5.2 Congruent Polygons

Learning Target: Understand congruence in terms of rigid motions.

Success Criteria:
- I can use rigid motions to show that two triangles are congruent.
- I can identify corresponding parts of congruent polygons.
- I can use congruent polygons to solve problems.

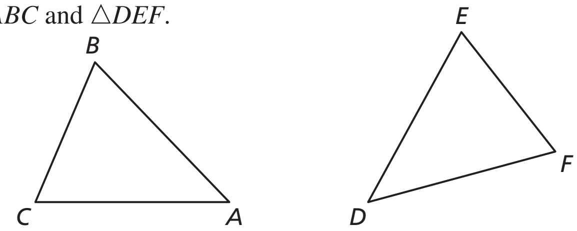
EXPLORE IT! Describing Rigid Motions

4 MTR COMMUNICATE CLEARLY

When the corresponding sides and corresponding angles of $\triangle ABC$ and $\triangle DEF$ are congruent, can you conclude that a rigid motion or a composition of rigid motions maps $\triangle ABC$ to $\triangle FED$? Explain.

Work with a partner.

a. Consider $\triangle ABC$ and $\triangle DEF$.



i. Explain why the statement is true.

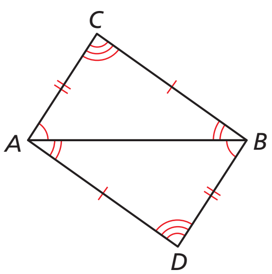
> If a rigid motion or a composition of rigid motions maps ΔABC to ΔDEF, then the corresponding sides and corresponding angles of ΔABC and ΔDEF are congruent.

ii. Show why the statement is true.

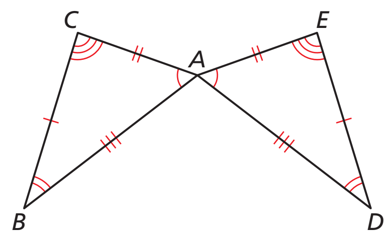
> If the corresponding sides and corresponding angles of ΔABC and ΔDEF are congruent, then a rigid motion or a composition of rigid motions maps ΔABC to ΔDEF.

b. For each pair of triangles, describe a rigid motion or a composition of rigid motions that maps one of the triangles to the other. Then explain what you can conclude about each pair of triangles.

i. $\triangle ABC$ and $\triangle BAD$



ii. $\triangle ABC$ and $\triangle ADE$



Geometric Reasoning

MA.912.GR.1.6 Solve mathematical and real-world problems involving congruence or similarity in two-dimensional figures.

MA.912.GR.2.6 Apply rigid transformations to map one figure onto another to justify that the two figures are congruent.

H Also MA.912.GR.2.7

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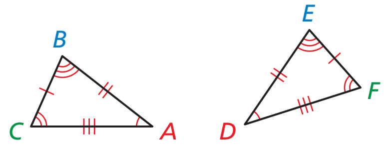
Identifying and Using Corresponding Parts

Vocabulary AZ VOCAB

corresponding parts, *p. 238*

Recall that two geometric figures are congruent if and only if a rigid motion or a composition of rigid motions maps one of the figures onto the other. A rigid motion maps each part of a figure to a **corresponding part** of its image. Because rigid motions preserve length and angle measure, corresponding parts of congruent figures are congruent. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

When $\triangle DEF$ is the image of $\triangle ABC$ after a rigid motion or a composition of rigid motions, you can write congruence statements for the corresponding angles and corresponding sides.



Corresponding angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Corresponding sides

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

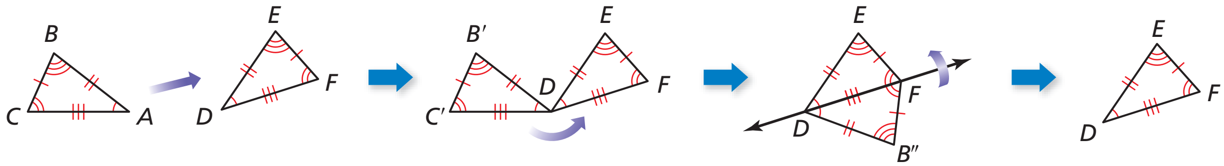
STUDY TIP

Notice that both of the following statements are true.

1. If two triangles are congruent, then all their corresponding parts are congruent.
2. If all the corresponding parts of two triangles are congruent, then the triangles are congruent.

When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles above are $\triangle ABC \cong \triangle DEF$ or $\triangle BCA \cong \triangle EFD$.

When all the corresponding parts of two triangles are congruent, you can show that the triangles are congruent. Using the triangles above, first translate $\triangle ABC$ so that point A maps to point D. This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$. Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$.

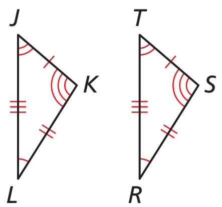


Now, reflect $\triangle DB''F$ in the line through points D and F. This reflection maps the sides and angles of $\triangle DB''F$ to the corresponding sides and corresponding angles of $\triangle DEF$, so $\triangle ABC \cong \triangle DEF$.

So, to show that two triangles are congruent, it is sufficient to show that their corresponding parts are congruent. In general, this is true for all polygons.

STUDY TIP

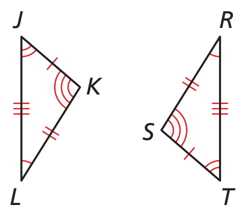
To help you identify corresponding parts, rotate $\triangle TSR$.



EXAMPLE 1 Identifying Corresponding Parts

WATCH

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



SOLUTION

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

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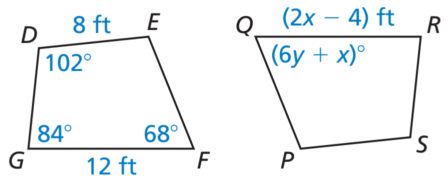
EXAMPLE 2 Using Properties of Congruent Figures

WATCH

In the diagram, $DEFG \cong SPQR$.

a. Find the value of x.

b. Find the value of y.



SOLUTION

a. You know that $\overline{FG} \cong \overline{QR}$.

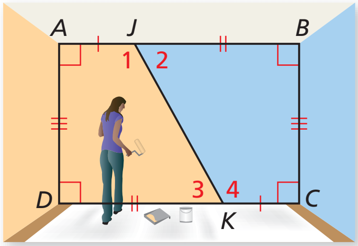
$$FG = QR$$
$$12 = 2x - 4$$
$$16 = 2x$$
$$8 = x$$

b. You know that $\angle F \cong \angle Q$.

$$m\angle F = m\angle Q$$
$$68^\circ = (6y + x)^\circ$$
$$68 = 6y + 8$$
$$10 = y$$

EXAMPLE 3 Showing that Figures Are Congruent

WATCH



You divide a wall into orange and blue sections along $\overline{JK}$. Will the sections of the wall be the same size and shape? Explain.

SOLUTION

From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{AB} \parallel \overline{DC}$. Then $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent. The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property of Segment Congruence, $\overline{JK} \cong \overline{KJ}$. So, all pairs of corresponding sides are congruent. Because all corresponding parts are congruent, $AJKD \cong CKJB$.

▶ Yes, the two sections will be the same size and shape.

STUDY TIP

The properties of congruence that are true for segments and angles are also true for triangles.

THEOREM

5.3 Properties of Triangle Congruence

Triangle congruence is reflexive, symmetric, and transitive.

Reflexive For any triangle $\triangle ABC$, $\triangle ABC \cong \triangle ABC$.

Symmetric If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

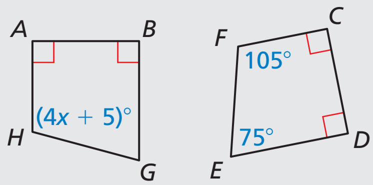
Transitive If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Proof *BigIdeasMath.com*

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

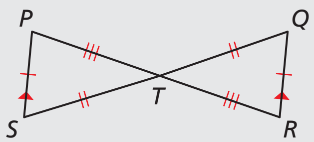
In the diagram, $ABGH \cong CDEF$.



1. Identify all pairs of congruent corresponding parts.
2. Find the value of x.

4 MTR

3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find *both* answers.



Is $\triangle PTS \cong \triangle RTQ$?	Is $\triangle TSP \cong \triangle TQR$?
Is $\triangle SPT \cong \triangle QRT$?	Is $\triangle TPS \cong \triangle TQR$?

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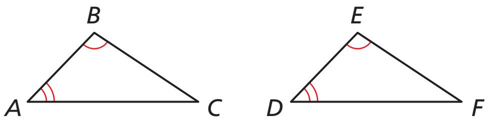
Using the Third Angles Theorem

THEOREM

5.4 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

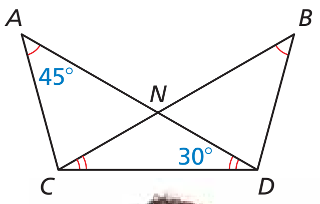


If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

Prove this Theorem Exercise 21, page 242

EXAMPLE 4 Using the Third Angles Theorem

WATCH



Find $m\angle BDC$.

SOLUTION

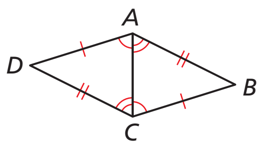
$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

▶ So, $m\angle BDC = m\angle ACD = 105°$ by the definition of congruent angles.

EXAMPLE 5 Proving that Triangles Are Congruent

WATCH

Use the information in the figure to prove that $\triangle ACD \cong \triangle CAB$.



SOLUTION

Given $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

Prove $\triangle ACD \cong \triangle CAB$

Plan for Proof

a. Use the Reflexive Property of Segment Congruence to show that $\overline{AC} \cong \overline{CA}$.

b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

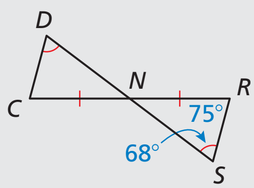
Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	**1.** Given
a.	**2.** $\overline{AC} \cong \overline{CA}$	**2.** Reflexive Property of Segment Congruence
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	**3.** Given
b.	**4.** $\angle B \cong \angle D$	**4.** Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	**5.** All corresponding parts are congruent.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram.



4. Find $m\angle DCN$.

5. REASONING What additional information is needed to conclude that $\triangle NDC \cong \triangle NSR$?

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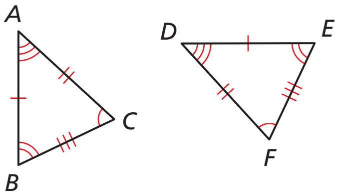
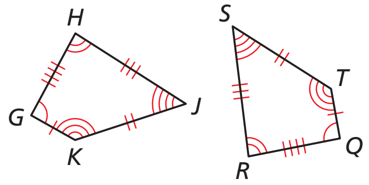
5.2 Practice WITH CalcChat® AND CalcView®

In Exercises 1 and 2, write a congruence statement for the polygons. Identify all pairs of congruent corresponding parts. *(See Example 1.)*

▶ **1.**



2.



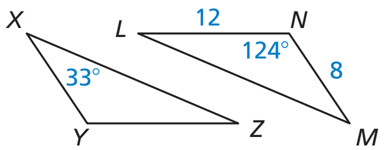
In Exercises 3–6, $\triangle XYZ \cong \triangle MNL$. Complete the statement.

3. $m\angle Y =$ ______

4. $m\angle M =$ ______

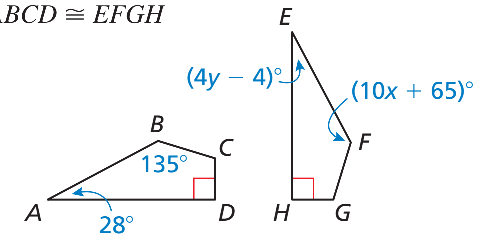
5. $m\angle Z =$ ______

6. $XY =$ ______

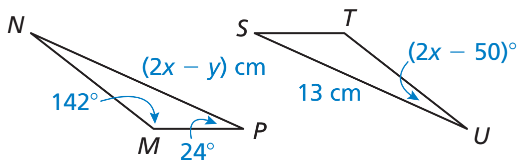


In Exercises 7 and 8, find the values of x and y. *(See Example 2.)*

▶ **7.** $ABCD \cong EFGH$

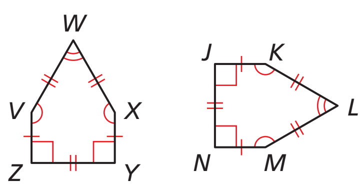


8. $\triangle MNP \cong \triangle TUS$

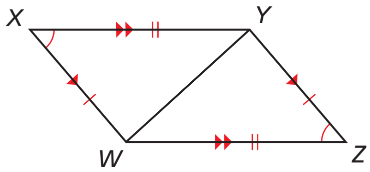


In Exercises 9 and 10, show that the polygons are congruent. Explain your reasoning. *(See Example 3.)*

▶ **9.**



10.



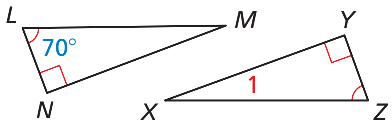
In Exercises 11 and 12, name the property that the statement illustrates.

11. If $\triangle QRS \cong \triangle TUV$ and $\triangle TUV \cong \triangle XYZ$, then $\triangle QRS \cong \triangle XYZ$.

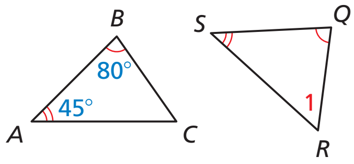
12. If $\triangle EFG \cong \triangle JKL$, then $\triangle JKL \cong \triangle EFG$.

In Exercises 13 and 14, find $m\angle 1$. *(See Example 4.)*

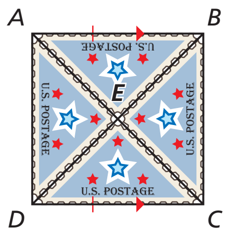
▶ **13.**



14.



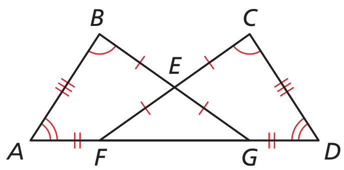
15. **PROOF** Triangular postage stamps, like the ones shown, are highly valued by stamp collectors. Prove that $\triangle AEB \cong \triangle CED$. *(See Example 5.)*



Given $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$, E is the midpoint of $\overline{AC}$ and $\overline{BD}$.

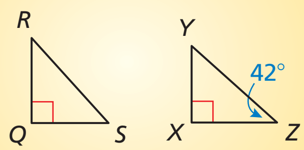
Prove $\triangle AEB \cong \triangle CED$

16. **PROOF** Use the information in the figure to prove that $\triangle ABG \cong \triangle DCF$.



17. **HELP A CLASSMATE** Explain to a classmate how to find $m\angle R$.

Given $\triangle QRS \cong \triangle XZY$



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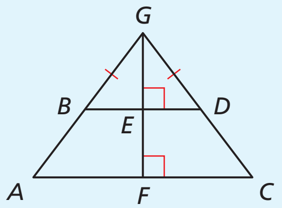
18. HOW DO YOU SEE IT?
In the diagram, $ABEF \cong CDEF$.



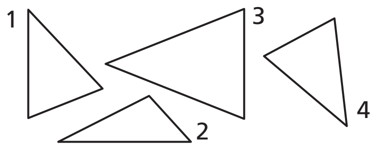
a. Explain how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.

b. Explain how you know that $\angle GBE \cong \angle GDE$.

c. Explain how you know that $\angle GEB \cong \angle GED$.

d. Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? Explain.

19. REASONING Which of the triangles appear congruent? Which of the triangles appear similar? Explain your reasoning. How can you justify your answer?



20. REASONING $\triangle JKL$ is congruent to $\triangle XYZ$. Identify all pairs of congruent corresponding parts.

21. PROVING A THEOREM Prove the Third Angles Theorem by using the Triangle Sum Theorem.

22. THOUGHT PROVOKING
Draw a triangle. Copy the triangle multiple times to create a rug design made of congruent triangles. Which property guarantees that all the triangles are congruent?

5 MTR **CONNECTING CONCEPTS** **In Exercises 23 and 24, use the given information to write and solve a system of linear equations to find the values of x and y.**

23. $\triangle LMN \cong \triangle PQR$, $m\angle L = 40°$, $m\angle M = 90°$, $m\angle P = (17x - y)°$, $m\angle R = (2x + 4y)°$

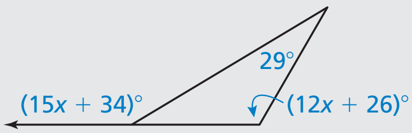
24. $\triangle STU \cong \triangle XYZ$, $m\angle T = 28°$, $m\angle U = (4x + y)°$, $m\angle X = 130°$, $m\angle Y = (8x - 6y)°$

25. PROOF Prove that the criteria for congruent triangles are equivalent to the definition of congruence in terms of rigid motions.

REVIEW & REFRESH

WATCH

26. Find the measure of the exterior angle.

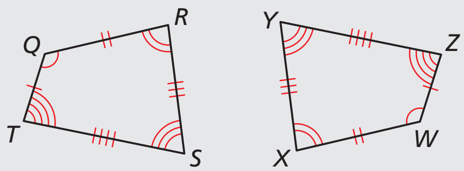


In Exercises 27 and 28, graph $\triangle FGH$ with vertices $F(-6, 3)$, $G(3, 0)$, and $H(3, -6)$ and its image after the similarity transformation.

27. **Translation:** $(x, y) \rightarrow (x + 2, y - 1)$
Dilation: $(x, y) \rightarrow (2x, 2y)$

28. **Dilation:** $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$
Reflection: in the y-axis

29. Write a congruence statement for the quadrilaterals. Identify all pairs of congruent corresponding parts.



7 MTR **30. MODELING REAL LIFE** You design a logo for your soccer team. The logo is 3 inches by 5 inches. You decide to dilate the logo to 1.5 inches by 2.5 inches. What is the scale factor of this dilation?

In Exercises 31 and 32, factor the polynomial.

31. $t^2 + 7t + 10$

32. $2x^2 + 5x - 12$

In Exercises 33 and 34, using f, graph g. Describe the transformation from the graph of f to the graph of g.

33. $f(x) = 2x$; $g(x) = 4f(x)$

34. $f(x) = x^2 + 3$; $g(x) = f(x - 2)$

In Exercises 35 and 36, determine whether the equation represents a *linear* or *nonlinear* function. Explain.

35. $y = 5 - 2x$

36. $18 = -y + x^2$

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