3.4 Proofs with Perpendicular Lines

Learning Target: Prove and use theorems about perpendicular lines.

Success Criteria:

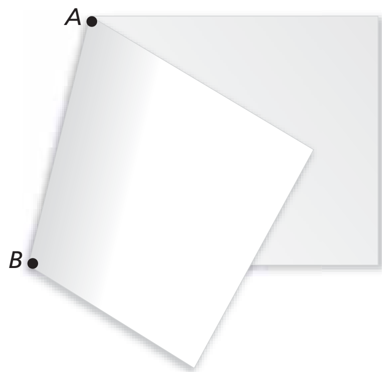
- I can find the distance from a point to a line.
- I can construct perpendicular lines and perpendicular bisectors.
- I can prove theorems about perpendicular lines.

EXPLORE IT! Constructing Perpendicular Lines

Work with a partner.

a. Use a piece of paper.

i. Fold and crease the piece of paper, as shown. Label the ends of the crease as *A* and *B*.

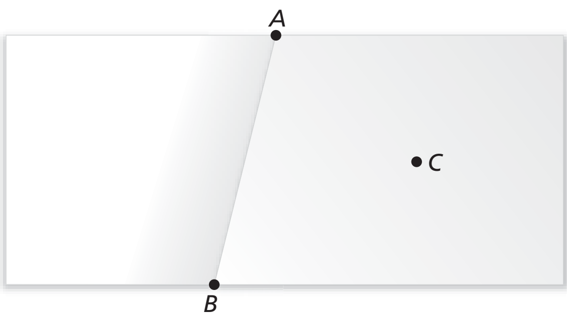


ii. Fold the paper again so that point *A* coincides with point *B*. Crease the paper on that fold.

iii. Unfold the paper and examine the four angles formed by the two creases. What can you conclude about the four angles?

b. Use a new piece of paper and repeat the first step of part (a).

i. Unfold the paper and draw a point not on the crease, as shown. Label the point *C*.



ii. Fold the paper so that the existing crease lies on top of itself and point *C* lies on the new fold. Crease the paper on the new fold.

iii. Unfold the paper and examine the four angles formed by the two creases. What can you conclude about the four angles?

iv. Can you find a line segment that connects $\overline{AB}$ and *C* that is shorter than the one on the new fold? Explain.

2 MTR USE ANOTHER METHOD

What other tools can you use to perform the construction in part (b)?

Geometric Reasoning

MA.912.GR.1.1 Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.

MA.912.GR.5.2 Construct the bisector of a segment or an angle, including the perpendicular bisector of a line segment.

Also MA.912.GR.3.3

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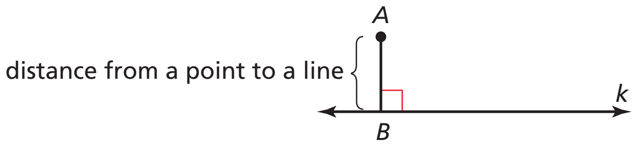
Finding the Distance from a Point to a Line

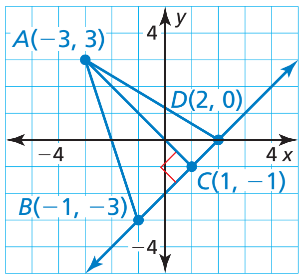
The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. The length of this segment is the shortest distance between the point and the line. For example, the distance between point A and line k is AB.



> **Vocabulary**
> distance from a point to a line, *p. 142*
> perpendicular bisector, *p. 143*

EXAMPLE 1 Finding the Distance from a Point to a Line

Find the distance from point A to $\overleftrightarrow{BD}$.



> **REMEMBER**
> Recall that the distance between $A(x_1, y_1)$ and $C(x_2, y_2)$ is
> $$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

SOLUTION

Because $\overline{AC} \perp \overleftrightarrow{BD}$, the distance from point A to $\overleftrightarrow{BD}$ is AC. Use the Distance Formula.

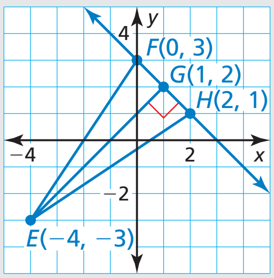
$$AC = \sqrt{[1 - (-3)]^2 + (-1 - 3)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{32} \approx 5.7$$

▶ So, the distance from point A to $\overleftrightarrow{BD}$ is about 5.7 units.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Find the distance from point E to $\overleftrightarrow{FH}$.



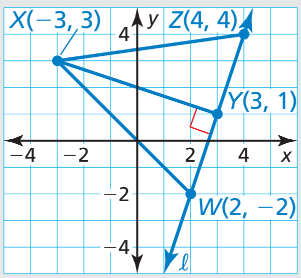
4 MTR **2. DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the distance from point X to line $\overleftrightarrow{WZ}$.

Find XZ.

Find the length of $\overline{XY}$.

Find the distance from line ℓ to point X.



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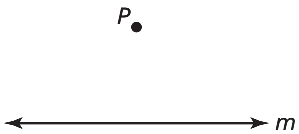
Constructing Perpendicular Lines

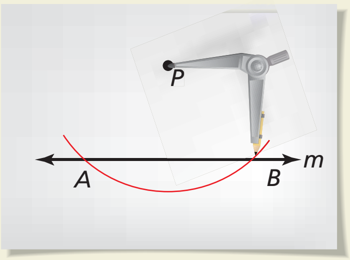
CONSTRUCTION **Constructing a Perpendicular Line** WATCH

Use a compass and straightedge to construct a line perpendicular to line *m* through point *P*, which is not on line *m*.



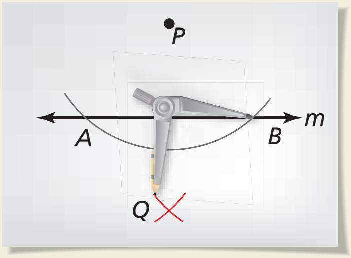
SOLUTION

Step 1



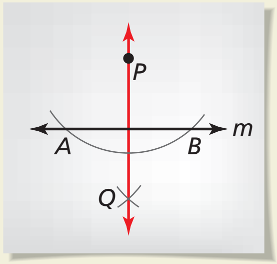
Draw arc with center *P* Place the compass at point *P* and draw an arc that intersects the line twice. Label the intersections *A* and *B*.

Step 2

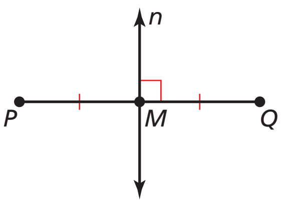


Draw intersecting arcs Draw an arc with center *A*. Using the same radius, draw an arc with center *B*. Label the intersection of the arcs *Q*.

Step 3



Draw perpendicular line Draw $\overleftrightarrow{PQ}$. This line is perpendicular to line *m*.



The **perpendicular bisector** of a line segment $\overline{PQ}$ is the line *n* with the following two properties.

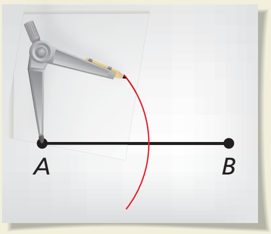
- $n \perp \overline{PQ}$
- *n* passes through the midpoint *M* of $\overline{PQ}$.

CONSTRUCTION **Constructing a Perpendicular Bisector** WATCH

Use a compass and straightedge to construct the perpendicular bisector of $\overline{AB}$.

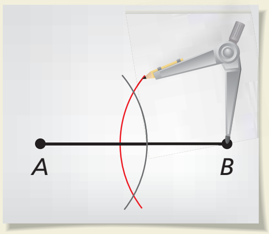
SOLUTION

Step 1



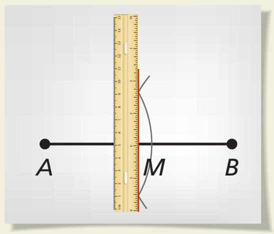
Draw an arc Place the compass at *A*. Use a compass setting that is greater than half the length of $\overline{AB}$. Draw an arc.

Step 2



Draw a second arc Keep the same compass setting. Place the compass at *B*. Draw an arc. It should intersect the other arc at two points.

Step 3



Bisect segment Draw a line through the two points of intersection. This line is the perpendicular bisector of $\overline{AB}$. It passes through *M*, the midpoint of $\overline{AB}$. So, $AM = MB$.

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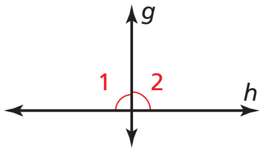
Proving Theorems about Perpendicular Lines

THEOREMS

3.10 Linear Pair Perpendicular Theorem

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.

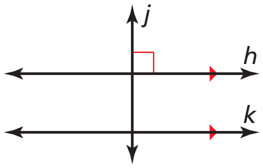


Prove this Theorem Exercise 9, page 146

3.11 Perpendicular Transversal Theorem

In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

If $h \parallel k$ and $j \perp h$, then $j \perp k$.

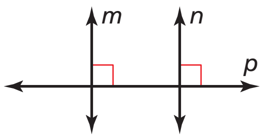


Prove this Theorem Exercise 3, page 144

3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If $m \perp p$ and $n \perp p$, then $m \parallel n$.

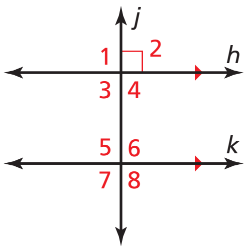


Prove this Theorem Exercise 10, page 146;
Exercise 47, page 156

EXAMPLE 2 Proving the Perpendicular Transversal Theorem

WATCH

Use the diagram to prove the Perpendicular Transversal Theorem.



SOLUTION

Given $h \parallel k$, $j \perp h$

Prove $j \perp k$

STATEMENTS	REASONS
1. $h \parallel k$, $j \perp h$	**1.** Given
2. $m\angle 2 = 90°$	**2.** Definition of perpendicular lines
3. $\angle 2 \cong \angle 6$	**3.** Corresponding Angles Theorem
4. $m\angle 2 = m\angle 6$	**4.** Definition of congruent angles
5. $m\angle 6 = 90°$	**5.** Transitive Property of Equality
6. $j \perp k$	**6.** Definition of perpendicular lines

SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

3. Prove the Perpendicular Transversal Theorem using the diagram in Example 2 and the Alternate Exterior Angles Theorem.

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Solving Real-Life Problems

7 MTR **EXAMPLE 3** **Modeling Real Life** WATCH

The photo shows the layout of a neighborhood. Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.

SOLUTION

Lines p and q are both perpendicular to s, so by the Lines Perpendicular to a Transversal Theorem, $p \parallel q$. Also, lines s and t are both perpendicular to q, so by the Lines Perpendicular to a Transversal Theorem, $s \parallel t$.

▶ So, from the diagram you can conclude $p \parallel q$ and $s \parallel t$.

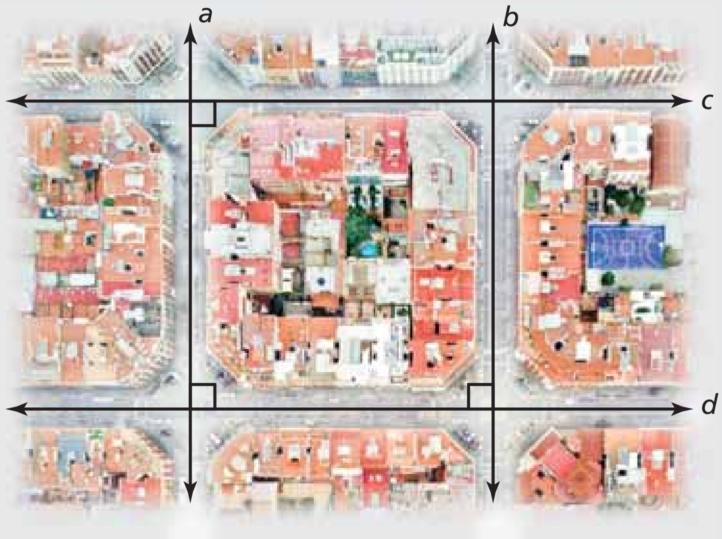
SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the lines marked in the photo.

4. Is $b \parallel a$? Explain your reasoning.

5. Is $b \perp c$? Explain your reasoning.



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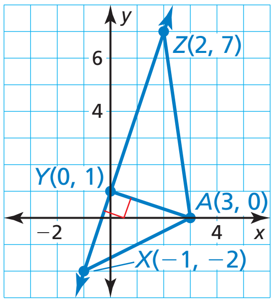
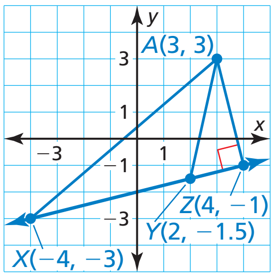
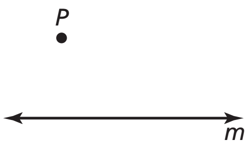
3.4 Practice WITH CalcChat® AND CalcView®

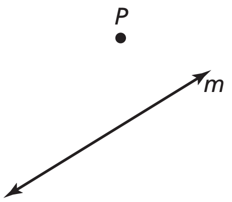
In Exercises 1 and 2, find the distance from point A to $\overleftrightarrow{XZ}$. *(See Example 1.)*

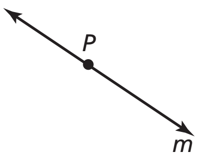
1. 

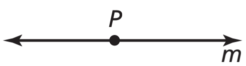
2. 

CONSTRUCTION In Exercises 3–6, trace line m and point P. Then use a compass and straightedge to construct a line perpendicular to line m through point P.

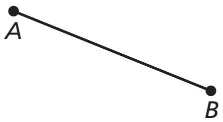
3. 

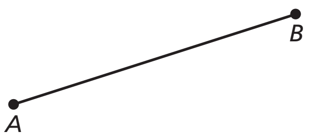
4. 

5. 

6. 

CONSTRUCTION In Exercises 7 and 8, trace $\overline{AB}$. Then use a compass and straightedge to construct the perpendicular bisector of $\overline{AB}$.

7. 

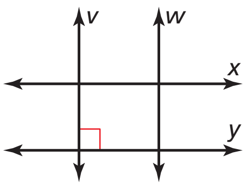
8. 

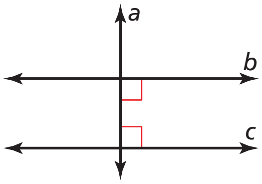
PROVING A THEOREM In Exercises 9 and 10, prove the theorem. *(See Example 2.)*

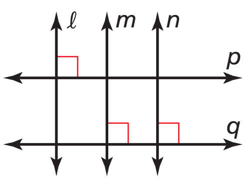
9. Linear Pair Perpendicular Theorem

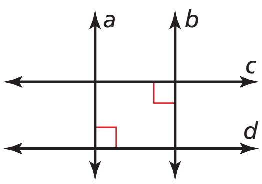
10. Lines Perpendicular to a Transversal Theorem

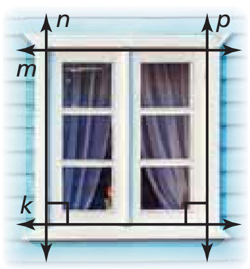
In Exercises 11–16, determine which lines, if any, must be parallel. Explain your reasoning. *(See Example 3.)*

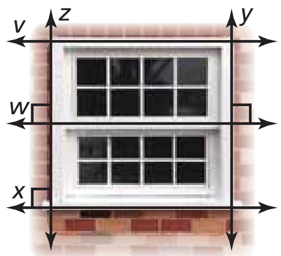
11. 

12. 

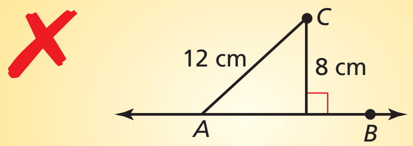
13. 

14. 

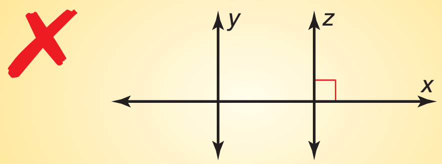
15. 

16. 

ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in the statement about the diagram.

17. 

The distance from point C to $\overleftrightarrow{AB}$ is 12 centimeters.

18. 

Lines y and z are parallel.

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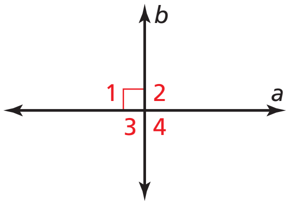
PROOF In Exercises 19 and 20, use the diagram to write a proof of the statement.

19. If two intersecting lines are perpendicular, then they intersect to form four right angles.

Given $a \perp b$

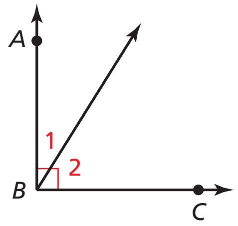
Prove $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.



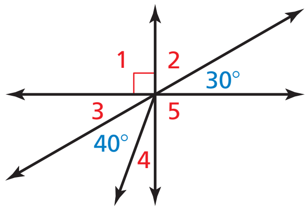
20. If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

Given $\overrightarrow{BA} \perp \overrightarrow{BC}$

Prove $\angle 1$ and $\angle 2$ are complementary.

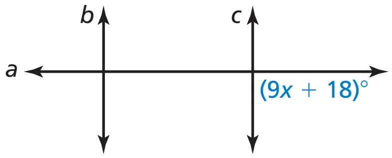


21. JUSTIFYING STEPS Find all the unknown angle measures in the diagram. Justify your answer for each angle measure.

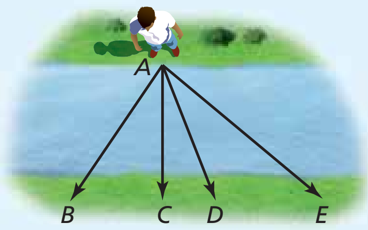


22. MAKING AN ARGUMENT Your friend claims that because you can find the distance from a point to a line, you should be able to find the distance between any two lines. Is your friend correct? Explain your reasoning.

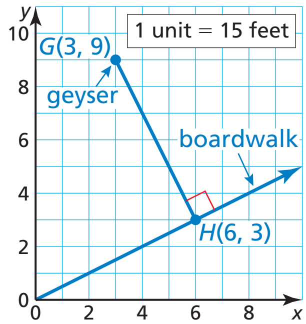
23. STRUCTURE In the diagram, $a \perp b$. Find the value of x that makes $b \parallel c$.



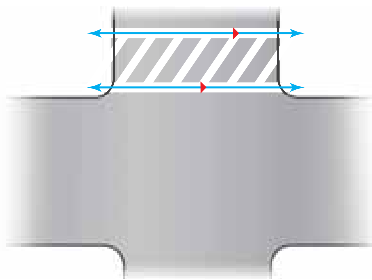
24. HOW DO YOU SEE IT?
You are trying to cross a stream from point A. Which point should you jump to in order to jump the shortest distance? Explain your reasoning.



25. MODELING REAL LIFE Park officials want to build a boardwalk that passes a geyser. A proposed design is shown. The boardwalk must be at least 100 feet from the center of the geyser at point G. Does the design meet this requirement? Explain.



26. MODELING REAL LIFE The painted line segments that form the path of a crosswalk can be painted as shown, or they can be perpendicular to the two parallel lines of the crosswalk. Which type of pattern requires less paint if both types use lines of equal thickness? Explain your reasoning.



27. CONSTRUCTION Construct a square of side length AB.

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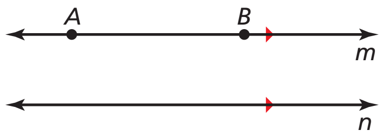
28. **REASONING** Two lines, a and b, are perpendicular to line c. Line d is parallel to line c. The distance between lines a and b is x meters. The distance between lines c and d is y meters. What shape is formed by the intersections of the four lines?

5 MTR **29.** **CONNECTING CONCEPTS** Let C be a point on line n. Does the area of $\triangle ABC$ depend on the location of C? Explain your reasoning.



30. **THOUGHT PROVOKING**
The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, how many right angles are formed by two perpendicular lines? Justify your answer.

31. **REASONING** Describe how you can find the distance from a point to a plane. Can you find the distance from a line to a plane? Explain your reasoning.

REVIEW & REFRESH

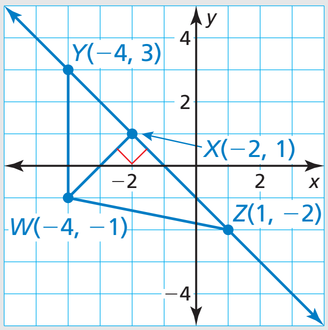
WATCH

In Exercises 32 and 33, find the slope and the *y*-intercept of the graph of the linear equation.

32. $y = \frac{1}{6}x - 8$ **33.** $-3x + y = 9$

34. Two angles form a linear pair. The measure of one angle is 77°. Find the measure of the other angle.

35. Find the distance from point W to $\overleftrightarrow{YZ}$.



36. Find the slope of the line that passes through $(-4, 3)$ and $(6, 8)$.

7 MTR **37.** **MODELING REAL LIFE** The post office and the grocery store are both on the same straight road between the school and your house. The distance from the school to the post office is 376 yards, the distance from the post office to your house is 929 yards, and the distance from the grocery store to your house is 513 yards.

a. What is the distance from the post office to the grocery store?

b. What is the distance from the school to your house?

3 MTR **38.** **CHOOSE A METHOD** Solve the system using any method. Explain your choice of method.

$$2x + y = 5$$
$$3x + 2y = 6$$

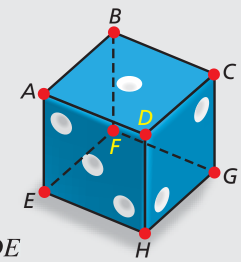
In Exercises 39–42, consider the lines that contain the segments in the figure and the planes that contain the faces of the figure. Which line(s) or plane(s) contain point *G* and appear to fit the description?

39. line(s) parallel to $\overleftrightarrow{EF}$

40. line(s) perpendicular to $\overleftrightarrow{EF}$

41. line(s) skew to $\overleftrightarrow{EF}$

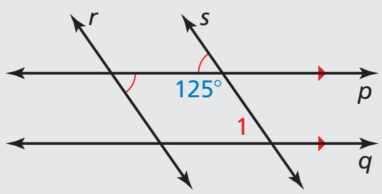
42. plane(s) parallel to plane ADE



In Exercises 43 and 44, graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

43. $g(x) = |x| + 9$ **44.** $h(x) = -\frac{3}{2}|x|$

In Exercises 45 and 46, use the diagram.



45. Find $m\angle 1$. Tell which theorem you use.

46. Is there enough information to prove that $r \parallel s$? If so, state the theorem you can use.

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