4.5 Dilations

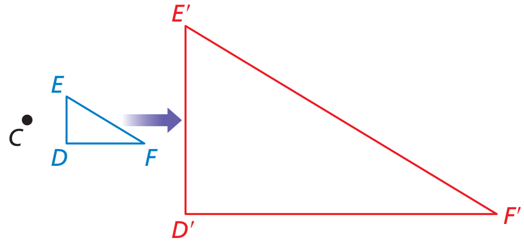
Learning Target: Understand dilations of figures.

Success Criteria:
- I can identify dilations.
- I can dilate figures.
- I can solve real-life problems involving scale factors and dilations.

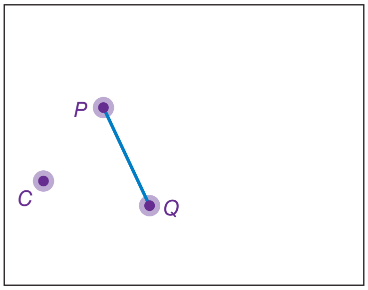
EXPLORE IT! Dilating Figures

Work with a partner.

a. The diagram shows a *dilation* of $\triangle DEF$ to $\triangle D'E'F'$. How would you define a dilation?



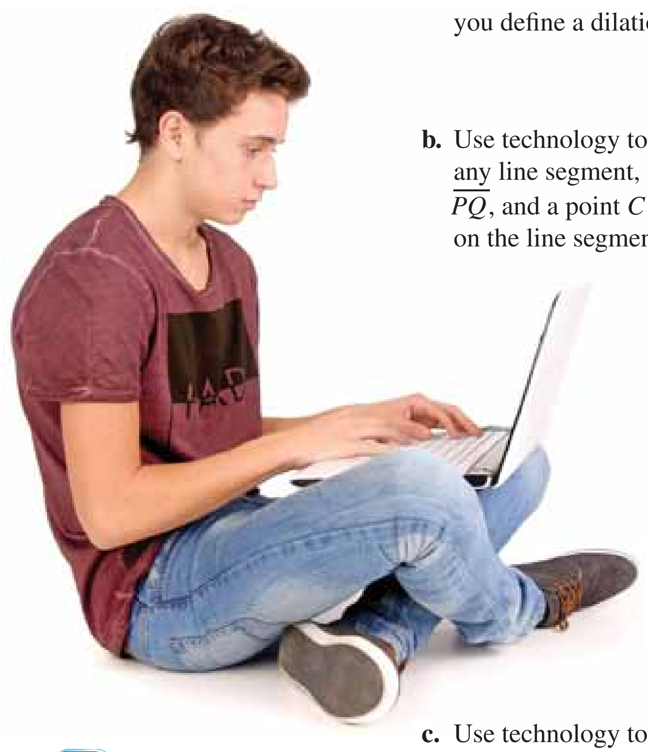
b. Use technology to draw any line segment, $\overline{PQ}$, and a point C not on the line segment.



i. *Dilate* $\overline{PQ}$ using a *scale factor* of 2 and the *center of dilation* C to form $\overline{P'Q'}$. What do you notice? Make several observations.

ii. Choose two other scale factors, where one scale factor is greater than 1 and the other scale factor is between 0 and 1. What conclusions can you make?

iii. What scale factor results in $\overline{PQ}$ and $\overline{P'Q'}$ being congruent?



c. Use technology to draw any $\triangle PQR$ and a point C not in $\triangle PQR$.

i. Dilate $\triangle PQR$ using the center of dilation C and several different scale factors to form $\triangle P'Q'R'$. Compare $\triangle PQR$ and $\triangle P'Q'R'$.

ii. Make a conjecture about the side lengths and angle measures of the image of $\triangle PQR$ after a dilation with a scale factor of k.

d. Based on your results in parts (b) and (c), is there anything you would like to change or include in your definition in part (a)? Explain.

e. Is a dilation a rigid motion? Explain your reasoning.

4 MTR CONSTRUCT AN ARGUMENT
What happens to a figure when a scale factor is less than 0?

Geometric Reasoning
MA.912.GR.2.1 Given a preimage and image, describe the transformation and represent the transformation algebraically using coordinates.
MA.912.GR.2.5 Given a geometric figure and a sequence of transformations, draw the transformed figure on a coordinate plane.
Also MA.912.GR.2.2

GO DIGITAL

Identifying and Performing Dilations

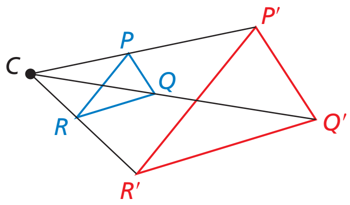
Vocabulary
dilation, *p. 204*
center of dilation, *p. 204*
scale factor, *p. 204*
enlargement, *p. 204*
reduction, *p. 204*

KEY IDEA
Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

- If P is the center of dilation C, then $P = P'$.
- If P is not the center of dilation C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \dfrac{CP'}{CP}$.
- Angle measures are preserved.



A dilation does not change any line that passes through the center of dilation. A dilation maps a line that does not pass through the center of dilation to a parallel line. In the figure above, $\overleftrightarrow{PR} \parallel \overleftrightarrow{P'R'}$, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{P'Q'}$, and $\overleftrightarrow{QR} \parallel \overleftrightarrow{Q'R'}$.

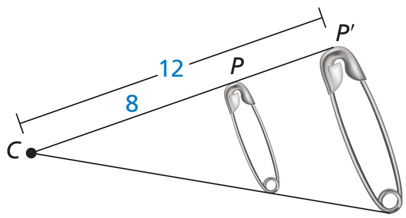
When the scale factor $k > 1$, a dilation is an **enlargement**. When $0 < k < 1$, a dilation is a **reduction**.

READING
The scale factor of a dilation can be written as a fraction, decimal, or percent.

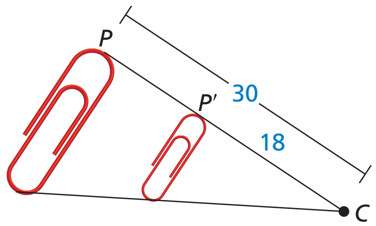
EXAMPLE 1 Identifying Dilations

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

a.



b.



SOLUTION

a. Because $\dfrac{CP'}{CP} = \dfrac{12}{8}$, the scale factor is $k = \dfrac{3}{2}$. So, the dilation is an enlargement.

b. Because $\dfrac{CP'}{CP} = \dfrac{18}{30}$, the scale factor is $k = \dfrac{3}{5}$. So, the dilation is a reduction.

SELF-ASSESSMENT
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. In a dilation, $CP' = 3$ and $CP = 12$. Find the scale factor. Then tell whether the dilation is a *reduction* or an *enlargement*.

4 MTR **2.** **WHICH ONE DOESN'T BELONG?** Which scale factor does *not* belong with the other three? Explain your reasoning.

$\frac{5}{4}$	60%	115%	2

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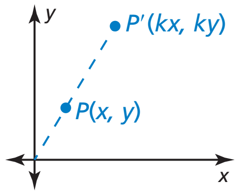
KEY IDEA

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.



$(x, y) \rightarrow (kx, ky)$

STUDY TIP

In this chapter, for all dilations in the coordinate plane, the center of dilation is the origin unless otherwise noted.

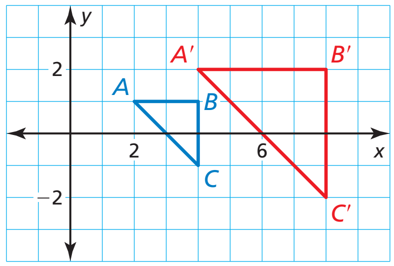
EXAMPLE 2 Dilating a Figure in the Coordinate Plane

WATCH

Graph $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 1)$, and $C(4, -1)$ and its image after a dilation with a scale factor of 2.

SOLUTION

Use the coordinate rule for a dilation with $k = 2$ to find the coordinates of the vertices of the image. Then graph $\triangle ABC$ and its image.



$$(x, y) \rightarrow (2x, 2y)$$

$$A(2, 1) \rightarrow A'(4, 2)$$

$$B(4, 1) \rightarrow B'(8, 2)$$

$$C(4, -1) \rightarrow C'(8, -2)$$

Notice the relationships between the lengths and slopes of the sides of the triangles in Example 2. Each side length of $\triangle A'B'C'$ is longer than its corresponding side by the scale factor. The corresponding sides are parallel because their slopes are the same.

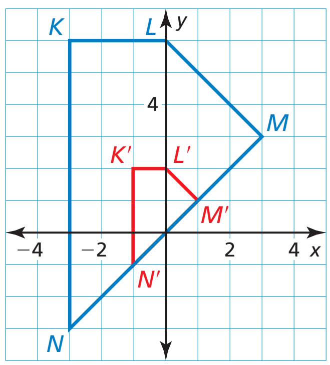
EXAMPLE 3 Dilating a Figure in the Coordinate Plane

WATCH

Graph quadrilateral $KLMN$ with vertices $K(-3, 6)$, $L(0, 6)$, $M(3, 3)$, and $N(-3, -3)$ and its image after a dilation with a scale factor of $\frac{1}{3}$.

SOLUTION

Use the coordinate rule for a dilation with $k = \frac{1}{3}$ to find the coordinates of the vertices of the image. Then graph quadrilateral $KLMN$ and its image.



$$(x, y) \rightarrow \left(\tfrac{1}{3}x, \tfrac{1}{3}y\right)$$

$$K(-3, 6) \rightarrow K'(-1, 2)$$

$$L(0, 6) \rightarrow L'(0, 2)$$

$$M(3, 3) \rightarrow M'(1, 1)$$

$$N(-3, -3) \rightarrow N'(-1, -1)$$

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph $\triangle PQR$ and its image after a dilation with scale factor k.

3. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$

4. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$

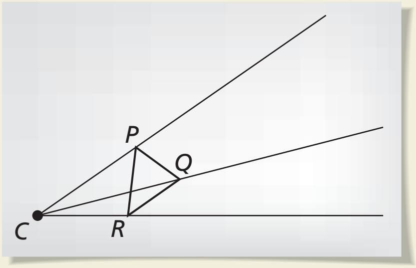
GO DIGITAL

CONSTRUCTION Constructing a Dilation

Use a compass and straightedge to construct a dilation of $\triangle PQR$ with a scale factor of 2. Use a point C outside the triangle as the center of dilation.

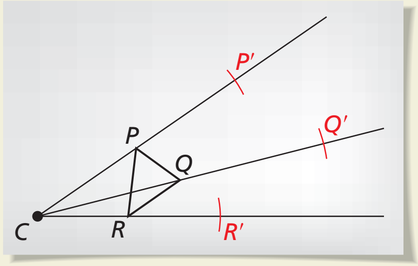
SOLUTION

Step 1



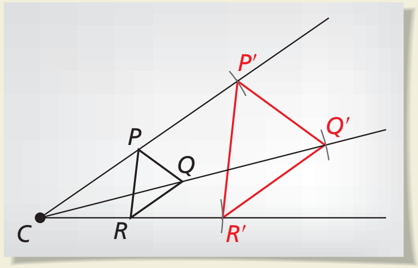
Draw a triangle Draw $\triangle PQR$ and choose the center of the dilation C outside the triangle. Draw rays from C through the vertices of the triangle.

Step 2

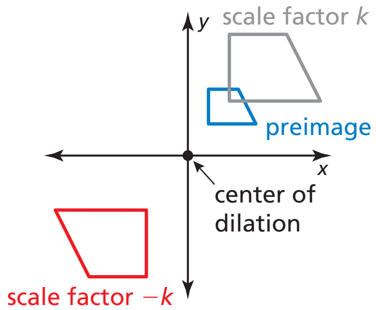


Use a compass Use a compass to locate P' on $\overrightarrow{CP}$ so that $CP' = 2(CP)$. Locate Q' and R' using the same method.

Step 3



Connect points Connect points P', Q', and R' to form $\triangle P'Q'R'$.



Scale factors can be negative numbers. When this occurs, the figure rotates 180°. So, when $k > 0$, a dilation with a scale factor of $-k$ is the same as the composition of a dilation with a scale factor of k followed by a rotation of 180° about the center of dilation. Using the coordinate rules for a dilation and a rotation of 180°, you can think of the notation as

$$(x, y) \rightarrow (kx, ky) \rightarrow (-kx, -ky).$$

EXAMPLE 4 Using a Negative Scale Factor

Graph $\triangle FGH$ with vertices $F(-4, -2)$, $G(-2, 4)$, and $H(-2, -2)$ and its image after a dilation with a scale factor of $-\frac{1}{2}$.

SOLUTION

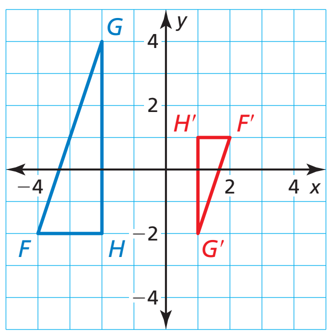
Use the coordinate rule for a dilation with $k = -\frac{1}{2}$ to find the coordinates of the vertices of the image. Then graph $\triangle FGH$ and its image.

$$(x, y) \rightarrow \left(-\tfrac{1}{2}x, -\tfrac{1}{2}y\right)$$

$F(-4, -2) \rightarrow F'(2, 1)$

$G(-2, 4) \rightarrow G'(1, -2)$

$H(-2, -2) \rightarrow H'(1, 1)$



SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

5. Graph $\triangle PQR$ with vertices $P(1, 2)$, $Q(3, 1)$, and $R(1, -3)$ and its image after a dilation with a scale factor of -2.

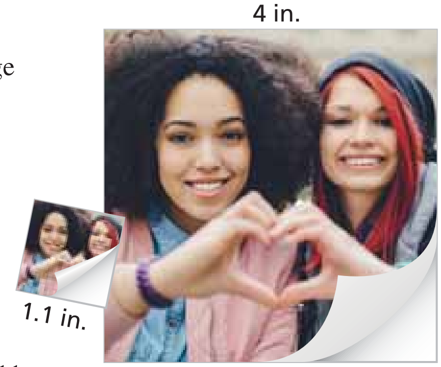
6. **REASONING** A polygon with a vertex at the origin is dilated. Explain why the corresponding vertex of the image is also at the origin.

GO DIGITAL

Solving Real-Life Problems

EXAMPLE 5 Finding a Scale Factor

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?



READING
A scale factor is written so that the units in the numerator and denominator divide out.

SOLUTION

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$.

▶ So, in simplest form, the scale factor is $\frac{11}{40}$.

EXAMPLE 6 Finding the Length of an Image

You are using a magnifying glass that shows the image of an object as six times the object's actual size. Determine the length of the image of the spider seen through the magnifying glass.

SOLUTION

(cm) (cm)

$$\frac{\text{image length}}{\text{actual length}} = k$$ Write ratio of corresponding lengths.

$$\frac{x}{1.5} = 6$$ Substitute values.

$$x = 9$$ Multiply each side by 1.5.

▶ So, the image length seen through the magnifying glass is 9 centimeters.

SELF-ASSESSMENT

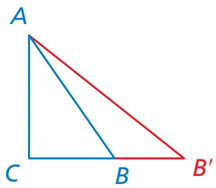
1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

7. An optometrist dilates the pupils of a patient's eyes to get a better look at the back of the eyes. A pupil dilates from 4.5 millimeters to 8 millimeters. What is the scale factor of this dilation?

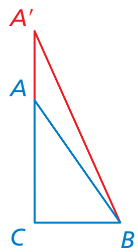
8. The image of another spider seen through the magnifying glass in Example 6 is shown at the right. Find the actual length of the spider.

When a transformation, such as a dilation, changes the shape or size of a figure, the transformation is *nonrigid*. In addition to dilations, there are many possible nonrigid transformations. Two examples are shown. It is important to pay close attention to whether a nonrigid transformation preserves lengths and angle measures.

Horizontal Stretch



Vertical Stretch

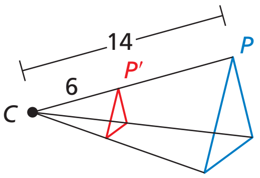


GO DIGITAL

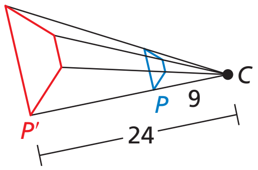
4.5 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*. *(See Example 1.)*

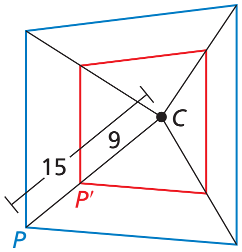
1.



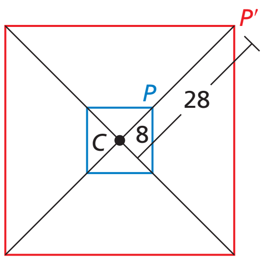
2.



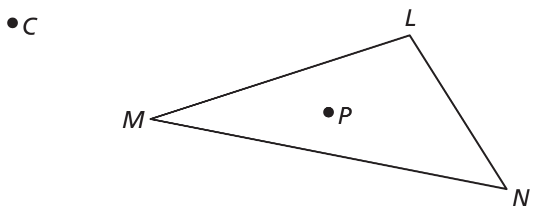
3.



4.



CONSTRUCTION In Exercises 5–8, copy the diagram. Then use a compass and straightedge to construct a dilation of $\triangle LMN$ with the given center and scale factor k.



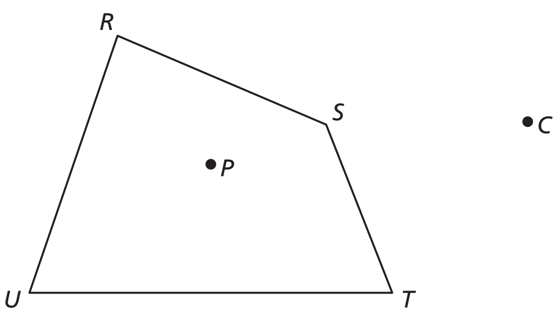
5. Center C, $k = 2$

6. Center P, $k = 3$

7. Center M, $k = \frac{1}{2}$

8. Center C, $k = 25\%$

CONSTRUCTION In Exercises 9–12, copy the diagram. Then use a compass and straightedge to construct a dilation of quadrilateral $RSTU$ with the given center and scale factor k.



9. Center P, $k = 2$

10. Center C, $k = 3$

11. Center C, $k = 75\%$

12. Center R, $k = 0.25$

In Exercises 13–18, graph the polygon with the given vertices and its image after a dilation with scale factor k. *(See Examples 2 and 3.)*

13. $X(6, -1)$, $Y(-2, -4)$, $Z(1, 2)$; $k = 3$

14. $Q(-3, -2)$, $R(1, 2)$, $S(4, 1)$; $k = 4$

15. $A(0, 5)$, $B(-10, -5)$, $C(5, -5)$; $k = 120\%$

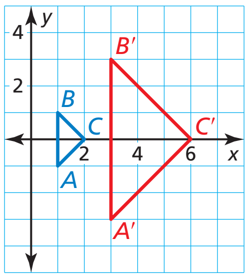
16. $D(-6, 4)$, $E(-4, -6)$, $F(4, 8)$; $k = 50\%$

17. $J(4, 0)$, $K(-8, 4)$, $L(0, -4)$, $M(12, -8)$; $k = 0.25$

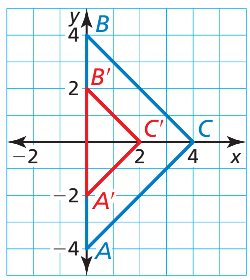
18. $T(9, -3)$, $U(6, 0)$, $V(3, 9)$, $W(0, 0)$; $k = \frac{2}{3}$

In Exercises 19 and 20, write a coordinate rule for the dilation.

19.



20.



In Exercises 21–24, graph the polygon with the given vertices and its image after a dilation with scale factor k. *(See Example 4.)*

21. $B(-5, -10)$, $C(-10, 15)$, $D(0, 5)$; $k = -\frac{1}{5}$

22. $L(0, 0)$, $M(-4, 1)$, $N(-3, -6)$; $k = -3$

23. $R(-7, -1)$, $S(2, 5)$, $T(-2, -3)$, $U(-3, -3)$; $k = -4$

24. $W(8, -2)$, $X(6, 0)$, $Y(-6, 4)$, $Z(-2, 2)$; $k = -0.5$

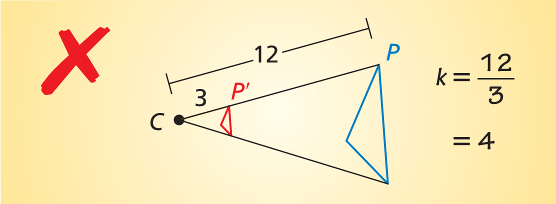
25. FINDING A SCALE FACTOR You receive wallet-sized photos of your school picture. The photo is 2.5 inches by 3.5 inches. You ask the photographer to dilate the photo to 5 inches by 7 inches. What is the scale factor of this dilation? *(See Example 5.)*

26. FINDING A SCALE FACTOR Your friend asks you to enlarge your notes to study because your writing is small. Your writing covers 7.5 inches by 10 inches on a piece of paper. The writing on the enlarged copy has a smaller side with a length of 9 inches. What is the scale factor of this dilation?

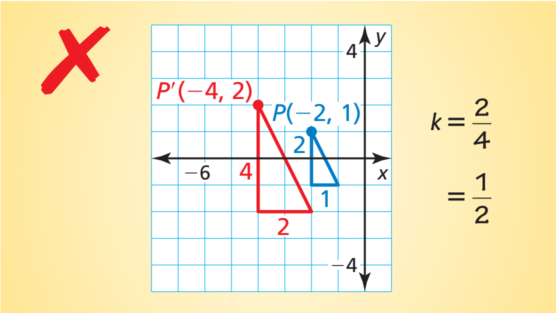
GO DIGITAL

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in finding the scale factor of the dilation.

27.



28.



In Exercises 29–32, determine whether the dilated figure or the original figure is closer to the center of dilation. Use the given location of the center of dilation and the scale factor *k*.

29. Center of dilation: inside the figure; $k = 3$

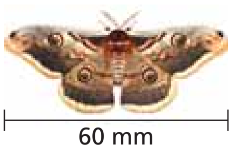
30. Center of dilation: inside the figure; $k = \frac{1}{2}$

31. Center of dilation: outside the figure; $k = 0.1$

32. Center of dilation: outside the figure; $k = 120\%$

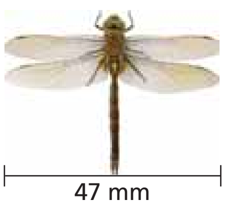
In Exercises 33–36, you are using a magnifying glass. Use the actual length of the insect and the magnification level to determine the length of the image of the insect seen through the magnifying glass. *(See Example 6.)*

33. emperor moth
Magnification: 5×

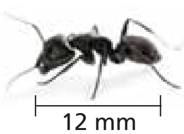


34. ladybug
Magnification: 10×

35. dragonfly
Magnification: 20×



36. carpenter ant
Magnification: 15×



GO DIGITAL

37. REASONING Use the given actual and magnified lengths to determine which of the following insects were seen using the same magnification level. Explain your reasoning.

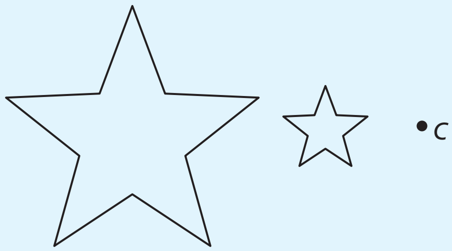
grasshopper
Actual: 2 in.
Magnified: 15 in.

black beetle
Actual: 0.6 in.
Magnified: 4.2 in.

honeybee
Actual: $\frac{5}{8}$ in.
Magnified: $\frac{75}{16}$ in.

monarch butterfly
Actual: 3.9 in.
Magnified: 29.25 in.

38. HOW DO YOU SEE IT? Point *C* is the center of dilation of the figures. The scale factor is $\frac{1}{3}$. Which figure is the preimage? Which figure is the image? Explain your reasoning.



39. REASONING You have a 4-inch by 6-inch photo from the school dance. You have an 8-inch by 10-inch frame. Can you enlarge the photo without cropping to fit the frame? Explain your reasoning.

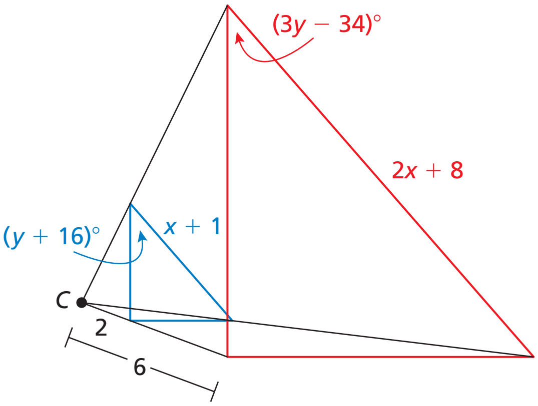
40. WRITING Is a scale factor of 2 the same as a scale factor of 200%? Explain your reasoning.

41. DISCUSS MATHEMATICAL THINKING Dilate the line through $O(0, 0)$ and $A(1, 2)$ using a scale factor of 2.

a. What do you notice about the lengths of $\overline{O'A'}$ and $\overline{OA}$?

b. What do you notice about $\overleftrightarrow{O'A'}$ and $\overleftrightarrow{OA}$?

5 MTR **42.** **CONNECTING CONCEPTS** The larger triangle is a dilation of the smaller triangle. Find the values of x and y.



4 MTR **DISCUSS MATHEMATICAL THINKING** **In Exercises 43 and 44, graph △*ABC* with vertices $A(-3, 4)$, $B(-4, 2)$, and $C(0, -4)$ and its image after the transformation. Then determine whether the transformation is a dilation. Explain your reasoning.**

43. $(x, y) \to (2x, y)$

44. $(x, y) \to (x, 3y)$

45. **REASONING** You put a reduction of a rectangle on the original rectangle. Explain why there is a point that is in the same location on both rectangles.

4 MTR **46.** **MAKING AN ARGUMENT** Your friend claims that dilating a figure by 1 is the same as dilating the figure by −1 because the original figure will not be enlarged or reduced. Is your friend correct? Explain your reasoning.

5 MTR **47.** **STRUCTURE** Rectangle *WXYZ* has vertices $W(-3, -1)$, $X(-3, 3)$, $Y(5, 3)$, and $Z(5, -1)$.

a. Find the perimeter and the area of the rectangle.

b. Dilate the rectangle using a scale factor of 3. Find the perimeter and the area of the image. Compare the perimeters and the areas of the rectangles. What do you notice?

c. Repeat part (b) using a scale factor of $\frac{1}{4}$.

d. Make conjectures for how the perimeter and area change when a figure is dilated.

48. **THOUGHT PROVOKING**
Explain why a dilation with a negative scale factor results in a rotation.

49. **DIG DEEPER** △*ABC* has vertices $A(4, 2)$, $B(4, 6)$, and $C(7, 2)$. Find the coordinates of the vertices of the image after a dilation with center $(4, 0)$ and scale factor 2.

REVIEW & REFRESH

WATCH

In Exercises 50–53, graph the polygon with the given vertices and its image after the indicated transformation.

50. $A(2, -1)$, $B(0, 4)$, $C(-3, 5)$
Translation: $(x, y) \to (x - 1, y + 3)$

51. $A(-5, 6)$, $B(-7, 8)$, $C(-3, 11)$
Reflection: in the x-axis

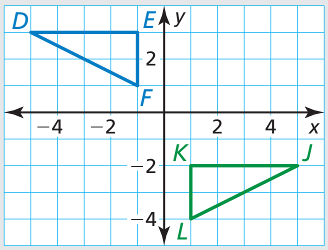
52. $D(-3, 2)$, $E(-1, 4)$, $F(1, 2)$, $G(1, -1)$
Rotation: 270° counterclockwise about the origin

53. $J(0, 4)$, $K(4, 4)$, $L(8, 0)$, $M(4, 0)$
Dilation: scale factor $k = 75\%$

54. Simplify $\dfrac{4^{-2}b^{-3}}{2^{-1}a^{0}b^{-4}}$.

7 MTR **55.** **MODELING REAL LIFE** You are painting a rectangular canvas that is 30 inches wide and 40 inches long. Your friend is painting a rectangular canvas, where the width and length are each x inches shorter. When $x = 3$, what is the area of your friend's canvas?

56. Describe a congruence transformation that maps the blue preimage to the green image.



57. Graph $g(x) = 3(x - 1)^2 + 7$. Compare the graph to the graph of $f(x) = x^2$.

In Exercises 58 and 59, find the product.

58. $(3x - 4)^2$

59. $(w - 5)(6 + 2w)$

3 MTR **CHOOSE A METHOD** **In Exercises 60 and 61, solve the system using any method. Explain your choice of method.**

60. $y = -3x + 5$
$2x + 4y = 15$

61. $0.6y + 0.5x = 1$
$0.25x = -0.5y + 2$

GO DIGITAL