10.3 Determinants and Cramer's Rule H

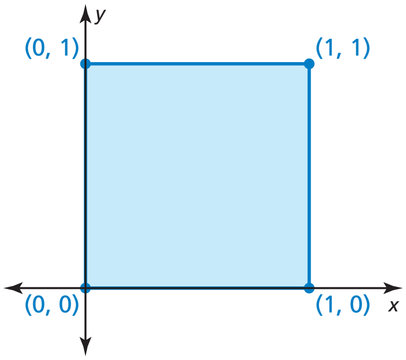
Learning Target: Find and use determinants of matrices.

Success Criteria:

- I can find the determinant of a square matrix.
- I can use determinants to find areas of triangles.
- I can use determinants to solve systems of equations.

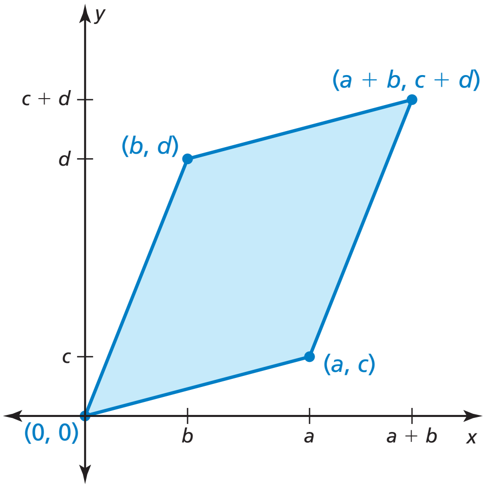
EXPLORE IT! Finding the Determinant

Work with a partner. Consider the unit square shown.



a. Form 2×1 matrices, $\begin{bmatrix} x \\ y \end{bmatrix}$, using the coordinates of each vertex.

b. Multiply $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ by each matrix in part (a). How does the diagram below correspond to your products?



c. The area of the shaded parallelogram in part (b) is called the *determinant* of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find the determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

5 MTR USE STRUCTURE

How can drawing a rectangle around the parallelogram help you find the determinant?

Number Sense and Operations

H MA.912.NSO.4.2 Given a mathematical or real-world context, represent and solve a system of two- or three-variable linear equations using matrices.

H MA.912.NSO.4.4 Solve mathematical and real-world problems using the inverse and determinant of matrices.

H Also MA.912.NSO.4.1

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Evaluating Determinants

Associated with each square ($n \times n$) matrix is a real number called its **determinant**. The determinant of a square matrix A is denoted by det A or by $|A|$.

Vocabulary

determinant, *p. 532*
Cramer's Rule, *p. 534*
coefficient matrix, *p. 534*
linear equation in three variables, *p. 535*
system of three linear equations, *p. 535*
solution of a system of three linear equations, *p. 535*
ordered triple, *p. 535*

KEY IDEA

The Determinant of a Matrix

Determinant of a 2 × 2 Matrix

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

The determinant of a 2 × 2 matrix is the difference of the products of the elements on the diagonals in the order shown.

Determinant of a 3 × 3 Matrix

Step 1 Repeat the first two columns to the right of the determinant.

Step 2 Subtract the sum of the red products from the sum of the blue products.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right| \begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

STUDY TIP

In the Explore It!, you viewed the determinant as the area of a transformed figure. In this lesson, you will use determinants to find areas of triangles and solutions of linear systems.

EXAMPLE 1 Evaluating Determinants

Evaluate the determinant of each matrix.

a. $\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & 6 \\ -2 & -3 & 1 \end{bmatrix}$

SOLUTION

a. $\begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix} = 2(4) - 1(5) = 8 - 5 = 3$

b. $$\left|\begin{matrix} 3 & 4 & -1 \\ 2 & 0 & 6 \\ -2 & -3 & 1 \end{matrix}\right| \begin{matrix} 3 & 4 \\ 2 & 0 \\ -2 & -3 \end{matrix} = (0 + (-48) + 6) - (0 + (-54) + 8)$$

$$= -42 - (-46)$$

$$= 4$$

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Evaluate the determinant of the matrix.

1. $\begin{bmatrix} 6 & -2 \\ 3 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 2 & -3 & 4 \\ 1 & 6 & 0 \\ 3 & -1 & 5 \end{bmatrix}$

3. $\begin{bmatrix} 5 & -1 & 6 \\ 1 & 2 & 4 \\ -3 & 0 & 2 \end{bmatrix}$

You can use a determinant to find the area of a triangle whose vertices are points in a coordinate plane.

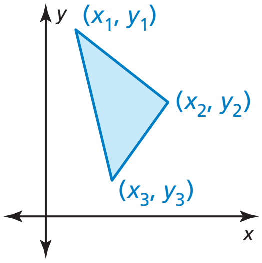
KEY IDEA

Area of a Triangle

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

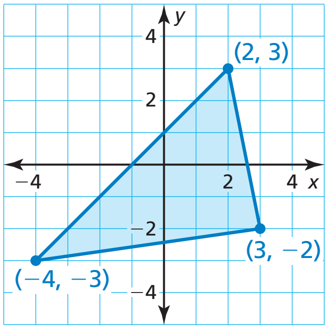
where the symbol ± indicates that the appropriate sign should be chosen to yield a positive value.



EXAMPLE 2 Finding the Area of a Triangle

WATCH

Find the area of the triangle.



SOLUTION

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 3 & -2 & 1 \\ -4 & -3 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left|\begin{array}{rrr|rr} 2 & 3 & 1 & 2 & 3 \\ 3 & -2 & 1 & 3 & -2 \\ -4 & -3 & 1 & -4 & -3 \end{array}\right.$$

$$= \pm\frac{1}{2}[(-4 - 12 - 9) - (8 - 6 + 9)]$$

$$= \pm\frac{1}{2}(-25 - 11)$$

$$= \pm(-18)$$

STUDY TIP

As stated in the Key Idea above, choose the sign that yields a positive value. Choose the negative sign because −(−18) = 18.

▶ So, the area of the triangle is 18 square units.

SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

4. Find the area of the triangle with vertices (3, 6), (8, −1), and (−4, 5).

Cramer's Rule

You can use determinants to solve a system of linear equations. The method, called **Cramer's Rule** and named after Swiss mathematician Gabriel Cramer (1704–1752), uses the **coefficient matrix** of the linear system.

Linear System	**Coefficient Matrix**
$ax + by = e$ $cx + dy = f$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

WORDS AND MATH

The term *determinant* was originally used to describe a property of linear systems before it was applied to matrices. You can *determine* whether a linear system has exactly one solution by calculating the determinant of its coefficient matrix.

KEY IDEA

Cramer's Rule for a 2 × 2 System

Let A be the coefficient matrix of the linear system

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}.$$

If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}.$$

Notice that the numerators for x and y are the determinants of the matrices formed by replacing the values of the x and y coefficient columns, respectively, with the column of the constant values.

EXAMPLE 3 Using Cramer's Rule for a 2 × 2 System

WATCH

Use Cramer's Rule to solve the system $\begin{aligned} 3x - 2y &= 7 \\ 5x + 4y &= -3 \end{aligned}$.

SOLUTION

Step 1 Evaluate the determinant of the coefficient matrix.

$$\begin{vmatrix} 3 & -2 \\ 5 & 4 \end{vmatrix} = 12 - (-10) = 22$$

Step 2 Apply Cramer's Rule because the determinant is not 0.

$$x = \frac{\begin{vmatrix} 7 & -2 \\ -3 & 4 \end{vmatrix}}{22} = \frac{28 - 6}{22} = \frac{22}{22} = 1$$

$$y = \frac{\begin{vmatrix} 3 & 7 \\ 5 & -3 \end{vmatrix}}{22} = \frac{-9 - 35}{22} = \frac{-44}{22} = -2$$

▶ The solution is (1, −2).

Check

$3x - 2y = 7$	Original equation	$5x + 4y = -3$
$3(1) - 2(-2) \overset{?}{=} 7$	Substitute.	$5(1) + 4(-2) \overset{?}{=} -3$
$3 + 4 \overset{?}{=} 7$	Simplify.	$5 - 8 \overset{?}{=} -3$
$7 = 7$ ✓	Simplify.	$-3 = -3$ ✓

2 MTR USE ANOTHER METHOD

Show how to solve Example 3 using elimination or substitution.

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$3x + 4y - 8z = -3$ Equation 1

$x + y + 5z = -12$ Equation 2

$4x - 2y + z = 10$ Equation 3

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$, where a, b, and c are not all zero. An example of a **system of three linear equations** in three variables is shown at the left. A **solution** of such a system is an **ordered triple** (x, y, z) whose coordinates make each equation true.

KEY IDEA

Cramer's Rule for a 3 × 3 System

Let A be the coefficient matrix of the linear system shown below.

Linear System	**Coefficient Matrix**
$ax + by + cz = j$ $dx + ey + fz = k$ $gx + hy + iz = l$	$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}, \quad \text{and} \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}.$$

STUDY TIP

The numerators for *x*, *y*, and *z* are the determinants of the matrices formed by replacing the values of the *x*, *y*, and *z* coefficient columns, respectively, with the column of the constant values.

EXAMPLE 4 Using Cramer's Rule for a 3 × 3 System

WATCH

Use Cramer's Rule to solve the system.

$-x - 3y + 5z = 21$ Equation 1

$x + 4y - 3z = -5$ Equation 2

$x - 2y - z = -17$ Equation 3

SOLUTION

Step 1 Evaluate the determinant of the coefficient matrix.

$$\begin{vmatrix} -1 & -3 & 5 \\ 1 & 4 & -3 \\ 1 & -2 & -1 \end{vmatrix}\begin{matrix} -1 & -3 \\ 1 & 4 \\ 1 & -2 \end{matrix} = (4 + 9 - 10) - (20 - 6 + 3) = -14$$

Step 2 Apply Cramer's Rule because the determinant is not 0.

$$x = \frac{\begin{vmatrix} 21 & -3 & 5 \\ -5 & 4 & -3 \\ -17 & -2 & -1 \end{vmatrix}}{-14} \qquad y = \frac{\begin{vmatrix} -1 & 21 & 5 \\ 1 & -5 & -3 \\ 1 & -17 & -1 \end{vmatrix}}{-14} \qquad z = \frac{\begin{vmatrix} -1 & -3 & 21 \\ 1 & 4 & -5 \\ 1 & -2 & -17 \end{vmatrix}}{-14}$$

$$= \frac{42}{-14} \qquad = \frac{-56}{-14} \qquad = \frac{-84}{-14}$$

$$= -3 \qquad = 4 \qquad = 6$$

▶ The solution is $(-3, 4, 6)$.

3 MTR ADAPT A PROCEDURE

Add Equation 1 and Equation 2 to produce *new Equation 1*. Then add original Equation 1 and Equation 3 to produce *new Equation 2*. Show how to use new Equation 1 and new Equation 2 to solve the system by elimination.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use Cramer's Rule to solve the linear system.

5. $7x - 2y = 20$
$3x + 4y = -6$

6. $5x + 6y = 14$
$-2x - 3y = -8$

7. $5x - 2y + 3z = -2$
$4x + 7y - z = 0$
$2x - 6y + 5z = -7$

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Solving Real-Life Problems

7 MTR **EXAMPLE 5** **Modeling Real Life** WATCH

The molecular mass of a compound is the sum of the atomic masses of the atoms it contains. The table shows the molecular masses (in atomic mass units) of three compounds. Use a linear system and Cramer's Rule to find the atomic masses of bromine (Br), nitrogen (N), and fluorine (F).

Compound	Formula	Molecular Mass
Bromine azide	BrN_3	122
Bromine trifluoride	BrF_3	137
Nitrogen trifluoride	NF_3	71

READING

Subscripts in a chemical formula indicate the number of atoms of each element in one molecule of the compound. No subscript indicates there is 1 atom in the molecule.

SOLUTION

Step 1 Write a linear system using the formula for each compound. Let *B*, *N*, and *F* represent the atomic masses of bromine, nitrogen, and fluorine.

$$B + 3N = 122 \qquad BrN_3\text{: 1 bromine atom and 3 nitrogen atoms}$$

$$B + 3F = 137 \qquad BrF_3\text{: 1 bromine atom and 3 fluorine atoms}$$

$$N + 3F = 71 \qquad NF_3\text{: 1 nitrogen atom and 3 fluorine atoms}$$

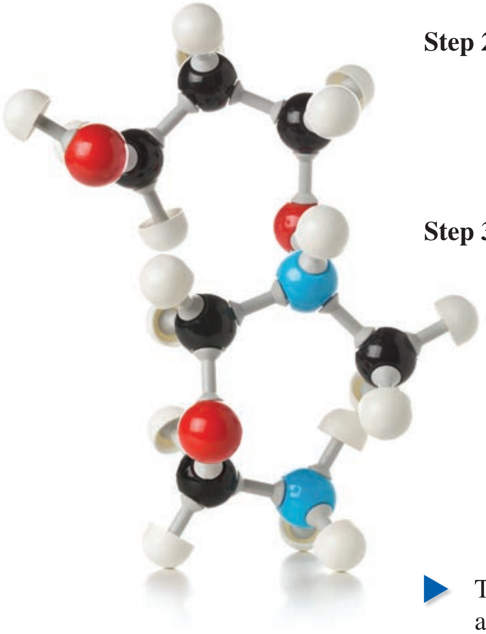
Step 2 Evaluate the determinant of the coefficient matrix.

$$\left|\begin{array}{ccc} 1 & 3 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & 3 \end{array}\right|\begin{array}{cc} 1 & 3 \\ 1 & 0 \\ 0 & 1 \end{array} = (0 + 0 + 0) - (0 + 3 + 9) = -12$$

Step 3 Apply Cramer's Rule because the determinant is not 0.

$$B = \frac{\begin{vmatrix} 122 & 3 & 0 \\ 137 & 0 & 3 \\ 71 & 1 & 3 \end{vmatrix}}{-12} = \frac{-960}{-12} = 80 \qquad N = \frac{\begin{vmatrix} 1 & 122 & 0 \\ 1 & 137 & 3 \\ 0 & 71 & 3 \end{vmatrix}}{-12} = \frac{-168}{-12} = 14 \qquad F = \frac{\begin{vmatrix} 1 & 3 & 122 \\ 1 & 0 & 137 \\ 0 & 1 & 71 \end{vmatrix}}{-12} = \frac{-228}{-12} = 19$$

▶ The atomic masses (in atomic mass units) of bromine, nitrogen, and fluorine are 80, 14, and 19, respectively.



SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

8. The table shows the molecular masses (in atomic mass units) of three compounds. Use a linear system and Cramer's Rule to find the atomic masses of carbon (C), hydrogen (H), and oxygen (O).

Compound	Formula	Molecular Mass
Acetone	C_3H_6O	58
Butanoic acid	$C_4H_8O_2$	88
Citric acid	$C_6H_8O_7$	192

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10.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–16, evaluate the determinant of the matrix. *(See Example 1.)*

▶ **1.** $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

3. $\begin{bmatrix} -1 & 3 \\ 2 & 6 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 5 \\ 8 & -4 \end{bmatrix}$

5. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

6. $\begin{bmatrix} -5 & 9 \\ 7 & -2 \end{bmatrix}$

7. $\begin{bmatrix} -2 & 4 \\ -8 & 10 \end{bmatrix}$

8. $\begin{bmatrix} -8 & -9 \\ 14 & 12 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 & 5 \\ 6 & 2 & 0 \\ 3 & 8 & 9 \end{bmatrix}$

10. $\begin{bmatrix} 6 & 1 & 2 \\ 5 & 0 & 4 \\ 9 & 3 & 0 \end{bmatrix}$

▶ **11.** $\begin{bmatrix} -2 & 7 & 0 \\ -3 & 1 & 4 \\ 5 & 0 & -6 \end{bmatrix}$

12. $\begin{bmatrix} 2 & -1 & 3 \\ 5 & 0 & 7 \\ 0 & -6 & -4 \end{bmatrix}$

13. $\begin{bmatrix} -9 & 0 & 1 \\ 2 & 4 & 8 \\ -3 & -4 & -6 \end{bmatrix}$

14. $\begin{bmatrix} -1 & 7 & 5 \\ 2 & -9 & -2 \\ 1 & 4 & 0 \end{bmatrix}$

15. $\begin{bmatrix} 4 & -1 & 7 \\ 2 & 8 & 5 \\ -3 & 1 & -7 \end{bmatrix}$

16. $\begin{bmatrix} 9 & 1 & 4 \\ -1 & -6 & -2 \\ -3 & 7 & 8 \end{bmatrix}$

4 MTR **ERROR ANALYSIS** **In Exercises 17 and 18, describe and correct the error in evaluating the determinant of the matrix.**

17.

$$\begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix} = 2(7) - 5(4) = -6$$

18.

$$\begin{array}{rr|rrr|} 2 & -2 & 2 & -2 & 4 \\ 3 & 0 & 3 & 0 & 7 \\ 1 & -1 & 1 & -1 & 6 \end{array}$$

$$= (0 + 6 + 0) - (0 + 6 + 0)$$
$$= 0$$

19. **NUMBER SENSE** Order the determinants from least to greatest.

A. $\begin{vmatrix} 5 & -2 \\ 1 & 5 \end{vmatrix}$

B. $\begin{vmatrix} 1 & 6 \\ 3 & 8 \end{vmatrix}$

C. $\begin{vmatrix} 5 & -3 \\ 7 & -1 \end{vmatrix}$

D. $\begin{vmatrix} -4 & 1 \\ 6 & 3 \end{vmatrix}$

20. **COLLEGE PREP** For what value of k is the determinant of the matrix below equal to 0?

$$\begin{bmatrix} 0 & 2 & 0 \\ k & 9 & 2 \\ 5 & 4 & 2 \end{bmatrix}$$

Ⓐ $k = -10$

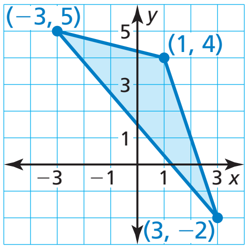
Ⓑ $k = -5$

Ⓒ $k = 5$

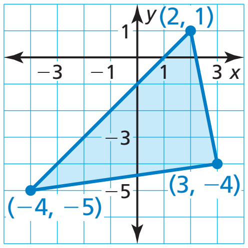
Ⓓ $k = 10$

In Exercises 21–24, find the area of the triangle. *(See Example 2.)*

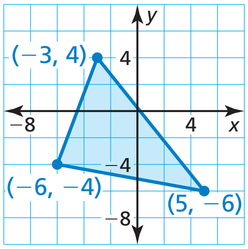
▶ **21.**



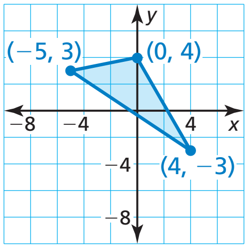
22.



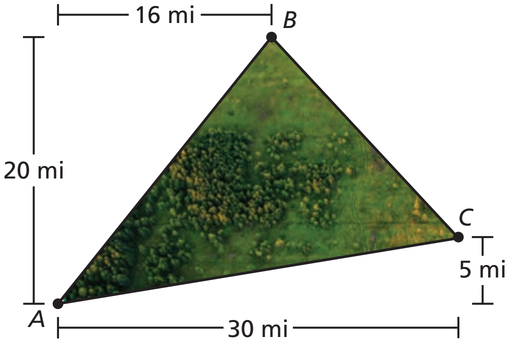
23.



24.



7 MTR **25.** **MODELING REAL LIFE** A researcher of Lyme disease tests ticks from the triangular region shown below. Find the area of the triangular region.



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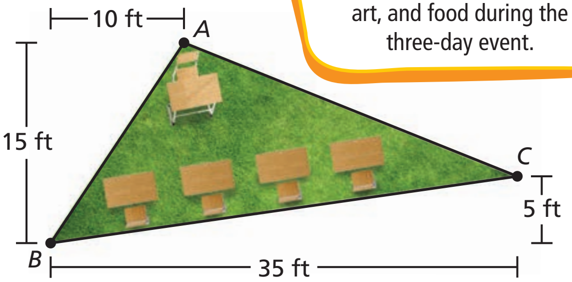
7 MTR **26.** **MODELING REAL LIFE** A volunteer for the Florida Folk Festival in White Springs sets up the space shown for a workshop. Find the area used for the workshop.

The Florida Folk Festival celebrates the state's cultural heritage through music, dance, art, and food during the three-day event.



5 MTR **STRUCTURE** **In Exercises 27 and 28, find a positive value of y such that a triangle with the given vertices has an area of 5 square units.**

27. $(-4, 3), (0, 1), (1, y)$

28. $(-3, -3), (0, -2), (-1, y)$

In Exercises 29–36, use Cramer's Rule to solve the linear system. *(See Examples 3 and 4.)*

▶ **29.** $\begin{aligned} 4x + 3y &= 5 \\ 3x + 2y &= 4 \end{aligned}$

30. $\begin{aligned} 2x + 5y &= 11 \\ 4x + 8y &= 20 \end{aligned}$

31. $\begin{aligned} 5x - y &= 12 \\ -x + 6y &= -14 \end{aligned}$

32. $\begin{aligned} -7x + 2y &= -13 \\ 8x + 3y &= -1 \end{aligned}$

▶ **33.** $\begin{aligned} -x + y - 4z &= -7 \\ 3x + 5y - 6z &= -5 \\ 2x - 3y + 5z &= 12 \end{aligned}$

34. $\begin{aligned} x + 3y + 2z &= 11 \\ 2x - 4y + 3z &= 7 \\ 5x + 2y - z &= -13 \end{aligned}$

35. $\begin{aligned} 2x - 4y + 7z &= -1 \\ -3x + 5y - z &= 26 \\ x - 6y + z &= -24 \end{aligned}$

36. $\begin{aligned} 3x + 2y + z &= 2 \\ 6x - 2y - 3z &= 23 \\ -2x + 3y - 2z &= -20 \end{aligned}$

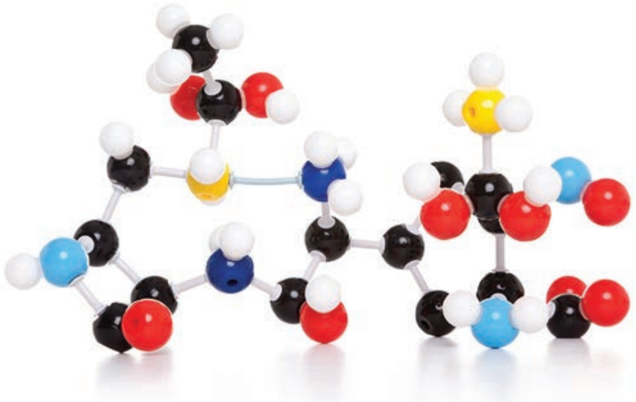
7 MTR **37.** **MODELING REAL LIFE** There are x occupied floor seats and y occupied balcony seats at a concert with 6700 total attendees. The tickets cost \$40 per floor seat and \$25 per balcony seat. The total revenue is \$185,500. Write and solve a system to determine the number of occupied seats of each type at the concert.

7 MTR **38.** **MODELING REAL LIFE** There are x beginners and y returning students in a coding camp with 31 students. The cost is \$200 per beginner and \$350 per returning student. The total cost is \$8000. Write and solve a system to determine the number of beginners and the number of returning students at the coding camp.

In Exercises 39 and 40, the table shows the molecular masses (in atomic mass units) of three compounds. Use a linear system and Cramer's Rule to find the atomic masses of the indicated elements. *(See Example 5.)*

▶ **39.** sulfur (S), chlorine (Cl), calcium (Ca)

Compound	Formula	Molecular Mass
Sulfur dichloride	SCl_2	103
Calcium chloride	$CaCl_2$	111
Calcium sulfide	CaS	72



40. phosphorus (P), selenium (Se), iodine (I)

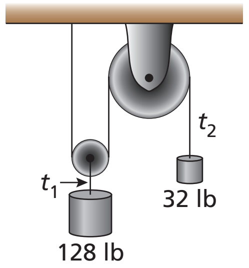
Compound	Formula	Molecular Mass
Diphosphorus pentaselenide	P_2Se_5	457
Phosphorus triiodide	PI_3	412
Selenium diiodide	SeI_2	333

7 MTR **41.** **MODELING REAL LIFE** The metal wire of a particular dental retainer is constructed using 40% nickel and 60% titanium. The price per ounce of nickel is \$0.50, and the price per ounce of titanium is \$1.80. A manufacturer buys a total \$102.40 of the metals to produce the wires. How many ounces of nickel does the manufacturer buy?

42. **MODELING REAL LIFE** Tin is usually found as an oxide called cassiterite, shown above. When tin is melted down, it can be combined with other metals. An artist creates bronze jewelry using aluminum, copper, and tin. The costs of each metal are shown in the table. The artist orders a total of 10 pounds of metals for a cost of \$32.70. The artist orders seven times as much copper as tin. How many pounds of tin does the artist order?

Metal	Price Per Pound
Aluminum	\$0.75
Copper	\$2.75
Tin	\$7.75

43. **MODELING REAL LIFE** A system of pulleys is loaded with 128-pound and 32-pound weights, as shown.



The tensions t_1 and t_2 (in pounds) in the ropes and the acceleration a (in feet per second squared) of the 32-pound weight are represented by the system

$$\begin{aligned} t_1 - 2t_2 &= 0 \\ t_1 - 2a &= 128. \\ t_2 + a &= 32 \end{aligned}$$

Find t_1, t_2, and a.

44. **MODELING REAL LIFE** Solar flares are classified based on their strength. In a study, the average intensities (in watts per square meter) of solar flares of class C, M, and X are solutions of the system

$$\begin{aligned} 37C + M + X &= 0.000705 \\ 15C + 14M + 4X &= 0.002775. \\ 5C + 9M + 4X &= 0.002475 \end{aligned}$$

Find the three intensities.

In Exercises 45 and 46, (x_1, y_1), (x_2, y_2), and (x_3, y_3) lie on the same line only when

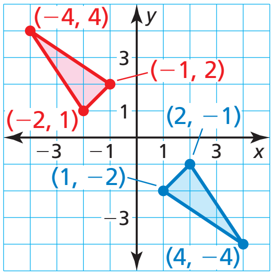
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

Determine whether the points lie on the same line.

45. (0, 1), (2, 4), (3, 6)

46. (−5, −4), (−2, −1), (3, 4)

47. **COMPARE METHODS** Compare the area of the red triangle to the area of the blue triangle using two methods. Which method do you prefer? Explain.



48. **STRUCTURE** Solve the equation

$$\begin{vmatrix} x+3 & x \\ 4 & x-1 \end{vmatrix} = 5.$$

49. **STRUCTURE** A community organizer requests a triangular plot of land with an area of 5000 square feet for an urban garden. On a blueprint, one vertex is at (0, 0) and another is at (100, 50). The y-coordinate of the final vertex is 120. Find the possible locations of the final vertex.

50. **HOW DO YOU SEE IT?**
Without calculating, explain why the determinant of the matrix below is 0.

$$\begin{bmatrix} 6 & -2 & 8 \\ 0 & 0 & 0 \\ 1 & 7 & 3 \end{bmatrix}$$

51. **REASONING** Consider 3 × 3 square matrices in which the elements are consecutive integers. An example is shown below.

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}$$

Find the determinants of four matrices of this type. Make a conjecture based on the results.

52. THOUGHT PROVOKING Consider the equation for the determinant of a 3×3 matrix,

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

How can you rewrite the determinant of a 3×3 matrix using determinants of 2×2 matrices? Show that your equation is equivalent to the equation above.

53. DIG DEEPER Consider the system

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}.$$

a. Solve the system by finding rational expressions for x and y.

b. What is the relationship between determinants and your answers in part (a)? Explain.

REVIEW & REFRESH

In Exercises 54 and 55, graph the function. Identify the x-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing.

54. $g(x) = \frac{1}{2}(x + 2)(x - 3)$

55. $f(x) = x^3 - 2x^2 - x + 2$

56. Solve the equation for a, b, c, and d.

$$\begin{bmatrix} 8 - a & 5b \\ c & -2d \end{bmatrix} = \begin{bmatrix} 4 & 20 \\ 22 & 14 \end{bmatrix}$$

57. Find the product $\begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 & -9 \\ 7 & 1 \end{bmatrix}$.

58. Determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

5, 10, 30, 120, . . .

59. Use Cramer's Rule to solve the system.

$$\begin{aligned} 8x + y &= 21 \\ x + 5y &= 27 \end{aligned}$$

60. Describe the pattern, write the next term, and write a rule for the nth term of the sequence.

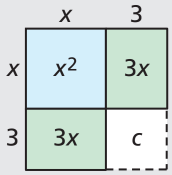
2.5, 6.5, 11.5, 17.5, . . .

61. MODELING REAL LIFE You are considering six full and three rimless frames for prescription glasses. You randomly select a frame, set it aside, and then randomly select another frame. Determine whether randomly selecting a full frame first and a rimless frame second are independent events.

62. Determine whether the function $h(x) = -2x^3 - 2x^2 + 18x + 18$ is *even*, *odd*, or *neither*.

63. Find the inverse of the function $f(x) = 9x^2$, $x \le 0$. Then graph the function and its inverse.

64. Find the value of c. Then write an expression represented by the diagram.



65. Graph the function $y = e^{x+4}$. Then find the domain and range.

66. Find the sum.

$$\frac{3}{x + 1} + \frac{2x}{x + 2}$$

67. Graph the system of quadratic inequalities.

$$\begin{aligned} y &\ge x^2 \\ y &< x^2 + 4x - 5 \end{aligned}$$

68. Describe the transformation of $f(x) = 2^x$ represented by $g(x) = 2^x - 5$. Then graph each function.

69. Determine whether the data show an exponential relationship. Then write a function that models the data.

x	−2	−1	−0	1	2	3
y	0.2	0.6	1.8	5.4	16.2	48.6

70. Use finite differences to determine the degree of the polynomial function that fits the data below. Then use technology to find the polynomial function.

(−2, −2), (−1, 0), (0, 4), (1, 10), (2, 18)