivalent to the expression?

$$72m - 60$$

F. $6(12m - 10)$

G. $4(18m - 15)$

H. $12m$

I. $12(6m - 5)$

11. You want to buy a bicycle with your friend. You have \$43.50 saved and plan to save an additional \$7.25 every week. Your friend has \$24.50 saved and plans to save an additional \$8.75 every week.

Think

Solve

Explain

Part A Simplify and interpret an expression that represents the amount of money you and your friend save after w weeks.

Part B After 10 weeks, you and your friend use all of the money and buy the bike. How much does the bike cost? Who pays more towards the cost of the bike? Explain your reasoning.

12. Your friend evaluated $3 + x^2 \div y$ when $x = -2$ and $y = 4$.

$$\begin{aligned} 3 + x^2 \div y &= 3 + (-2^2) \div 4 \\ &= 3 - 4 \div 4 \\ &= 3 - 1 \\ &= 2 \end{aligned}$$

What should your friend do to correct his error?

- A. Divide 3 by 4 before subtracting.
- B. Square -2 , then divide.
- C. Divide -2 by 4, then square.
- D. Subtract 4 from 3 before dividing.