5.3 Properties of Addition and Multiplication

Learning Target: Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria:
- I can explain the meaning of equivalent expressions.
- I can use properties of addition to generate equivalent expressions.
- I can use properties of multiplication to generate equivalent expressions.

EXPLORATION 1

Identifying Equivalent Expressions

Work with a partner.

a. Choose four values for a variable x. Then evaluate each expression for each value of x. Are any of the expressions *equivalent*? Explain your reasoning.

x				
$4 + x + 4$				

x				
$16x$				

x				
$4 \cdot (x \cdot 4)$				

x				
$x + 4 + 4$				

x				
$x + 8$				

x				
$(4 \cdot x) \cdot 4$				

b. You have used the following properties in a previous course. Use the examples to explain the meaning of each property.

Commutative Property of Addition: $3 + 5 = 5 + 3$

Commutative Property of Multiplication: $9 \cdot 3 = 3 \cdot 9$

Associative Property of Addition: $8 + (3 + 1) = (8 + 3) + 1$

Associative Property of Multiplication: $12 \cdot (6 \cdot 2) = (12 \cdot 6) \cdot 2$

Are these properties true for algebraic expressions? Explain your reasoning.

Math Practice

Use Counterexamples

Use a counterexample to show that the Commutative Property is not true for division.

Key Vocabulary 🔊
equivalent expressions,
p. 216

Expressions that result in the same number for any value of each variable are **equivalent expressions**. You can use the Commutative and Associative Properties to write equivalent expressions.

🔑 Key Ideas

Commutative Properties

Words Changing the order of addends or factors does not change the sum or product.

Numbers $5 + 8 = 8 + 5$ 　　　　　**Algebra** $a + b = b + a$

$5 \cdot 8 = 8 \cdot 5$ 　　　　　　　　　　$a \cdot b = b \cdot a$

Associative Properties

Words Changing the grouping of addends or factors does not change the sum or product.

Numbers $(7 + 4) + 2 = 7 + (4 + 2)$

$(7 \cdot 4) \cdot 2 = 7 \cdot (4 \cdot 2)$

Algebra $(a + b) + c = a + (b + c)$

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

EXAMPLE 1 **Using Properties to Write Equivalent Expressions**

One way to check whether expressions are equivalent is to evaluate each expression for any value of the variable. In Example 1(a), use $x = 2$.

$7 + (12 + x) \overset{?}{=} 19 + x$

$7 + (12 + 2) \overset{?}{=} 19 + 2$

$21 = 21$ ✓

a. **Simplify the expression $7 + (12 + x)$.**

$7 + (12 + x) = (7 + 12) + x$ 　　　Associative Property of Addition

$= 19 + x$ 　　　　　　　Add 7 and 12.

b. **Simplify the expression $(6.1 + x) + 8.4$.**

$(6.1 + x) + 8.4 = (x + 6.1) + 8.4$ 　　Commutative Property of Addition

$= x + (6.1 + 8.4)$ 　　Associative Property of Addition

$= x + 14.5$ 　　　　Add 6.1 and 8.4.

c. **Simplify the expression $5(11y)$.**

$5(11y) = (5 \cdot 11)y$ 　　　　　　Associative Property of Multiplication

$= 55y$ 　　　　　　　Multiply 5 and 11.

Try It **Simplify the expression. Explain each step.**

1. $10 + (a + 9)$ 　　　　**2.** $\left(c + \dfrac{2}{3} \right) + \dfrac{1}{2}$ 　　　　**3.** $5(4n)$

Key Ideas

Addition Property of Zero

Words The sum of any number and 0 is that number.

Numbers $7 + 0 = 7$ **Algebra** $a + 0 = a$

Multiplication Properties of Zero and One

Words The product of any number and 0 is 0.

The product of any number and 1 is that number.

Numbers $9 \cdot 0 = 0$ **Algebra** $a \cdot 0 = 0$

$4 \cdot 1 = 4$ $a \cdot 1 = a$

EXAMPLE 2 **Using Properties to Write Equivalent Expressions**

a. **Simplify the expression $9 \cdot 0 \cdot p$.**

$9 \cdot 0 \cdot p = (9 \cdot 0) \cdot p$ — Associative Property of Multiplication

$= 0 \cdot p$ — Multiplication Property of Zero

$= 0$ — Multiplication Property of Zero

b. **Simplify the expression $4.5 \cdot r \cdot 1$.**

$4.5 \cdot r \cdot 1 = 4.5 \cdot (r \cdot 1)$ — Associative Property of Multiplication

$= 4.5 \cdot r$ — Multiplication Property of One

$= 4.5r$ — Rewrite.

Try It Simplify the expression. Explain each step.

4. $12 \cdot b \cdot 0$ **5.** $1 \cdot m \cdot 24$ **6.** $(t + 15) + 0$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

USING PROPERTIES Simplify the expression. Explain each step.

7. $(7 + c) + 4$ **8.** $4(b \cdot 6)$ **9.** $0 \cdot b \cdot 9$

10. **WRITING** Explain what it means for expressions to be equivalent. Then give an example of equivalent expressions.

11. **OPEN-ENDED** Write an algebraic expression that can be simplified using the Associative Property of Multiplication and the Multiplication Property of One.

EXAMPLE 3 **Modeling Real Life**

You and six friends play on a basketball team. A sponsor paid $100 for the league fee, x dollars for each player's T-shirt, and $68.25 for basketballs. Write an expression that represents the total amount (in dollars) the sponsor paid. Then find the total amount paid when each T-shirt costs $14.50.

Use a verbal model to write an expression that represents the sum of the league fee, the cost of the T-shirts, and the cost of the basketballs. Then evaluate the expression when $x = 14.5$.

> **Common Error**
>
> You **and** six friends are on the team, so use 7, not 6, to represent the number of T-shirts.

League fee (dollars)		Number of T-shirts		Cost per T-shirt (dollars)		Cost of basketballs (dollars)
$100	+	7	·	x	+	$68.25

$$100 + 7x + 68.25 = 7x + 100 + 68.25 \quad \text{Commutative Property of Addition}$$
$$= 7x + (100 + 68.25) \quad \text{Associative Property of Addition}$$
$$= 7x + 168.25 \quad \text{Add 100 and 68.25.}$$

Evaluate $7x + 168.25$ when $x = 14.5$.

$$7x + 168.25 = 7(14.5) + 168.25 = 101.5 + 168.25 = 269.75$$

▶ An expression that represents the total amount (in dollars) is $7x + 168.25$. When each T-shirt costs $14.50, the sponsor pays $269.75.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. You and five friends form a team for an outdoor adventure race. Your team needs to raise money to pay for $130 of travel fees, x dollars for each team member's entry fee, and $85.50 for food. Use an algebraic expression to find the total amount your team needs to raise when the entry fee is $25.50 per person.

13. You have $50 and a $15 gift card to spend online. You purchase a pair of headphones for $34.99 and 8 songs for x dollars each. Use an algebraic expression to find the amount you have left when each song costs $1.10.

5.3 Practice

? Go to **BigIdeasMath.com** to get HELP with solving the exercises.

▶ Review & Refresh

Write the phrase as an expression.

1. 10 added to a number p

2. the product of 6 and a number m

3. the quotient of a number b and 15

4. 7 fewer than a number s

Write the prime factorization of the number.

5. 36

6. 144

7. 147

8. 205

Evaluate the expression.

9. $8.092 + 3.5$

10. $16.78 - 12.237$

11. $9.17 + 1.83 + 2.641$

12. $8.43 - 6.218 + 4.2$

Represent the ratio relationship using a graph.

13.

Oil (teaspoons)	8	16	24
Flour (cups)	1	2	3

14.

Atoms	4	8	12
Protons	64	128	192

▶ Concepts, Skills, & Problem Solving

MATCHING Match the expression with an equivalent expression.
(See Exploration 1, p. 215.)

15. $3 + 3 + y$

16. $(y \cdot y) \cdot 3$

17. $3 \cdot 1 \cdot y$

18. $(3 + 0) + (y + y)$

A. $y \cdot 3$

B. $y + 3 + 3$

C. $y(3 \cdot y)$

D. $(3 + y) + y$

IDENTIFYING PROPERTIES Tell which property the statement illustrates.

19. $5 \cdot p = p \cdot 5$

20. $2 + (12 + r) = (2 + 12) + r$

21. $4 \cdot (x \cdot 10) = (4 \cdot x) \cdot 10$

22. $x + 7.5 = 7.5 + x$

23. $(c + 2) + 0 = c + 2$

24. $a \cdot 1 = a$

25. YOU BE THE TEACHER Your friend states the property that the statement illustrates. Is your friend correct? Explain your reasoning.

$(7 + x) + 3 = (x + 7) + 3$

Associative Property of Addition

USING PROPERTIES Simplify the expression. Explain each step.

26. $6 + (5 + x)$

27. $(14 + y) + 3$

28. $6(2b)$

29. $7(9w)$

30. $3.2 + (x + 5.1)$

31. $(0 + a) + 8$

32. $9 \cdot c \cdot 4$

33. $(18.6 \cdot d) \cdot 1$

34. $\left(3k + 4\frac{1}{5}\right) + 8\frac{3}{5}$

35. $(2.4 + 4n) + 9$

36. $(3s) \cdot 8$

37. $z \cdot 0 \cdot 12$

38. GEOMETRY The expression $12 + x + 4$ represents the perimeter of a triangle. Simplify the expression.

39. MP PRECISION A case of scout cookies has 10 cartons. A carton has 12 boxes. The amount you earn on a whole case is $10(12x)$ dollars.

    a. What does x represent?

    b. Simplify the expression.

40. MODELING REAL LIFE A government estimates the cost to design new radar technology over a period of m months. The government estimates \$840,000 for equipment, \$15,000 for software, and \$40,000 per month for wages. Use an algebraic expression to find the total cost the government estimates when the project takes 16 months to complete.

**WRITING EXPRESSIONS** Write the phrase as an expression. Then simplify the expression.

41. 7 plus the sum of a number x and 5

42. the product of 8 and a number y, multiplied by 9

USING PROPERTIES Copy and complete the statement using the specified property.

	Property	Statement
43.	Associative Property of Multiplication	$7(2y) =$
44.	Commutative Property of Multiplication	$13.2 \cdot (x \cdot 1) =$
45.	Associative Property of Addition	$17 + (6 + 2x) =$
46.	Addition Property of Zero	$2 + (c + 0) =$
47.	Multiplication Property of One	$1 \cdot w \cdot 16 =$

48. GEOMETRY Five identical triangles form the trapezoid shown.

    a. What is the perimeter of the trapezoid?

    b. How can you use some or all of the triangles to form a new trapezoid with a perimeter of $3x + 14$? Explain your reasoning.
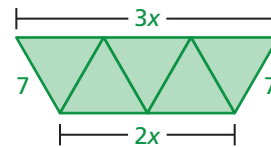
49. DIG DEEPER! You and a friend sell hats at a fair booth. You sell 16 hats on the first shift and 21 hats on the third shift. Your friend sells x hats on the second shift.

    a. The expression $37(14) + 10x$ represents the amount (in dollars) that you both earn. How can you tell that your friend is selling the hats for a lower price?

    b. You earn more money than your friend. What can you say about the value of x?