# 9.2 Special Right Triangles

**Learning Target:** Understand and use special right triangles.

**Success Criteria:**
- I can find side lengths in 45°-45°-90° triangles.
- I can find side lengths in 30°-60°-90° triangles.
- I can use special right triangles to solve real-life problems.

## EXPLORE IT! Finding Side Ratios of Special Right Triangles

**Work with a partner.**

**a.** One type of special right triangle is a 45°-45°-90° triangle.

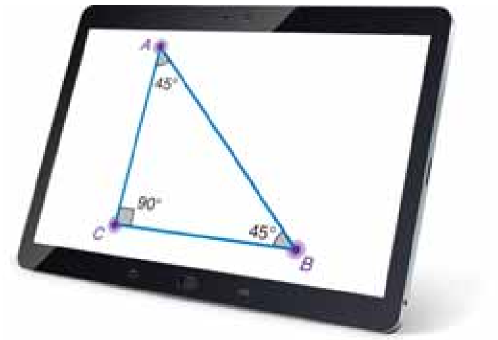
**i.** Construct a right triangle with acute angle measures of 45°.

**ii.** Find the exact ratios of the side lengths.

$\frac{AB}{BC} =$ ____

$\frac{AB}{AC} =$ ____

$\frac{AC}{BC} =$ ____




**iii.** Repeat parts (i) and (ii) for several other 45°-45°-90° triangles. Use your results to write a conjecture about the ratios of the side lengths of 45°-45°-90° triangles.

**b.** Another type of special right triangle is a 30°-60°-90° triangle.

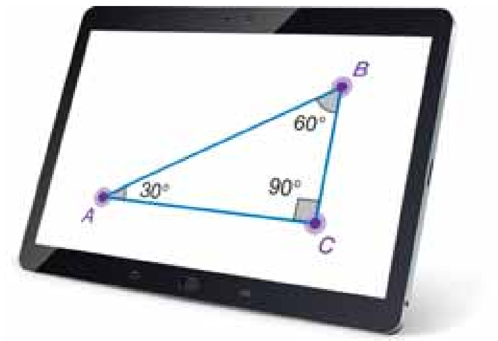
**i.** Construct a right triangle with acute angle measures of 30° and 60°.

**ii.** Find the exact ratios of the side lengths.

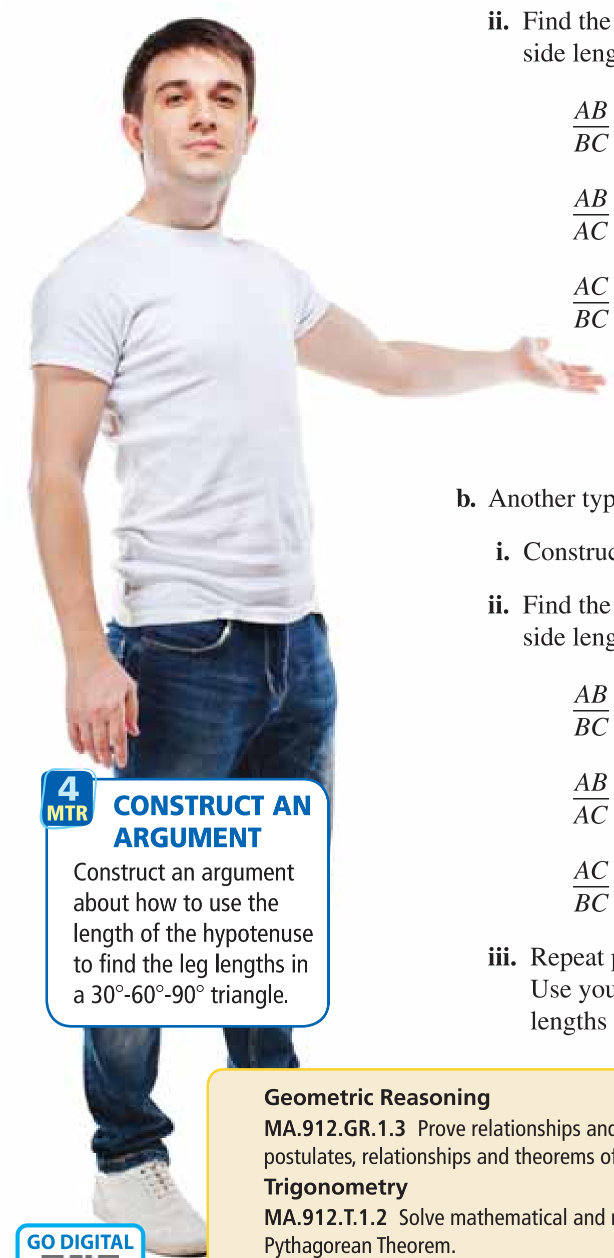
$\frac{AB}{BC} =$ ____

$\frac{AB}{AC} =$ ____

$\frac{AC}{BC} =$ ____




**iii.** Repeat parts (i) and (ii) for several other 30°-60°-90° triangles. Use your results to write a conjecture about the ratios of the side lengths of 30°-60°-90° triangles.



**4 MTR CONSTRUCT AN ARGUMENT**
Construct an argument about how to use the length of the hypotenuse to find the leg lengths in a 30°-60°-90° triangle.

**Geometric Reasoning**
**MA.912.GR.1.3** Prove relationships and theorems about triangles. Solve mathematical and real-world problems involving postulates, relationships and theorems of triangles.
**Trigonometry**
**MA.912.T.1.2** Solve mathematical and real-world problems involving right triangles using trigonometric ratios and the Pythagorean Theorem.

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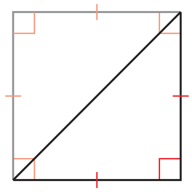
## Finding Side Lengths in Special Right Triangles

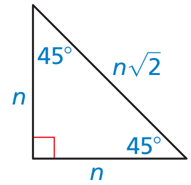
A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half diagonally.



## THEOREM

**9.4 45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



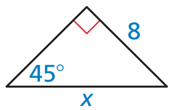

hypotenuse = leg • $\sqrt{2}$

*Prove this Theorem* Exercise 17, page 465

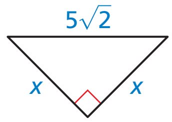
**EXAMPLE 1** **Finding Side Lengths in 45°-45°-90° Triangles** WATCH

Find the value of $x$. Write your answer in simplest form.

**a.**




**b.**




> **REMEMBER**
> An expression involving a radical with index 2 is in simplest form when no radicands have perfect squares as factors other than 1, no radicands contain fractions, and no radicals appear in the denominator of a fraction.

### SOLUTION

**a.** By the Triangle Sum Theorem, the measure of the third angle must be 45°, so the triangle is a 45°-45°-90° triangle.

| | |
|---|---|
| hypotenuse = leg • $\sqrt{2}$ | 45°-45°-90° Triangle Theorem |
| $x = 8 \cdot \sqrt{2}$ | Substitute. |
| $x = 8\sqrt{2}$ | Simplify. |

▶ The value of $x$ is $8\sqrt{2}$.

**b.** By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

| | |
|---|---|
| hypotenuse = leg • $\sqrt{2}$ | 45°-45°-90° Triangle Theorem |
| $5\sqrt{2} = x \cdot \sqrt{2}$ | Substitute. |
| $\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$ | Division Property of Equality |
| $5 = x$ | Simplify. |

▶ The value of $x$ is 5.

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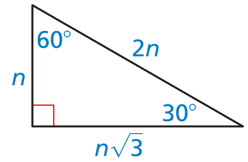
# THEOREM

### 9.5 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



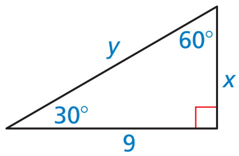

hypotenuse = shorter leg • 2
longer leg = shorter leg • $\sqrt{3}$

*Prove this Theorem* Exercise 19, page 466

## EXAMPLE 2 Finding Side Lengths in a 30°-60°-90° Triangle

WATCH

Find the values of $x$ and $y$. Write your answers in simplest form.




### SOLUTION

**Step 1** Find the value of $x$.

| | |
|---|---|
| longer leg = shorter leg • $\sqrt{3}$ | 30°-60°-90° Triangle Theorem |
| $9 = x \cdot \sqrt{3}$ | Substitute. |
| $\frac{9}{\sqrt{3}} = x$ | Divide each side by $\sqrt{3}$. |
| $\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$ | Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$. |
| $\frac{9\sqrt{3}}{3} = x$ | Multiply fractions. |
| $3\sqrt{3} = x$ | Simplify. |

▶ The value of $x$ is $3\sqrt{3}$.

**REMEMBER**
Because the angle opposite 9 is larger than the angle opposite $x$, the leg with length 9 is longer than the leg with length $x$ by the Triangle Larger Angle Theorem.

**Step 2** Find the value of $y$.

| | |
|---|---|
| hypotenuse = shorter leg • 2 | 30°-60°-90° Triangle Theorem |
| $y = 3\sqrt{3} \cdot 2$ | Substitute. |
| $y = 6\sqrt{3}$ | Simplify. |

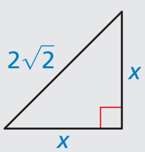
▶ The value of $y$ is $6\sqrt{3}$.

## SELF-ASSESSMENT

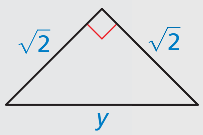
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

**Find the missing side length(s). Write your answer(s) in simplest form.**

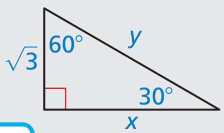
**1.**



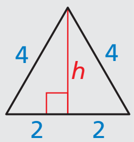

**2.**




**3.**




**4.**

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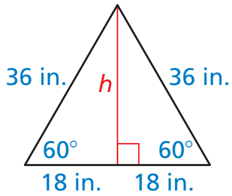
## Solving Real-Life Problems

7 MTR **EXAMPLE 3** **Modeling Real Life** WATCH

The biohazard sign is shaped like an equilateral triangle. Estimate the area of the sign.


### SOLUTION

First find the height $h$ of the triangle by dividing it into two 30°-60°-90° triangles. The length of the longer leg of one of these triangles is $h$. The length of the shorter leg is 18 inches.



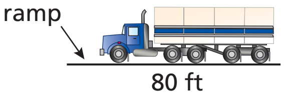

$$h = 18 \cdot \sqrt{3} = 18\sqrt{3}$$ 30°-60°-90° Triangle Theorem

Use $h = 18\sqrt{3}$ to find the area of the equilateral triangle.

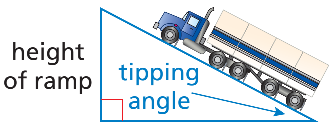
$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(36)\left(18\sqrt{3}\right) \approx 561.18$$

▶ The area of the sign is about 561 square inches.

7 MTR **EXAMPLE 4** **Modeling Real Life** WATCH




A tipping platform is a ramp used to unload trucks. How high is the end of an 80-foot ramp when the tipping angle is 30°? 45°?




### SOLUTION

When the tipping angle is 30°, the height $h$ of the ramp is the length of the shorter leg of a 30°-60°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = 2h$ — 30°-60°-90° Triangle Theorem

$40 = h$ — Divide each side by 2.

When the tipping angle is 45°, the height is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = h \cdot \sqrt{2}$ — 45°-45°-90° Triangle Theorem

$\frac{80}{\sqrt{2}} = h$ — Divide each side by $\sqrt{2}$.

$56.6 \approx h$ — Use technology.

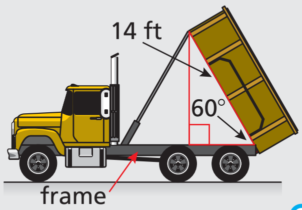
▶ When the tipping angle is 30°, the ramp height is 40 feet. When the tipping angle is 45°, the height is about 56 feet 7 inches.

## SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. The logo on a recycling bin resembles an equilateral triangle with side lengths of 6 centimeters. Approximate the area of the logo.

6. The body of a dump truck rests on a frame. The body is raised to empty a load of sand. How far from the frame is the front of the 14-foot-long body when it is tipped upward by a 60° angle?

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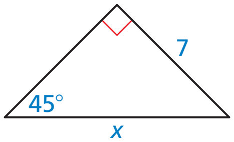
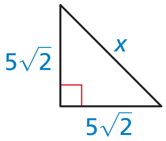
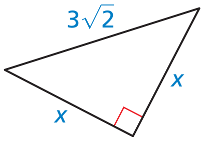
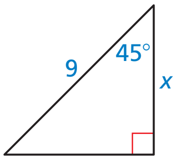
# 9.2 Practice WITH CalcChat® AND CalcView®

**In Exercises 1–4, find the value of *x*. Write your answer in simplest form.** *(See Example 1.)*

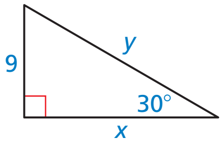
**1.**


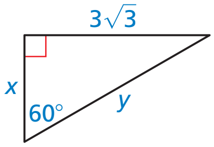

**2.**


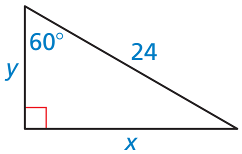

**3.**


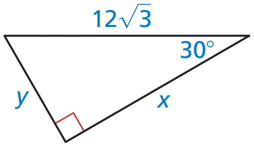

**4.**



**In Exercises 5–8, find the values of *x* and *y*. Write your answers in simplest form.** *(See Example 2.)*

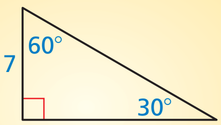
**5.**



**6.**



**7.**


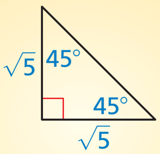

**8.**



**ERROR ANALYSIS** **In Exercises 9 and 10, describe and correct the error in finding the length of the hypotenuse in the special right triangle.**

**9.**



hypotenuse = shorter leg • $\sqrt{3} = 7\sqrt{3}$

So, the length of the hypotenuse is $7\sqrt{3}$ units.

**10.**



hypotenuse = leg • 2 = $2\sqrt{5}$

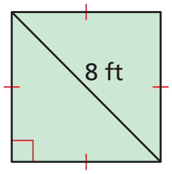
So, the length of the hypotenuse is $2\sqrt{5}$ units.

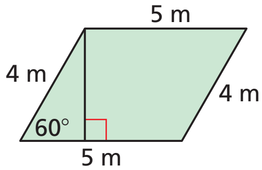
**In Exercises 11 and 12, sketch the figure that is described. Find the indicated length. Write your answer in simplest form.**

**11.** The perimeter of a square is 36 inches. Find the length of a diagonal.

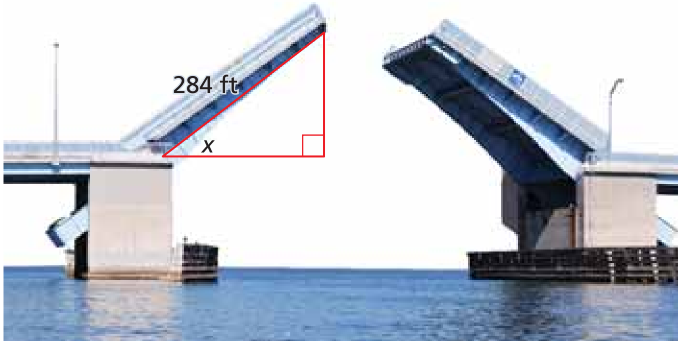
**12.** The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude.

**In Exercises 13 and 14, find the area of the figure. Write your answer in simplest form.** *(See Example 3.)*

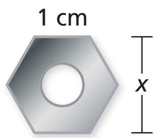
**13.**



**14.**



**15.** **MODELING REAL LIFE** Each half of the drawbridge is about 284 feet long. How high does the drawbridge rise when *x* is 30°? 45°? 60°? *(See Example 4.)*



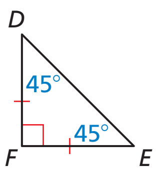

**16.** **MODELING REAL LIFE** A nut is shaped like a regular hexagon with side lengths of 1 centimeter. Find the value of *x*.




**17.** **PROVING A THEOREM** Write a paragraph proof of the 45°-45°-90° Triangle Theorem.

**Given** $\triangle DEF$ is a 45°-45°-90° triangle.

**Prove** The hypotenuse is $\sqrt{2}$ times as long as each leg.

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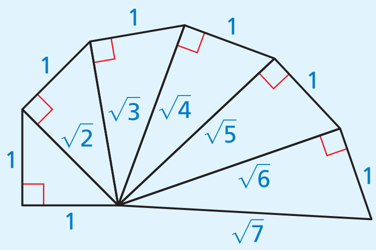
**18. HOW DO YOU SEE IT?**
The diagram shows part of the *Wheel of Theodorus*.



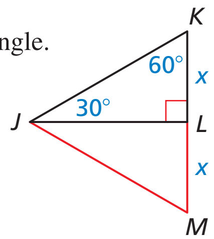

**a.** Which triangles, if any, are 45°-45°-90° triangles?

**b.** Which triangles, if any, are 30°-60°-90° triangles?

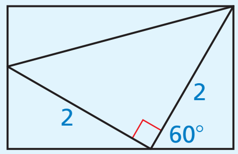
**19. PROVING A THEOREM** Write a paragraph proof of the 30°-60°-90° Triangle Theorem. (*Hint*: Construct $\triangle JML$ congruent to $\triangle JKL$.)

**Given** $\triangle JKL$ is a 30°-60°-90° triangle.

**Prove** The hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.




**20. THOUGHT PROVOKING**
The diagram below is called the *Ailles rectangle*. Each triangle in the diagram has rational angle measures, and each side length contains at most one square root. Label the sides and angles in the diagram. Describe the triangles.




**21. WRITING** Describe two ways to show that all isosceles right triangles are similar to each other.

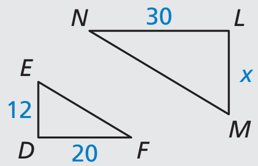
**22. REASONING** The area of an equilateral triangle is $3\sqrt{3}$ square units. Find the side length of the triangle. Justify your answer.

**23. DIG DEEPER** $\triangle TUV$ is a 30°-60°-90° triangle, where two vertices are $U(3, -1)$ and $V(-3, -1)$, $\overline{UV}$ is the hypotenuse, and point $T$ is in Quadrant I. Find the coordinates of $T$.

## REVIEW & REFRESH

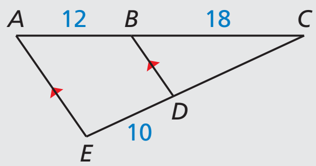
WATCH

**24.** In the diagram, $\triangle DEF \sim \triangle LMN$. Find the value of $x$.

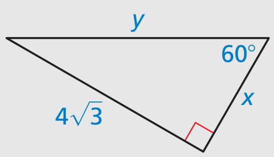



**25.** Determine whether segments with lengths of 2.6 feet, 4.8 feet, and 6.0 feet form a triangle. If so, is the triangle *acute*, *right*, or *obtuse*?

**26.** Find the length of $\overline{DC}$.




**27.** Find the values of $x$ and $y$. Write your answers in simplest form.

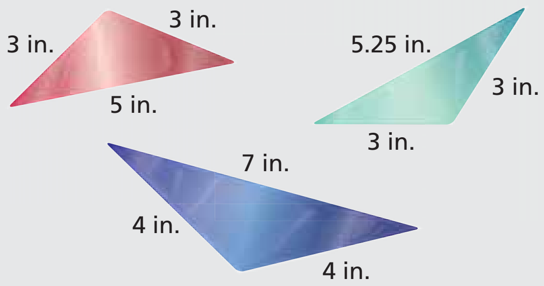



**28.** The endpoints of $\overline{CD}$ are $C(-2, 9)$ and $D(3, -1)$. Find the coordinates of the midpoint $M$.

**29.** Determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.

$D(3, 5)$, $E(8, 0)$, $F(4, -3)$ and
$M(-5, 3)$, $N(0, 8)$, $P(3, 4)$

7 MTR **30. MODELING REAL LIFE** Which pieces of stained glass, if any, are similar? Explain.




**31.** Three vertices of $\square JKLM$ are $J(0, 5)$, $K(4, 5)$, and $M(3, 0)$. Find the coordinates of vertex $L$.

**32.** Rewrite the definition as a biconditional statement.

**Definition** A *parallelogram* is a quadrilateral with both pairs of opposite sides parallel.

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