2.5 Systems of Linear Inequalities

GO DIGITAL

Learning Target Graph and write systems of linear inequalities.

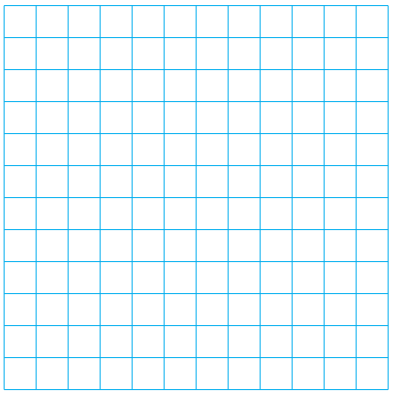
Success Criteria

- I can determine whether an ordered pair is a solution of a system of linear inequalities.
- I can graph systems of linear inequalities.
- I can write systems of linear inequalities from a graph.
- I can solve real-life problems using systems of linear inequalities.

EXPLORE IT! Writing Systems of Linear Inequalities

Work with a partner.

a. Write two linear equations: one with a positive slope and a nonzero y-intercept, and one with a negative slope and a nonzero y-intercept. Graph the equations in the same coordinate plane.

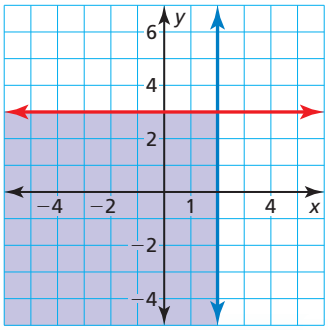


b. How many regions are formed by the graphs of the equations? Shade and label each region a different color.

c. How can you represent a region algebraically? Use your method to represent each region in part (b).

d. Use your method to represent the shaded region of each graph algebraically.

i.



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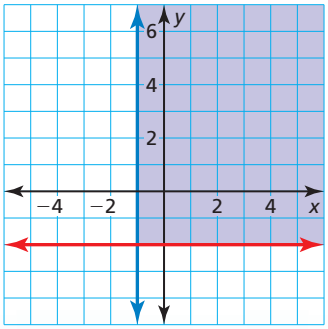
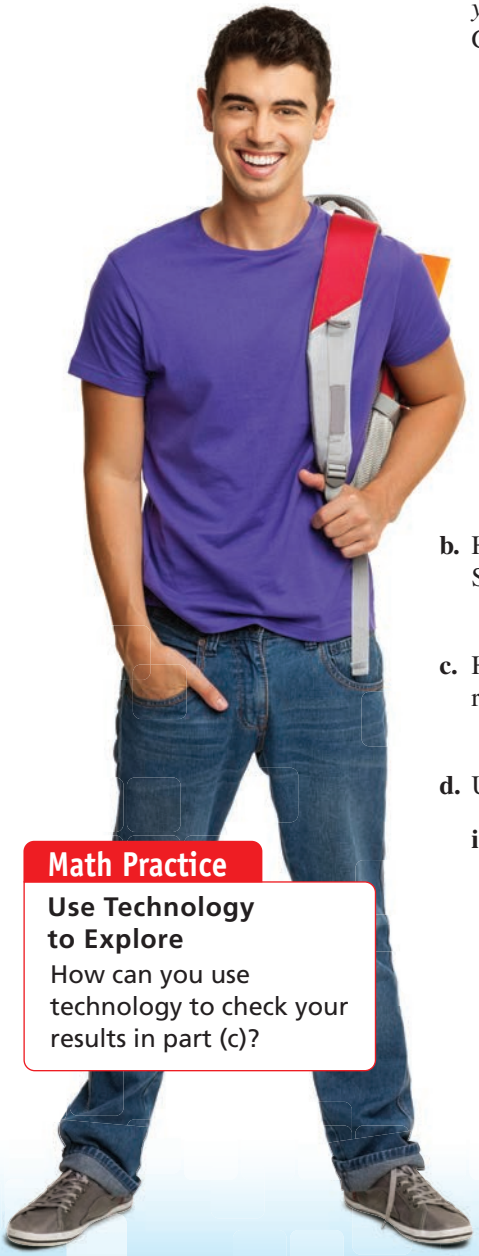




Math Practice

Use Technology to Explore

How can you use technology to check your results in part (c)?

Vocabulary

system of linear inequalities, *p. 106*
solution of a system of linear inequalities, *p. 106*
graph of a system of linear inequalities, *p. 107*

Systems of Linear Inequalities

A **system of linear inequalities** is a set of two or more linear inequalities in the same variables. An example is shown below.

$y < x + 2$ Inequality 1
$y \geq 2x - 1$ Inequality 2

A **solution of a system of linear inequalities** in two variables is an ordered pair that is a solution of each inequality in the system.

EXAMPLE 1 Checking Solutions

WATCH

Tell whether each ordered pair is a solution of the system of linear inequalities.

$y < 2x$ Inequality 1
$y \geq x + 1$ Inequality 2

a. $(3, 5)$ **b.** $(-2, 0)$

SOLUTION

a. Substitute 3 for x and 5 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$5 \stackrel{?}{<} 2(3)$	$5 \stackrel{?}{\geq} 3 + 1$
$5 < 6$ ✓	$5 \geq 4$ ✓

▶ Because the ordered pair $(3, 5)$ is a solution of each inequality, it is a solution of the system.

b. Substitute -2 for x and 0 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$0 \stackrel{?}{<} 2(-2)$	$0 \stackrel{?}{\geq} -2 + 1$
$0 \not< -4$ ✗	$0 \geq -1$ ✓

▶ Because $(-2, 0)$ is not a solution of each inequality, it is *not* a solution of the system.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. **VOCABULARY** How many linear inequalities are in a system of linear inequalities?
2. **WRITING** How can you verify that an ordered pair is a solution of a system of linear inequalities?

Tell whether the ordered pair is a solution of the system of linear inequalities.

3. $(-1, 5)$; $y < 5$, $y > x - 4$
4. $(1, 4)$; $y \geq 3x + 1$, $y > x - 1$
5. $\left(0, -\frac{5}{2}\right)$; $y > \frac{1}{2}x - 8$, $y < 6 - x$
6. **OPEN-ENDED** One inequality in a system is $y \geq 2x + 3$. Write another possible inequality in the system so that $(-2, 6)$ is a solution of the system.

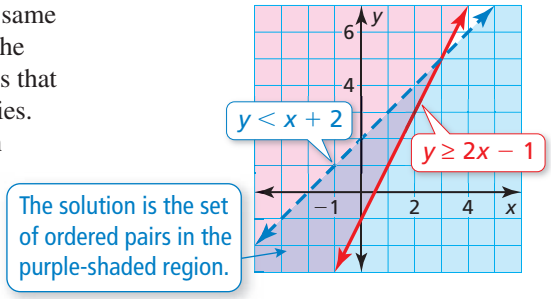
Graphing Systems of Linear Inequalities

The **graph of a system of linear inequalities** is the graph of all the solutions of the system.

KEY IDEA

Graphing a System of Linear Inequalities

Graph each inequality in the same coordinate plane. Then find the intersection of the half-planes that are solutions of the inequalities. This intersection is the graph of the system.



EXAMPLE 2 Graphing a System of Linear Inequalities

WATCH

Graph the system. Identify a solution.

$y \leq 3$ Inequality 1

$y > x + 2$ Inequality 2

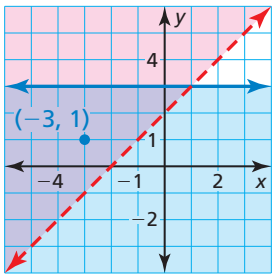
Check

Verify that $(-3, 1)$ is a solution of each inequality.

Inequality 1	Inequality 2
$y \leq 3$	$y > x + 2$
$1 \leq 3$ ✓	$1 \stackrel{?}{>} -3 + 2$
	$1 > -1$ ✓

SOLUTION

Graph each inequality. Then find the intersection of the half-planes.



▶ One solution is $(-3, 1)$.

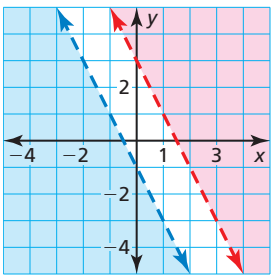
EXAMPLE 3 Graphing a System of Linear Inequalities: No Solution

WATCH

Graph the system.

$2x + y < -1$ Inequality 1

$2x + y > 3$ Inequality 2



SOLUTION

Graph each inequality. Then find the intersection of the half-planes. Notice that the lines are parallel, and the half-planes do not intersect.

▶ So, the system has no solution.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the system. Identify a solution, if possible.

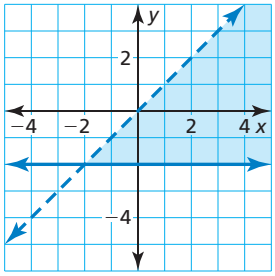
7. $y \geq -x + 4$
$x + y \leq 0$

8. $y > 2x - 3$
$y \geq \frac{1}{2}x + 1$

9. $-2x + y < 4$
$2x + y > 4$

Writing Systems of Linear Inequalities

EXAMPLE 4 Writing a System of Linear Inequalities



Write a system of linear inequalities represented by the graph.

SOLUTION

Inequality 1 The horizontal boundary line passes through $(0, -2)$. So, an equation of the line is $y = -2$. The shaded region is *above* the *solid* boundary line, so the inequality is $y \geq -2$.

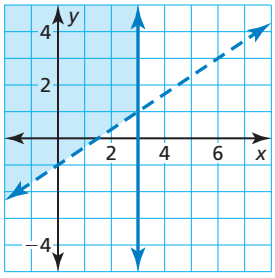
Inequality 2 The slope of the other boundary line is 1, and the y-intercept is 0. So, an equation of the line is $y = x$. The shaded region is *below* the *dashed* boundary line, so the inequality is $y < x$.

▶ The system of linear inequalities represented by the graph is

$y \geq -2$ Inequality 1

$y < x$. Inequality 2

EXAMPLE 5 Writing a System of Linear Inequalities



Write a system of linear inequalities represented by the graph.

SOLUTION

Inequality 1 The vertical boundary line passes through $(3, 0)$. So, an equation of the line is $x = 3$. The shaded region is to the *left* of the *solid* boundary line, so the inequality is $x \leq 3$.

Inequality 2 The slope of the other boundary line is $\frac{2}{3}$, and the y-intercept is -1. So, an equation of the line is $y = \frac{2}{3}x - 1$. The shaded region is *above* the *dashed* boundary line, so the inequality is $y > \frac{2}{3}x - 1$.

▶ The system of linear inequalities represented by the graph is

$x \leq 3$ Inequality 1

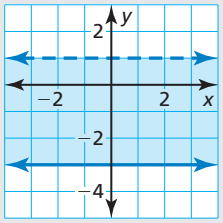
$y > \frac{2}{3}x - 1$. Inequality 2

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Write a system of linear inequalities represented by the graph.

10.



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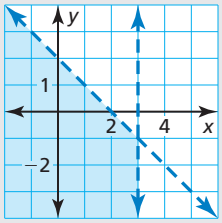

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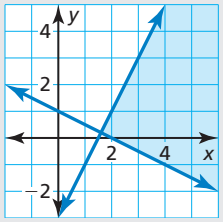


13. OPEN-ENDED Write a system of linear inequalities whose graph can be represented by a rectangular region.

14. MP REASONING Is it possible to write a system of three linear inequalities that has no solution? If so, give an example. If not, explain why not.

Solving Real-Life Problems

EXAMPLE 6 **Modeling Real Life** WATCH

A company is loading recliners and sofas onto a trailer that has a volume of about 3800 cubic feet. Each recliner takes up about 40 cubic feet and each sofa takes up about 80 cubic feet. The company wants the shipment to have at least 30 recliners and more than 25 sofas. Write and graph a system that represents the situation. Give one example of the numbers of recliners and sofas the company can have in the shipment.

SOLUTION

1. **Understand the Problem** You know the volume of the trailer and the volume of each recliner and sofa. You also know how many recliners and sofas the company wants in the shipment. You are asked to give an example of the numbers of recliners and sofas the company can have in the shipment.

2. **Make a Plan** Use the given information to write a system of linear inequalities. Then graph the system and identify and interpret an ordered pair in the solution region.

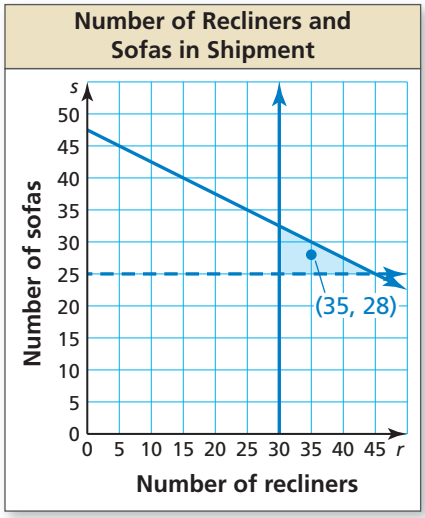
3. **Solve and Check** Let r be the number of recliners and let s be the number of sofas in the shipment.

$40r + 80s \leq 3800$ at most 3800 cubic feet is available in the shipment

$r \geq 30$ at least 30 recliners in the shipment

$s > 25$ more than 25 sofas in the shipment

Graph the system.



One ordered pair in the solution region is (35, 28).

▶ So, the company can have 35 recliners and 28 sofas in the shipment.

Check Check your solution by substituting it into the inequalities in the system.

$40r + 80s \leq 3800$ $\quad r \geq 30$ $\quad s > 25$

$40(35) + 80(28) \stackrel{?}{\leq} 3800$ $\quad 35 \geq 30$ ✓ $\quad 28 > 25$ ✓

$3640 \leq 3800$ ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

15. Identify and interpret another ordered pair in the solution region in Example 6.

16. You have at most 8 hours to spend at the mall and at the beach. You want to spend more than 2 hours at the mall and more than 4 hours at the beach.

a. Write and graph a system that represents the situation. Give one example of the amount of time you can spend at each location.

b. You want to spend at least 45 more minutes at the mall. How does this change the system in part (a)? Is your example still valid? Explain.

2.5 Practice WITH CalcChat® AND CalcView®

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In Exercises 1–4, tell whether the ordered pair is a solution of the system of linear inequalities.
Example 1

1. $(-5, 2)$; $y < 4$, $y > x + 3$

2. $(1, -1)$; $y > -2$, $y > x - 5$

3. $(0, 0)$; $y \leq x + 7$, $y \geq 2x + 3$

4. $(4, -3)$; $y \leq -x + 1$, $y \leq 5x - 2$

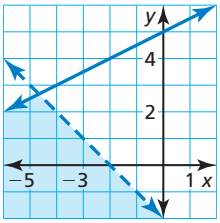
In Exercises 5–8, tell whether the ordered pair is a solution of the system of linear inequalities.

5. $(-4, 3)$

6. $(-3, -1)$

7. $(-2, 0)$

8. $(1, 0.5)$



In Exercises 9–18, graph the system. Identify a solution, if possible. *Examples 2 and 3*

9. $y > -3$, $y \geq 5x$

10. $y < -1$, $x > 4$

11. $y < -2$, $y > 2$

12. $y < x - 1$, $y \geq x + 1$

13. $y \geq -5$, $y - 1 < 3x$

14. $x + y > 4$, $y \geq \frac{3}{2}x - 9$

15. $x + y > 1$, $-x - y < -3$

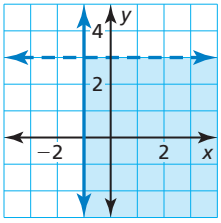
16. $2x + y \leq 5$, $y + 2 \geq -2x$

17. $x < 3.5$, $y > 1$, $y \geq -x + 1$

18. $\frac{3}{4}x + y \leq 3$, $\frac{3}{4}x - y \geq 1$, $y > -1\frac{1}{2}$

In Exercises 19–24, write a system of linear inequalities represented by the graph. *Examples 4 and 5*

19.



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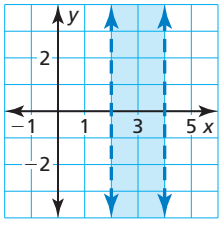

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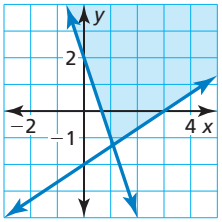

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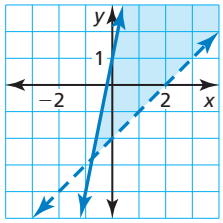

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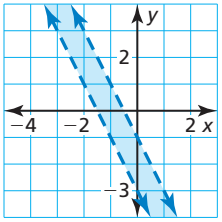

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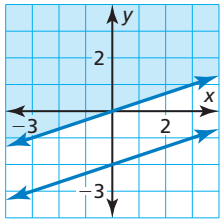
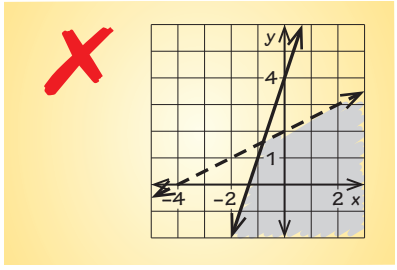
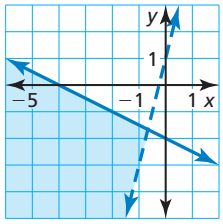


25. ERROR ANALYSIS Describe and correct the error in graphing the system $y \leq 3x + 4$ and $y > \frac{1}{2}x + 2$.



26. COLLEGE PREP Which of the following systems is represented by the graph?



(A) $y \leq -\frac{1}{2}x - 2$, $y > 4x + 1$

(B) $y \leq -\frac{1}{2}x - 2$, $y < 4x + 1$

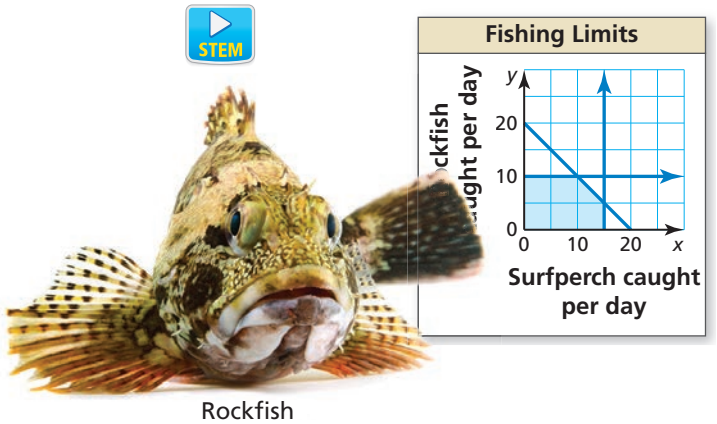
(C) $y < -\frac{1}{2}x - 2$, $y \geq 4x + 1$

(D) $y < -\frac{1}{2}x - 2$, $y \leq 4x + 1$

27. MODELING REAL LIFE A group of scientists have at most 7 hours to spend on an expedition to one of the deepest areas of the ocean. They expect the total travel time to be more than 3 hours and want to spend at least $2\frac{1}{2}$ hours exploring. Write and graph a system that represents the situation. Give one example of the amount of time they can spend on each part of the expedition. *Example 6*

28. **MODELING REAL LIFE** You earn $10 per hour working at a grocery store and must work there at least 8 hours per week. You also teach music lessons for $15 per hour. Between the two jobs, you need to earn at least $120 per week and work no more than 20 hours per week. Write and graph a system that represents the situation. Give one example of the number of hours you can work at each job.

29. **MODELING REAL LIFE** You are fishing for surfperch and rockfish. The graph shows limits on the numbers of fish you are allowed to catch per day.



Rockfish

a. Write and interpret a system of linear inequalities that represents the situation.

b. Can you catch 11 surfperch and 9 rockfish in 1 day? Explain.

30. MP **REASONING** Describe the graph of the system shown.

$$x - y \leq 4$$
$$x - y \geq 4$$

31. MP **PROBLEM SOLVING** Your cousin plans to spend less than half of her monthly $2000 paycheck on housing and savings. She wants to spend at least 10% of her paycheck on savings and at most 30% of it on housing. Give one example of the amount of money your cousin can spend on savings and housing. Justify your answer using a system of linear inequalities.

32. MP **PROBLEM SOLVING** A travel club can spend at most 10 nights in two cities on a trip. The club needs to reserve four rooms each night and wants to spend no more than $4200 on hotels and fuel. The estimated fuel cost is $200. Can the club spend 3 nights in City A and 6 nights in City B? 7 nights in City A and 3 nights in City B? Justify your answers using a system of linear inequalities.

Nightly Hotel Rates	
City A hotel	City B hotel
$85 per room	$130 per room

33. **CONNECTING CONCEPTS** The vertices of a shaded rectangle are $(-1, 1)$, $(6, 1)$, $(6, -3)$, and $(-1, -3)$.

a. Write a system of linear inequalities with a graph that can be represented by the shaded rectangle.

b. Find the area of the rectangle.

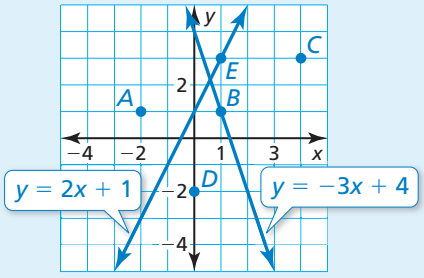
34. **CONNECTING CONCEPTS** The vertices of a shaded triangle are $(2, 5)$, $(6, -3)$, and $(-2, -3)$.

a. Write a system of linear inequalities with a graph that can be represented by the shaded triangle.

b. Find the area of the triangle.

35. MP **STRUCTURE** Write a system of linear inequalities that has the same solutions as $|y| < x$, where $x > 0$. Graph the system.

36. **HOW DO YOU SEE IT?**
The graphs of two linear equations are shown.



Replace the equal signs with inequality symbols to create a system of linear inequalities that has points C and E as solutions, but not points A, B, and D. Explain your reasoning.

$$y \square -3x + 4$$
$$y \square 2x + 1$$

37. **MAKING AN ARGUMENT** Your friend says that a system of linear inequalities in which the boundary lines are parallel must have no solution. Is your friend correct? Explain.

38. **CRITICAL THINKING** Is it possible for the solution set of a system of linear inequalities to be all points in the coordinate plane? Explain your reasoning.

OPEN-ENDED **In Exercises 39–41, write a system of linear inequalities with the given characteristic.**

39. All solutions are in Quadrant I.

40. All solutions have one positive coordinate and one negative coordinate.

41. There are no solutions.

42. **OPEN-ENDED** One inequality in a system is $-4x + 2y > 6$. Write another possible inequality in the system so that the system has no solution.

43. **PERFORMANCE TASK** The table shows three systems.

System A	System B	System C
$y \geq x$ $y \leq x + 4$ $y \geq -2$ $y \leq 6$	$y > x + 4$ $x \geq -6$ $y \leq 6$	$y < x$ $x \leq 6$ $y \geq -2$

a. Graph each system in the same coordinate plane. Color the solutions of System A yellow, the solutions of System B green, and the solutions of System C red. Erase all other shading.

b. The directions in part (a) produce the flag of a country. Determine which country the flag represents.

c. Choose the flag of a different country, or design your own flag. Describe how to draw the flag using systems of linear inequalities.

44. **THOUGHT PROVOKING**
Write a system of linear inequalities that has exactly one solution.

45. DIG DEEPER You make necklaces and key chains to sell at a craft fair. The table shows the amounts of time and money it takes to make a necklace and a key chain, and the amounts of time and money you have available for making them.

	Necklace	Key chain	Available
Time (hours)	0.5	0.25	20
Cost (dollars)	2	3	120

a. Give three examples of the number of each item you can make. Justify your answers using a system of linear inequalities.

b. You sell each necklace for \$10 and each key chain for \$8. How many necklaces and key chains should you sell to maximize your revenue? What is the maximum revenue? (*Hint:* The maximum revenue occurs at one of the vertices of the graph of the system in part (a).)

REVIEW & REFRESH

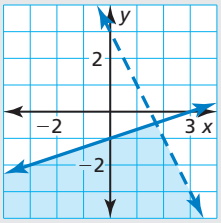
46. Graph $-\frac{3}{4}x - y < 6$ in a coordinate plane.

In Exercises 47 and 48, write the product as a power.

47. $(-13) \cdot (-13) \cdot (-13)$

48. $x \cdot x \cdot x \cdot x \cdot x \cdot x$

49. Write a system of linear inequalities represented by the graph.



50. Write an equation of the line with a slope of $-\frac{1}{4}$ and a y-intercept of -1.

In Exercises 51 and 52, solve the equation.

51. $\frac{2}{3}(x - 6) = 4$

52. $6x - 7 = -2x - 9$

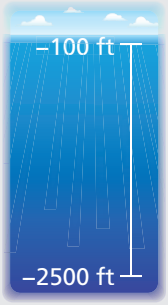
In Exercises 53 and 54, solve the system using any method. Explain your choice of method.

53. $5x + 10y = 8$
$3x + 6y = 4$

54. $-11x + 2y = 12$
$y = 4x + 3$

WATCH

55. **MODELING REAL LIFE** Slitsnails are large mollusks that live in deep waters. They have been found in the range of elevations shown. Write and graph a compound inequality that represents this range.



56. Write an equation for the nth term of the arithmetic sequence shown. Then find a_{15}.

$-14, -5, 4, 13, \ldots$

57. MP **NUMBER SENSE** The sixth term of an arithmetic sequence is 8. The common difference of the sequence is $-\frac{1}{3}$ times the first term. Graph the sequence.

58. Use intercepts to graph the equation $-3x + 6y = 24$. Label the points corresponding to the intercepts.

59. The points represented by the table lie on a line. Find the slope of the line.

x	4	4	4	4
y	−6	−3	0	3