7.4 Properties of Special Parallelograms

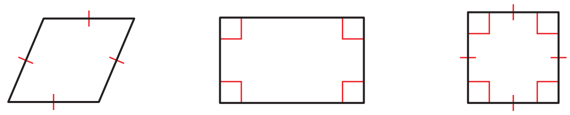
Learning Target: Explain the properties of special parallelograms.

Success Criteria:

- I can identify special quadrilaterals.
- I can explain how special parallelograms are related.
- I can find missing measures of special parallelograms.
- I can identify special parallelograms in a coordinate plane.

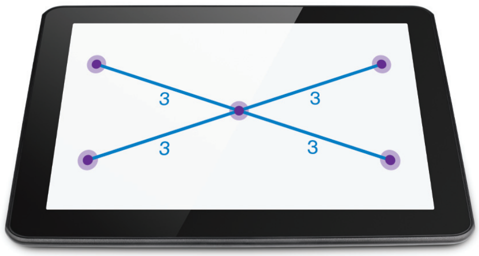
EXPLORE IT! Analyzing Diagonals of Quadrilaterals

Work with a partner. Recall the three types of parallelograms shown below.



a. Use the diagrams to define each type of quadrilateral.

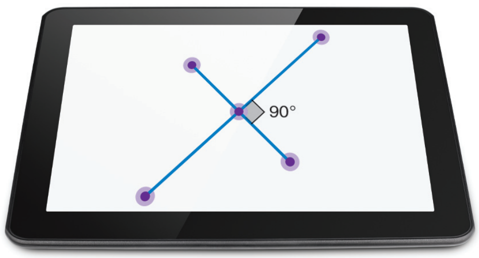
b. Use technology to construct two congruent line segments that bisect each other. Draw a quadrilateral by connecting the endpoints.



c. Is the quadrilateral you drew a parallelogram? a rectangle? a rhombus? a square? Explain your reasoning.

d. Repeat parts (b) and (c) for several pairs of congruent line segments that bisect each other. Make conjectures based on your results.

e. Use technology to construct two line segments that are perpendicular bisectors of each other. Draw a quadrilateral by connecting the endpoints.

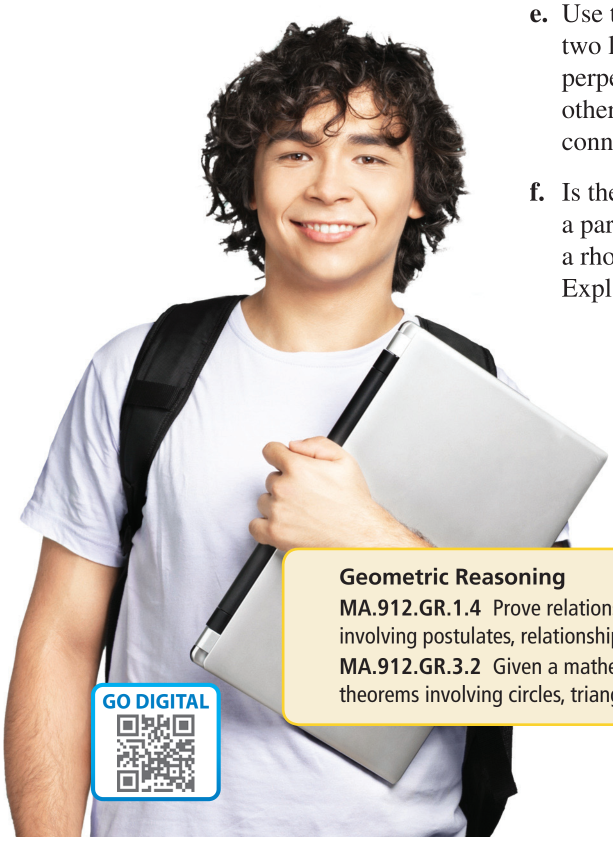


f. Is the quadrilateral you drew a parallelogram? a rectangle? a rhombus? a square? Explain your reasoning.

g. Repeat parts (e) and (f) for several other line segments that are perpendicular bisectors of each other. Make conjectures based on your results.

h. What are some properties of the diagonals of rectangles, rhombuses, and squares?

4 MTR CONSTRUCT AN ARGUMENT
What other quadrilaterals can you form using similar methods? Explain your reasoning.



GO DIGITAL

Geometric Reasoning

MA.912.GR.1.4 Prove relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms.

MA.912.GR.3.2 Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals.

Vocabulary

AZ VOCAB

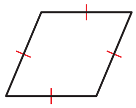
rhombus, *p. 380*
rectangle, *p. 380*
square, *p. 380*

Using Properties of Special Parallelograms

In this lesson, you will learn about corollaries and theorems that correspond to three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.

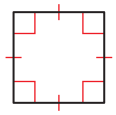
KEY IDEAS

Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.



A **square** is a parallelogram with four congruent sides and four right angles.

You can use the corollaries below to prove that a quadrilateral is a rhombus, rectangle, or square, without first proving that the quadrilateral is a parallelogram.

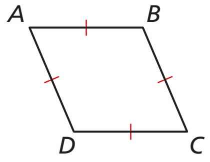
COROLLARIES

7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Prove this Corollary Exercise 77, page 387

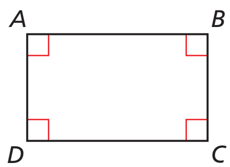


7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Prove this Corollary Exercise 78, page 387

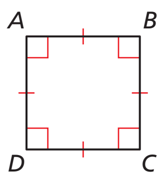


7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Prove this Corollary Exercise 79, page 387



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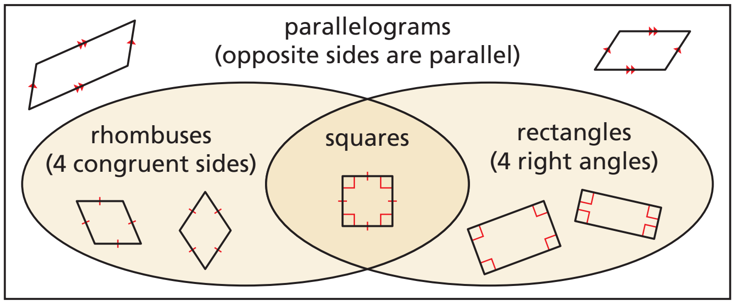
The Venn diagram below illustrates some important relationships among parallelograms, rhombuses, rectangles, and squares. For example, you can see that a square is a rhombus because it is a parallelogram with four congruent sides. Because it has four right angles, a square is also a rectangle.



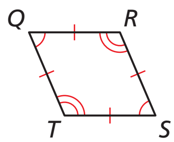
EXAMPLE 1 Using Properties of Special Quadrilaterals

For any rhombus $QRST$, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

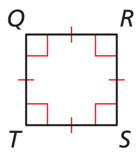
a. $\angle Q \cong \angle S$ **b.** $\angle Q \cong \angle R$

SOLUTION

a. By definition, a rhombus is a parallelogram with four congruent sides. By the Parallelogram Opposite Angles Theorem, opposite angles of a parallelogram are congruent. So, $\angle Q \cong \angle S$. The statement is *always* true.

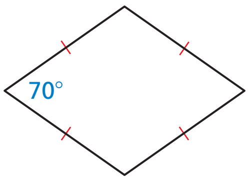


b. If rhombus $QRST$ is a square, then all four angles are congruent right angles. So, $\angle Q \cong \angle R$ when $QRST$ is a square. Because not all rhombuses are also squares, the statement is *sometimes* true.



EXAMPLE 2 Classifying Special Quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



SOLUTION

The quadrilateral has four congruent sides. By the Rhombus Corollary, the quadrilateral is a rhombus. Because one of the angles is not a right angle, the rhombus cannot be a square.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. VOCABULARY What is another name for an equilateral rectangle?

2. For any square $JKLM$, is it *always* or *sometimes* true that $\overline{JK} \perp \overline{KL}$? Explain your reasoning.

3. For any rectangle $EFGH$, is it *always* or *sometimes* true that $\overline{FG} \cong \overline{GH}$? Explain your reasoning.

4. A quadrilateral has four congruent sides and three angles that measure 90°. Sketch the quadrilateral and classify it.

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Using Properties of Diagonals

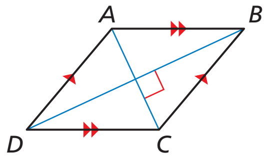
THEOREMS

7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Prove this Theorem Exercise 72, page 387

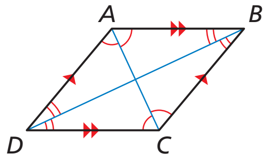


7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

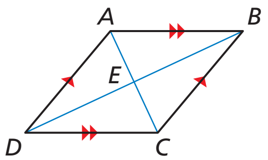
Prove this Theorem Exercises 75 and 76, page 387



READING

Recall that biconditionals, such as the Rhombus Diagonals Theorem, can be rewritten as two parts. To prove a biconditional, you must prove both parts.

PROOF Part of Rhombus Diagonals Theorem



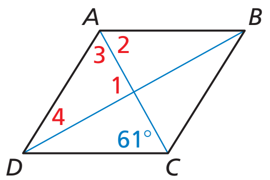
Given $ABCD$ is a rhombus.

Prove $\overline{AC} \perp \overline{BD}$

$ABCD$ is a rhombus. By the definition of a rhombus, $\overline{AB} \cong \overline{BC}$. Because a rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, $\overline{BD}$ bisects $\overline{AC}$ at E. So, $\overline{AE} \cong \overline{EC}$. $\overline{BE} \cong \overline{BE}$ by the Reflexive Property of Segment Congruence. So, $\triangle AEB \cong \triangle CEB$ by the SSS Congruence Theorem. $\angle AEB \cong \angle CEB$ because corresponding parts of congruent triangles are congruent. Then by the Linear Pair Postulate, $\angle AEB$ and $\angle CEB$ are supplementary. Two congruent angles that form a linear pair are right angles, so $m\angle AEB = m\angle CEB = 90°$ by the definition of a right angle. So, $\overline{AC} \perp \overline{BD}$ by the definition of perpendicular lines.

EXAMPLE 3 Finding Angle Measures in a Rhombus

WATCH



Find the measures of the numbered angles in rhombus $ABCD$.

SOLUTION

Use the Rhombus Diagonals Theorem and the Rhombus Opposite Angles Theorem to find the angle measures.

$m\angle 1 = 90°$	The diagonals of a rhombus are perpendicular.
$m\angle 2 = 61°$	Alternate Interior Angles Theorem
$m\angle 3 = 61°$	Each diagonal of a rhombus bisects a pair of opposite angles, and $m\angle 2 = 61°$.
$m\angle 1 + m\angle 3 + m\angle 4 = 180°$	Triangle Sum Theorem
$90° + 61° + m\angle 4 = 180°$	Substitute 90° for $m\angle 1$ and 61° for $m\angle 3$.
$m\angle 4 = 29°$	Solve for $m\angle 4$.

▶ So, $m\angle 1 = 90°$, $m\angle 2 = 61°$, $m\angle 3 = 61°$, and $m\angle 4 = 29°$.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. In Example 3, find $m\angle ADC$ and $m\angle BCD$.

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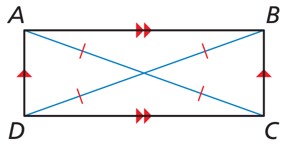
THEOREM

7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

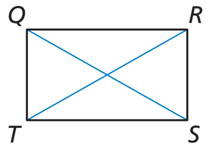


Prove this Theorem Exercises 86 and 87, page 388

EXAMPLE 4 Finding Diagonal Lengths in a Rectangle

WATCH

In rectangle $QRST$, $QS = 5x - 31$ and $RT = 2x + 11$. Find the lengths of the diagonals of $QRST$.



SOLUTION

By the Rectangle Diagonals Theorem, the diagonals of a rectangle are congruent. Find x so that $\overline{QS} \cong \overline{RT}$.

$QS = RT$	Set the diagonal lengths equal.
$5x - 31 = 2x + 11$	Substitute $5x - 31$ for QS and $2x + 11$ for RT.
$x = 14$	Simplify.

When $x = 14$, $QS = 5(14) - 31 = 39$ and $RT = 2(14) + 11 = 39$.

▶ Each diagonal has a length of 39 units.

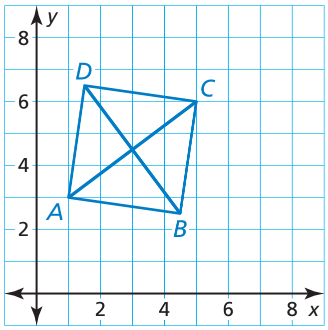
EXAMPLE 5 Finding Coordinates of Missing Vertices

WATCH

Square $ABCD$ has a diagonal $\overline{AC}$ with vertices $A(1, 3)$ and $C(5, 6)$. Find the coordinates of the remaining vertices.

SOLUTION

By the Parallelogram Diagonals Theorem, $\overline{AC}$ and $\overline{BD}$ bisect each other. So, they have the same midpoint. Using the Midpoint Formula, the midpoint is $M(3, 4.5)$. By the Rhombus Diagonals Theorem, $\overline{AC} \perp \overline{BD}$. By the Rectangle Diagonals Theorem, $\overline{AC} \cong \overline{BD}$. So, a 90° counterclockwise rotation about $M(3, 4.5)$ maps $\overline{AC}$ to $\overline{BD}$.



Step 1 Translate $\overline{AC}$ along $\overrightarrow{MO}$.

$$(a, b) \rightarrow (a - 3, b - 4.5)$$

$$A(1, 3) \rightarrow A'(1 - 3, 3 - 4.5) = A'(-2, -1.5)$$

$$C(5, 6) \rightarrow C'(5 - 3, 6 - 4.5) = C'(2, 1.5)$$

Step 2 Use the coordinate rule for a 90° counterclockwise rotation about the origin.

$$(a, b) \rightarrow (-b, a)$$

$$A'(-2, -1.5) \rightarrow A''(1.5, -2)$$

$$C'(2, 1.5) \rightarrow C''(-1.5, 2)$$

Step 3 Translate back along $\overrightarrow{OM}$.

$$(a, b) \rightarrow (a + 3, b + 4.5)$$

$$A''(1.5, -2) \rightarrow A'''(1.5 + 3, -2 + 4.5) = A'''(4.5, 2.5) = B$$

$$C''(-1.5, 2) \rightarrow C'''(-1.5 + 3, 2 + 4.5) = C'''(1.5, 6.5) = D$$

▶ So, the remaining vertices are $B(4.5, 2.5)$ and $D(1.5, 6.5)$.

Check Check that the diagonals are perpendicular.

$$m_{\overline{AC}} = \frac{6 - 3}{5 - 1} = \frac{3}{4}$$

$$m_{\overline{BD}} = \frac{6.5 - 2.5}{1.5 - 4.5} = -\frac{4}{3} \checkmark$$

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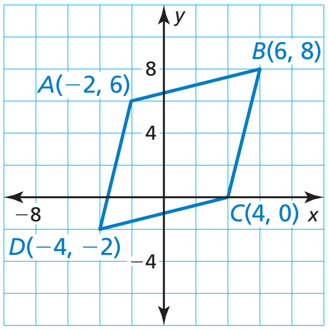
EXAMPLE 6 Identifying a Parallelogram in the Coordinate Plane

WATCH

Decide whether ▱*ABCD* with vertices $A(-2, 6)$, $B(6, 8)$, $C(4, 0)$, and $D(-4, -2)$ is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

SOLUTION

1. Understand the Problem You know the vertices of ▱*ABCD*. You need to identify the type of parallelogram.



2. Make a Plan Begin by graphing the vertices. From the graph, it appears that all four sides are congruent and there are no right angles.

Check the lengths and slopes of the diagonals of ▱*ABCD*. If the diagonals are congruent, then ▱*ABCD* is a rectangle. If the diagonals are perpendicular, then ▱*ABCD* is a rhombus. If they are both congruent and perpendicular, then ▱*ABCD* is a rectangle, a rhombus, and a square.

3. Solve and Check Use the Distance Formula to find *AC* and *BD*.

$$AC = \sqrt{(-2-4)^2 + (6-0)^2} = \sqrt{72} = 6\sqrt{2}$$

$$BD = \sqrt{[6-(-4)]^2 + [8-(-2)]^2} = \sqrt{200} = 10\sqrt{2}$$

Because $6\sqrt{2} \neq 10\sqrt{2}$, the diagonals are not congruent. So, ▱*ABCD* is not a rectangle. Because it is not a rectangle, it also cannot be a square.

Use the slope formula to find the slopes of the diagonals $\overline{AC}$ and $\overline{BD}$.

$$\text{slope of } \overline{AC} = \frac{6-0}{-2-4} = \frac{6}{-6} = -1 \qquad \text{slope of } \overline{BD} = \frac{8-(-2)}{6-(-4)} = \frac{10}{10} = 1$$

Because the product of the slopes of the diagonals is −1, the diagonals are perpendicular.

▶ So, ▱*ABCD* is a rhombus.

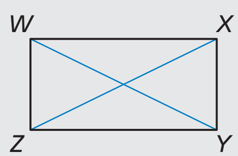
Check Check the side lengths of ▱*ABCD*. Each side has a length of $2\sqrt{17}$ units, so ▱*ABCD* is a rhombus. Check the slopes of two consecutive sides.

$$\text{slope of } \overline{AB} = \frac{8-6}{6-(-2)} = \frac{2}{8} = \frac{1}{4} \qquad \text{slope of } \overline{BC} = \frac{8-0}{6-4} = \frac{8}{2} = 4$$

Because the product of these slopes is not −1, $\overline{AB}$ is not perpendicular to $\overline{BC}$. So, ∠*ABC* is not a right angle, and ▱*ABCD* cannot be a rectangle or a square. ✓

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.



6. In rectangle *WXYZ*, $WY = 4x - 15$ and $XZ = 3x + 8$. Find the lengths of the diagonals of *WXYZ*.

7. Square *ABCD* has a diagonal $\overline{AC}$ with vertices $A(-2, 1)$ and $C(2, 4)$. Find the coordinates of the remaining vertices.

8. Decide whether ▱*PQRS* with vertices $P(-5, 2)$, $Q(0, 4)$, $R(2, -1)$, and $S(-3, -3)$ is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

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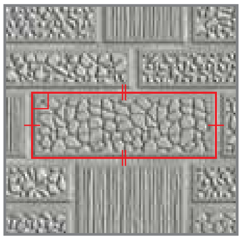
7.4 Practice WITH CalcChat® AND CalcView®

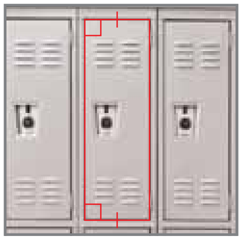
In Exercises 1–6, for any rhombus *JKLM*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning. *(See Example 1.)*

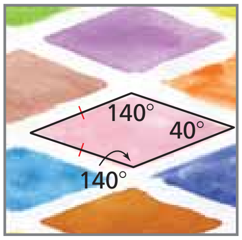
1. $\angle L \cong \angle M$
2. $\angle K \cong \angle M$
3. $\overline{JM} \cong \overline{KL}$
4. $\overline{JK} \cong \overline{KL}$
5. $\overline{JL} \cong \overline{KM}$
6. $\angle JKM \cong \angle LKM$

In Exercises 7–10, classify the quadrilateral. Explain your reasoning. *(See Example 2.)*

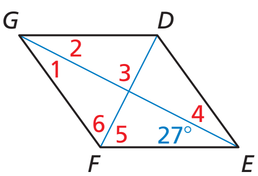
7. 

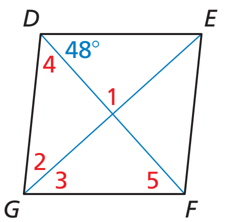
8. 

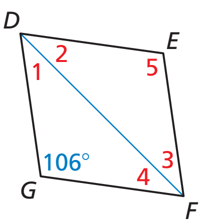
9. 

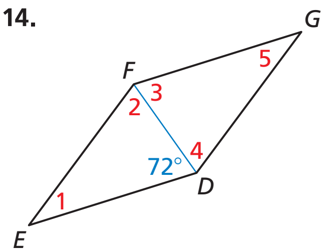
10. 

In Exercises 11–14, find the measures of the numbered angles in rhombus *DEFG*. *(See Example 3.)*

11. 

12. 

13. 

14. 

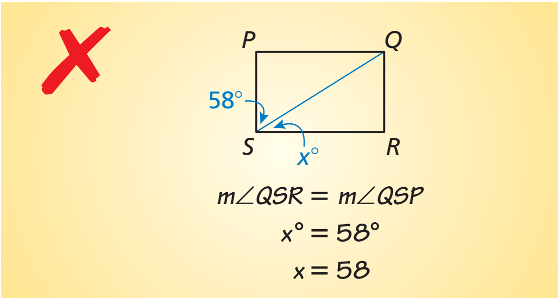
In Exercises 15–20, for any rectangle *WXYZ*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

15. $\angle W \cong \angle X$
16. $\overline{WX} \cong \overline{YZ}$
17. $\overline{WX} \cong \overline{XY}$
18. $\overline{WY} \cong \overline{XZ}$
19. $\overline{WY} \perp \overline{XZ}$
20. $\angle WXZ \cong \angle YXZ$

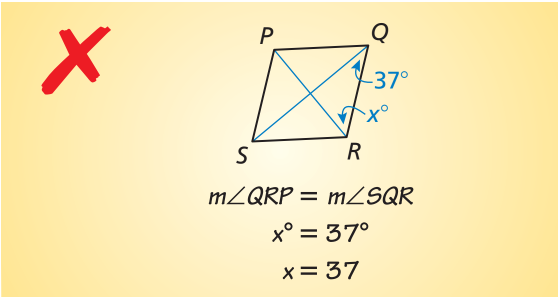
In Exercises 21–24, find the lengths of the diagonals of rectangle *WXYZ*. *(See Example 4.)*

21. $WY = 6x - 7$
    $XZ = 3x + 2$
22. $WY = 14x + 10$
    $XZ = 11x + 22$
23. $WY = 24x - 8$
    $XZ = -18x + 13$
24. $WY = 16x + 2$
    $XZ = 36x - 6$

4 MTR 25. **ERROR ANALYSIS** Quadrilateral *PQRS* is a rectangle. Describe and correct the error in finding the value of *x*.



4 MTR 26. **ERROR ANALYSIS** Quadrilateral *PQRS* is a rhombus. Describe and correct the error in finding the value of *x*.



In Exercises 27–30, square *ABCD* has a diagonal $\overline{AC}$ with the given vertices. Find the coordinates of the remaining vertices. *(See Example 5.)*

27. $A(1, 3)$ and $C(6, -2)$
28. $A(-3, 2)$ and $C(0, 5)$
29. $A(-4, -1)$ and $C(2, 5)$
30. $A(2, 2)$ and $C(6, -2)$

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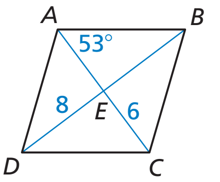
In Exercises 31–36, the diagonals of rhombus $ABCD$ intersect at E. Given that $m\angle BAC = 53°$, $DE = 8$, and $EC = 6$, find the indicated measure.



31. $m\angle DAC$

32. $m\angle AED$

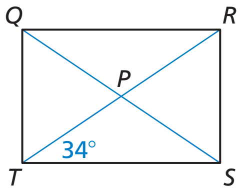
33. $m\angle ADC$

34. DB

35. AE

36. AC

In Exercises 37–42, the diagonals of rectangle $QRST$ intersect at P. Given that $m\angle PTS = 34°$ and $QS = 10$, find the indicated measure.



37. $m\angle QTR$

38. $m\angle QRT$

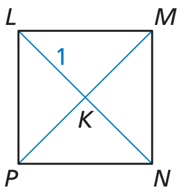
39. $m\angle SRT$

40. QP

41. RT

42. RP

In Exercises 43–48, the diagonals of square $LMNP$ intersect at K. Given that $LK = 1$, find the indicated measure.



43. $m\angle MKN$

44. $m\angle LMK$

45. $m\angle LPK$

46. KN

47. LN

48. MP

In Exercises 49–54, name each quadrilateral—*parallelogram*, *rectangle*, *rhombus*, or *square*—for which the statement is always true.

49. It is equiangular.

50. It is equiangular and equilateral.

51. The diagonals are perpendicular.

52. Opposite sides are congruent.

53. The diagonals bisect each other.

54. The diagonals bisect opposite angles.

In Exercises 55–60, decide whether ▱$JKLM$ is a rectangle, a rhombus, or a square. Give all names that apply. Explain your reasoning. *(See Example 6.)*

55. $J(-4, 2)$, $K(0, 3)$, $L(1, -1)$, $M(-3, -2)$

56. $J(-2, 7)$, $K(7, 2)$, $L(-2, -3)$, $M(-11, 2)$

57. $J(3, 1)$, $K(3, -3)$, $L(-2, -3)$, $M(-2, 1)$

58. $J(-1, 4)$, $K(-3, 2)$, $L(2, -3)$, $M(4, -1)$

59. $J(5, 2)$, $K(1, 9)$, $L(-3, 2)$, $M(1, -5)$

60. $J(5, 2)$, $K(2, 5)$, $L(-1, 2)$, $M(2, -1)$

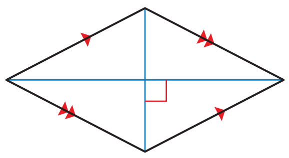
61. **COLLEGE PREP** Which name can be used to classify the quadrilateral? Select all that apply.

Ⓐ parallelogram

Ⓑ rectangle

Ⓒ rhombus

Ⓓ square



62. **REASONING** Order the terms in a diagram so that each term builds off the previous term(s). Explain why each term is in the location you chose.

quadrilateral | square

rectangle | rhombus

parallelogram

DISCUSS MATHEMATICAL THINKING (4 MTR) **In Exercises 63–68, complete each statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

63. A square is _________ a rhombus.

64. A rectangle is _________ a square.

65. A rectangle _________ has congruent diagonals.

66. The diagonals of a square _________ bisect its angles.

67. A rhombus _________ has four congruent angles.

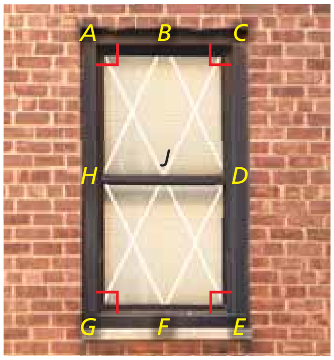
68. A rectangle _________ has perpendicular diagonals.

69. **REASONING** Which quadrilateral can be called a regular quadrilateral? Explain your reasoning.

GO DIGITAL

70. USING TOOLS You want to mark off a square region for a garden at school. You use a tape measure to mark off a quadrilateral on the ground. Each side of the quadrilateral is 2.5 meters long. Explain how you can use the tape measure to determine whether the quadrilateral is a square.

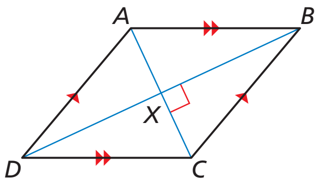
7 MTR **71. MODELING REAL LIFE** In the window, $\overline{BD} \cong \overline{DF} \cong \overline{BH} \cong \overline{HF}$. Also, $\angle HAB$, $\angle BCD$, $\angle DEF$, and $\angle FGH$ are right angles.



a. Classify *HBDF* and *ACEG*. Explain your reasoning.

b. $BD = 25$ inches and $AE = 50$ inches. What is the total length of the material used to create the white grid on the window? Explain.

72. PROVING A THEOREM Use the plan for proof to write a paragraph proof for one part of the Rhombus Diagonals Theorem.



Given *ABCD* is a parallelogram.
$\overline{AC} \perp \overline{BD}$

Prove *ABCD* is a rhombus.

Plan for Proof Because *ABCD* is a parallelogram, its diagonals bisect each other at *X*. Use $\overline{AC} \perp \overline{BD}$ to show that $\triangle BXC \cong \triangle DXC$. Then show that $\overline{BC} \cong \overline{DC}$. Use the properties of a parallelogram to show that *ABCD* is a rhombus.

73. REASONING Determine whether it is possible for a diagonal of the given quadrilateral to divide the quadrilateral into two equilateral triangles. Explain your reasoning.

a. square **b.** rhombus

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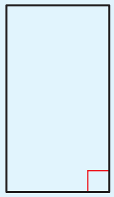
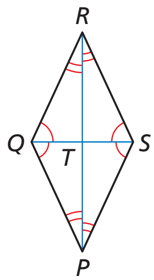
74. HOW DO YOU SEE IT?
What additional information do you need to determine whether the figure is a rectangle?



PROVING A THEOREM In Exercises 75 and 76, write a proof for part of the Rhombus Opposite Angles Theorem.

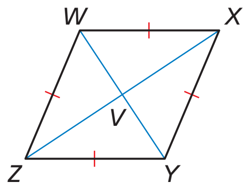
75. Given *PQRS* is a parallelogram.
$\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$.
$\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.

Prove *PQRS* is a rhombus.



76. Given *WXYZ* is a rhombus.

Prove $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$.
$\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.



PROVING A COROLLARY In Exercises 77–79, write the corollary as a conditional statement and its converse. Then explain why each statement is true.

77. Rhombus Corollary **78.** Rectangle Corollary

79. Square Corollary

4 MTR **80. MAKING AN ARGUMENT** Is it possible for a rhombus to have congruent diagonals? Explain your reasoning.

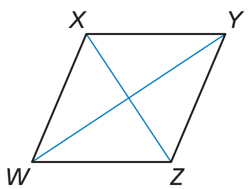
4 MTR **81. DISCUSS MATHEMATICAL THINKING** Are all rhombuses similar? Are all squares similar? Explain your reasoning.

82. THOUGHT PROVOKING
Explain why every rhombus has at least two lines of symmetry.

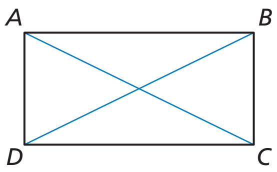
83. PROOF Write a proof.

Given $\triangle XYZ \cong \triangle XWZ$,
$\angle XYW \cong \angle ZWY$

Prove *WXYZ* is a rhombus.



84. **PROOF** Write a proof.



Given $\overline{BC} \cong \overline{AD}$,
$\overline{BC} \perp \overline{DC}$,
$\overline{AD} \perp \overline{DC}$

Prove $ABCD$ is a rectangle.

85. **DIG DEEPER** The length of one diagonal of a rhombus is 4 times the length of the other diagonal. Write an expression that represents the perimeter of the rhombus.

PROVING A THEOREM **In Exercises 86 and 87, write a proof for part of the Rectangle Diagonals Theorem.**

86. **Given** $PQRS$ is a rectangle.

Prove $\overline{PR} \cong \overline{SQ}$

87. **Given** $PQRS$ is a parallelogram.
$\overline{PR} \cong \overline{SQ}$

Prove $PQRS$ is a rectangle.

REVIEW & REFRESH

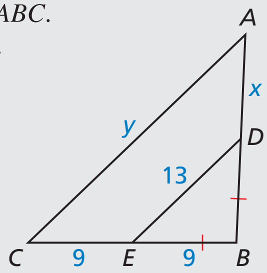
WATCH

In Exercises 88 and 89, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

88. $f(x) = -4x + 7$, $g(x) = f(x - 2)$

89. $f(x) = 6x - 4$, $g(x) = \frac{1}{2}f(x)$

90. $\overline{DE}$ is a midsegment of $\triangle ABC$. Find the values of x and y.



91. Rewrite the definition as a biconditional statement.

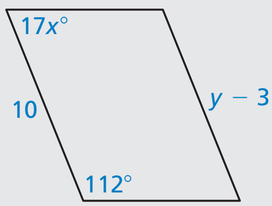
Definition A *diagonal* of a polygon is a segment that joins two nonconsecutive vertices.

In Exercises 92 and 93, solve the inequality. Graph the solution, if possible.

92. $|2h - 7| + 10 \leq 6$

93. $2(g - 4) > 3(3g + 2)$

94. Find the values of x and y in the parallelogram.



95. Determine whether the relation is a function. Explain.

Input, x	-2	-1	0	1	2
Output, y	7	4	3	4	7

96. Find the measure of each interior angle and each exterior angle of a regular 24-gon.

7 MTR **97.** **MODELING REAL LIFE** Classify the quadrilateral. Explain your reasoning.

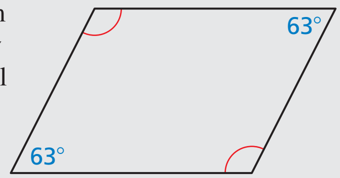
98. Find the perimeter and area of $\triangle PQR$ with vertices $P(3, -2)$, $Q(3, 4)$, and $R(6, 2)$.

In Exercises 99 and 100, decide whether you can use the given information to prove that $\triangle JKL \cong \triangle XYZ$. Explain your reasoning.

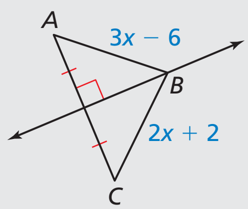
99. $\angle K \cong \angle Y$, $\overline{JL} \cong \overline{XZ}$, $\overline{JK} \cong \overline{XY}$

100. $\angle L \cong \angle Z$, $\angle J \cong \angle X$, $\overline{JL} \cong \overline{YZ}$

101. State which theorem you can use to show that the quadrilateral is a parallelogram.



102. Find the length of $\overline{AB}$. Explain your reasoning.



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