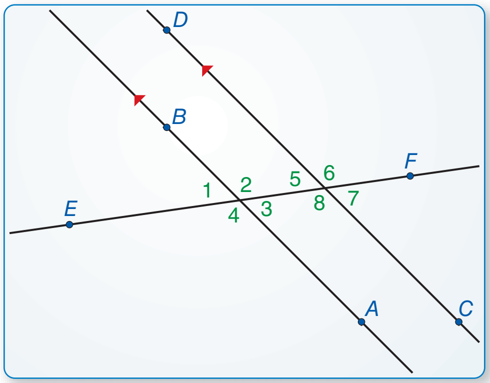
3.2 Parallel Lines and Transversals

Learning Target: Prove and use theorems about parallel lines.

Success Criteria:
- I can use properties of parallel lines to find angle measures.
- I can prove theorems about parallel lines.

EXPLORE IT! Making Conjectures about Parallel Lines

Work with a partner. Draw two parallel lines. Draw a transversal that intersects both parallel lines.

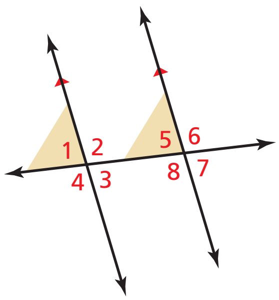


a. Find the measures of the eight angles that are formed. What do you notice?

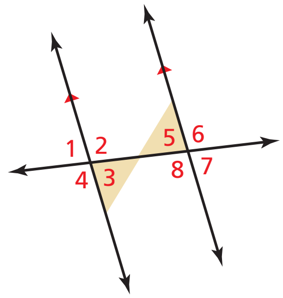
b. Adjust the parallel lines and transversal so they intersect at different angles. Repeat part (a). How do your results compare to part (a)?

c. Write conjectures about each pair of angles formed by two parallel lines and a transversal.

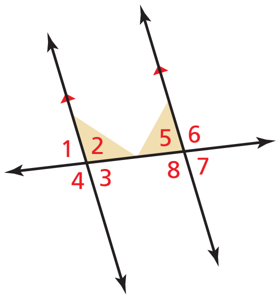
i. corresponding angles



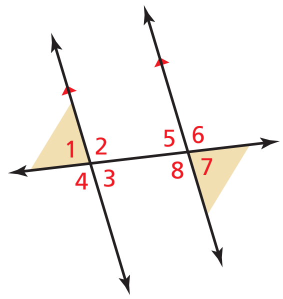
ii. alternate interior angles



iii. consecutive interior angles



iv. alternate exterior angles



1 MTR ANALYZE A PROBLEM
What is the sum of the measures of a pair of consecutive interior angles? Does it stay the same or change when you adjust the lines?

Geometric Reasoning
MA.912.GR.1.1 Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.

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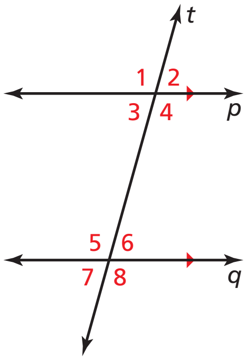
Using Properties of Parallel Lines

THEOREMS



3.1 Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.

Prove this Theorem Exercise 35, page 174

3.2 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the left, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Prove this Theorem Exercise 17, page 131

3.3 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

Proof Example 4, page 130

3.4 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

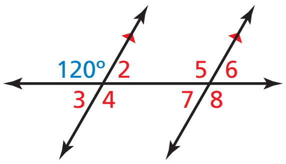
Examples In the diagram at the left, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

Prove this Theorem Exercise 18, page 131

EXAMPLE 1 Identifying Angles

WATCH

The measures of three of the numbered angles are 120°. Identify the angles. Explain your reasoning.



SOLUTION

Using the Alternate Exterior Angles Theorem, $m\angle 8 = 120°$.

$\angle 5$ and $\angle 8$ are vertical angles. Using the Vertical Angles Congruence Theorem, $m\angle 5 = 120°$.

$\angle 5$ and $\angle 4$ are alternate interior angles. Using the Alternate Interior Angles Theorem, $m\angle 4 = 120°$.

▶ So, the three angles that each have a measure of 120° are $\angle 4$, $\angle 5$, and $\angle 8$.

2 MTR USE ANOTHER METHOD

Solve Example 1 another way by finding the measure of a different angle first.

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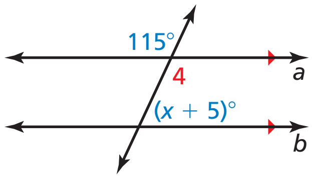
EXAMPLE 2 Using Properties of Parallel Lines

WATCH



Find the value of x.

SOLUTION

By the Vertical Angles Congruence Theorem, $m\angle 4 = 115°$. Lines a and b are parallel, so you can use theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

▶ So, the value of x is 60.

Check

$115° + (x + 5)° = 180°$

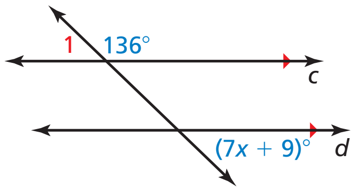
$115 + (60 + 5) \stackrel{?}{=} 180$

$180 = 180$ ✓

EXAMPLE 3 Using Properties of Parallel Lines

WATCH

Find the value of x.



SOLUTION

By the Linear Pair Postulate, $m\angle 1 = 180° - 136° = 44°$. Lines c and d are parallel, so you can use theorems about parallel lines.

$m\angle 1 = (7x + 9)°$	Alternate Exterior Angles Theorem
$44° = (7x + 9)°$	Substitute 44° for $m\angle 1$.
$35 = 7x$	Subtract 9 from each side.
$5 = x$	Divide each side by 7.

▶ So, the value of x is 5.

Check

$44° = (7x + 9)°$

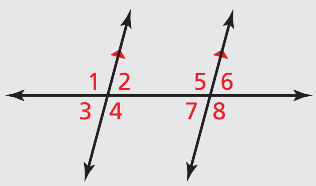
$44 \stackrel{?}{=} 7(5) + 9$

$44 = 44$ ✓

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram.

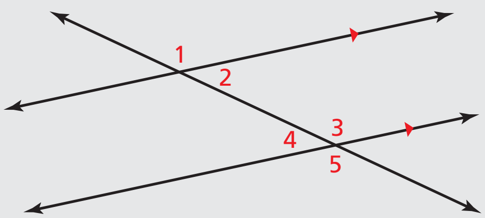


1. Given $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which theorem you use in each case.

2. Given $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps.

4 MTR **3.** **WHICH ONE DOESN'T BELONG?** Which pair of angle measures does *not* belong with the other three? Explain.

$m\angle 1$ and $m\angle 3$	$m\angle 2$ and $m\angle 4$
$m\angle 2$ and $m\angle 3$	$m\angle 1$ and $m\angle 5$



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Proving Theorems about Parallel Lines

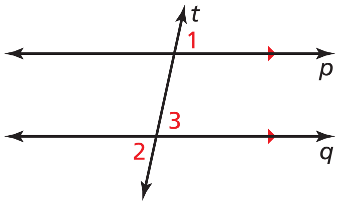
EXAMPLE 4 **Proving the Alternate Exterior Angles Theorem** WATCH

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

4 MTR COMPARE METHODS
Rewrite the proof in Example 4 as a flowchart proof. Explain which version of the proof you prefer and why.

SOLUTION

Draw a diagram. Label a pair of alternate exterior angles as ∠1 and ∠2. You are looking for an angle that is related to both ∠1 and ∠2. Notice that one angle is a vertical angle with ∠2 and a corresponding angle with ∠1. Label it ∠3.



REMEMBER
Before you write a proof, identify the **Given** and **Prove** statements for the situation described or for any diagram you draw.

Given $p \parallel q$

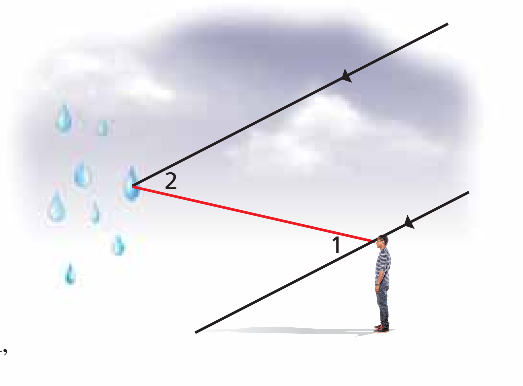
Prove $\angle 1 \cong \angle 2$

STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Theorem
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Angle Congruence

Solving Real-Life Problems

7 MTR **EXAMPLE 5** **Modeling Real Life** WATCH

When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?



SOLUTION

The Sun's rays are parallel, and ∠1 and ∠2 are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$.

▶ So, by the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. Write an alternative proof for the Alternate Exterior Angles Theorem.

5. **WHAT IF?** In Example 5, yellow light leaves a drop at an angle of $m\angle 2 = 41°$. What is $m\angle 1$? How do you know?

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3.2 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case. *(See Example 1.)*

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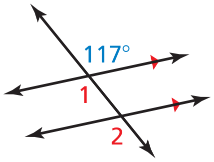

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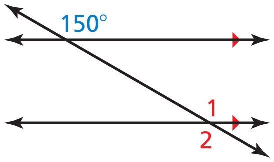

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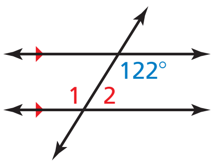

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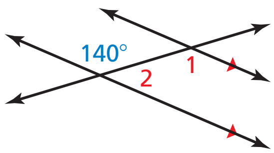
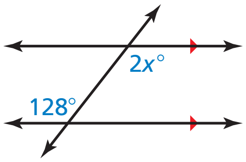


In Exercises 5–10, find the value of x. Show your steps. *(See Examples 2 and 3.)*

5.



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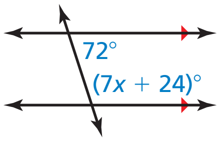

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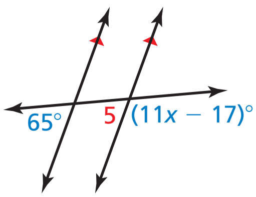

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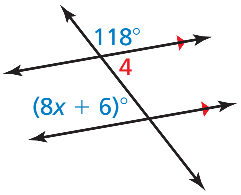

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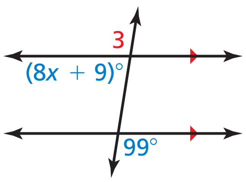

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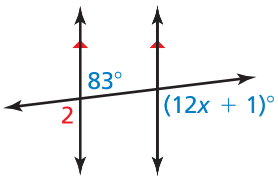
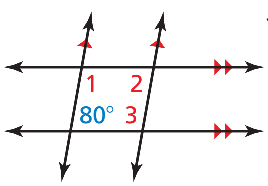


In Exercises 11–14, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. Explain your reasoning.

11.



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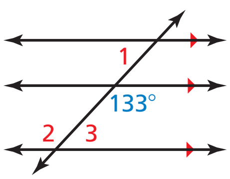

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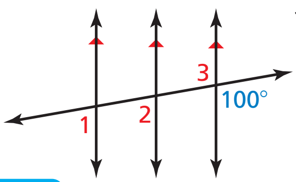

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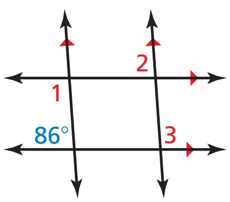

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ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in the student's reasoning.**

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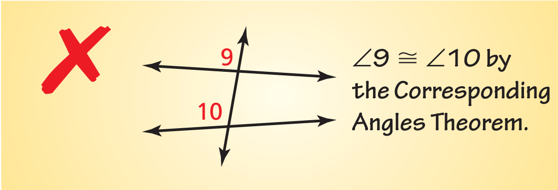

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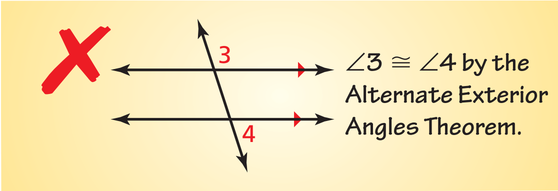
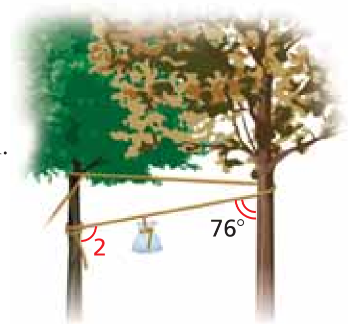


PROVING A THEOREM **In Exercises 17 and 18, prove the theorem.** *(See Example 4.)*

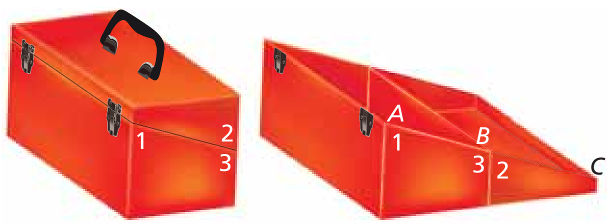
17. Alternate Interior Angles Theorem

18. Consecutive Interior Angles Theorem

19. **MODELING REAL LIFE** A group of campers at Ocala National Forest tie up their food in a bear bag between two parallel trees, as shown. The rope is pulled taut, forming a straight line. Find $m\angle 2$. Explain your reasoning. *(See Example 5.)*



20. **MODELING REAL LIFE** You are designing a box like the one shown.



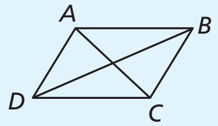
a. The measure of $\angle 1$ is 70°. Find $m\angle 2$ and $m\angle 3$.

b. Explain why $\angle ABC$ is a straight angle.

c. If $m\angle 1$ is 60°, will $\angle ABC$ still be a straight angle? Will the opening of the box be *more steep* or *less steep*? Explain.

21. **REASONING** Is it possible for consecutive interior angles to be congruent? Explain.

22. HOW DO YOU SEE IT?
Use the diagram.

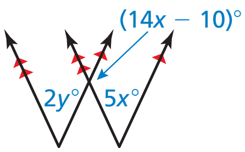


a. Name two pairs of congruent angles when $\overline{AD}$ and $\overline{BC}$ are parallel. Explain your reasoning.

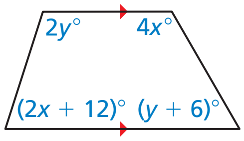
b. Name two pairs of supplementary angles when $\overline{AB}$ and $\overline{DC}$ are parallel. Explain your reasoning.

5 MTR **CONNECTING CONCEPTS** **In Exercises 23 and 24, write and solve a system of linear equations to find the values of x and y.**

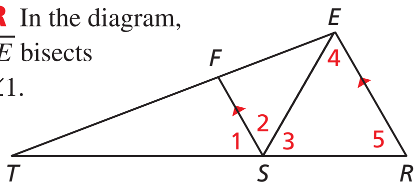
23.



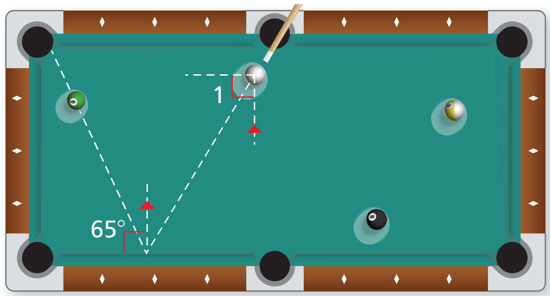
24.



25. DIG DEEPER In the diagram, $\angle 4 \cong \angle 5$ and $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. Explain your reasoning.



4 MTR **26. MAKING AN ARGUMENT** During a game of pool, your friend claims to be able to make the shot shown in the diagram by hitting the cue ball so that $m\angle 1 = 25°$. Is your friend correct? Explain your reasoning.



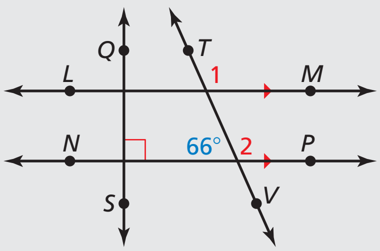
27. OPEN-ENDED Draw a real-life situation modeled by parallel lines and transversals. Describe the relationships between the angles.

28. THOUGHT PROVOKING
The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, is it possible that a transversal intersects two parallel lines? Explain your reasoning.

REVIEW & REFRESH

WATCH

In Exercises 29–31, use the diagram.



29. Name a pair of perpendicular lines.

30. Name a pair of parallel lines.

31. Find $m\angle 1$ and $m\angle 2$. Tell which postulates or theorems you used.

In Exercises 32 and 33, name the property that the statement illustrates.

32. If $\angle F \cong \angle G$ and $\angle G \cong \angle H$, then $\angle F \cong \angle H$.

33. If $\overline{WX} \cong \overline{YZ}$, then $\overline{YZ} \cong \overline{WX}$.

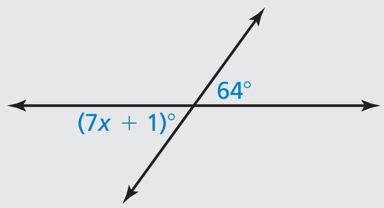
In Exercises 34 and 35, factor the polynomial completely.

34. $t^3 - 5t^2 + 3t - 15$ **35.** $4x^4 - 36x^2$

36. Find the x- and y-intercepts of the graph of $7x - 4y = 28$.

7 MTR **37. MODELING REAL LIFE** A square painting is surrounded by a frame with uniform width. The painting has a side length of $(x - 3)$ inches. The side length of the frame is $(x + 2)$ inches. Write an expression for the area of the square frame. Then find the area of the frame when $x = 12$.

38. Find the value of x in the diagram.



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