5.6 Performing Function Operations

Learning Target: Perform arithmetic operations on two functions.

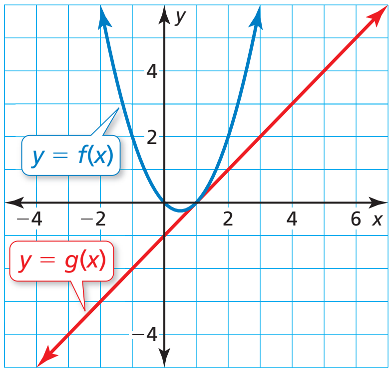
Success Criteria:
- I can explain what it means to perform an arithmetic operation on two functions.
- I can find arithmetic combinations of two functions.
- I can state the domain of an arithmetic combination of two functions.
- I can evaluate an arithmetic combination of two functions for a given input.

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions can be combined to form other functions.

EXPLORE IT! Graphing Arithmetic Combinations of Two Functions

1 MTR ANALYZE A PROBLEM
In part (b), when is each function positive and negative? How does this help you determine which graphs could represent p and q?

Work with a partner. Consider the graphs of f and g.



a. Describe what it means to add two functions. Then describe what it means to subtract one function from another function.

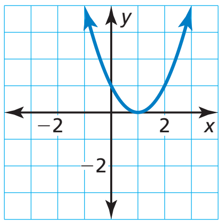
b. Match each function with its graph. Explain your reasoning.

i. $m(x) = f(x) + g(x)$

ii. $n(x) = f(x) - g(x)$

iii. $p(x) = f(x) \cdot g(x)$

iv. $q(x) = f(x) \div g(x)$

A.


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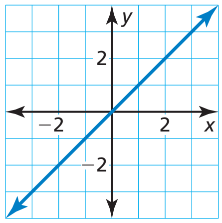

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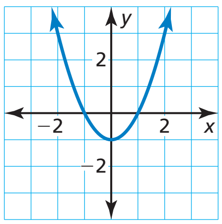

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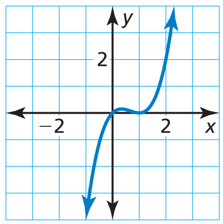

c. What is the domain of each function in part (b)? How do you know?

Functions
MA.912.F.3.2 Given a mathematical or real-world context, combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the new function.

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Operations on Functions

You have learned how to add, subtract, multiply, and divide polynomial expressions. These operations are also defined for functions.

KEY IDEA

Operations on Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g.

Operation	**Definition**	**Example: $f(x) = 5x$, $g(x) = x + 2$**
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of both f and g. Additionally, the domain of the quotient does not include x-values for which $g(x) = 0$.

EXAMPLE 1 Adding Two Functions

WATCH

Let $f(x) = 4(2^x)$ and $g(x) = -3(2^x)$. Find $(f + g)(x)$ and state the domain. Then evaluate $(f + g)(4)$.

SOLUTION

$$
\begin{aligned}
(f + g)(x) &= f(x) + g(x) && \text{Definition of function addition} \\
&= 4(2^x) + [-3(2^x)] && \text{Write sum of } f(x) \text{ and } g(x). \\
&= (4 - 3) \cdot (2^x) && \text{Distributive Property} \\
&= 2^x && \text{Subtract.}
\end{aligned}
$$

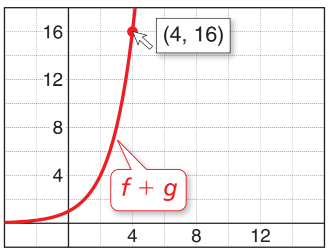
The functions f and g each have the same domain: all real numbers. So, the domain of $f + g$ also consists of all real numbers. To evaluate $f + g$ when $x = 4$, you can use several methods. Here are two:

Method 1 Use an algebraic approach.

$$(f + g)(4) = 2^4 = 16$$

Method 2 Use a graphical approach.

Use technology to graph the sum of the functions. The graph shows that $(f + g)(4) = 16$.



3 MTR SELECT METHODS

Show how to find $(f + g)(4)$ by adding $f(4) + g(4)$. Do you prefer this method or the methods in Example 1?

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EXAMPLE 2 Subtracting Two Functions

WATCH

Let $f(x) = 3x^3 - 2x^2 + 5$ and $g(x) = x^3 - 3x^2 + 4x - 2$. Find $(f - g)(x)$ and state the domain. Then evaluate $(f - g)(-2)$.

SOLUTION

$$
\begin{aligned}
(f - g)(x) &= f(x) - g(x) \\
&= 3x^3 - 2x^2 + 5 - (x^3 - 3x^2 + 4x - 2) \\
&= 2x^3 + x^2 - 4x + 7
\end{aligned}
$$

The functions f and g each have the same domain: all real numbers. So, the domain of $f - g$ also consists of all real numbers.

$$(f - g)(-2) = 2(-2)^3 + (-2)^2 - 4(-2) + 7 = 3$$

EXAMPLE 3 Multiplying Two Functions

WATCH

Let $f(x) = x^2$ and $g(x) = x - 4$. Find $(fg)(x)$ and state the domain. Then evaluate $(fg)(3)$.

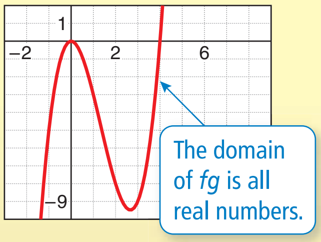
SOLUTION

$$(fg)(x) = f(x) \cdot g(x) = x^2(x - 4) = x^3 - 4x^2$$

The functions f and g each have the same domain: all real numbers. So, the domain of fg also consists of all real numbers.

$$(fg)(3) = 3^3 - 4(3^2) = 27 - 36 = -9$$

Check



EXAMPLE 4 Dividing Two Functions

WATCH

Let $f(x) = 2x + 10$ and $g(x) = 6x$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate $\left(\frac{f}{g}\right)(16)$.

SOLUTION

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x + 10}{6x} = \frac{x + 5}{3x}$$

The functions f and g each have a domain of all real numbers, and $g(0) = 0$. So, the domain of $\frac{f}{g}$ is $\{x \mid x \neq 0\}$.

$$\left(\frac{f}{g}\right)(16) = \frac{16 + 5}{3(16)} = \frac{21}{48} = \frac{7}{16}$$

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Let $f(x) = -2x + 4$ and $g(x) = 3x^2$. Find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $(f + g)(-3)$ and $(f - g)(-3)$.

2. Let $f(x) = 3x$ and $g(x) = 3(x + 1)^2$. Find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate $(fg)(5)$ and $\left(\frac{f}{g}\right)(5)$.

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$f(x) = 2^x$
$g(x) = (x+2)^2$
$f(2) + g(2)$ $= 20$
$f(2) - g(2)$ $= -12$
$f(2) \cdot g(2)$ $= 64$
$\dfrac{f(2)}{g(2)}$ $= 0.25$

EXAMPLE 5 Performing Function Operations Using Technology

WATCH

Let $f(x) = 2^x$ and $g(x) = (x + 2)^2$. Use technology to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\dfrac{f}{g}\right)(x)$ when $x = 2$.

SOLUTION

Enter f and g. From the screen, you can see that $f(2) + g(2) = 20$, so $(f + g)(2) = 20$. Similarly,

$$(f - g)(2) = -12,\ (fg)(2) = 64, \text{ and } \left(\frac{f}{g}\right)(2) = 0.25.$$

7 MTR

EXAMPLE 6 Modeling Real Life

WATCH STEM

The number of active annual memberships at a gym x years after it opens is represented by $n(x) = x^2 + 17x + 60$. The cost (in dollars) of an annual membership in year x is represented by $c(x) = 3x + 34$.

a. Find $(nc)(x)$.

b. Explain what $(nc)(x)$ represents.

SOLUTION

a. $(nc)(x) = (x^2 + 17x + 60)(3x + 34)$

$$\begin{array}{rrrrl}
 & x^2 & +\ 17x & +\ 60 & \\
\times & & 3x & +\ 34 & \text{Align like terms vertically.} \\
\hline
 & 34x^2 & +\ 578x & +\ 2040 & \text{Distributive Property} \\
3x^3 & +\ 51x^2 & +\ 180x & & \text{Distributive Property} \\
\hline
3x^3 & +\ 85x^2 & +\ 758x & +\ 2040 & \text{Combine like terms.}
\end{array}$$

Multiply $34(x^2 + 17x + 60)$.

Multiply $3x(x^2 + 17x + 60)$.

b. Multiplying the number of active annual memberships by the cost of each membership gives the total revenue (in dollars) earned from sales of annual memberships in year x.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

3. Let $f(x) = 8x$ and $g(x) = 2x^{5/6}$. Use technology to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\dfrac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places.

4. **WRITING** In Example 5, explain why you can evaluate $(f + g)(-2)$, $(f - g)(-2)$, and $(fg)(-2)$, but not $\left(\dfrac{f}{g}\right)(-2)$.

5. Use the answer in Example 6(a) to find the revenue earned from sales of annual memberships 3 years after the gym opens.

6. The cost (in dollars) to rent a scooter for x minutes in City A is represented by $A(x) = 0.15x + 1$. The cost (in dollars) in City B is represented by $B(x) = 0.29x + 1$. Find $(B - A)(x)$ and explain what it represents.

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5.6 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f + g$ and $f - g$ for the given value of x. *(See Examples 1 and 2.)*

1. $f(x) = -2(4^x)$, $g(x) = 5(4^x)$; $x = 2$

2. $f(x) = 2(3^x)$, $g(x) = -6(3^x)$; $x = 3$

3. $f(x) = 6x - 4x^2 - 7x^3$, $g(x) = 9x^2 - 5x$; $x = -1$

4. $f(x) = 11x + 2x^2$, $g(x) = -7x - 3x^2 + 4$; $x = 2$

5. $f(x) = 2x - 3$, $g(x) = x^4 - 2x^3 + x$; $x = -2$

6. $f(x) = 3x^3 - 2x^4$, $g(x) = 2x^2 - 3x^4 - 8$; $x = -3$

In Exercises 7–14, find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ for the given value of x. *(See Examples 3 and 4.)*

7. $f(x) = 2x^2$, $g(x) = x - 1$; $x = 3$

8. $f(x) = x^4$, $g(x) = x + 2$; $x = 2$

9. $f(x) = 2^x$, $g(x) = 3x$; $x = 4$

10. $f(x) = -5x$, $g(x) = 4^x$; $x = 1$

11. $f(x) = x^2 - 2x - 3$, $g(x) = x^2 - 4$; $x = -3$

12. $f(x) = x^3 - x$, $g(x) = x^2 + 2x - 8$; $x = -2$

13. $f(x) = x^2 - 14x + 48$, $g(x) = x^2 - x^3$; $x = -4$

14. $f(x) = x^2 + 9x + 20$, $g(x) = x^3 - 4x$; $x = -1$

USING TOOLS In Exercises 15–18, use technology to evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ when $x = 5$. Round your answers to two decimal places. *(See Example 5.)*

15. $f(x) = x^3 + 2x - 8$; $g(x) = 3^x$

16. $f(x) = x^3 + 5x^2 + x$; $g(x) = 4^x$

17. $f(x) = x^4 + 2x^3 + 6$; $g(x) = x^3 - x^2 + 9$

18. $f(x) = x^2 + 4$; $g(x) = x^4 + 8x^3 - 6x^2$

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4 MTR **ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in stating the domain.**

19.

$f(x) = x^3$ and $g(x) = x^2 - 4$

The domain of $\left(\frac{f}{g}\right)(x) = \frac{x^3}{x^2 - 4}$

is $\{x \mid x \neq 2\}$.

20.

$f(x) = x^2$ and $g(x) = x^3$

The domain of $(fg)(x) = x^5$

is $\{x \mid x \neq 0\}$.

7 MTR 21. **MODELING REAL LIFE** Over a period of 9 years, the female population of Okaloosa County, Florida, can be modeled by

$$F(t) = 13.856t^3 - 199.64t^2 + 2265.4t + 88.601$$

and the male population can be modeled by

$$M(t) = 21.262t^3 - 373.27t^2 + 3532.9t + 88.706$$

where t is the number of years since 2010. *(See Example 6.)*

a. Find $(F + M)(t)$.

b. Explain what $(F + M)(t)$ represents.

7 MTR 22. **MODELING REAL LIFE** From 2010 to 2019, the numbers of cruise ship departures from Florida can be modeled by

$$F(t) = 1.886t^3 - 12.63t^2 + 231.1t + 5725$$

and the numbers of cruise ship departures from Port Canaveral can be modeled by

$$C(t) = 0.960t^3 - 18.98t^2 + 195.2t + 1305$$

where t is the number of years since 2010.

a. Find $(F - C)(t)$.

b. Explain what $(F - C)(t)$ represents.

23. **COLLEGE PREP** For which of the functions f and g is $(f + g)(1) = 3$? Select all that apply.

A. $f(x) = x^2 + 2$
$g(x) = 2x^3 + x$

B. $f(x) = 3x^3 + 1$
$g(x) = x^2 - 2x$

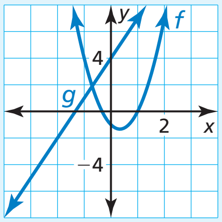
C. $f(x) = 2x + 7$
$g(x) = -x^4 - 5x$

D. $f(x) = x^2 + 3x - 1$
$g(x) = x^2 + 2$

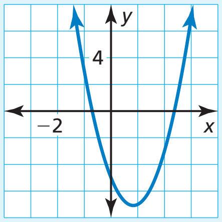
24. **OPEN-ENDED** Write a cubic function f and a quadratic function g such that $(fg)(2) = 10$.

25. **MAKING AN ARGUMENT** Is the addition of functions commutative? the multiplication of functions? Explain your reasoning.

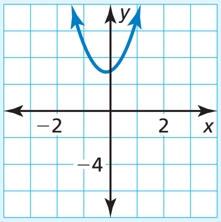
26. **HOW DO YOU SEE IT?** The graphs of the functions $f(x) = 3x^2 - 2x - 1$ and $g(x) = 3x + 4$ are shown. Which graph represents the function $f + g$? the function $f - g$? Explain.



A.



B.



27. **REASONING** The table shows the outputs of the two functions f and g. Use the table to find each value.

x	0	1	2	3	4
$f(x)$	-2	-4	0	10	26
$g(x)$	-1	-3	-13	-31	-57

a. $(f + g)(3)$ **b.** $(f - g)(1)$

c. $(fg)(2)$ **d.** $\left(\frac{f}{g}\right)(0)$

28. **THOUGHT PROVOKING**
Is it possible to write two functions whose sum contains radicals but whose product does not? Justify your answers.

29. **DIG DEEPER** For the functions f and g, $(f + g)(-1) = 4$ and $\left(\frac{f}{g}\right)(-1) = -\frac{3}{2}$. Find $f(-1)$ and $g(-1)$.

REVIEW & REFRESH

WATCH

In Exercises 30 and 31, solve the equation.

30. $3\sqrt{2x - 5} = 9$ **31.** $\sqrt{-x - 3} = x + 5$

In Exercises 32 and 33, solve the literal equation for *n*.

32. $3xn - 9 = 6y$ **33.** $\frac{3 + 4n}{n} = 7b$

In Exercises 34 and 35, determine whether the relation is a function. Explain.

34. $(1, 6), (7, -3), (4, 0), (3, 0)$

35. $(3, 8), (2, 5), (9, 5), (2, -3)$

36. Let $f(x) = 8x^3$ and $g(x) = 4 - x$. Find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ when $x = 2$.

In Exercises 37 and 38, perform the indicated operation and write the answer in simplest form. Assume all variables are positive.

37. $10\sqrt[3]{x} + 8\sqrt[3]{x}$ **38.** $\sqrt[3]{27ab^5c^3} \cdot \sqrt[3]{a^6b^2c^4}$

In Exercises 39 and 40, graph the functions. Then describe the transformation of *f* represented by *g*.

39. $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x} + 2$

40. $f(x) = \sqrt[3]{x}$, $g(x) = 4\sqrt[3]{x} - 6$

41. Determine whether the table represents a *linear* or *nonlinear* function. Explain.

x	12	9	6	3
y	-1	0	1	2

42. **MODELING REAL LIFE** The number A of commercial drones sold (in thousands) can be modeled by the function $A = 19t^2 + 30t + 110$, where t represents the number of years after 2016.

a. In what year did commercial drone sales reach 200,000?

b. Find and interpret the average rate of change from 2016 to 2018.

c. Do you think this model will be accurate after 20 years? Explain your reasoning.

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