1.3 Using Midpoint and Distance Formulas

Learning Target: Find midpoints and lengths of segments.

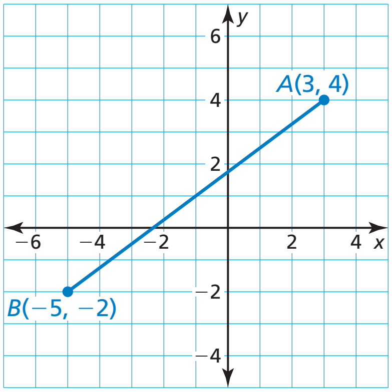
Success Criteria:
- I can find lengths of segments.
- I can construct a segment bisector.
- I can find the weighted average of two or more points on a number line.
- I can find the midpoint of a segment.

EXPLORE IT! Finding Midpoints of Line Segments

1 MTR ANALYZE A PROBLEM
Draw a right triangle with hypoteneuse $\overline{AB}$. How can you find the length of $\overline{AB}$?

Work with a partner.

a. Plot any two points A and B. Then graph $\overline{AB}$. Identify the point M on $\overline{AB}$ that is halfway between points A and B, called the *midpoint* of $\overline{AB}$. Explain how you found the midpoint.



b. Repeat part (a) five times and complete the table.

Coordinates of A	Coordinates of B	Coordinates of M

c. Compare the x-coordinates of A, B, and M. Compare the y-coordinates of A, B, and M. How are the coordinates of the midpoint M related to the coordinates of A and B?

Geometric Reasoning
MA.912.GR.3.1 Determine the weighted average of two or more points on a line.
MA.912.GR.3.3 Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals.
Also MA.912.GR.5.2

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Midpoints and Segment Bisectors

Vocabulary VOCAB

midpoint, *p. 20*
segment bisector, *p. 20*
weighted average, *p. 22*

READING

The word *bisect* means "to cut into two equal parts."

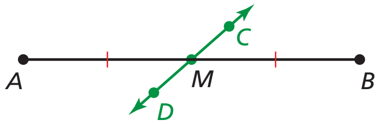
KEY IDEAS

Midpoints and Segment Bisectors

The **midpoint** of a segment is the point that divides the segment into two congruent segments.

M is the midpoint of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

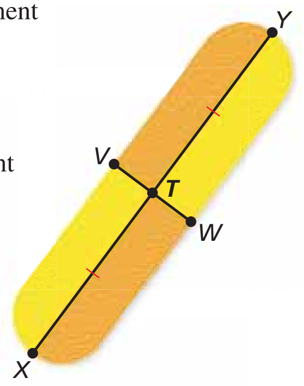
A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.



$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

EXAMPLE 1 **Finding Segment Lengths** WATCH

In the skateboard design, $XT = 39.9$ cm. Identify the segment bisector of $\overline{XY}$. Then find XY.



SOLUTION

The design shows that $\overline{XT} \cong \overline{TY}$. So, point T is the midpoint of $\overline{XY}$, and $XT = TY = 39.9$ cm. Because $\overline{VW}$ intersects $\overline{XY}$ at its midpoint T, $\overline{VW}$ bisects $\overline{XY}$. Find XY.

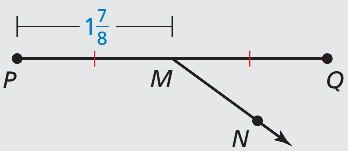
$XY = XT + TY$	Segment Addition Postulate
$= 39.9 + 39.9$	Substitute.
$= 79.8$	Add.

▶ $\overline{VW}$ is the segment bisector of $\overline{XY}$, and XY is 79.8 centimeters.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Identify the segment bisector of $\overline{PQ}$. Then find PQ.

1.



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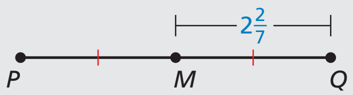

3. VOCABULARY If a point, ray, line, line segment, or plane intersects a segment at its midpoint, then what does it do to the segment?

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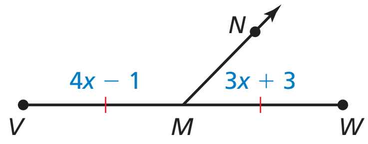
EXAMPLE 2 Using Algebra with Segment Lengths

WATCH



Identify the segment bisector of $\overline{VW}$. Then find VM.

SOLUTION

The figure shows that $\overline{VM} \cong \overline{MW}$. So, point M is the midpoint of $\overline{VW}$, and $VM = MW$. Because $\overrightarrow{MN}$ intersects $\overline{VW}$ at its midpoint M, $\overrightarrow{MN}$ bisects $\overline{VW}$. Find VM.

Step 1 Write and solve an equation to find VM.

$VM = MW$	Write the equation.
$4x - 1 = 3x + 3$	Substitute.
$x - 1 = 3$	Subtract $3x$ from each side.
$x = 4$	Add 1 to each side.

Step 2 Evaluate the expression for VM when $x = 4$.

$$VM = 4x - 1 = 4(4) - 1 = 15$$

▶ $\overrightarrow{MN}$ is the segment bisector of $\overline{VW}$, and VM is 15.

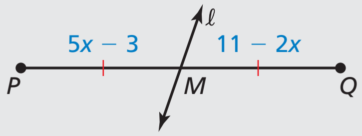
Check Because $VM = MW$, the length of $\overline{MW}$ should be 15.

$$MW = 3x + 3$$
$$= 3(4) + 3$$
$$= 15 \checkmark$$

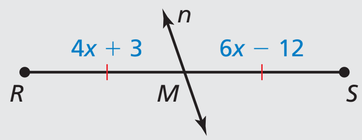
SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. Identify the segment bisector of $\overline{PQ}$. Then find MQ.



5. Identify the segment bisector of $\overline{RS}$. Then find RS.



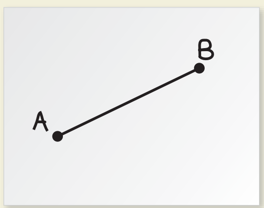
CONSTRUCTION Bisecting a Segment

WATCH

Construct a segment bisector of $\overline{AB}$ by paper folding. Then label the midpoint M of $\overline{AB}$.

SOLUTION

Step 1

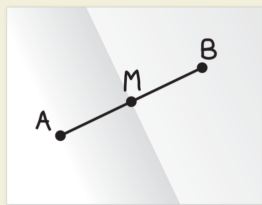


Draw a segment
Use a straightedge to draw $\overline{AB}$ on a piece of paper.

Step 2

Fold the paper
Fold the paper so that B is on top of A.

Step 3



Label the midpoint
Label point M. Compare AM, MB, and AB.

$$AM = MB = \tfrac{1}{2}AB$$

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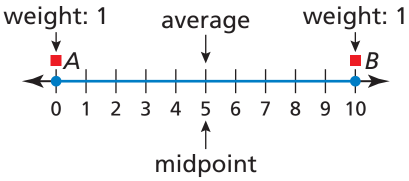
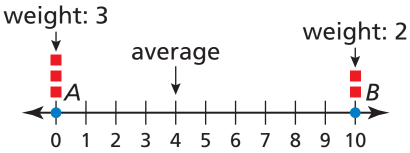
Finding Weighted Averages on a Number Line

When two distinct points A and B are weighted equally, the average is the midpoint of $\overline{AB}$. When points are weighted unequally, the average is a *weighted average*.





KEY IDEA

Weighted Averages

To find the **weighted average** of points on a number line, multiply the coordinate of each point by its weight, and divide the sum of the weighted values by the sum of the weights.

$$W = \frac{\text{sum of the weighted values}}{\text{sum of the weights}}$$

EXAMPLE 3 Finding Weighted Averages

WATCH

4 MTR CONSTRUCT AN ARGUMENT

How does increasing or decreasing the weight on a point affect the weighted average? Explain.

Find each weighted average.

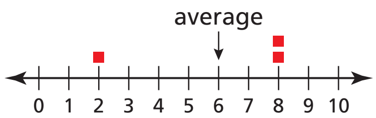
a. The coordinate 2 has a weight of 1, and the coordinate 8 has a weight of 2.

b. The coordinate 3 has a weight of 1, the coordinate 5 has a weight of 3, and the coordinate 9 has a weight of 2.

SOLUTION

a. Multiply each coordinate by its weight. Divide the sum of the weighted values by the sum of the weights.

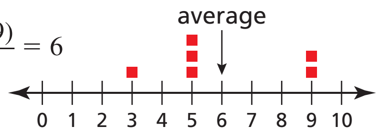
$$\frac{\text{sum of the weighted values}}{\text{sum of the weights}} = \frac{1(2) + 2(8)}{1 + 2} = 6$$



▶ The weighted average is 6.

b. Multiply each coordinate by its weight. Divide the sum of the weighted values by the sum of the weights.

$$\frac{\text{sum of the weighted values}}{\text{sum of the weights}} = \frac{1(3) + 3(5) + 2(9)}{1 + 3 + 2} = 6$$



▶ The weighted average is 6.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. The coordinate 3 has a weight of 2, and the coordinate 9 has a weight of 1. Find the weighted average.

7. The coordinate 2 has a weight of 2, the coordinate 3 has a weight of 2, and the coordinate 10 has a weight of 1. Find the weighted average.

8. Your final grade in a class is based on your score on two tests and the final exam. You scored 86% and 94% on the tests and 92% on the final exam. Find your final grade when the final exam has twice the weight of each test.

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Using the Midpoint Formula

You can use the coordinates of the endpoints of a segment in the coordinate plane to find the coordinates of the midpoint.

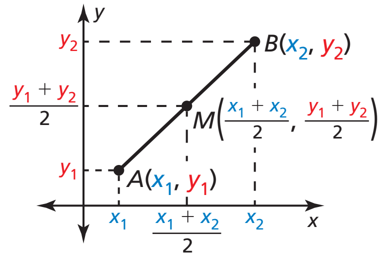
KEY IDEA

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



EXAMPLE 4 Using the Midpoint Formula

WATCH

a. The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M.

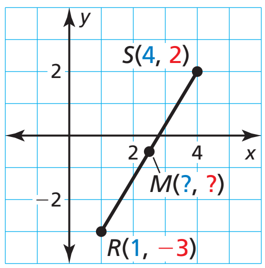
b. The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K.

SOLUTION

a. Use the Midpoint Formula.

$$M\left(\frac{1 + 4}{2}, \frac{-3 + 2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.



b. Let (x, y) be the coordinates of endpoint K. Use the Midpoint Formula.

Step 1 Find x.

$$\frac{1 + x}{2} = 2$$

$$1 + x = 4$$

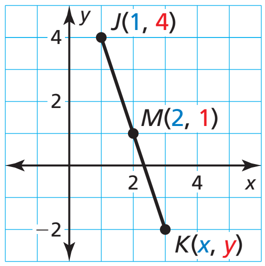
$$x = 3$$

Step 2 Find y.

$$\frac{4 + y}{2} = 1$$

$$4 + y = 2$$

$$y = -2$$



▶ The coordinates of endpoint K are $(3, -2)$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

The endpoints of $\overline{AB}$ are given. Find the coordinates of the midpoint M.

9. $A(1, 2)$ and $B(7, 8)$

10. $A(-4, 3)$ and $B(-6, 5)$

The midpoint M and one endpoint of $\overline{TU}$ are given. Find the coordinates of the other endpoint.

11. $T(1, 1)$ and $M(2, 4)$

12. $U(4, 4)$ and $M(-1, -2)$

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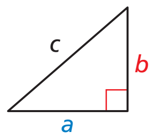
Using the Distance Formula

You can use the Distance Formula to find the distance between two points in a coordinate plane. You can derive the Distance Formula from the *Pythagorean Theorem*, which you will see again when you work with right triangles.

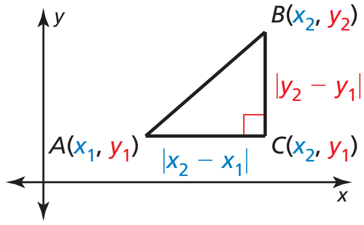
Pythagorean Theorem

$c^2 = a^2 + b^2$



Distance Formula

$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



READING

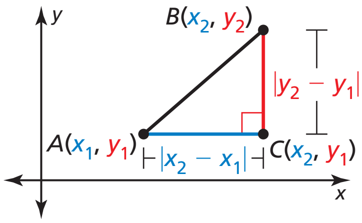
The red mark at the corner of the triangle that makes a right angle indicates a right triangle.

KEY IDEA

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



7 MTR

EXAMPLE 5 Modeling Real Life

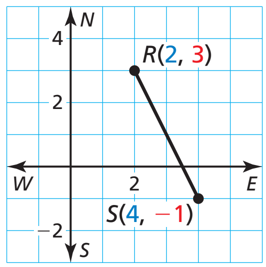
WATCH

Your school is 4 miles east and 1 mile south of your apartment. A recycling center, where your class is going on a field trip, is 2 miles east and 3 miles north of your apartment. Estimate the distance between the recycling center and your school.

SOLUTION

You can model the situation using a coordinate plane with your apartment at the origin (0, 0). The coordinates of the recycling center and the school are $R(2, 3)$ and $S(4, -1)$, respectively. Use the Distance Formula. Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, -1)$.

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(4 - 2)^2 + (-1 - 3)^2}$	Substitute.
$= \sqrt{2^2 + (-4)^2}$	Subtract.
$= \sqrt{4 + 16}$	Evaluate powers.
$= \sqrt{20}$	Add.
≈ 4.5	Use technology.



READING

The symbol ≈ means "is approximately equal to."

So, the distance between the recycling center and your school is about 4.5 miles.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

13. In Example 5, a park is 3 miles east and 4 miles south of your apartment. Estimate the distance between the park and your school.

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1.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, identify the segment bisector of $\overline{RS}$. Then find RS. *(See Example 1.)*

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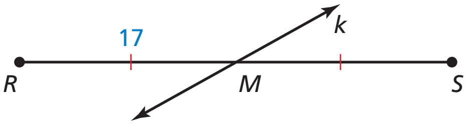

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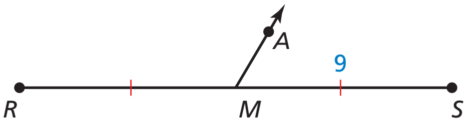

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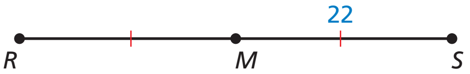

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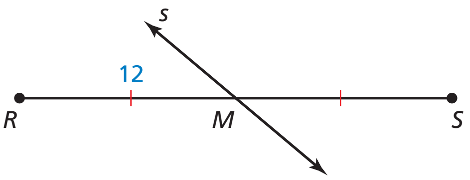
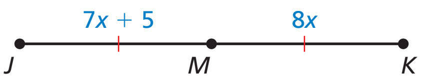


In Exercises 5 and 6, identify the segment bisector of $\overline{JK}$. Then find JM. *(See Example 2.)*

5.


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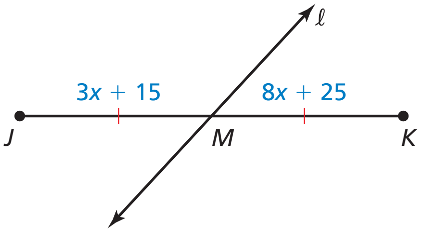
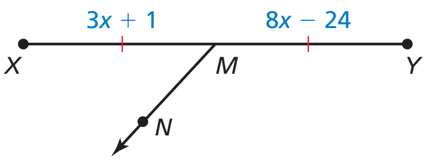


In Exercises 7 and 8, identify the segment bisector of $\overline{XY}$. Then find XY. *(See Example 2.)*

7.


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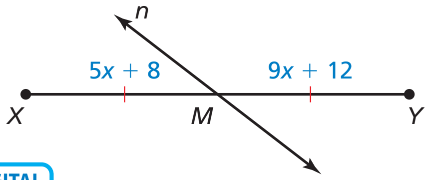

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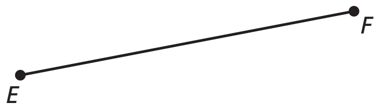
CONSTRUCTION In Exercises 9–12, copy the segment and construct a segment bisector by paper folding. Then label the midpoint M.

9.


10.


11.


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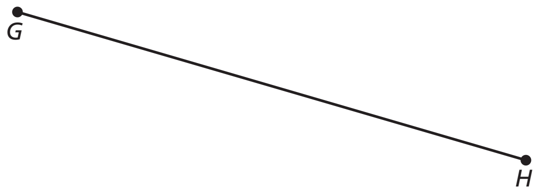


In Exercises 13–20, find the weighted average. *(See Example 3.)*

13. The coordinate 3 has a weight of 2, and the coordinate 8 has a weight of 3.

14. The coordinate −6 has a weight of 3, and the coordinate 2 has a weight of 1.

15. The coordinate −3 has a weight of 2, and the coordinate 4 has a weight of 5.

16. The coordinate 1 has a weight of 4, and the coordinate 4 has a weight of 2.

17. The coordinate −1 has a weight of 1, the coordinate 4 has a weight of 2, and the coordinate 9 has a weight of 2.

18. The coordinate 2 has a weight of 2, the coordinate 5 has a weight of 1, and the coordinate 7 has a weight of 3.

19. The coordinate −2 has a weight of 1, the coordinate 0 has a weight of 1, and the coordinate 3 has a weight of 2.

20. The coordinate 6 has a weight of 1, the coordinate 8 has a weight of 3, and the coordinate 10 has a weight of 1.

In Exercises 21–26, the endpoints of $\overline{CD}$ are given. Find the coordinates of the midpoint M. *(See Example 4.)*

▶ **21.** $C(3, -5)$ and $D(7, 9)$

22. $C(-4, 7)$ and $D(0, -3)$

23. $C(-2, 0)$ and $D(4, 9)$

24. $C(-8, -6)$ and $D(-4, 10)$

25. $C(-3, 5)$ and $D(4, -2)$

26. $C(-7, -3)$ and $D(2, 1)$

In Exercises 27–32, the midpoint M and one endpoint of $\overline{GH}$ are given. Find the coordinates of the other endpoint. *(See Example 4.)*

▶ **27.** $G(5, -6)$ and $M(4, 3)$

28. $H(-3, 7)$ and $M(-2, 5)$

29. $G(-7, 2)$ and $M(-1, 3)$

30. $H(4, -4)$ and $M(-2, 0)$

31. $H(-2, 9)$ and $M(8, 0)$

32. $G(-4, 1)$ and $M\left(-\frac{13}{2}, -6\right)$

In Exercises 33–40, find the distance between the two points. *(See Example 5.)*

▶ **33.** $A(13, 2)$ and $B(7, 10)$

34. $C(-6, 5)$ and $D(-3, 1)$

▶ **35.** $E(3, 7)$ and $F(6, 5)$

36. $G(-5, 4)$ and $H(2, 6)$

37. $J(-8, 0)$ and $K(1, 4)$

38. $L(7, -1)$ and $M(-2, 4)$

39. $R(0, 1)$ and $S(6, 3.5)$

40. $T(13, 1.6)$ and $V(5.4, 3.7)$

4 MTR **ERROR ANALYSIS** **In Exercises 41 and 42, describe and correct the error in finding the distance between $A(6, 2)$ and $B(1, -4)$.**

41.

$$\begin{aligned} AB &= (6-1)^2 + [2-(-4)]^2 \\ &= 5^2 + 6^2 \\ &= 25 + 36 \\ &= 61 \end{aligned}$$

42.

$$\begin{aligned} AB &= \sqrt{(6-2)^2 + [1-(-4)]^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \end{aligned}$$

In Exercises 43–46, the endpoints of two segments are given. Find the length of each segment. Tell whether the segments are congruent. If they are not congruent, tell which segment is longer.

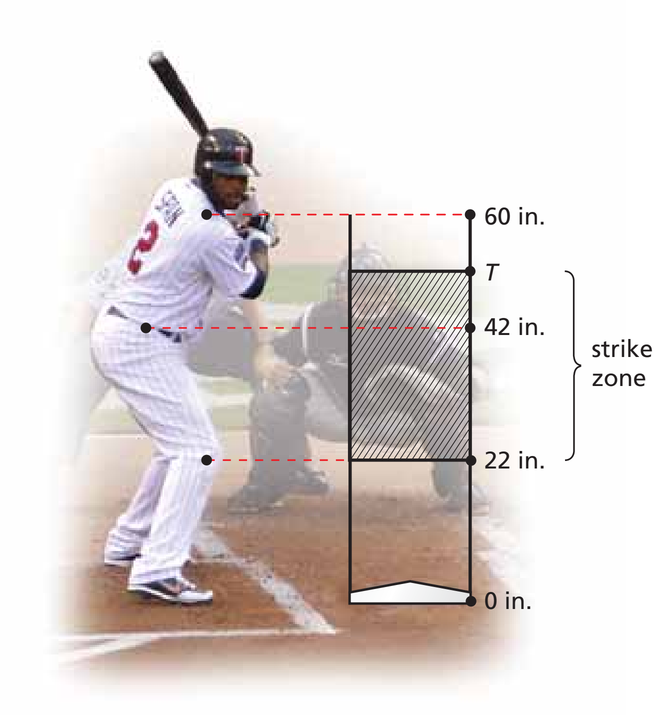
43. $\overline{AB}$: $A(0, 2)$, $B(-3, 8)$ and $\overline{CD}$: $C(-2, 2)$, $D(0, -4)$

44. $\overline{EF}$: $E(1, 4)$, $F(5, 1)$ and $\overline{GH}$: $G(-3, 1)$, $H(1, 6)$

45. $\overline{WX}$: $W(-3, -1)$, $X(3, 4)$ and $\overline{YZ}$: $Y(2, -6)$, $Z(7, -1)$

46. $\overline{LM}$: $L(-5, 1)$, $M(2, -2)$ and $\overline{NP}$: $N(-1, -3)$, $P(2, 4)$

7 MTR **47.** **MODELING REAL LIFE** In baseball, the strike zone is the region a baseball needs to pass through for the umpire to declare it a strike when the batter does not swing. The bottom of the strike zone is a horizontal plane passing through a point just below the kneecap. The top of the strike zone is a horizontal plane passing through the midpoint of the top of the batter's shoulders and the top of the uniform pants when the player is in a batting stance. Find the height of T.



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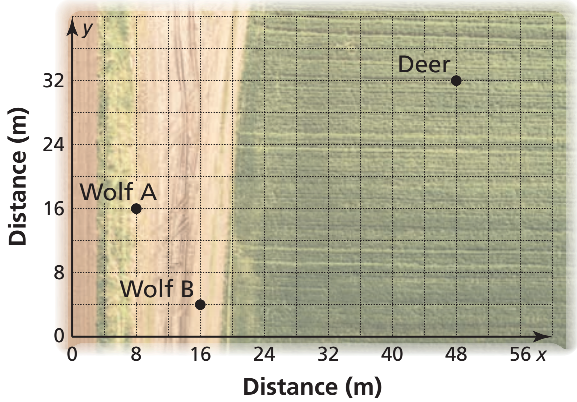
48. **MODELING REAL LIFE** Two wolves spot a deer in a field. The positions of the animals are shown. Which wolf is closer to the deer?



49. **MODELING REAL LIFE** A theater is 3 miles east and 1 mile north of a bus stop. A museum is 4 miles west and 3 miles south of the bus stop. Estimate the distance between the theater and the museum.

50. **MODELING REAL LIFE** Your school is 20 blocks east and 12 blocks south of your home. The mall, where you plan to go after school, is 7 blocks west and 10 blocks north of your home. One block is 0.1 mile. Estimate the distance in miles between your school and the mall.

51. **MAKING AN ARGUMENT** Your friend claims there is an easier way to find the length of a segment than using the Distance Formula when the *x*-coordinates of the endpoints are equal. He claims all you have to do is subtract the *y*-coordinates. Do you agree with his statement? Explain your reasoning.

52. **STRUCTURE** The endpoints of a segment are located at (a, c) and (b, c). Find the coordinates of the midpoint and the length of the segment in terms of a, b, and c.

53. **MODELING REAL LIFE** The chart shows the weights of the assignments in your math class and your corresponding scores.

Assignment	Percent of grade	Score
Homework	10%	95%
Quizzes	20%	75%
Midterm	30%	85%
Final Exam	40%	90%

a. Find your final grade.

b. Your friend scores 90% on homework, 70% on quizzes, and 80% on the midterm. Is it possible for your friend to obtain a higher final grade than you? Explain.

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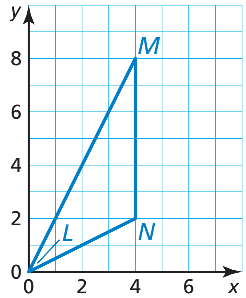
54. **ASSESS REASONABLENESS** Panama City is about a 375-mile drive from Tampa. Your friend claims that the midpoint between Panama City and Tampa is about 187.5 miles between the two cities. Is your friend's claim reasonable? Explain.

55. **CONNECTING CONCEPTS** Triangle *LMN* is shown.

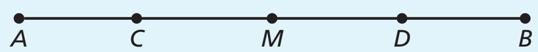


a. Label point M' as the midpoint of segment *LM*.

b. Label point N' as the midpoint of segment *LN*.

c. Is $LM'N'$ a dilation of *LMN*? Justify your answer.

56. **HOW DO YOU SEE IT?**
$\overline{AB}$ contains midpoint *M* and points *C* and *D*, as shown. Compare the lengths. If you cannot draw a conclusion, write *impossible to tell*. Explain your reasoning.



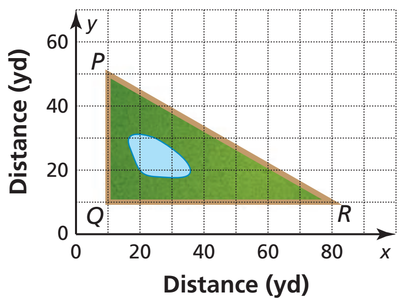
a. *AM* and *MB*

b. *AC* and *MB*

c. *CM* and *MD*

d. *MB* and *DB*

57. **PROBLEM SOLVING** A new bridge is constructed in the triangular park shown. The bridge spans from point *Q* to the midpoint *M* of $\overline{PR}$. A person jogs from *P* to *Q* to *M* to *R* to *Q* and back to *P* at an average speed of 150 yards per minute. About how many minutes does it take? Explain your reasoning.



58. **STRUCTURE** The length of $\overline{XY}$ is 24 centimeters. The midpoint of $\overline{XY}$ is *M*, and point *C* lies on $\overline{XM}$ so that *XC* is $\frac{2}{3}$ of *XM*. Point *D* lies on $\overline{MY}$ so that *MD* is $\frac{3}{4}$ of *MY*. What is the length of $\overline{CD}$?

59. **DIG DEEPER** The endpoints of $\overline{AB}$ are $A(2x, y - 1)$ and $B(y + 3, 3x + 1)$. The midpoint of $\overline{AB}$ is $M\left(-\frac{7}{2}, -8\right)$. What is the length of $\overline{AB}$?

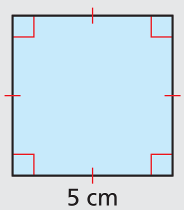
60. **THOUGHT PROVOKING**
The distance between $K(1, -5)$ and a point L with integer coordinates is $\sqrt{58}$ units. Find all the possible coordinates of point L.

REVIEW & REFRESH

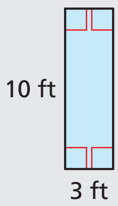
WATCH

In Exercises 61–64, find the perimeter and area of the figure.

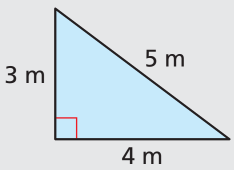
61.



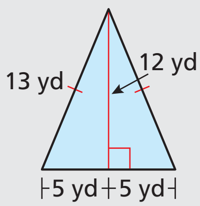
62.



63.



64.



In Exercises 65–68, solve the inequality. Graph the solution.

65. $a + 18 < 7$

66. $y - 5 \geq 8$

67. $-3x > 24$

68. $\frac{z}{4} \leq 12$

69. The endpoints of $\overline{YZ}$ are $Y(1, -6)$ and $Z(-2, 8)$. Find the coordinates of the midpoint M. Then find YZ.

70. Solve the literal equation $5x + 15y = -30$ for y.

71. Find the average rate of change of $f(x) = 3^x$ from $x = 1$ to $x = 3$.

In Exercises 72–75, factor the polynomial.

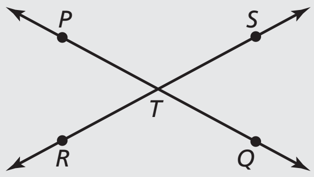
72. $3x^2 - 36x$

73. $n^2 + 3n - 70$

74. $121p^2 - 100$

75. $15y^2 + 4y - 4$

76. Name two pairs of opposite rays in the diagram.



In Exercises 77 and 78, simplify the expression. Write your answer using only positive exponents.

77. $\frac{b^4 \cdot b^{-2}}{b^{10}}$

78. $\left(\frac{2}{5t^4}\right)^{-3}$

79. Plot $A(-3, 3)$, $B(1, 3)$, $C(3, 2)$, and $D(3, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

7 MTR **80.** **MODELING REAL LIFE** The function $p(x) = 80 - 2x$ represents the number of points earned on a test with x incorrect answers.

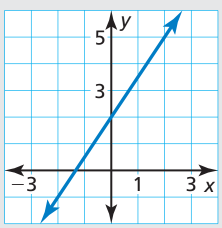
a. How many points are earned with 2 incorrect answers?

b. How many incorrect answers are there when 68 points are earned?

81. Convert 320 fluid ounces to gallons.

82. Determine when the function $y = -|x| - 3$ is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

83. Write an equation of the line in slope-intercept form.



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