2.3 Postulates and Diagrams

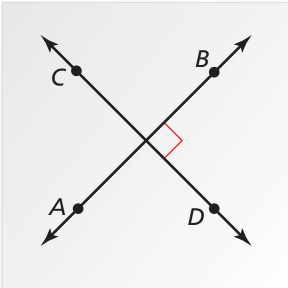
Learning Target: Interpret and sketch diagrams.

Success Criteria:
- I can identify postulates represented by diagrams.
- I can sketch a diagram given a verbal description.
- I can interpret a diagram.

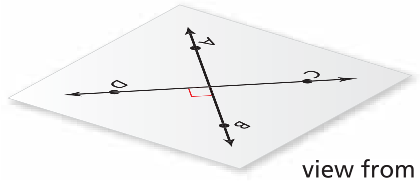
EXPLORE IT! Interpreting Diagrams

Work with a partner.

a. On a piece of paper, draw two perpendicular lines. Label them $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Look at the diagram from different angles. Do the lines appear perpendicular regardless of the angle at which you look at them? Describe *all* the angles at which you can look at the lines and have them appear perpendicular.



view from above


view from upper right

4 MTR COMMUNICATE CLEARLY
From the diagram, can you assume that the lines intersect? Without the right-angle symbol, can you assume the lines are perpendicular?

b. When you draw a diagram, you are communicating with others. It is important that you include sufficient information in the diagram. Use the diagram below to determine which of the following statements you can assume to be true. Explain your reasoning.

i. Points D, G, and I are collinear.

ii. Points A, C, and H are collinear.

iii. $\overleftrightarrow{EG}$ and $\overleftrightarrow{AH}$ are perpendicular.

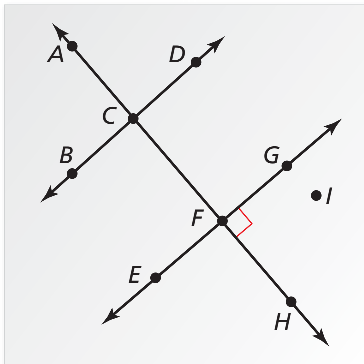
iv. $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are perpendicular.

v. $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are coplanar.

vi. $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ do not intersect.

vii. $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ intersect.

viii. $\overleftrightarrow{AC}$ and $\overleftrightarrow{FH}$ are the same line.



c. Use the diagram in part (b) to write two statements you can assume to be true and two statements you cannot assume to be true. Your statements should be different from those given in part (b). Explain your reasoning.

Geometric Reasoning
preparing for MA.912.GR.1.1 Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.

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Vocabulary

line perpendicular to a plane, *p. 84*

Identifying Postulates

Here are seven more postulates involving points, lines, and planes.

POSTULATES

Point, Line, and Plane Postulates

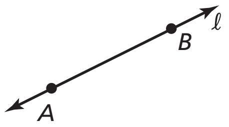
Postulate | **Example**

2.1 Two Point Postulate

Through any two points, there exists exactly one line.

2.2 Line-Point Postulate

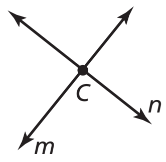
A line contains at least two points.



Through points A and B, there is exactly one line ℓ. Line ℓ contains at least two points.

2.3 Line Intersection Postulate

If two lines intersect, then their intersection is exactly one point.



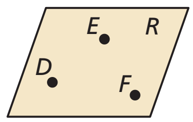
The intersection of line m and line n is point C.

2.4 Three Point Postulate

Through any three noncollinear points, there exists exactly one plane.

2.5 Plane-Point Postulate

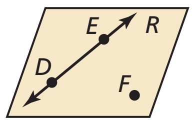
A plane contains at least three noncollinear points.



Through points D, E, and F, there is exactly one plane, plane R. Plane R contains at least three noncollinear points.

2.6 Plane-Line Postulate

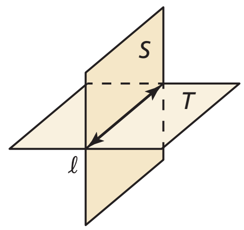
If two points lie in a plane, then the line containing them lies in the plane.



Points D and E lie in plane R, so $\overleftrightarrow{DE}$ lies in plane R.

2.7 Plane Intersection Postulate

If two planes intersect, then their intersection is a line.



The intersection of plane S and plane T is line ℓ.

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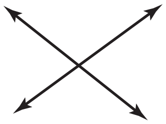
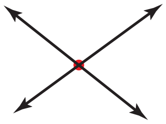
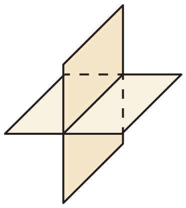
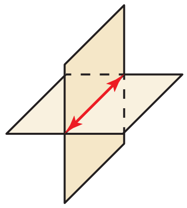
EXAMPLE 1 Identifying a Postulate Using a Diagram

State the postulate illustrated by the diagram.

a. If  then 

b. If  then 

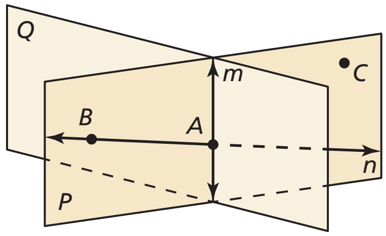
SOLUTION

a. Line Intersection Postulate If two lines intersect, then their intersection is exactly one point.

b. Plane Intersection Postulate If two planes intersect, then their intersection is a line.

EXAMPLE 2 Identifying Postulates from a Diagram

Use the diagram to write examples of the Plane-Point Postulate and the Plane-Line Postulate.



SOLUTION

Plane-Point Postulate Plane P contains at least three noncollinear points, A, B, and C.

Plane-Line Postulate Point A and point B lie in plane P. So, line n containing points A and B also lies in plane P.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. In the diagram in Example 2, which postulate allows you to say that the intersection of plane P and plane Q is a line?

2. Use the diagram in Example 2 to write an example of each postulate.

a. Two Point Postulate

b. Line-Point Postulate

c. Line Intersection Postulate

3. WRITING Explain why at least three noncollinear points are needed to determine a plane.

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Sketching and Interpreting Diagrams

EXAMPLE 3 **Sketching a Diagram** WATCH

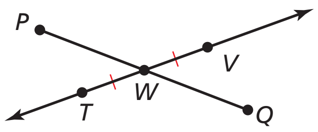
Sketch a diagram showing $\overleftrightarrow{TV}$ intersecting $\overline{PQ}$ at point W, so that $\overline{TW} \cong \overline{WV}$.

SOLUTION

Step 1 Draw $\overleftrightarrow{TV}$ and label points T and V.

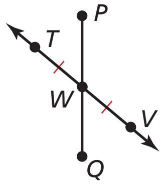
Step 2 Draw point W at the midpoint of $\overline{TV}$. Mark the congruent segments.

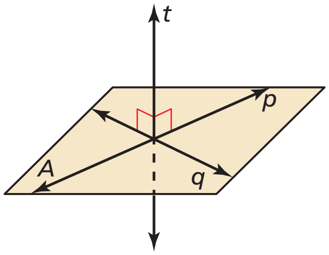
Step 3 Draw $\overline{PQ}$ through W.



ANOTHER WAY

In Example 3, there are many ways you can sketch the diagram. Another way is shown below.





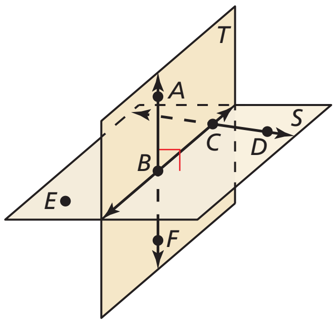
A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

In a diagram, a line perpendicular to a plane must be marked with a right-angle symbol, as shown.

EXAMPLE 4 **Interpreting a Diagram** WATCH

Which of the following statements *cannot* be assumed from the diagram?

- Points A, B, and F are collinear.
- Points E, B, and D are collinear.
- $\overleftrightarrow{AB} \perp$ plane S
- $\overleftrightarrow{CD} \perp$ plane T
- $\overleftrightarrow{AF}$ intersects $\overleftrightarrow{BC}$ at point B.



SOLUTION

No drawn line connects points E, B, and D. So, you cannot assume they are collinear. With no right angle marked, you cannot assume $\overleftrightarrow{CD} \perp$ plane T.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram in Example 3.

4. If it is given that $\overline{PW}$ and $\overline{QW}$ are congruent, how can you indicate this in the diagram?
5. Name a pair of supplementary angles in the diagram. Explain.

Use the diagram in Example 4.

6. Can you assume that plane S intersects plane T at $\overleftrightarrow{BC}$? Explain.
7. Explain how you know that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

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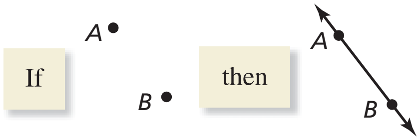
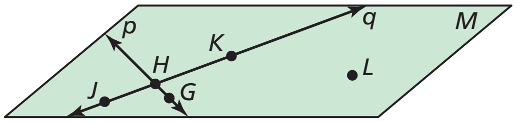
2.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1 and 2, state the postulate illustrated by the diagram. *(See Example 1.)*

▶ **1.** 

2. 

In Exercises 3–6, use the diagram to write an example of the postulate. *(See Example 2.)*



▶ **3.** Line-Point Postulate

4. Line Intersection Postulate

5. Three Point Postulate

6. Plane-Line Postulate

In Exercises 7–10, sketch a diagram of the description. *(See Example 3.)*

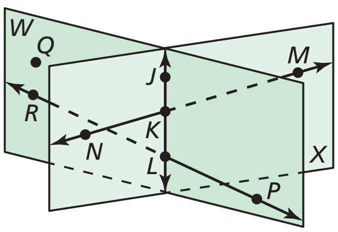
▶ **7.** plane P and line m perpendicular to plane P

8. $\overline{XY}$ in plane P, $\overline{XY}$ bisected by point A, and point C not on $\overline{XY}$

9. $\overline{XY}$ intersecting $\overline{WV}$ at point A, so that $XA = VA$

10. $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are all in plane P, and point X is the midpoint of all three segments.

In Exercises 11–18, use the diagram to determine whether you can assume the statement. *(See Example 4.)*



▶ **11.** Planes W and X intersect at $\overleftrightarrow{KL}$.

12. Points K, L, M, and N are coplanar.

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13. Points Q, J, and M are collinear.

14. $\overleftrightarrow{MN}$ and $\overleftrightarrow{RP}$ intersect.

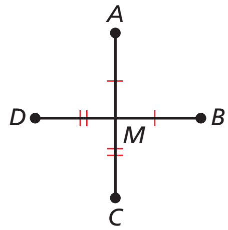
15. $\overleftrightarrow{JK}$ lies in plane X.

16. $\angle PLK$ is a right angle.

17. $\angle NKL$ and $\angle JKM$ are vertical angles.

18. $\angle NKJ$ and $\angle JKM$ are supplementary angles.

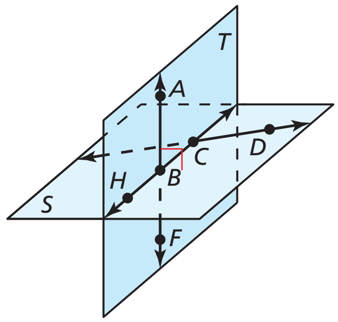
4 MTR **ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in the statement made about the diagram.**



19. ✗ M is the midpoint of $\overline{AC}$ and $\overline{BD}$.

20. ✗ $\overline{AC}$ intersects $\overline{BD}$ at a $90°$ angle, so $\overline{AC} \perp \overline{BD}$.

21. **COLLEGE PREP** Select all the statements about the diagram that you *cannot* conclude.



Ⓐ A, B, and C are coplanar.

Ⓑ Plane T intersects plane S in $\overleftrightarrow{BC}$.

Ⓒ $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$.

Ⓓ $\overleftrightarrow{HC} \perp \overleftrightarrow{CD}$

Ⓔ Plane $T \perp$ plane S

Ⓕ Point B bisects $\overline{HC}$.

Ⓖ $\angle ABH$ and $\angle HBF$ are a linear pair.

Ⓗ $\overleftrightarrow{AF} \perp \overleftrightarrow{CD}$

22. HOW DO YOU SEE IT?
Use the diagram of line m and point C. Make a conjecture about how many planes can be drawn so that line m and point C lie in the same plane. Use postulates to justify your conjecture.

5 MTR **23. CONNECTING CONCEPTS** One way to graph a linear equation is to plot two points whose coordinates satisfy the equation and then connect them with a line. Which postulate guarantees this process works for any linear equation?

5 MTR **24. CONNECTING CONCEPTS** The graph of a system of two linear equations consists of two different lines that intersect. A solution of the system is a point of intersection of the graphs of the equations. Which postulate guarantees that the system has exactly one solution?

In Exercises 25 and 26, rewrite the postulate in if-then form.

25. Two Point Postulate

26. Plane-Point Postulate

4 MTR **27. MAKING AN ARGUMENT** Your friend claims that by the Plane Intersection Postulate, any two planes intersect in a line. Is your friend's interpretation of the Plane Intersection Postulate correct? Explain your reasoning.

28. DIG DEEPER If two lines intersect, then they intersect in exactly one point by the Line Intersection Postulate. Do the two lines have to be in the same plane? Draw a picture to support your answer. Then explain your reasoning.

29. REASONING Points E, F, and G all lie in plane P and in plane Q. What must be true about points E, F, and G so that planes P and Q are different planes? What must be true about points E, F, and G to force planes P and Q to be the same plane? Make sketches to support your answers.

30. THOUGHT PROVOKING
The postulates in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. A plane is the surface of the sphere. Find a postulate on page 82 that is not true in spherical geometry. Explain your reasoning.

REVIEW & REFRESH

WATCH

31. Find a counterexample to show that the conjecture is false.

If a figure has four sides, then it is a rectangle.

In Exercises 32–35, solve the equation. Justify each step.

32. $t - 6 = -4$

33. $3x = 21$

34. $9 + x = 13$

35. $5 = \frac{x}{7}$

36. $\angle 1$ is a supplement of $\angle 2$, and $m\angle 2 = 27°$. Find $m\angle 1$.

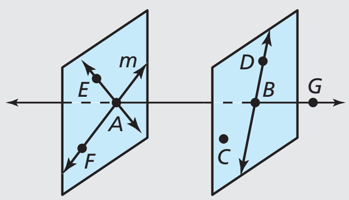
7 MTR **37. MODELING REAL LIFE** A locker in the shape of a rectangular prism has a width of 12 inches. Its height is four times its depth. The volume of the locker is 10,800 cubic inches. Find the height and depth of the locker.

38. Write an equation in point-slope form of the line that passes through $(-8, 2)$ and has a slope of $\frac{3}{4}$.

39. Rewrite the statements as a single biconditional statement.

If you can vote, then you are at least 18 years old.
If you are at least 18 years old, then you can vote.

In Exercises 40–43, use the diagram to determine whether you can assume the statement.



40. Points D, B, and C are coplanar.

41. $m\angle DBG = 90°$

42. Line m intersects $\overleftrightarrow{AB}$ at point A.

43. $\overleftrightarrow{DC}$ lies in plane DBC.

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