10.3 Determinants and Cramer’s Rule

**EXPERIENCE IT! Finding the Determinant**

Work with a partner. Consider the unit square shown.

![Unit Square Diagram]

**a.** Form $2 \times 1$ matrices, \[
\begin{bmatrix}
y \\
x
\end{bmatrix},
\]
using the coordinates of each vertex.

**b.** Multiply \[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\]
by each matrix in part (a). How does the diagram below correspond to your products?

![Diagonal Quadrilateral Diagram]

**c.** The area of the shaded parallelogram in part (b) is called the **determinant** of \[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}.
\]
Find the determinant of \[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}.
\]

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**Learning Target:** Find and use determinants of matrices.

**Success Criteria:**
- I can find the determinant of a square matrix.
- I can use determinants to find areas of triangles.
- I can use determinants to solve systems of equations.

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**Number Sense and Operations**

- MA.912.NSO.4.2 Given a mathematical or real-world context, represent and solve a system of two- or three-variable linear equations using matrices.
- MA.912.NSO.4.4 Solve mathematical and real-world problems using the inverse and determinant of matrices.
- Also MA.912.NSO.4.1
Evaluating Determinants

Associated with each square \((n \times n)\) matrix is a real number called its **determinant**. The determinant of a square matrix \(A\) is denoted by \(\det A\) or by \(\mid A\mid\).

**KEY IDEA**
The Determinant of a Matrix

**Determinant of a 2 \times 2 Matrix**

\[
\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \mid a & b \\ c & d \mid = ad - cb
\]

**Determinant of a 3 \times 3 Matrix**

Step 1 Repeat the first two columns to the right of the determinant.

Step 2 Subtract the sum of the red products from the sum of the blue products.

\[
\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \mid a & b & c \\ d & e & f \\ g & h & i \mid = (aei + bfg + cdh) - (gec + hfa + idb)
\]

**STUDY TIP**
In the Explore It!, you viewed the determinant as the area of a transformed figure. In this lesson, you will use determinants to find areas of triangles and solutions of linear systems.

**EXAMPLE 1** Evaluating Determinants

Evaluate the determinant of each matrix.

\[
a. \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} \\
b. \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & 6 \\ -2 & -3 & 1 \end{bmatrix}
\]

**SOLUTION**

\[
a. \det = 2(4) - 1(5) = 8 - 5 = 3 \\
b. \det = (0 + (-48) + 6) - (0 + (-54) + 8) = -42 - (-46) = 4
\]

**SELF-ASSESSMENT**

Evaluate the determinant of the matrix.

1. \[
\begin{bmatrix} 6 & -2 \\ 3 & 5 \end{bmatrix}
\]
2. \[
\begin{bmatrix} 2 & -3 & 4 \\ 1 & 6 & 0 \\ 3 & -1 & 5 \end{bmatrix}
\]
3. \[
\begin{bmatrix} 5 & -1 & 6 \\ 1 & 2 & 4 \\ -3 & 0 & 2 \end{bmatrix}
\]
You can use a determinant to find the area of a triangle whose vertices are points in a coordinate plane.

**KEY IDEA**

**Area of a Triangle**

The area of a triangle with vertices \((x_1, y_1), (x_2, y_2),\) and \((x_3, y_3)\) is given by

\[
\text{Area} = \pm \frac{1}{2} \begin{vmatrix}
  x_1 & y_1 & 1 \\
  x_2 & y_2 & 1 \\
  x_3 & y_3 & 1 \\
\end{vmatrix}
\]

where the symbol \(\pm\) indicates that the appropriate sign should be chosen to yield a positive value.

**EXAMPLE 2**  
**Finding the Area of a Triangle**

Find the area of the triangle.

**SOLUTION**

\[
\text{Area} = \pm \frac{1}{2} \begin{vmatrix}
  2 & 3 & 1 \\
  3 & -2 & 1 \\
  -4 & -3 & 1 \\
\end{vmatrix}
\]

\[
= \pm \frac{1}{2} \begin{vmatrix}
  2 & 3 & 2 \\
  3 & -2 & 3 \\
  -4 & -3 & -4 \\
\end{vmatrix}
\]

\[
= \pm \frac{1}{2} \left[ (-4 - 12 - 9) - (8 - 6 + 9) \right]
\]

\[
= \pm \frac{1}{2} (-25 - 11)
\]

\[
= \pm (-18)
\]

So, the area of the triangle is 18 square units.

**SELF-ASSESSMENT**  
1. I don't understand yet.  
2. I can do it with help.  
3. I can do it on my own.  
4. I can teach someone else.

4. Find the area of the triangle with vertices \((3, 6), (8, -1),\) and \((-4, 5).\)
Cramer’s Rule

You can use determinants to solve a system of linear equations. The method, called Cramer’s Rule and named after Swiss mathematician Gabriel Cramer (1704–1752), uses the coefficient matrix of the linear system.

### Key Idea

**Cramer’s Rule for a $2 \times 2$ System**

Let $A$ be the coefficient matrix of the linear system

\[
\begin{align*}
ax + by &= e \\
\, cx + dy &= f
\end{align*}
\]

If $\det A \neq 0$, then the system has exactly one solution. The solution is

\[
\begin{align*}
x &= \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \\
y &= \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}
\end{align*}
\]

Notice that the numerators for $x$ and $y$ are the determinants of the matrices formed by replacing the values of the $x$ and $y$ coefficient columns, respectively, with the column of the constant values.

### Example 3  Using Cramer’s Rule for a $2 \times 2$ System

Use Cramer’s Rule to solve the system

\[
\begin{align*}
3x - 2y &= 7 \\
5x + 4y &= -3
\end{align*}
\]

**Solution**

**Step 1** Evaluate the determinant of the coefficient matrix.

\[
\begin{vmatrix} 3 & -2 \\ 5 & -4 \end{vmatrix} = 12 - (-10) = 22
\]

**Step 2** Apply Cramer’s Rule because the determinant is not 0.

\[
x = \frac{\begin{vmatrix} 7 & -2 \\ -3 & 4 \end{vmatrix}}{22} = \frac{28 - 6}{22} = \frac{22}{22} = 1
\]

\[
y = \frac{\begin{vmatrix} 3 & 7 \\ 5 & -3 \end{vmatrix}}{22} = \frac{-9 - 35}{22} = \frac{-44}{22} = -2
\]

The solution is $(1, -2)$.
A **linear equation in three variables** \(x, y,\) and \(z\) is an equation of the form
\[ax + by + cz = d,\]
where \(a, b,\) and \(c\) are not all zero. An example of a **system of three linear equations** in three variables is shown at the left. A **solution** of such a system is an **ordered triple** \((x, y, z)\) whose coordinates make each equation true.

**STUDY TIP**
The numerators for \(x, y,\) and \(z\) are the determinants of the matrices formed by replacing the values of the \(x, y,\) and \(z\) coefficient columns, respectively, with the column of the constant values.

**EXAMPLE 4 Using Cramer’s Rule for a 3 \(\times\) 3 System**

Use Cramer’s Rule to solve the system.

\[\begin{align*}
-x - 3y + 5z &= 21 \\
x + 4y - 3z &= -5 \\
x - 2y - z &= -17
\end{align*}\]

**SOLUTION**

**Step 1** Evaluate the determinant of the coefficient matrix.

\[
\begin{vmatrix}
3 & -3 & 5 \\
1 & 4 & -3 \\
1 & -2 & -1
\end{vmatrix} = (4 + 9 - 10) - (20 - 6 + 3) = -14
\]

**Step 2** Apply Cramer’s Rule because the determinant is not 0.

\[
x = \frac{21 - (-5) - 17}{-14} = \frac{42}{-14} = -3
\]

\[
y = \frac{-1 - 21 + 17}{-14} = \frac{-14}{-14} = 1
\]

\[
z = \frac{-1 - 3 + 17}{-14} = \frac{14}{-14} = -1
\]

The solution is \((-3, 1, -1)\).

**SELF-ASSESSMENT**

1. I don’t understand yet.
2. I can do it with help.
3. I can do it on my own.
4. I can teach someone else.

**Use Cramer’s Rule to solve the linear system.**

5. \[\begin{align*}
7x - 2y &= 20 \\
3x + 4y &= -6
\end{align*}\]

6. \[\begin{align*}
5x + 6y &= 14 \\
-2x - 3y &= -8
\end{align*}\]

7. \[\begin{align*}
5x - 2y + 3z &= -2 \\
4x + 7y - z &= 0 \\
2x - 6y + 5z &= -7
\end{align*}\]
EXAMPLE 5  Modeling Real Life

The molecular mass of a compound is the sum of the atomic masses of the atoms it contains. The table shows the molecular masses (in atomic mass units) of three compounds. Use a linear system and Cramer’s Rule to find the atomic masses of bromine (Br), nitrogen (N), and fluorine (F).

<table>
<thead>
<tr>
<th>Compound</th>
<th>Formula</th>
<th>Molecular Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bromine azide</td>
<td>BrN₃</td>
<td>122</td>
</tr>
<tr>
<td>Bromine trifluoride</td>
<td>BrF₃</td>
<td>137</td>
</tr>
<tr>
<td>Nitrogen trifluoride</td>
<td>NF₃</td>
<td>71</td>
</tr>
</tbody>
</table>

**SOLUTION**

Step 1  Write a linear system using the formula for each compound. Let \( B \), \( N \), and \( F \) represent the atomic masses of bromine, nitrogen, and fluorine.

\[
\begin{align*}
B + 3N &= 122 & \text{BrN₃: 1 bromine atom and 3 nitrogen atoms} \\
B + 3F &= 137 & \text{BrF₃: 1 bromine atom and 3 fluorine atoms} \\
N + 3F &= 71 & \text{NF₃: 1 nitrogen atom and 3 fluorine atoms}
\end{align*}
\]

Step 2  Evaluate the determinant of the coefficient matrix.

\[
\begin{vmatrix}
1 & 3 & 0 \\
1 & 0 & 3 \\
0 & 1 & 3
\end{vmatrix} = (0 + 0 + 0) - (0 + 3 + 9) = -12
\]

Step 3  Apply Cramer’s Rule because the determinant is not 0.

\[
B = \frac{122}{1(-12) - 0(-12)} = \frac{122}{-12} = -10.1667
\]

\[
N = \frac{112}{1(-12) - 3(-12)} = \frac{112}{-12} = -9.3333
\]

\[
F = \frac{171}{1(-12) - 3(1)} = \frac{171}{-12} = -14.25
\]

The atomic masses (in atomic mass units) of bromine, nitrogen, and fluorine are 80, 14, and 19, respectively.
10.3 Practice with CalcChat* and CalcView®

In Exercises 1–16, evaluate the determinant of the matrix. (See Example 1.)

1. \[
\begin{bmatrix}
3 & 1 \\
2 & 4
\end{bmatrix}
\]
2. \[
\begin{bmatrix}
5 & 2 \\
7 & 3
\end{bmatrix}
\]
3. \[
\begin{bmatrix}
-1 & 3 \\
2 & 6
\end{bmatrix}
\]
4. \[
\begin{bmatrix}
2 & 5 \\
8 & -4
\end{bmatrix}
\]
5. \[
\begin{bmatrix}
1 & -2 \\
-3 & 4
\end{bmatrix}
\]
6. \[
\begin{bmatrix}
-5 & 9 \\
7 & -2
\end{bmatrix}
\]
7. \[
\begin{bmatrix}
-2 & 4 \\
-8 & 10
\end{bmatrix}
\]
8. \[
\begin{bmatrix}
-8 & -9 \\
14 & 12
\end{bmatrix}
\]
9. \[
\begin{bmatrix}
1 & 0 & 5 \\
6 & 2 & 0 \\
3 & 8 & 9
\end{bmatrix}
\]
10. \[
\begin{bmatrix}
-2 & -7 & 0 \\
-3 & 1 & 4 \\
5 & 0 & -6
\end{bmatrix}
\]
11. \[
\begin{bmatrix}
-9 & 0 & 1 \\
2 & 4 & 8 \\
-3 & -4 & -6
\end{bmatrix}
\]
12. \[
\begin{bmatrix}
2 & -1 & 3 \\
5 & 0 & 7 \\
0 & -6 & -4
\end{bmatrix}
\]

ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in evaluating the determinant of the matrix.

17. \[
\begin{vmatrix}
5 & 2 \\
7 & 4
\end{vmatrix} = 2(7) - 5(4) = -6
\]

18. \[
\begin{vmatrix}
2 & -2 \\
3 & -3
\end{vmatrix} = (0 + 6 + 0) - (0 + 6 + 0) = 0
\]

GO DIGITAL

19. NUMBER SENSE Order the determinants from least to greatest.

A. \[
\begin{vmatrix}
5 & -2 \\
1 & 5
\end{vmatrix}
\]
B. \[
\begin{vmatrix}
1 & 6 \\
3 & 8
\end{vmatrix}
\]
C. \[
\begin{vmatrix}
5 & -3 \\
7 & -1
\end{vmatrix}
\]
D. \[
\begin{vmatrix}
-4 & 1 \\
6 & 3
\end{vmatrix}
\]

20. COLLEGE PREP For what value of \(k\) is the determinant of the matrix below equal to 0?

\[
\begin{bmatrix}
0 & 2 & 0 \\
9 & 9 & 2 \\
5 & 4 & 2
\end{bmatrix}
\]

A. \(k = -10\)
B. \(k = -5\)
C. \(k = 5\)
D. \(k = 10\)

In Exercises 21–24, find the area of the triangle. (See Example 2.)

21. \[
\begin{array}{c}
A(3, -2) \\
B(1, 4) \\
C(-3, 5)
\end{array}
\]

22. \[
\begin{array}{c}
A(-3, -1) \\
B(1, 3) \\
C(-4, -5)
\end{array}
\]

23. \[
\begin{array}{c}
A(-3, 4) \\
B(4, 0) \\
C(-6, -4)
\end{array}
\]

24. \[
\begin{array}{c}
A(-5, 3) \\
B(0, 4) \\
C(-4, -3)
\end{array}
\]

25. MODELING REAL LIFE A researcher of Lyme disease tests ticks from the triangular region shown below. Find the area of the triangular region.

In Exercises 21–24, find the area of the triangle. (See Example 2.)

10.3 Determinants and Cramer’s Rule 537
26. **MODELING REAL LIFE** A volunteer for the Florida Folk Festival in White Springs sets up the space shown for a workshop. Find the area used for the workshop.

The Florida Folk Festival celebrates the state’s cultural heritage through music, dance, art, and food during the three-day event.

![Diagram of a triangle with sides 10 ft, 15 ft, and 35 ft and a right angle.]

A volunteer for the Florida Folk Festival in White Springs sets up the space shown for a workshop. Find the area used for the workshop.

In Exercises 27 and 28, find a positive value of $y$ such that a triangle with the given vertices has an area of 5 square units.

27. $(-4, 3), (0, 1), (1, y)$
28. $(-3, -3), (0, -2), (-1, y)$

STRUCTURE In Exercises 27 and 28, find a positive value of $y$ such that a triangle with the given vertices has an area of 5 square units.

In Exercises 27 and 28, find a positive value of $y$ such that a triangle with the given vertices has an area of 5 square units.

In Exercises 27 and 28, find a positive value of $y$ such that a triangle with the given vertices has an area of 5 square units.

27. $(-4, 3), (0, 1), (1, y)$
28. $(-3, -3), (0, -2), (-1, y)$

In Exercises 29–36, use Cramer’s Rule to solve the linear system. (See Examples 3 and 4.)

29. $4x + 3y = 5$
   $3x + 2y = 4$
30. $2x + 5y = 11$
   $4x + 8y = 20$
31. $5x - y = 12$
   $-x + 6y = -14$
32. $-7x + 2y = -13$
   $8x + 3y = -1$
33. $-x + y - 4z = -7$
   $3x + 5y - 6z = -5$
   $2x - 3y + 5z = 12$
34. $x + 3y + 2z = 11$
   $2x - 4y + 3z = 7$
   $5x + 2y - z = -13$
35. $2x - 4y + 7z = -1$
   $-3x + 5y - z = 26$
   $x - 6y + z = -24$
36. $3x + 2y + z = 2$
   $6x - 2y - 3z = 23$
   $-2x + 3y - 2z = -20$

In Exercises 39 and 40, the table shows the molecular masses (in atomic mass units) of three compounds. Use a linear system and Cramer’s Rule to find the atomic masses of the indicated elements. (See Example 5.)

39. sulfur (S), chlorine (Cl), calcium (Ca)

<table>
<thead>
<tr>
<th>Compound</th>
<th>Formula</th>
<th>Molecular Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sulfur dichloride</td>
<td>SCl₂</td>
<td>103</td>
</tr>
<tr>
<td>Calcium chloride</td>
<td>CaCl₂</td>
<td>111</td>
</tr>
<tr>
<td>Calcium sulfide</td>
<td>CaS</td>
<td>72</td>
</tr>
</tbody>
</table>

40. phosphorus (P), selenium (Se), iodine (I)

<table>
<thead>
<tr>
<th>Compound</th>
<th>Formula</th>
<th>Molecular Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diphosphorus pentaselenide</td>
<td>P₂Se₅</td>
<td>457</td>
</tr>
<tr>
<td>Phosphorus triiodide</td>
<td>PI₃</td>
<td>412</td>
</tr>
<tr>
<td>Selenium diiodide</td>
<td>SeI₂</td>
<td>333</td>
</tr>
</tbody>
</table>

In Exercises 39 and 40, the table shows the molecular masses (in atomic mass units) of three compounds. Use a linear system and Cramer’s Rule to find the atomic masses of the indicated elements. (See Example 5.)

41. **MODELING REAL LIFE** The metal wire of a particular dental retainer is constructed using 40% nickel and 60% titanium. The price per ounce of nickel is $0.50, and the price per ounce of titanium is $1.80. A manufacturer buys a total $102.40 of the metals to produce the wires. How many ounces of nickel does the manufacturer buy?

**GO DIGITAL**
10.3 Determinants and Cramer’s Rule

42. **MODELING REAL LIFE** Tin is usually found as an oxide called cassiterite, shown above. When tin is melted down, it can be combined with other metals. An artist creates bronze jewelry using aluminum, copper, and tin. The costs of each metal are shown in the table. The artist orders a total of 10 pounds of metals for a cost of $32.70. The artist orders seven times as much copper as tin. How many pounds of tin does the artist order?

<table>
<thead>
<tr>
<th>Metal</th>
<th>Price Per Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$0.75</td>
</tr>
<tr>
<td>Copper</td>
<td>$2.75</td>
</tr>
<tr>
<td>Tin</td>
<td>$7.75</td>
</tr>
</tbody>
</table>

43. **MODELING REAL LIFE** A system of pulleys is loaded with 128-pound and 32-pound weights, as shown.

The tensions $t_1$ and $t_2$ (in pounds) in the ropes and the acceleration $a$ (in feet per second squared) of the 32-pound weight are represented by the system:

\[
\begin{align*}
    t_1 - 2t_2 &= 0 \\
    t_1 - 2a &= 128 \\
    t_2 + a &= 32
\end{align*}
\]

Find $t_1$, $t_2$, and $a$.

44. **MODELING REAL LIFE** Solar flares are classified based on their strength. In a study, the average intensities (in watts per square meter) of solar flares of class C, M, and X are solutions of the system:

\[
\begin{align*}
    37C + M + X &= 0.000705 \\
    15C + 14M + 4X &= 0.002775 \\
    5C + 9M + 4X &= 0.002475
\end{align*}
\]

Find the three intensities.

45. In Exercises 45 and 46, $(x_1, y_1)$, $(x_2, y_2)$, and $(x_3, y_3)$ lie on the same line only when

\[
\begin{vmatrix}
    x_1 & y_1 & 1 \\
    x_2 & y_2 & 1 \\
    x_3 & y_3 & 1
\end{vmatrix} = 0.
\]

Determine whether the points lie on the same line.

45. $(0, 1)$, $(2, 4)$, $(3, 6)$

46. $(-5, -4)$, $(-2, -1)$, $(3, 4)$

47. **COMPARE METHODS** Compare the area of the red triangle to the area of the blue triangle using two methods. Which method do you prefer? Explain.

48. **STRUCTURE** Solve the equation

\[
\begin{vmatrix}
    x + 3 & x \\
    4 & x - 1
\end{vmatrix} = 5.
\]

49. **STRUCTURE** A community organizer requests a triangular plot of land with an area of 5000 square feet for an urban garden. On a blueprint, one vertex is at $(0, 0)$ and another is at $(100, 50)$. The y-coordinate of the final vertex is 120. Find the possible locations of the final vertex.

50. **HOW DO YOU SEE IT?** Without calculating, explain why the determinant of the matrix below is 0.

\[
\begin{bmatrix}
    6 & -2 & 8 \\
    0 & 0 & 0 \\
    1 & 7 & 3
\end{bmatrix}
\]

51. **REASONING** Consider $3 \times 3$ square matrices in which the elements are consecutive integers. An example is shown below.

\[
\begin{bmatrix}
    2 & 3 & 4 \\
    5 & 6 & 7 \\
    8 & 9 & 10
\end{bmatrix}
\]

Find the determinants of four matrices of this type. Make a conjecture based on the results.
52. **THOUGHT PROVOKING**

Consider the equation for the determinant of a \(3 \times 3\) matrix,
\[
\begin{vmatrix}
 a & b & c \\
 d & e & f \\
 g & h & i \\
\end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).
\]

How can you rewrite the determinant of a \(3 \times 3\) matrix using determinants of \(2 \times 2\) matrices? Show that your equation is equivalent to the equation above.

53. **DIG DEEPER**

Consider the system
\[
\begin{align*}
ax + by &= e \\
 cx + dy &= f \\
\end{align*}
\]

a. Solve the system by finding rational expressions for \(x\) and \(y\).

b. What is the relationship between determinants and your answers in part (a)? Explain.

54. **REVIEW & REFRESH**

In Exercises 54 and 55, graph the function. Identify the \(x\)-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing.

54. \(g(x) = \frac{1}{2}(x + 2)(x - 3)\)

55. \(f(x) = x^3 - 2x^2 - x + 2\)

56. Solve the equation for \(a, b, c,\) and \(d\).
\[
\begin{pmatrix}
 8 - a & 5b \\
 c & -2d \\
\end{pmatrix} = \begin{pmatrix}
 4 & 20 \\
 22 & 14 \\
\end{pmatrix}
\]

57. Find the product
\[
\begin{pmatrix}
 0 & 2 \\
 -1 & 3 \\
\end{pmatrix} \begin{pmatrix}
 0 & -9 \\
 17 & 1 \\
\end{pmatrix}
\]

58. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

59. Use Cramer’s Rule to solve the system.
\[
\begin{align*}
8x + y &= 21 \\
x + 5y &= 27 \\
\end{align*}
\]

60. Describe the pattern, write the next term, and write a rule for the \(n\)th term of the sequence.

61. **MODELING REAL LIFE** You are considering six full and three rimless frames for prescription glasses. You randomly select a frame, set it aside, and then randomly select another frame. Determine whether randomly selecting a full frame first and a rimless frame second are independent events.

62. Determine whether the function \(h(x) = -2x^3 - 2x^2 + 18x + 18\) is *even*, *odd*, or *neither*.

63. Find the inverse of the function \(f(x) = 9x^2, x \leq 0\). Then graph the function and its inverse.

64. Find the value of \(c\). Then write an expression represented by the diagram.

65. Graph the function \(y = e^x + 4\). Then find the domain and range.

66. Find the sum.
\[
\frac{3}{x + 1} + \frac{2x}{x + 2}
\]

67. Graph the system of quadratic inequalities.
\[
\begin{align*}
y &\geq x^2 \\
y &< x^2 + 4x - 5 \\
\end{align*}
\]

68. Describe the transformation of \(f(x) = 2^x\) represented by \(g(x) = 2^x - 5\). Then graph each function.

69. Determine whether the data show an exponential relationship. Then write a function that models the data.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(-0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0.2</td>
<td>0.6</td>
<td>1.8</td>
<td>5.4</td>
<td>16.2</td>
<td>48.6</td>
</tr>
</tbody>
</table>

70. Use finite differences to determine the degree of the polynomial function that fits the data below. Then use technology to find the polynomial function.

\((-2, -2), (-1, 0), (0, 4), (1, 10), (2, 18)\)