2.6 Proving Geometric Relationships

Learning Target: Prove geometric relationships.

Success Criteria:
- I can prove geometric relationships by writing flowchart proofs.
- I can prove geometric relationships by writing paragraph proofs.

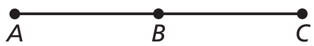
EXPLORE IT! Completing Flowchart Proofs

Work with a partner.

a. Complete the flowchart to prove that $AB = BC$.

Given $AC = AB + AB$

Prove $AB = BC$



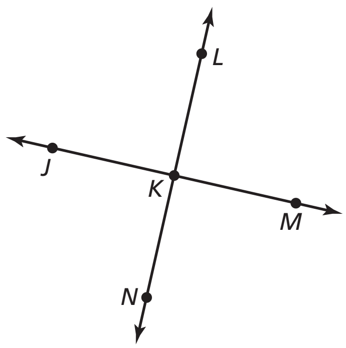
$AC = AB + AB$ →

$AB + BC = AC$ → $AB + BC = AB + AB$ → $AB = BC$

b. Complete the flowchart to prove that $\overleftrightarrow{JM} \perp \overleftrightarrow{LN}$.

Given $\angle JKL \cong \angle MKL$

Prove $\overleftrightarrow{JM} \perp \overleftrightarrow{LN}$



$\angle JKL \cong \angle MKL$
↓
$m\angle JKL = m\angle MKL$
↓
$m\angle JKL + m\angle MKL = 180°$ → $m\angle JKL + m\angle JKL = 180°$ → $2(m\angle JKL) = 180°$
↓
$m\angle JKL = 90°$
↓
$\overleftrightarrow{JM} \perp \overleftrightarrow{LN}$

c. How can you use a flowchart to prove a mathematical statement?

d. Compare the flowchart proofs above with the proofs in the 2.5 Explore It! Explain the advantages and disadvantages of each.

3 MTR REFLECT ON YOUR METHOD
Why is it helpful to sketch a diagram when writing certain proofs?

Geometric Reasoning
MA.912.GR.1.1 Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.

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Writing Flowchart Proofs

Another proof format is a **flowchart proof**, or **flow proof**, which uses boxes and arrows to show the flow of a logical argument. Each reason is below the statement it justifies. A flowchart proof of the *Right Angles Congruence Theorem* is shown in Example 1.

Vocabulary (AZ VOCAB)

flowchart proof, or flow proof, *p. 102*
paragraph proof or narrative proof, *p. 104*

THEOREM

2.3 Right Angles Congruence Theorem

All right angles are congruent.

EXAMPLE 1 Proving the Right Angles Congruence Theorem

Use the given flowchart proof to write a two-column proof of the Right Angles Congruence Theorem.

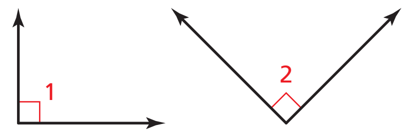
WATCH

STUDY TIP

When you prove a theorem, write the hypothesis of the theorem as the **Given** statement. The conclusion is what you must **Prove**.

Given $\angle 1$ and $\angle 2$ are right angles.

Prove $\angle 1 \cong \angle 2$



Flowchart Proof

$\angle 1$ and $\angle 2$ are right angles.
Given
↓
$m\angle 1 = 90°, m\angle 2 = 90°$ → $m\angle 1 = m\angle 2$ → $\angle 1 \cong \angle 2$
Definition of right angle | Transitive Property of Equality | Definition of congruent angles

Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are right angles.	**1.** Given
2. $m\angle 1 = 90°, m\angle 2 = 90°$	**2.** Definition of right angle
3. $m\angle 1 = m\angle 2$	**3.** Transitive Property of Equality
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

3 MTR SELECT METHODS

Which type of proof is easier to write when there are many statements?

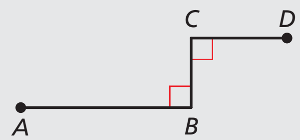
SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

1. Complete the flowchart proof. Then write a two-column proof.

Given $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$

Prove $\angle B \cong \angle C$



$\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$ → ______ → $\angle B \cong \angle C$
Given | Definition of $\perp$ lines | ______

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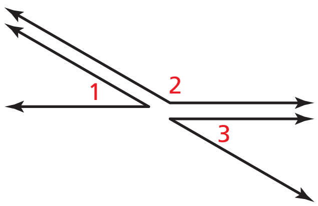
THEOREMS

2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

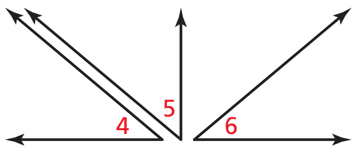


Prove this Theorem Exercise 20 (case 2), page 109

2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.



Prove this Theorem Exercise 19 (case 1), page 108; Exercise 24 (case 2), page 110

STUDY TIP

To prove the Congruent Supplements Theorem, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of the Congruent Complements Theorem also requires two cases.

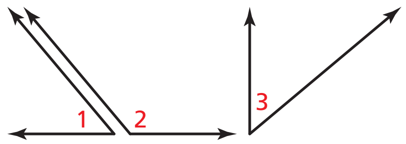
EXAMPLE 2 Proving a Case of the Congruent Supplements Theorem

WATCH

Use the given two-column proof to write a flowchart proof that proves that two angles supplementary to the same angle are congruent.

Given $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 2$ are supplementary.

Prove $\angle 1 \cong \angle 3$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 2$ are supplementary.	**1.** Given
2. $m\angle 1 + m\angle 2 = 180^\circ$, $m\angle 3 + m\angle 2 = 180^\circ$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**3.** Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	**5.** Definition of congruent angles

Flowchart Proof

$\angle 1$ and $\angle 2$ are supplementary. → $m\angle 1 + m\angle 2 = 180^\circ$ → $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$

Given | Definition of supplementary angles | Transitive Property of Equality

$\angle 3$ and $\angle 2$ are supplementary. → $m\angle 3 + m\angle 2 = 180^\circ$ ↗

Given | Definition of supplementary angles

↓ $m\angle 1 = m\angle 3$

Subtraction Property of Equality

↓ $\angle 1 \cong \angle 3$

Definition of congruent angles

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Writing Paragraph Proofs

Another proof format is a **paragraph proof**, or **narrative proof**, which presents the statements and reasons of a proof as sentences in a paragraph. It uses words to explain the logical flow of the argument.

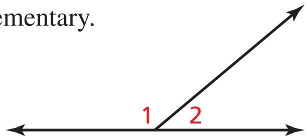
Two intersecting lines form pairs of vertical angles and linear pairs. The *Linear Pair Postulate* formally states the relationship between angles that form linear pairs. You can use this postulate to prove the *Vertical Angles Congruence Theorem*.

POSTULATE AND THEOREM

2.8 Linear Pair Postulate

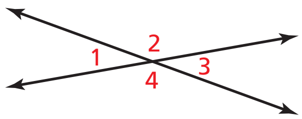
If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

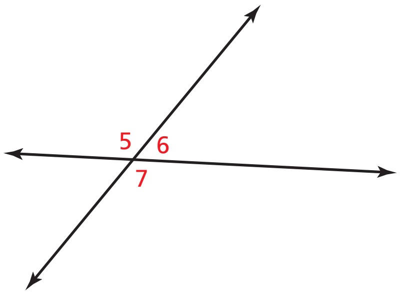
EXAMPLE 3 Proving the Vertical Angles Congruence Theorem

WATCH

Use the given paragraph proof to write a two-column proof of the Vertical Angles Congruence Theorem.

Given $\angle 5$ and $\angle 7$ are vertical angles.

Prove $\angle 5 \cong \angle 7$



STUDY TIP

In paragraph proofs, *transitional words* such as *so*, *then*, and *therefore* help make the logic clear.

Paragraph Proof

$\angle 5$ and $\angle 7$ are vertical angles formed by intersecting lines. As shown in the diagram, $\angle 5$ and $\angle 6$ are a linear pair, and $\angle 6$ and $\angle 7$ are a linear pair. Then, by the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are supplementary and $\angle 6$ and $\angle 7$ are supplementary. So, by the Congruent Supplements Theorem, $\angle 5 \cong \angle 7$.

Two-Column Proof

STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	**1.** Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	**2.** Definition of linear pair, as shown in the diagram
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	**3.** Linear Pair Postulate
4. $\angle 5 \cong \angle 7$	**4.** Congruent Supplements Theorem

STUDY TIP

Your proof can use information that is labeled in a diagram.

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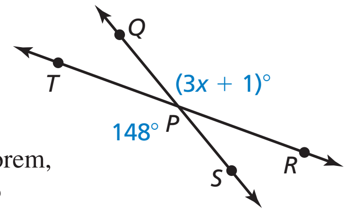
EXAMPLE 4 Using Angle Relationships

Find the value of x.



SOLUTION

$\angle TPS$ and $\angle QPR$ are vertical angles. By the Vertical Angles Congruence Theorem, the angles are congruent. Use this fact to write and solve an equation.

$m\angle TPS = m\angle QPR$	Definition of congruent angles
$148° = (3x + 1)°$	Substitute angle measures.
$147 = 3x$	Subtract 1 from each side.
$49 = x$	Divide each side by 3.

▶ So, the value of x is 49.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

2. Complete the two-column proof. Then write a flowchart proof.

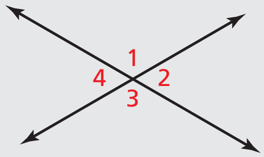
Given $AB = DE$, $BC = CD$

Prove $\overline{AC} \cong \overline{CE}$

STATEMENTS	REASONS
1. $AB = DE$, $BC = CD$	**1.** Given
2. $AB + BC = BC + DE$	**2.** Addition Property of Equality
3. ________________	**3.** Substitution Property of Equality
4. $AB + BC = AC$, $CD + DE = CE$	**4.** ________________
5. ________________	**5.** Substitution Property of Equality
6. $\overline{AC} \cong \overline{CE}$	**6.** ________________

3. Write a two-column proof of the Vertical Angle Congruence Theorem without using the Congruent Supplements Theorem. Compare your proof with the proof in Example 3.

Use the diagram and the given angle measure to find the other three angle measures.

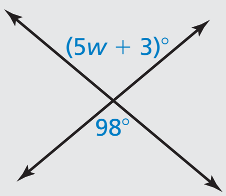


4. $m\angle 1 = 117°$

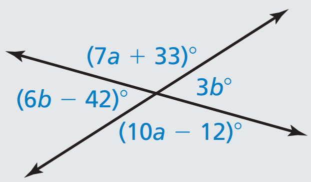
5. $m\angle 2 = 59°$

6. $m\angle 4 = 88°$

7. Find the value of w.



8. Find the values of a and b.



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EXAMPLE 5 Using the Vertical Angles Congruence Theorem

WATCH

Write a paragraph proof.

Given $\angle 1 \cong \angle 4$

Prove $\angle 2 \cong \angle 3$

Paragraph Proof

$\angle 1$ and $\angle 4$ are congruent. By the Vertical Angles Congruence Theorem, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. By the Transitive Property of Angle Congruence, $\angle 2 \cong \angle 4$. Using the Transitive Property of Angle Congruence once more, $\angle 2 \cong \angle 3$.

4 MTR COMPARE METHODS

Rewrite the proof as a flowchart proof and a two-column proof. Compare the proofs. Which format do you prefer? Explain.

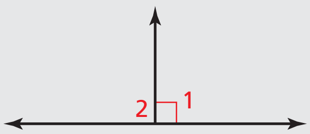
SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

9. Write a paragraph proof.

Given $\angle 1$ is a right angle.

Prove $\angle 2$ is a right angle.

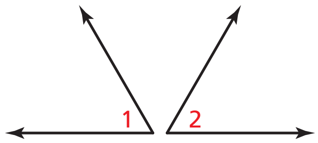


CONCEPT SUMMARY

Types of Proofs for the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

Flowchart Proof

$\angle 1 \cong \angle 2$ → $m\angle 1 = m\angle 2$ → $m\angle 2 = m\angle 1$ → $\angle 2 \cong \angle 1$

Given; Definition of congruent angles; Symmetric Property of Equality; Definition of congruent angles

Paragraph Proof

$\angle 1$ is congruent to $\angle 2$. By the definition of congruent angles, the measure of $\angle 1$ is equal to the measure of $\angle 2$. The measure of $\angle 2$ is equal to the measure of $\angle 1$ by the Symmetric Property of Equality. Then by the definition of congruent angles, $\angle 2$ is congruent to $\angle 1$.

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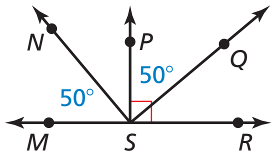
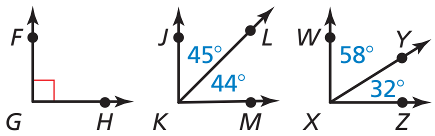
2.6 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, identify the pair(s) of congruent angles in the figures. Explain how you know they are congruent.

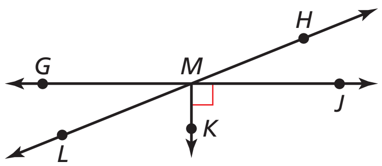
▶ **1.**



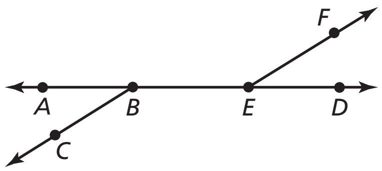
2.



3.



4. $\angle ABC$ is supplementary to $\angle CBD$. $\angle CBD$ is supplementary to $\angle DEF$.



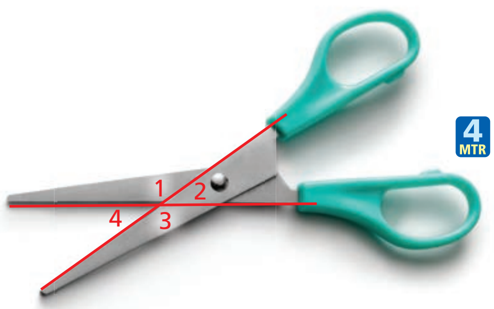
In Exercises 5–8, use the diagram and the given angle measure to find the other three measures.

▶ **5.** $m\angle 1 = 143°$

6. $m\angle 3 = 159°$

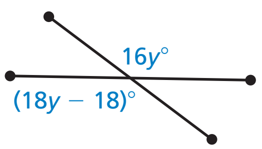
7. $m\angle 2 = 34°$

8. $m\angle 4 = 29°$

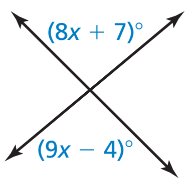


In Exercises 9–12, find the value of each variable. *(See Example 4.)*

▶ **9.**



10.



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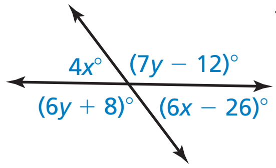

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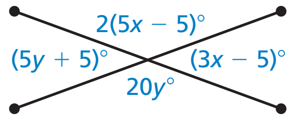
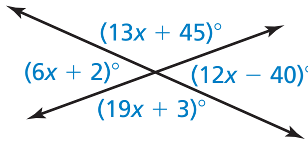
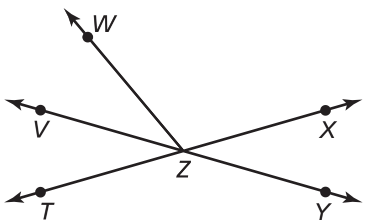


13. **ERROR ANALYSIS** Describe and correct the error in using the diagram to find the value of x.



✗
$$(13x + 45)° + (19x + 3)° = 180°$$
$$32x + 48 = 180$$
$$32x = 132$$
$$x = 4.125$$

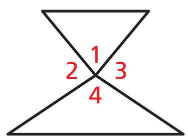
14. **COLLEGE PREP** Which statements can you conclude from the diagram? Select all that apply.



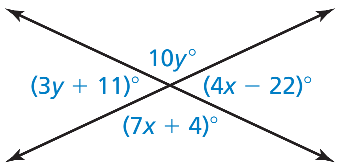
Ⓐ $\angle VZT \cong \angle XZY$ Ⓒ $\angle VZX \cong \angle TZY$

Ⓑ $\angle VZT \cong \angle VZW$ Ⓓ $\angle VZW \cong \angle XZY$

15. **MAKING AN ARGUMENT** Your friend claims that $\angle 1 \cong \angle 4$ because they are vertical angles. Is your friend correct? Support your answer with definitions or theorems.



16. **STRUCTURE** Find the measure of each angle in the diagram.



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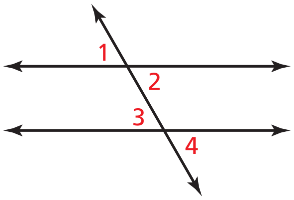
▶ **17. PROOF** Complete the flowchart proof. Then write a two-column proof. *(See Example 1.)*

Given $\angle 1 \cong \angle 3$

Prove $\angle 2 \cong \angle 4$



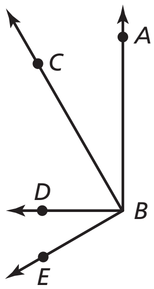
$\angle 1 \cong \angle 3$ → $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ → $\angle 2 \cong \angle 3$ → $\angle 2 \cong \angle 4$

Given | Vertical Angles Congruence Theorem | ______ | ______

18. PROOF Complete the two-column proof. Then write a flowchart proof. *(See Example 2.)*

Given $\angle ABD$ is a right angle.
$\angle CBE$ is a right angle.

Prove $\angle ABC \cong \angle DBE$

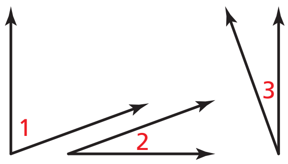


STATEMENTS	REASONS
1. $\angle ABD$ is a right angle. $\angle CBE$ is a right angle.	**1.** ______
2. $\angle ABC$ and $\angle CBD$ are complementary.	**2.** Definition of complementary angles
3. $\angle DBE$ and $\angle CBD$ are complementary.	**3.** ______
4. $\angle ABC \cong \angle DBE$	**4.** ______

▶ **19. PROVING A THEOREM** Complete the paragraph proof for the Congruent Complements Theorem. Then write a two-column proof. *(See Example 3.)*

Given $\angle 1$ and $\angle 2$ are complementary.
$\angle 1$ and $\angle 3$ are complementary.

Prove $\angle 2 \cong \angle 3$



$\angle 1$ and $\angle 2$ are complementary, and $\angle 1$ and $\angle 3$ are complementary. By the definition of ______ angles, $m\angle 1 + m\angle 2 = 90°$ and ______ $= 90°$. By the ______, $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$. By the Subtraction Property of Equality, ______. So, $\angle 2 \cong \angle 3$ by the definition of ______.

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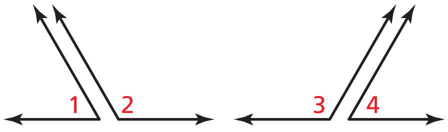
20. PROVING A THEOREM Complete the two-column proof for the Congruent Supplements Theorem. Then write a paragraph proof. *(See Example 5.)*

Given $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 4$ are supplementary.
$\angle 1 \cong \angle 4$



Prove $\angle 2 \cong \angle 3$

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary. $\angle 1 \cong \angle 4$	**1.** Given
2. $m\angle 1 + m\angle 2 = 180°$, $m\angle 3 + m\angle 4 = 180°$	**2.** ______________
3. ________ $= m\angle 3 + m\angle 4$	**3.** Transitive Property of Equality
4. $m\angle 1 = m\angle 4$	**4.** Definition of congruent angles
5. $m\angle 1 + m\angle 2 =$ ________	**5.** Substitution Property of Equality
6. $m\angle 2 = m\angle 3$	**6.** ______________
7. ______________	**7.** ______________

21. WRITING Explain why you do not use inductive reasoning when writing a proof.

22. HOW DO YOU SEE IT?
Consider the two-column proof. What is the writer trying to prove?

Given $\angle 1 \cong \angle 2$
$\angle 1$ and $\angle 2$ are supplementary.

Prove ______________

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 1 + m\angle 2 = 180°$	**3.** Definition of supplementary angles
4. $m\angle 1 + m\angle 1 = 180°$	**4.** Substitution Property of Equality
5. $2(m\angle 1) = 180°$	**5.** Simplify.
6. $m\angle 1 = 90°$	**6.** Division Property of Equality
7. $m\angle 2 = 90°$	**7.** Transitive Property of Equality
8. ______________	**8.** ______________

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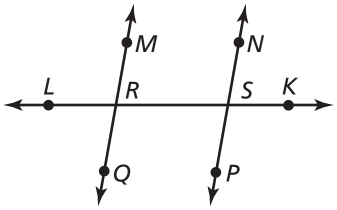
3 MTR **CHOOSE A METHOD** **In Exercises 23–26, write a proof using any format. Explain your choice.**

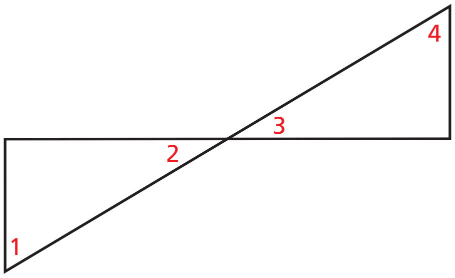
23. **Given** $\angle QRS$ and $\angle PSR$ are supplementary.

Prove $\angle QRL \cong \angle PSR$



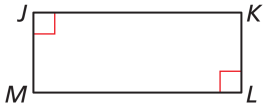
24. **Given** $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 4$ are complementary.

Prove $\angle 1 \cong \angle 4$



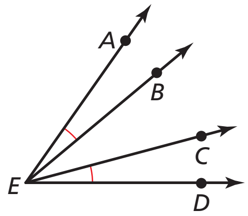
25. **Given** $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$, $\angle J \cong \angle M$, $\angle K \cong \angle L$

Prove $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



26. **Given** $\angle AEB \cong \angle DEC$

Prove $\angle AEC \cong \angle DEB$



27. **REASONING** Is the converse of the Linear Pair Postulate true? If so, write a biconditional statement. If not, explain why not.

28. **THOUGHT PROVOKING**
Draw three lines all intersecting at the same point. Label two of the angle measures so that you can find the remaining four angle measures. Explain how you chose which angle measures to label.

REVIEW & REFRESH

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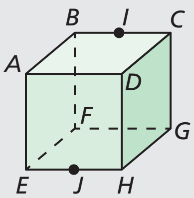
29. Complete the statement. Name the property you use.

If $\overline{RS} \cong \overline{TU}$ and $\overline{TU} \cong \overline{VW}$, then ____ $\cong$ ____.

In Exercises 30–32, use the cube.



30. Name three collinear points.

31. Write an example of the Three Point Postulate.

32. Name two planes containing $\overline{BC}$.

7 MTR **33.** **MODELING REAL LIFE** The final velocity v_f of an object is given by the formula $v_f = v_i + at$, where v_i is the initial velocity, a is the acceleration, and t is the time.

a. Solve the formula for t. Justify each step.

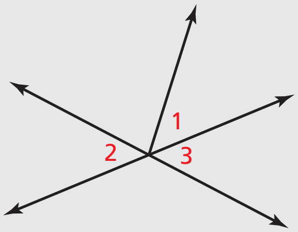
b. A car with an initial velocity of 14 meters per second accelerates at a constant rate of 2.5 meters per second squared. How many seconds does it take the car to reach a final velocity of 29 meters per second?

34. Complete the square for $x^2 - 14x$. Then factor the trinomial.

35. Complete the two-column proof. Then write a flowchart proof.

Given $\angle 1 \cong \angle 2$

Prove $\angle 1 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** ______________
3. ________	**3.** Transitive Property of Angle Congruence

In Exercises 36–39, graph the function.

36. $f(x) = -x^2 - 6$

37. $g(x) = 2x^2 - 2x + 3$

38. $y = (x + 2)(x - 4)$

39. $y = -3(x - 1)^2 + 4$

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