4.3 Dividing Polynomials

Learning Target: Divide polynomials by other polynomials and use the Remainder Theorem.

Success Criteria:
- I can use long division to divide polynomials by other polynomials.
- I can divide polynomials by binomials of the form $x - k$ using synthetic division.
- H I can explain the Remainder Theorem.

EXPLORE IT! Dividing Polynomials

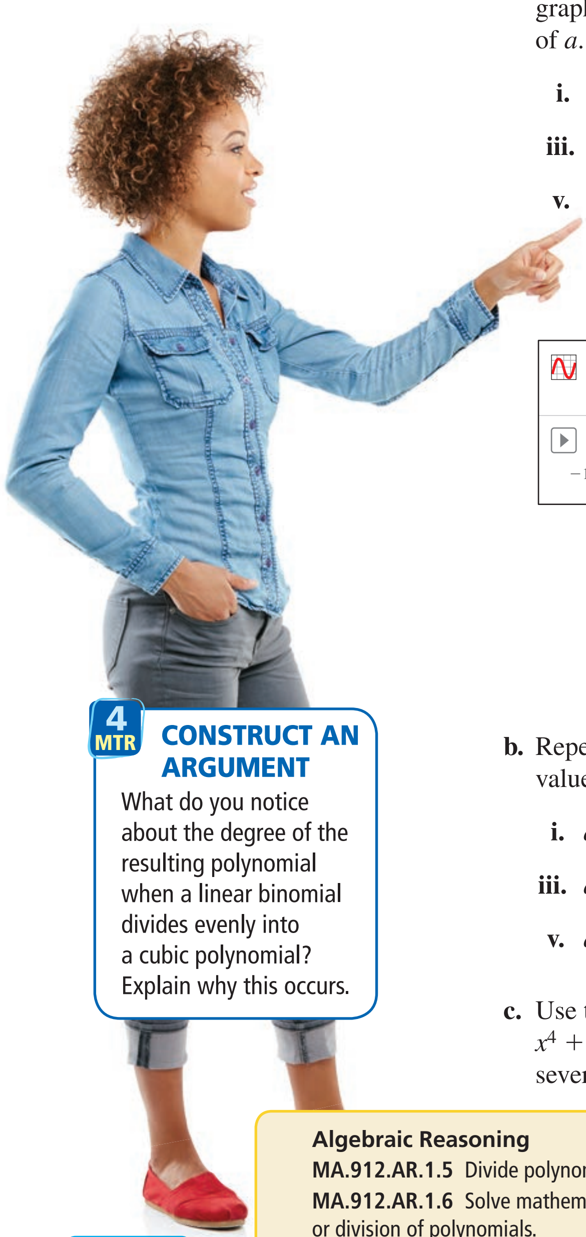
Work with a partner.

a. Consider the polynomial $x^3 + 2x^2 - x - 2$. Use technology to explore the graph of the polynomial divided by the binomial $x + a$ for the given values of a. What do you notice? What can you conclude?

i. $a = 1$ **ii.** $a = 2$

iii. $a = 3$ **iv.** $a = -1$

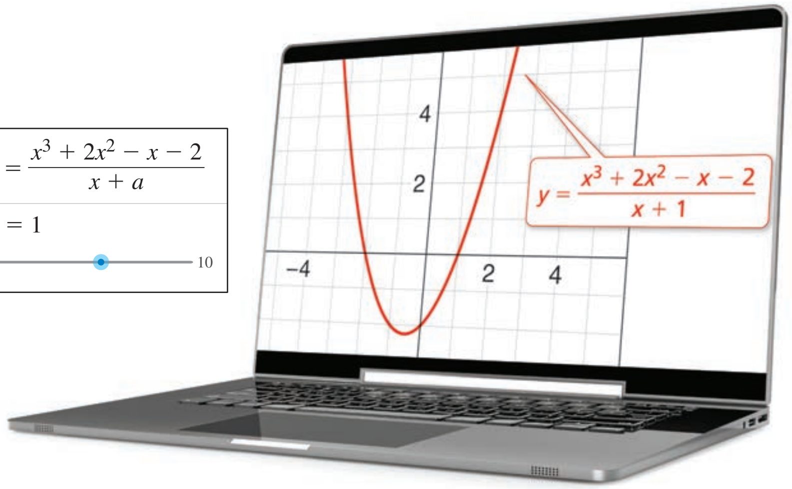
v. $a = -2$ **vi.** $a = 4$



$$y = \frac{x^3 + 2x^2 - x - 2}{x + a}$$

$a = 1$

−10 10



4 MTR CONSTRUCT AN ARGUMENT
What do you notice about the degree of the resulting polynomial when a linear binomial divides evenly into a cubic polynomial? Explain why this occurs.

b. Repeat part (a) for the polynomial $x^3 - 3x^2 - 10x + 24$ and the given values of a. What do you notice? What can you conclude?

i. $a = -1$ **ii.** $a = 2$

iii. $a = -2$ **iv.** $a = 3$

v. $a = -3$ **vi.** $a = 4$

c. Use technology to explore the graph of the polynomial $x^4 + 7x^3 + 9x^2 - 7x - 10$ divided by the binomial $x + a$ for several values of a. Make several observations about the graphs.

Algebraic Reasoning
MA.912.AR.1.5 Divide polynomial expressions using long division, synthetic division or algebraic manipulation.
MA.912.AR.1.6 Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of polynomials.
H Also MA.912.AR.6.2

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Vocabulary AZ VOCAB

polynomial long division, *p. 190*
synthetic division, *p. 191*

Long Division of Polynomials

When you divide a polynomial $f(x)$ by a nonzero polynomial divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the divisor $d(x)$ is less than or equal to the degree of the dividend $f(x)$. Also, the degree of the remainder $r(x)$ must be less than the degree of the divisor. When the remainder is 0, the divisor *divides evenly* into the dividend. One way to divide polynomials is called **polynomial long division**.

EXAMPLE 1 Using Polynomial Long Division

WATCH

Divide $2x^4 + 3x^3 + 5x - 1$ by $x^2 + 3x + 2$.

SOLUTION

Write polynomial division in the same format you use when dividing numbers. Include a "0" as the coefficient of x^2 in the dividend. At each step, divide the term with the highest power in what is left of the dividend by the first term of the divisor. This gives the next term of the quotient.

$$\begin{array}{rrrrrl}
 & 2x^2 & - 3x & + 5 & & \leftarrow \text{quotient} \\
x^2 + 3x + 2 \;\big)\; 2x^4 + 3x^3 & + 0x^2 & + 5x & - 1 & & \\
2x^4 + 6x^3 & + 4x^2 & & & & \text{Multiply divisor by } \frac{2x^4}{x^2} = 2x^2. \\
-3x^3 & - 4x^2 & + 5x & & & \text{Subtract. Bring down next term.} \\
-3x^3 & - 9x^2 & - 6x & & & \text{Multiply divisor by } \frac{-3x^3}{x^2} = -3x. \\
 & 5x^2 & + 11x & - 1 & & \text{Subtract. Bring down next term.} \\
 & 5x^2 & + 15x & + 10 & & \text{Multiply divisor by } \frac{5x^2}{x^2} = 5. \\
 & & -4x & - 11 & & \leftarrow \text{remainder}
\end{array}$$

COMMON ERROR
When there is a remainder, add the expression $\frac{r(x)}{d(x)}$, not just $r(x)$.

$$\frac{2x^4 + 3x^3 + 5x - 1}{x^2 + 3x + 2} = 2x^2 - 3x + 5 + \frac{-4x - 11}{x^2 + 3x + 2}$$

Check You can check the result of a division problem by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\begin{aligned}
&(2x^2 - 3x + 5)(x^2 + 3x + 2) + (-4x - 11) \\
&= (2x^2)(x^2 + 3x + 2) - (3x)(x^2 + 3x + 2) + (5)(x^2 + 3x + 2) - 4x - 11 \\
&= 2x^4 + 6x^3 + 4x^2 - 3x^3 - 9x^2 - 6x + 5x^2 + 15x + 10 - 4x - 11 \\
&= 2x^4 + 3x^3 + 5x - 1 \checkmark
\end{aligned}$$

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Divide using polynomial long division.

1. $(2x^2 - 5x - 3) \div (x - 3)$
2. $(4x^2 - 7x - 11) \div (x + 1)$
3. $(x^3 - x^2 - 2x + 8) \div (x - 1)$
4. $(x^4 + 2x^2 - x + 5) \div (x^2 - x + 1)$
5. **REASONING** Write a trinomial that can be divided evenly by $x - 4$. Explain how you found your answer.

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Synthetic Division

Synthetic division is a shortcut for dividing polynomials by binomials of the form $x - k$.

EXAMPLE 2 Using Synthetic Division

WATCH

Divide $-x^3 + 4x^2 + 9$ by $x - 3$.

SOLUTION

Step 1 Write the coefficients of the dividend in order of descending exponents. Include a "0" for the missing x-term. Because the divisor is $x - 3$, $k = 3$. Write the k-value to the left of the vertical bar.

k-value → 3 | −1 4 0 9 ← coefficients of $-x^3 + 4x^2 + 9$

3	−1	4	0	9

Step 2 Bring down the leading coefficient. Multiply the leading coefficient by k. Write the product under the second coefficient. Add.

3	−1	4	0	9
		−3		
	−1	1		

Step 3 Multiply the previous sum by k. Write the product under the third coefficient. Add. Repeat this process for the remaining coefficient. The first three numbers in the bottom row are the coefficients of the quotient, and the last number is the remainder.

3	−1	4	0	9
		−3	3	9
coefficients of quotient →	−1	1	3	18 ← remainder

$$\frac{-x^3 + 4x^2 + 9}{x - 3} = -x^2 + x + 3 + \frac{18}{x - 3}$$

EXAMPLE 3 Using Synthetic Division

WATCH

Divide $-9x^3 - 2x^2 + 2x - 5$ by $x + 1$.

SOLUTION

Use synthetic division. Because the divisor is $x + 1 = x - (-1)$, $k = -1$.

−1	−9	−2	2	−5
		9	−7	5
	−9	7	−5	0

$$\frac{-9x^3 - 2x^2 + 2x - 5}{x + 1} = -9x^2 + 7x - 5$$

3 MTR REFLECT ON YOUR METHOD

Explain why you can simply "bring down" the leading coefficient of the dividend when using synthetic division.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. Divide (a) $(x^3 - 3x^2 - 7x + 6) \div (x - 2)$ and (b) $(2x^3 - 16x + 6) \div (x + 3)$ using synthetic division.

7. **REASONING** Is the set of polynomials closed under division? Justify your answer.

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The Remainder Theorem

The remainder in the synthetic division process has an important interpretation. When you divide a polynomial $f(x)$ by $d(x) = x - k$, the result is

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$ Polynomial division

$$\frac{f(x)}{x - k} = q(x) + \frac{r(x)}{x - k}$$ Substitute $x - k$ for $d(x)$.

$$f(x) = (x - k)q(x) + r(x).$$ Multiply both sides by $x - k$.

Because either $r(x) = 0$ *or* the degree of $r(x)$ is less than the degree of $x - k$, you know that $r(x)$ is a constant function. So, let $r(x) = r$, where r is a real number, and evaluate $f(x)$ when $x = k$.

$$f(k) = (k - k)q(k) + r$$ Substitute k for x and r for $r(x)$.

$$f(k) = r$$ Simplify.

This result is stated in the *Remainder Theorem*.

5 MTR RELATE CONCEPTS

A classmate says that when the remainder is zero, the Remainder Theorem indicates that k is a zero of f. Do you agree? Explain your reasoning.

KEY IDEA

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

The Remainder Theorem tells you that synthetic division can be used to evaluate a polynomial function. So, to evaluate $f(x)$ when $x = k$, divide $f(x)$ by $x - k$. The remainder will be $f(k)$.

7 MTR

EXAMPLE 4 Modeling Real Life

WATCH H

A tachometer measures the speed (in revolutions per minute, or RPMs) at which an engine shaft rotates. For a certain boat, the speed x (in hundreds of RPMs) of the engine shaft and the speed s (in miles per hour) of the boat are modeled by

$$s(x) = 0.00547x^3 - 0.225x^2 + 3.62x - 11.0.$$

Find and interpret $s(10)$.

SOLUTION

Use synthetic division to evaluate $s(x) = 0.00547x^3 - 0.225x^2 + 3.62x - 11.0$ when $x = 10$.

10	0.00547	−0.225	3.62	−11
		0.0547	−1.703	19.17
	0.00547	−0.1703	1.917	8.17

▶ Because the remainder is 8.17, you can conclude from the Remainder Theorem that $s(10) = 8.17$. So, when the speed of the engine shaft is 1000 RPMs, the speed of the boat is about 8 miles per hour.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use synthetic division to evaluate the function for the indicated value of x.

8. $f(x) = 4x^2 - 10x - 21; x = 5$

9. $f(x) = 5x^4 + 2x^3 - 20x - 6; x = 2$

10. In Example 4, find and interpret $s(15)$.

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4.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–8, divide using polynomial long division. *(See Example 1.)*

1. $(x^2 + x - 17) \div (x - 4)$
2. $(3x^2 - 14x - 5) \div (x - 5)$
3. $(x^3 + x^2 + x + 2) \div (x^2 - 1)$
4. $(7x^3 + x^2 + x) \div (x^2 + 1)$
5. $(8x^3 - 3x + 1) \div (4x^3 + x^2 - 2x - 3)$
6. $(10x^3 + 5x^2 - 1) \div (2x^3 - 4x^2 - x + 2)$
7. $(4x^4 + 0.4x^2 - 3x + 0.8) \div (x^2 - x - 1)$
8. $(0.5x^4 + 5x - 1) \div (x^2 - 3x - 2)$

In Exercises 9–16, divide using synthetic division. *(See Examples 2 and 3.)*

9. $(x^2 + 8x + 1) \div (x - 4)$
10. $(4x^2 - 13x - 5) \div (x - 2)$
11. $(2x^2 + 7x + 6) \div (x + 2)$
12. $(x^2 + 1) \div \left(x + \frac{1}{4}\right)$
13. $(x^3 - 4x + 6) \div (x + 3)$
14. $(2x^3 - 5x^2 - 8x + 15) \div (x - 3)$
15. $(x^4 - 3x^3 - 7x^2 - 11x - 20) \div (x - 5)$
16. $(x^4 + 4x^3 + 16x - 35) \div (x + 5)$

REASONING In Exercises 17–20, match the equivalent expressions. Justify your answers.

17. $(x^2 + x - 3) \div (x - 2)$
18. $(x^2 - x - 3) \div (x - 2)$
19. $(x^2 - x + 3) \div (x - 2)$
20. $(x^2 + x + 3) \div (x - 2)$

A. $x + 1 - \dfrac{1}{x - 2}$

B. $x + 3 + \dfrac{9}{x - 2}$

C. $x + 1 + \dfrac{5}{x - 2}$

D. $x + 3 + \dfrac{3}{x - 2}$

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4 MTR **ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in using synthetic division to divide $x^3 - 5x + 3$ by $x - 2$.**

21.
$$\begin{array}{r|rrrr} 2 & 1 & 0 & -5 & 3 \\ & & 2 & 4 & -2 \\ \hline & 1 & 2 & -1 & 1 \end{array}$$
$$\frac{x^3 - 5x + 3}{x - 2} = x^3 + 2x^2 - x + 1$$

22.
$$\begin{array}{r|rrr} 2 & 1 & -5 & 3 \\ & & 2 & -6 \\ \hline & 1 & -3 & -3 \end{array}$$
$$\frac{x^3 - 5x + 3}{x - 2} = x^2 - 3x - \frac{3}{x - 2}$$

In Exercises 23–30, use synthetic division to evaluate the function for the indicated value of x.

23. $f(x) = -x^2 - 8x + 30;\ x = -1$
24. $f(x) = 3x^2 + 2x - 20;\ x = 3$
25. $f(x) = x^3 - 2x^2 + 4x + 3;\ x = 2$
26. $f(x) = x^3 + x^2 - 3x + 9;\ x = -\frac{1}{2}$

7 MTR
27. **MODELING REAL LIFE** From 2010 to 2018, the population R (in hundreds) of people of Puerto Rican origin in Florida can be modeled by

$$R = 2.474t^3 - 28.40t^2 + 493.5t + 8415$$

where t is the number of years since 2010. Find and interpret $R(7)$. *(See Example 4.)*

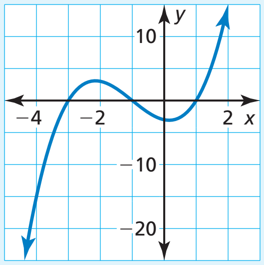
7 MTR
28. **MODELING REAL LIFE** The profit P (in millions of dollars) earned by a company x years since 2014 can be modeled by

$$P = 0.1x^3 - x^2 + 2.5x + 1.7$$

where $0 < x < 6$. Find and interpret $P(4)$.

4 MTR
29. **MAKING AN ARGUMENT** You use synthetic division to divide $f(x)$ by $x - a$ and find that the remainder is 25. Your friend concludes that $f(25) = a$. Is your friend correct? Explain your reasoning.

30. HOW DO YOU SEE IT?
The graph represents the polynomial function $f(x) = x^3 + 3x^2 - x - 3$.



a. When $f(x)$ is divided by $x - k$, the remainder is -15. What is the value of k?

b. Use the graph to determine the remainders of $(x^3 + 3x^2 - x - 3) \div (x + 3)$ and $(x^3 + 3x^2 - x - 3) \div (x + 1)$.

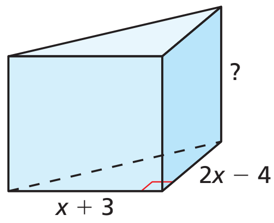
5 MTR **31. STRUCTURE** You divide two polynomials and obtain the result $5x^2 - 13x + 47 - \frac{102}{x + 2}$. What is the dividend? How did you find it?

32. COLLEGE PREP What is the value of k such that $(x^3 - x^2 + kx - 30) \div (x - 5)$ has a remainder of 0?

Ⓐ -14 Ⓒ 26

Ⓑ -2 Ⓓ 32

5 MTR **33. CONNECTING CONCEPTS** The volume V of the triangular prism is given by $V = 2x^3 - 5x^2 - 19x + 42$. Find an expression for the missing dimension.



34. THOUGHT PROVOKING
Explain how to use synthetic division to divide $4x^3 - 3x^2 + 3x - 7$ by $4x + 1$. Then find the quotient.

REVIEW & REFRESH

WATCH

In Exercises 35–38, find the zero(s) of the function.

35. $f(x) = x^2 - 6x + 9$

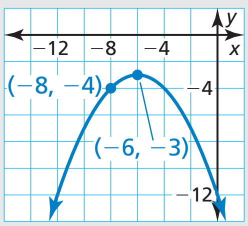
36. $g(x) = 3(x + 6)(x - 2)$

37. $g(x) = x^2 + 14x + 45$

38. $h(x) = 4x^2 + 36$

39. Use synthetic division to evaluate $f(x) = x^3 - 3x + 12$ when $x = -5$.

40. Write an equation of the parabola in vertex form.



5 MTR **41. STRUCTURE** For what value of k is the expression $(x + 3)(kx^2 - 4x + 1) - (2x^3 + 9x^2 - 7)$ equal to $4x^3 + 5x^2 - 11x + 10$?

42. Graph $f(x) = x^3 - 3x + 1$.

43. Divide $4x^4 - 2x^3 + x^2 - 5x + 8$ by $x^2 - 2x - 1$.

44. The table shows the results of flipping a penny 50 times. What is the experimental probability of flipping heads?

Heads	Tails
23	27

In Exercises 45–48, simplify the expression.

45. $\sqrt{300}$

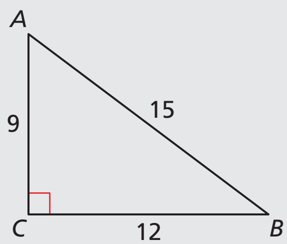
46. $\sqrt{\frac{10}{49}}$

47. $2\sqrt{3} + 7\sqrt{3}$

48. $5\sqrt{2} - \sqrt{8}$

7 MTR **49. MODELING REAL LIFE** City officials want to build a rectangular flower bed in a downtown park. The flower bed must have a perimeter of 50 feet and an area of at least 100 square feet. Describe the possible lengths of the flower bed.

50. The sine of which acute angle is $\frac{3}{5}$?



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