2.5 Proving Statements about Segments and Angles

Learning Target: Prove statements about segments and angles.

Success Criteria:
- I can explain the structure of a two-column proof.
- I can write a two-column proof.
- I can identify properties of congruence.

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true.

EXPLORE IT! Completing Proofs

Work with a partner.

a. Complete the statements to prove that $AB = BC$.
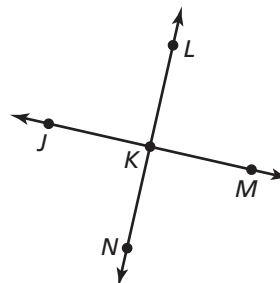
Given $AC = AB + AB$

Prove $AB = BC$

You are given that $AC = $ _____ . By the _____ , $AB + BC = AC$.

$AB + BC = AB + AB$ by the _____ . Then by the _____ , $AB = BC$.

b. Seven steps of a proof are shown. Complete the statements to prove that $\overleftrightarrow{JM} \perp \overleftrightarrow{LN}$.

Given $\angle JKL \cong \angle MKL$

Prove $\overleftrightarrow{JM} \perp \overleftrightarrow{LN}$

STATEMENTS	REASONS
1. $\angle JKL \cong \angle MKL$	1. Given
2. $m\angle JKL = m\angle MKL$	2. _____
3. $m\angle JKL + m\angle MKL = 180°$	3. _____
4. $m\angle JKL + $ _____ $= 180°$	4. Substitution Property of Equality
5. $2(m\angle JKL) = 180°$	5. _____
6. _____	6. Division Property of Equality
7. $\overleftrightarrow{JM} \perp \overleftrightarrow{LN}$	7. _____

Geometric Reasoning
MA.912.GR.1.1 Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.

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Writing Two-Column Proofs

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is an explanation for the corresponding statement.
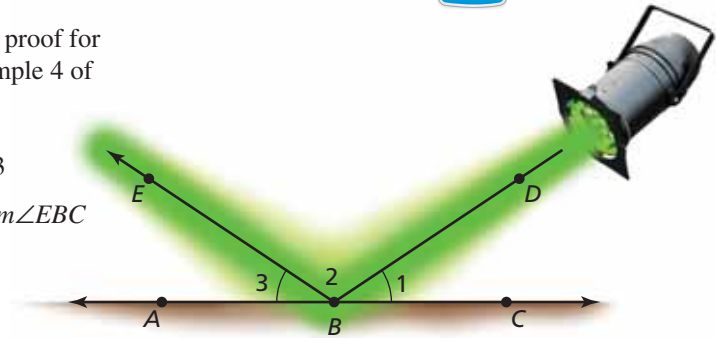
EXAMPLE 1   Writing a Two-Column Proof ▶ WATCH

Write a two-column proof for the situation in Example 4 of Section 2.4.

Given $m\angle 1 = m\angle 3$

Prove $m\angle DBA = m\angle EBC$

STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle DBA = m\angle 3 + m\angle 2$	2. Angle Addition Postulate
3. $m\angle DBA = m\angle 1 + m\angle 2$	3. Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle EBC$	4. Angle Addition Postulate
5. $m\angle DBA = m\angle EBC$	5. Transitive Property of Equality

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SELF-ASSESSMENT **1** I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

1. Complete the proof.

Given T is the midpoint of $\overline{SU}$.

Prove $x = 5$

S ———— 7x ———— T ———— 3x + 20 ———— U

STATEMENTS	REASONS
1. T is the midpoint of $\overline{SU}$.	1. _____
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. _____
5. _____	5. Subtraction Property of Equality
6. $x = 5$	6. _____

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Using Properties of Congruence

The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

THEOREMS

2.1 Properties of Segment Congruence

Segment congruence is reflexive, symmetric, and transitive.

Reflexive   For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric   If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive   If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Prove this Theorem Exercises 7 and 9, page 99

2.2 Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

Reflexive   For any angle A, $\angle A \cong \angle A$.

Symmetric   If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive   If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proof Concept Summary, page 98

Prove this Theorem Exercise 8, page 99; Exercise 35, page 113

EXAMPLE 2   Naming Properties of Congruence   ▶ WATCH

Name the property that each statement illustrates.

a. If $\angle T \cong \angle V$ and $\angle V \cong \angle R$, then $\angle T \cong \angle R$.

b. If $\overline{JL} \cong \overline{YZ}$, then $\overline{YZ} \cong \overline{JL}$.

SOLUTION

a. Transitive Property of Angle Congruence

b. Symmetric Property of Segment Congruence

EXAMPLE 3   Proving a Symmetric Property of Congruence   ▶ WATCH

Write a two-column proof for the Symmetric Property of Segment Congruence.

Given $\overline{LM} \cong \overline{NP}$

Prove $\overline{NP} \cong \overline{LM}$

STATEMENTS	REASONS
1. $\overline{LM} \cong \overline{NP}$	1. Given
2. $LM = NP$	2. Definition of congruent segments
3. $NP = LM$	3. Symmetric Property of Equality
4. $\overline{NP} \cong \overline{LM}$	4. Definition of congruent segments

EXAMPLE 4 **Writing a Two-Column Proof**

Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one-half AB.

Given M is the midpoint of $\overline{AB}$.

Prove $AB = 2AM$, $AM = \frac{1}{2}AB$

A ————————— M ————————— B

STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	**1.** Given
2. $\overline{AM} \cong \overline{MB}$	**2.** Definition of midpoint
3. $AM = MB$	**3.** Definition of congruent segments
4. $AM + MB = AB$	**4.** Segment Addition Postulate
5. $AM + AM = AB$	**5.** Substitution Property of Equality
6. $2AM = AB$	**6.** Distributive Property
7. $AM = \frac{1}{2}AB$	**7.** Division Property of Equality

SELF-ASSESSMENT **1** I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

2. **WRITING** How is a theorem different from a postulate?

Name the property that the statement illustrates.

3. $\overline{GH} \cong \overline{GH}$

4. If $\angle K \cong \angle P$, then $\angle P \cong \angle K$.

5. **WHAT IF?** In Example 4, you want to prove that $AB = 2MB$ and that $MB = \frac{1}{2}AB$ instead. How would the proof be different?
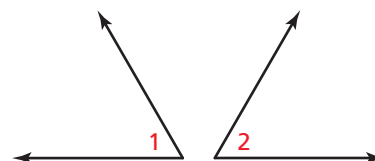
CONCEPT SUMMARY

Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

1   2

Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information from the Prove statement.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$

statements based on facts that you know or on conclusions from deductive reasoning

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

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2.5 Practice WITH CalcChat® AND CalcView®

In Exercises 1 and 2, complete the proof. *(See Example 1.)*

1. Given $PQ = RS$

Prove $PR = QS$

STATEMENTS	REASONS
1. $PQ = RS$	1. _____
2. $PQ + QR = RS + QR$	2. _____
3. _____	3. Segment Addition Postulate
4. $RS + QR = QS$	4. Segment Addition Postulate
5. $PR = QS$	5. _____

2. Given $\angle 1$ is a complement of $\angle 2$.
$\angle 2 \cong \angle 3$

Prove $\angle 1$ is a complement of $\angle 3$.
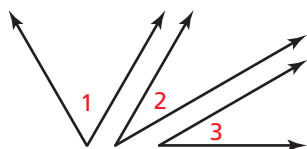
STATEMENTS	REASONS
1. $\angle 1$ is a complement of $\angle 2$.	1. Given
2. $\angle 2 \cong \angle 3$	2. _____
3. $m\angle 1 + m\angle 2 = 90°$	3. _____
4. $m\angle 2 = m\angle 3$	4. Definition of congruent angles
5. _____	5. Substitution Property of Equality
6. $\angle 1$ is a complement of $\angle 3$.	6. _____

In Exercises 3–6, name the property that the statement illustrates. *(See Example 2.)*

3. If $\overline{PQ} \cong \overline{ST}$ and $\overline{ST} \cong \overline{UV}$, then $\overline{PQ} \cong \overline{UV}$.

4. $\angle F \cong \angle F$

5. If $\overline{XY} \cong \overline{UV}$, then $\overline{UV} \cong \overline{XY}$.

6. If $\angle L \cong \angle M$ and $\angle M \cong \angle N$, then $\angle L \cong \angle N$.

PROVING A THEOREM In Exercises 7–9, write a two-column proof for the property. *(See Example 3.)*

7. Reflexive Property of Segment Congruence

8. Transitive Property of Angle Congruence
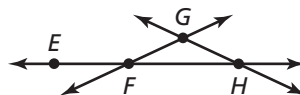
9. Transitive Property of Segment Congruence

10. ERROR ANALYSIS In the diagram, $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$. Describe and correct the error in the reasoning.

Because $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Reflexive Property of Segment Congruence.
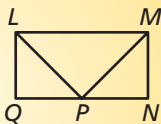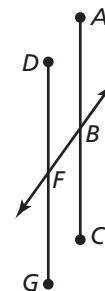
PROOF In Exercises 11 and 12, write a two-column proof. *(See Example 4.)*

11. Given $\angle GFH \cong \angle GHF$

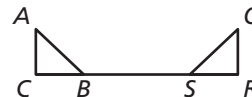**Prove** $\angle EFG$ and $\angle GHF$ are supplementary.

12. Given $\overline{AB} \cong \overline{FG}$,
$\overleftrightarrow{BF}$ bisects $\overline{AC}$ and $\overline{DG}$.

Prove $\overline{BC} \cong \overline{DF}$

13. MAKING AN ARGUMENT In the figure, $\overline{SR} \cong \overline{CB}$ and $\overline{AC} \cong \overline{QR}$. Your friend claims that, because of this, $\overline{CB} \cong \overline{AC}$ by the Transitive Property of Segment Congruence. Is your friend correct? Explain your reasoning.
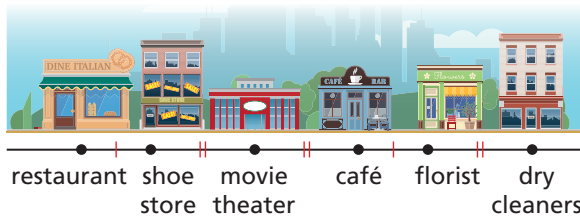

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14. REASONING
In the sign, ∠1 ≅ ∠2 and ∠2 ≅ ∠3. Classify the triangle and justify your answer.

15. MODELING REAL LIFE Use the diagram and the steps below to prove that the distance from the restaurant to the movie theater is the same as the distance from the café to the dry cleaners.

restaurant   shoe store   movie theater   café   florist   dry cleaners

a. State what is given and what is to be proven for the situation.

b. Write a two-column proof.

16. THOUGHT PROVOKING
The distance from Springfield to Lake City is equal to the distance from Springfield to Bettsville. Janisburg is 50 miles farther from Springfield than Bettsville is. Moon Valley is 50 miles farther from Springfield than Lake City is. Use line segments to draw a diagram that represents this situation. Then prove that the distance from Springfield to Janisburg is equal to the distance from Moon Valley to Springfield.

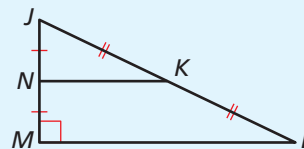**17. CONNECTING CONCEPTS** Write a two-column proof. Then solve for x. Justify each step.

Given $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$

Prove $\overline{QR} \cong \overline{RS}$

P S Q $2x + 5$ R $10 - 3x$
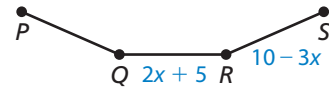
18. HOW DO YOU SEE IT?
Use the figure to write Given and Prove statements for the conclusion "A segment connecting the midpoints of two sides of a triangle is half as long as the third side."

J K N M L

19. DIG DEEPER Fold two corners of a piece of paper so their edges match, as shown.

a. What do you notice about the angle formed at the top of the page where the folds meet?

b. Write a two-column proof to show that the angle measure is always the same no matter how you make the folds.
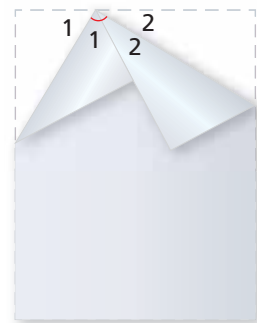
REVIEW & REFRESH

WATCH

In Exercises 20 and 21, solve the equation using any method. Explain your choice of method.

20. $4x^2 - 87 = 109$

21. $3x^2 - 2x - 7 = 0$

22. Does the table represent a *linear* or *nonlinear* function? Explain.

x	2	4	6	8	10
y	$\frac{1}{2}$	1	2	4	8

In Exercises 23 and 24, find the angle measure.

23. ∠1 is a complement of ∠4, and $m\angle 1 = 33°$. Find $m\angle 4$.

24. ∠3 is a supplement of ∠2, and $m\angle 2 = 147°$. Find $m\angle 3$.

25. Use inductive reasoning to make a conjecture about the sum of two negative integers. Then use deductive reasoning to show that the conjecture is true.

26. Solve $-3(6x - 1) = 6x - 9$. Justify each step.

In Exercises 27 and 28, name the property that the statement illustrates.

27. $\overline{JK} \cong \overline{JK}$

28. $\angle C \cong \angle C$

29. MODELING REAL LIFE A fitness center charges members an initial fee of $10 and a monthly fee of $21.99. Find the total cost of 1 year of membership.

30. Sketch a diagram showing $\overrightarrow{AB}$ intersecting $\overleftrightarrow{CD}$ at point K, so that $\overline{AK} \cong \overline{KB}$ and $\overline{CK} \cong \overline{KD}$.

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