3.2 Complex Numbers

Learning Target: Understand the imaginary unit i and perform operations with complex numbers.

Success Criteria:

- I can define the imaginary unit i and use it to rewrite the square root of a negative number.
- I can add, subtract, multiply, and divide complex numbers.
- I can find complex solutions of quadratic equations and complex zeros of quadratic functions.

EXPLORE IT! Using Complex Numbers

Work with a partner.

a. A student solves the equations below as shown. Justify each solution step.

i.

$x^2 = 36$	Original equation
$x = \pm\sqrt{36}$	______
$x = \pm 6$	______

ii.

$x^2 = -9$	Original equation
$x = \pm\sqrt{-9}$	______
$x = \pm\sqrt{9}\sqrt{-1}$	______
$x = \pm 3\sqrt{-1}$	______

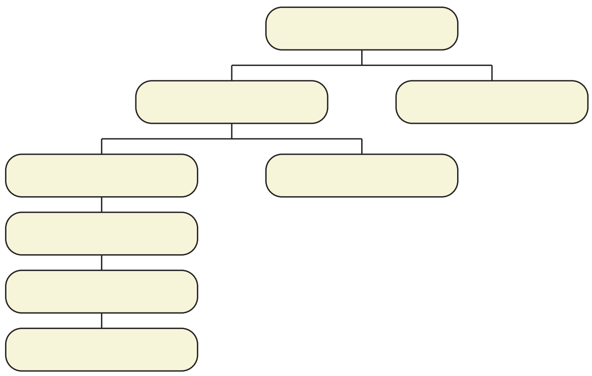
5 MTR USE STRUCTURE
How can you recognize when a quadratic equation of the form $x^2 + c = 0$ will have solutions that are not real numbers?

b. In your study of mathematics, you have probably worked with only real numbers, which can be represented graphically on the real number line. Describe the solutions of the equation $x^2 = c$ when $c > 0$, when $c = 0$, and when $c < 0$.

c. The solutions of the equation $x^2 = -9$ are *imaginary numbers*, and are typically written as $3i$ and $-3i$. Explain what i represents. What is the value of i^2?

d. In this lesson, the system of numbers is expanded to include imaginary numbers. The real numbers and imaginary numbers compose the set of *complex numbers*. Complete the diagram that shows the relationships among the number systems shown at the left.

Integers
Natural Numbers
Rational Numbers
Whole Numbers
Real Numbers
Complex Numbers
Irrational Numbers
Imaginary Numbers



e. Determine which subsets of numbers in part (d) contain each number.

i. $\sqrt{9}$ **ii.** $\sqrt{0}$ **iii.** $-\sqrt{4}$ **iv.** $\sqrt{\frac{4}{9}}$ **v.** $\sqrt{2}$ **vi.** $\sqrt{-1}$

Number Sense and Operations
MA.912.NSO.2.1 Extend previous understanding of the real number system to include the complex number system. Add, subtract, multiply and divide complex numbers.
Algebraic Reasoning
MA.912.AR.3.2 Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real and complex number systems.

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The Imaginary Unit *i*

Vocabulary

imaginary unit i, *p. 114*
complex number, *p. 114*
imaginary number, *p. 114*
pure imaginary number, *p. 114*
complex conjugates, *p. 116*

Not all quadratic equations have real-number solutions. For example, $x^2 = -3$ has no real-number solutions because the square of any real number is never a negative number.

To overcome this problem, mathematicians created an expanded system of numbers using the **imaginary unit *i***, defined as $i = \sqrt{-1}$. Note that $i^2 = -1$. The imaginary unit i can be used to write the square root of *any* negative number.

KEY IDEA

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = \sqrt{-1}\sqrt{r} = i\sqrt{r}$.	$\sqrt{-3} = \sqrt{-1}\sqrt{3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = i^2 \cdot r = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -1 \cdot 3 = -3$

EXAMPLE 1 Finding Square Roots of Negative Numbers

WATCH

Find the square root of each number.

a. $\sqrt{-25}$ **b.** $\sqrt{-72}$ **c.** $-5\sqrt{-9}$

SOLUTION

a. $\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1} = 5i$

b. $\sqrt{-72} = \sqrt{72} \cdot \sqrt{-1} = \sqrt{36} \cdot \sqrt{2} \cdot i = 6\sqrt{2}\, i = 6i\sqrt{2}$

c. $-5\sqrt{-9} = -5\sqrt{9} \cdot \sqrt{-1} = -5 \cdot 3 \cdot i = -15i$

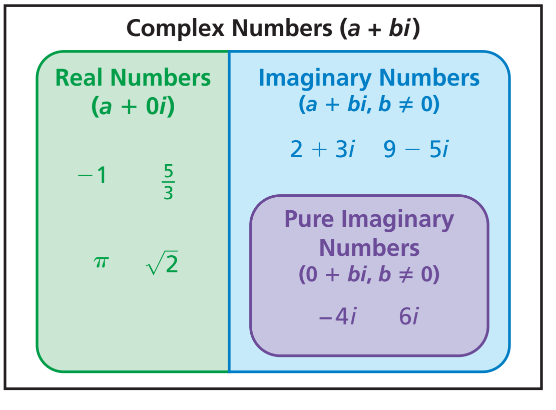
WORDS AND MATH

Numbers of the form $a + bi$ are called *complex* because they have more than one part.

A **complex number** written in *standard form* is a number $a + bi$, where a and b are real numbers. The number a is the *real part*, and the number bi is the *imaginary part*.

$$\underbrace{a}_{\text{real part}} + \underbrace{bi}_{\text{imaginary part}}$$

If $b \neq 0$, then $a + bi$ is an **imaginary number**. If $a = 0$ and $b \neq 0$, then $a + bi$ is a **pure imaginary number**. The diagram shows how different types of complex numbers are related.



1 MTR HELP A CLASSMATE

Help a classmate classify the number i^2.

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Two complex numbers $a + bi$ and $c + di$ are equal if and only if $a = c$ and $b = d$.

EXAMPLE 2 **Equality of Two Complex Numbers** WATCH

Find the values of x and y that satisfy the equation $2x - 7i = 10 + yi$.

SOLUTION

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x = 10$ Equate the real parts. $-7i = yi$ Equate the imaginary parts.

$x = 5$ Solve for x. $-7 = y$ Solve for y.

▶ So, $x = 5$ and $y = -7$.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the square root of the number.

1. $\sqrt{-4}$ **2.** $\sqrt{-12}$ **3.** $-\sqrt{-36}$ **4.** $2\sqrt{-54}$

Find the values of x and y that satisfy the equation.

5. $x + 3i = 9 - yi$ **6.** $5x + 4i = 20 + 2yi$ **7.** $9 + 4yi = -2x + 3i$

4 MTR **8.** **WHICH ONE DOESN'T BELONG?** Which number does *not* belong with the other three? Explain your reasoning.

$3 + 0i$ $2 + 5i$ $\sqrt{3} + 6i$ $0 - 7i$

Operations with Complex Numbers

KEY IDEA

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

EXAMPLE 3 **Adding and Subtracting Complex Numbers** WATCH

Add or subtract. Write the answer in standard form.

a. $(8 - i) + (5 + 4i)$ **b.** $(7 - 6i) - (3 - 6i)$

SOLUTION

a. $(8 - i) + (5 + 4i) = (8 + 5) + (-1 + 4)i$ Definition of complex addition

$= 13 + 3i$ Write in standard form.

b. $(7 - 6i) - (3 - 6i) = (7 - 3) + (-6 + 6)i$ Definition of complex subtraction

$= 4 + 0i$ Simplify.

$= 4$ Write in standard form.

5 MTR **RELATE CONCEPTS**

Explain the connection between adding and subtracting complex numbers and adding and subtracting radical expressions like $2 + 3\sqrt{2}$ and $1 - \sqrt{2}$.

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To multiply two complex numbers, use the Distributive Property, or the FOIL Method, just as you do when multiplying real numbers or algebraic expressions.

EXAMPLE 4 Multiplying Complex Numbers

WATCH

Multiply. Write the answer in standard form.

a. $4i(-6 + i)$

b. $(9 - 2i)(-4 + 7i)$

SOLUTION

a. $4i(-6 + i) = -24i + 4i^2$ — Distributive Property

$= -24i + 4(-1)$ — Use $i^2 = -1$.

$= -4 - 24i$ — Write in standard form.

b. $(9 - 2i)(-4 + 7i) = -36 + 63i + 8i - 14i^2$ — Multiply using FOIL.

$= -36 + 71i - 14(-1)$ — Simplify and use $i^2 = -1$.

$= -36 + 71i + 14$ — Simplify.

$= -22 + 71i$ — Write in standard form.

STUDY TIP

When simplifying an expression that involves complex numbers, be sure to simplify i^2 as -1.

Pairs of complex numbers of the forms $a + bi$ and $a - bi$, where $b \neq 0$, are called **complex conjugates**. Consider the product of complex conjugates below.

$(a + bi)(a - bi) = a^2 - (ab)i + (ab)i - b^2i^2$ — Multiply using FOIL.

$= a^2 - b^2(-1)$ — Simplify and use $i^2 = -1$.

$= a^2 + b^2$ — Simplify.

Because a and b are real numbers, $a^2 + b^2$ is a real number. So, the product of complex conjugates is a real number. You can use this fact to write the quotient of two complex numbers in standard form.

EXAMPLE 5 Dividing Complex Numbers

WATCH

Write the quotient $\frac{4 + 3i}{2 - i}$ in standard form.

SOLUTION

$\frac{4 + 3i}{2 - i} = \frac{4 + 3i}{2 - i} \cdot \frac{2 + i}{2 + i}$ — Multiply numerator and denominator by $2 + i$, the complex conjugate of $2 - i$.

$= \frac{8 + 4i + 6i + 3i^2}{4 + 2i - 2i - i^2}$ — Multiply using FOIL.

$= \frac{8 + 10i + 3(-1)}{4 - (-1)}$ — Simplify and use $i^2 = -1$.

$= \frac{5 + 10i}{5}$ — Simplify.

$= 1 + 2i$ — Write in standard form.

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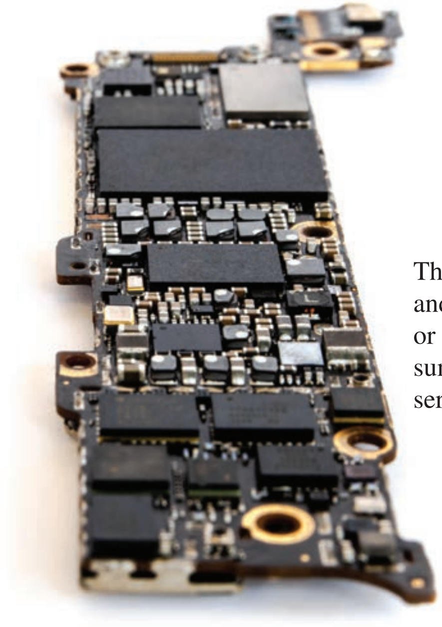
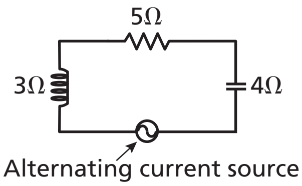
EXAMPLE 6 Modeling Real Life

Electrical circuit components, such as resistors, inductors, and capacitors, all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. Each of these quantities is measured in ohms. The symbol used for ohms is Ω, the uppercase Greek letter omega.



Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance (in ohms)	R	L	C
Impedance (in ohms)	R	Li	$-Ci$

The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled. The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the series circuit.



SOLUTION

The resistor has a resistance of 5 ohms, so its impedance is 5 ohms. The inductor has a reactance of 3 ohms, so its impedance is $3i$ ohms. The capacitor has a reactance of 4 ohms, so its impedance is $-4i$ ohms.

$$\begin{aligned}\text{Impedance of circuit} &= 5 + 3i + (-4i)\\ &= 5 - i\end{aligned}$$

▶ The impedance of the circuit is $(5 - i)$ ohms.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Add or subtract. Write the answer in standard form.

9. $(9 - i) + (-6 + 7i)$ **10.** $(3 + 7i) - (8 - 2i)$ **11.** $-4 - (1 + i) - (5 + 9i)$

12. $5 + (-9 + 3i) + 6i$ **13.** $(2 - i) + (1 + i) - 7i$ **14.** $8i - (6 - 3i) + (4 - 4i)$

15. OPEN-ENDED Write two complex numbers with a difference of 9.

Multiply. Write the answer in standard form.

16. $(-3i)(10i)$ **17.** $i(8 - i)$ **18.** $(3 + i)(5 - i)$

Divide. Write the answer in standard form.

19. $\dfrac{2}{1 - i}$ **20.** $\dfrac{7 + i}{1 - 2i}$ **21.** $\dfrac{2 - 4i}{3 + i}$

22. WHAT IF? In Example 6, what is the impedance of the circuit when the capacitor is replaced with one having a reactance of 7 ohms?

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Complex Solutions and Zeros

EXAMPLE 7 Solving Quadratic Equations

WATCH

Solve (a) $x^2 + 4 = 0$ and (b) $2x^2 - 11 = -47$.

SOLUTION

a. $x^2 + 4 = 0$ — Write original equation.

$x^2 = -4$ — Subtract 4 from each side.

$x = \pm\sqrt{-4}$ — Take square root of each side.

$x = \pm 2i$ — Write in terms of i.

▶ The solutions are $2i$ and $-2i$.

b. $2x^2 - 11 = -47$ — Write original equation.

$2x^2 = -36$ — Add 11 to each side.

$x^2 = -18$ — Divide each side by 2.

$x = \pm\sqrt{-18}$ — Take square root of each side.

$x = \pm i\sqrt{18}$ — Write in terms of i.

$x = \pm 3i\sqrt{2}$ — Simplify radical.

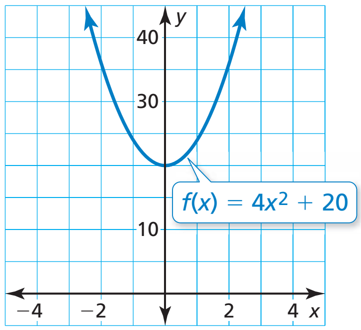
▶ The solutions are $3i\sqrt{2}$ and $-3i\sqrt{2}$.

5 MTR USE STRUCTURE
How can you use the solutions in Example 7(a) to factor $x^2 + 4$?

EXAMPLE 8 Finding Zeros of a Quadratic Function

WATCH

Find the zeros of the function.



SOLUTION

$4x^2 + 20 = 0$ — Set $f(x)$ equal to 0.

$4x^2 = -20$ — Subtract 20 from each side.

$x^2 = -5$ — Divide each side by 4.

$x = \pm\sqrt{-5}$ — Take square root of each side.

$x = \pm i\sqrt{5}$ — Write in terms of i.

▶ So, the zeros of f are $i\sqrt{5}$ and $-i\sqrt{5}$.

Check

$f(i\sqrt{5}) = 4(i\sqrt{5})^2 + 20 = 4 \cdot 5i^2 + 20 = 4(-5) + 20 = 0$ ✓

$f(-i\sqrt{5}) = 4(-i\sqrt{5})^2 + 20 = 4 \cdot 5i^2 + 20 = 4(-5) + 20 = 0$ ✓

2 MTR MAKE A CONNECTION
A quadratic function without real zeros must have imaginary zeros. How can you use a graph to determine whether a quadratic function has imaginary zeros?

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation.

23. $x^2 = -13$ **24.** $x^2 - 8 = -36$ **25.** $3x^2 - 7 = -31$ **26.** $5x^2 + 33 = 3$

Find the zeros of the function.

27. $f(x) = x^2 + 7$ **28.** $f(x) = -x^2 - 4$ **29.** $f(x) = 9x^2 + 1$

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3.2 Practice WITH CalcChat® AND CalcView®

In Exercises 1–8, find the square root of the number. *(See Example 1.)*

1. $\sqrt{-36}$
2. $\sqrt{-64}$
3. $\sqrt{-18}$
4. $\sqrt{-24}$
5. $2\sqrt{-16}$
6. $-3\sqrt{-49}$
7. $-4\sqrt{-32}$
8. $6\sqrt{-63}$

In Exercises 9–16, find the values of x and y that satisfy the equation. *(See Example 2.)*

9. $4x + 2i = 8 + yi$
10. $3x + 6i = 27 + yi$
11. $-10x + 12i = 20 + 3yi$
12. $9x - 18i = -36 + 6yi$
13. $2x - yi = 14 + 12i$
14. $-12x + yi = 60 - 13i$
15. $54 - \frac{1}{7}yi = 9x - 4i$
16. $15 - 3yi = \frac{1}{2}x + 2i$

In Exercises 17–26, add or subtract. Write the answer in standard form. *(See Example 3.)*

17. $(6 - i) + (7 + 3i)$
18. $(9 + 5i) + (11 + 2i)$
19. $(12 + 4i) - (3 - 7i)$
20. $(2 - 15i) - (4 + 5i)$
21. $(12 - 3i) + (7 + 3i)$
22. $(16 - 9i) - (2 - 9i)$
23. $7 - (3 + 4i) + 6i$
24. $16 - (2 - 3i) - i$
25. $-10 + (6 - 5i) - 9i$
26. $-3 + (8 + 2i) + 7i$

27. **STRUCTURE** Write each expression as a complex number in standard form.
    a. $\sqrt{-9} + \sqrt{-4} - \sqrt{16}$
    b. $\sqrt{-16} + \sqrt{8} + \sqrt{-36}$

28. **REASONING** The additive inverse of a complex number z is a complex number z_a such that $z + z_a = 0$. Find the additive inverse of each complex number.
    a. $z = 1 + i$
    b. $z = 3 - i$
    c. $z = -2 + 8i$

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In Exercises 29–36, multiply. Write the answer in standard form. *(See Example 4.)*

29. $3i(-5 + i)$
30. $2i(7 - i)$
31. $(3 - 2i)(4 + i)$
32. $(7 + 5i)(8 - 6i)$
33. $(5 - 2i)(-2 - 3i)$
34. $(-1 + 8i)(9 + 3i)$
35. $(3 - 6i)^2$
36. $(8 + 3i)^2$

ERROR ANALYSIS In Exercises 37 and 38, describe and correct the error in performing the operation and writing the answer in standard form.

37.
$$\begin{aligned}(3 + 2i)(5 - i) &= 15 - 3i + 10i - 2i^2\\ &= 15 + 7i - 2i^2\\ &= -2i^2 + 7i + 15\end{aligned}$$

38.
$$\begin{aligned}(4 + 6i)^2 &= (4)^2 + (6i)^2\\ &= 16 + 36i^2\\ &= 16 + (36)(-1)\\ &= -20\end{aligned}$$

In Exercises 39–46, divide. Write the answer in standard form. *(See Example 5.)*

39. $\dfrac{10}{2 + i}$
40. $\dfrac{2}{1 - i}$
41. $\dfrac{4 + 2i}{3 - i}$
42. $\dfrac{3 + 4i}{1 + i}$
43. $\dfrac{1 - 3i}{1 + 2i}$
44. $\dfrac{-7 + 6i}{-2 - 4i}$
45. $\dfrac{6 + 2i}{6 - 3i}$
46. $\dfrac{-3 - 2i}{-7 + 4i}$

NUMBER SENSE In Exercises 47 and 48, use the given numbers to complete the equation.

47. (____ − ____i) − (____ − ____i) = $2 - 4i$

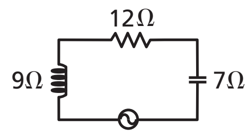
7   4   3   6

48. ____i(____ + ____i) = $-18 - 10i$

−5   9   2

MODELING REAL LIFE **In Exercises 49–52, find the impedance of the series circuit.** *(See Example 6.)*

49.



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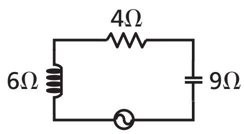

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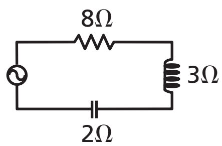

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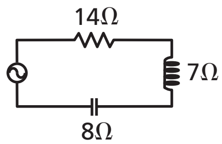


JUSTIFYING STEPS **In Exercises 53 and 54, justify each step in performing the operation.**

53. $11 - (4 + 3i) + 5i$

$= [(11 - 4) - 3i] + 5i$

$= (7 - 3i) + 5i$

$= 7 + (-3 + 5)i$

$= 7 + 2i$

54. $(3 + 2i)(7 - 4i)$

$= 21 - 12i + 14i - 8i^2$

$= 21 + 2i - 8(-1)$

$= 21 + 2i + 8$

$= 29 + 2i$

In Exercises 55–60, solve the equation. *(See Example 7.)*

55. $x^2 + 9 = 0$

56. $x^2 + 49 = 0$

57. $x^2 - 4 = -11$

58. $x^2 - 9 = -15$

59. $2x^2 + 6 = -34$

60. $x^2 + 7 = -47$

In Exercises 61–68, find the zeros of the function. *(See Example 8.)*

61. $f(x) = 3x^2 + 6$

62. $g(x) = 7x^2 + 21$

63. $h(x) = 2x^2 + 72$

64. $k(x) = -5x^2 - 125$

65. $m(x) = -x^2 - 27$

66. $p(x) = x^2 + 98$

67. $r(x) = -\frac{1}{2}x^2 - 24$

68. $f(x) = -\frac{1}{5}x^2 - 10$

69. **STRUCTURE** Expand $(a - bi)^2$ and write the result in standard form. Use your result to check your answer to Exercise 35.

70. **STRUCTURE** Expand $(a + bi)^2$ and write the result in standard form. Use your result to check your answer to Exercise 36.

71. **NUMBER SENSE** Write the complex conjugate of $1 - \sqrt{-12}$. Then find the product of the complex conjugates.

72. **NUMBER SENSE** Simplify each expression. Then classify your results in the table below.

a. $(-4 + 7i) + (-4 - 7i)$

b. $(2 - 6i) - (-10 + 4i)$

c. $(25 + 15i) - (25 - 6i)$

d. $(5 + i)(8 - i)$

e. $(17 - 3i) + (-17 - 6i)$

f. $(-1 + 2i)(11 - i)$

g. $\dfrac{-8 + 10i}{-10 - 8i}$

h. $\dfrac{-3 + 6i}{-4 + 8i}$

Real numbers	Imaginary numbers	Pure imaginary numbers

73. **STRUCTURE** The coordinate system shown below is called the *complex plane*. In the complex plane, the point that corresponds to the complex number $a + bi$ is (a, b). Match each complex number with its corresponding point.

a. 2

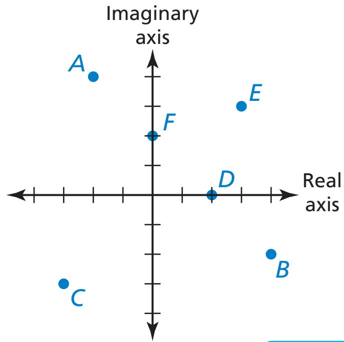
b. $2i$

c. $4 - 2i$

d. $3 + 3i$

e. $-2 + 4i$

f. $-3 - 3i$



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74. CHOOSE A METHOD Describe the methods shown for writing the complex expression in standard form. Which method do you prefer? Explain.

Method 1

$$4i(2-3i)+4i(1-2i) = 8i-12i^2+4i-8i^2$$
$$= 8i-12(-1)+4i-8(-1)$$
$$= 20+12i$$

Method 2

$$4i(2-3i)+4i(1-2i) = 4i[(2-3i)+(1-2i)]$$
$$= 4i[3-5i]$$
$$= 12i-20i^2$$
$$= 12i-20(-1)$$
$$= 20+12i$$

In Exercises 75–80, write the expression as a complex number in standard form.

75. $(3+4i)-(7-5i)+2i(9+12i)$

76. $3i(2+5i)+(6-7i)-(9+i)$

77. $(3+5i)(2-7i^4)$

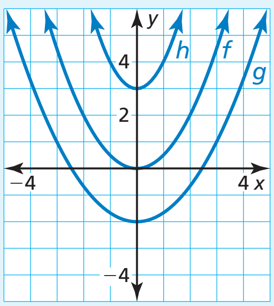
78. $2i^3(5-12i)$

79. $\dfrac{5+i}{3-i}-(-6+4i)$

80. $(4-3i)+\dfrac{6+8i}{2+2i}$

81. PATTERNS Make a table that shows the powers of i from i^1 to i^8 in the first row and the simplified forms of these powers in the second row. Describe the pattern you observe in the table. Then use the pattern to evaluate i^{25}, i^{50}, i^{75}, and i^{100}.

82. HOW DO YOU SEE IT?
The graphs of three functions are shown. Which function(s) have real zeros? imaginary zeros? Explain your reasoning.



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83. NUMBER SENSE Write each product as a complex number in standard form.

a. $(2-3i)^3$

b. $(3+i)^4$

84. REASONING Is it possible for a quadratic equation to have one real solution and one imaginary solution? Explain your reasoning.

85. COLLEGE PREP Which expressions are equivalent to $1+i$? Select all that apply.

Ⓐ $(4-i)-(3-2i)$

Ⓑ $\dfrac{-2}{1+i}$

Ⓒ $(2-i)(i+1)$

Ⓓ $i^{20}-i^{21}$

86. REASONING Rewrite the expression below with a real denominator.

$$\frac{1+3i}{2i}$$

87. JUSTIFYING STEPS Justify each step in the simplification of i^2.

Algebraic Step	Justification
$i^2 = (\sqrt{-1})^2$	
$= -1$	

88. MAKING AN ARGUMENT Your friend claims that the conclusion in Exercise 87 is incorrect because $i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{-1(-1)} = \sqrt{1} = 1$. Is your friend correct? Explain.

89. NUMBER SENSE Write a pair of complex numbers whose sum is -4 and whose product is 53.

90. PROBLEM SOLVING The sum of two complex numbers is $7+8i$. The difference of the numbers is $1-2i$. What is the product of the numbers?

91. DISCUSS MATHEMATICAL THINKING Determine whether each statement is true or false. If it is true, give an example. If it is false, give a counterexample.

a. The sum of two imaginary numbers is always an imaginary number.

b. The product of two pure imaginary numbers is always a real number.

c. A pure imaginary number is an imaginary number.

d. A complex number is a real number.

92. **DISCUSS MATHEMATICAL THINKING** The zeros of a quadratic function are $3 \pm 4i$.

a. What do you know about the vertex of the function? Explain.

b. Write and graph a quadratic function that has these zeros.

93. **DIG DEEPER** Write $\sqrt{i}$ as a complex number in standard form. (*Hint:* Use the equation $\sqrt{i} = a + bi$ to write a system of equations in terms of a and b.)

94. **THOUGHT PROVOKING**
Create a circuit that has an impedance of $14 - 3i$.

REVIEW & REFRESH

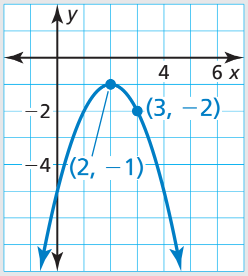
In Exercises 95 and 96, graph the function and its parent function. Then describe the transformation.

95. $f(x) = \frac{1}{4}x^2 + 1$ **96.** $f(x) = -\frac{1}{2}x - 4$

In Exercises 97 and 98, simplify the expression. Write your answer using only positive exponents.

97. $\left(\dfrac{4c^4d^{-5}}{8c^0d^4}\right)^2$ **98.** $\left(\dfrac{3m^{-5}}{m^{-6}n}\right)^3 \bullet \left(\dfrac{-4m^3n^{-1}}{2mn^{-7}}\right)^4$

99. Write a quadratic function in standard form whose graph is shown.



100. **REASONING** Find the zeros of the function $f(x) = 9x^2 + 2$. Does the graph of the function intersect the x-axis? Explain your reasoning.

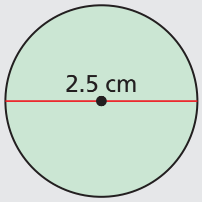
101. **MODELING REAL LIFE** The table shows the heights y (in feet) of a baseball x seconds after it is hit. Write an equation that represents the path of the baseball. What is the greatest height the baseball reaches?

Time, x	0	0.25	0.5	0.75	1
Height, y	5	8	9	8	5

102. Make a scatter plot of the data. Then describe the relationship between the data.

x	4	2	7	5	2	6	3	5
y	37	62	45	79	29	18	55	64

103. Find the circumference of the circle.



104. **MODELING REAL LIFE** A screen printing shop sells long-sleeved shirts and short-sleeved shirts. Order A includes 3 long-sleeved shirts and 12 short-sleeved shirts for a total cost of \$165. Order B includes 8 long-sleeved shirts and 2 short-sleeved shirts for a total cost of \$140. What is the cost of each type of shirt?

In Exercises 105–108, add, subtract, multiply, or divide. Write the answer in standard form.

105. $-3i(9 - 4i)$ **106.** $(7 + 8i) - (-4 + 9i)$

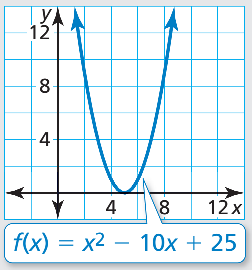
107. $\dfrac{1 - 12i}{5 - 2i}$ **108.** $(-15 - i) + (6 + 3i)$

In Exercises 109 and 110, write an inequality that represents the graph.

109.

110.

111. Use the graph to solve $x^2 - 10x + 25 = 0$.



In Exercises 112 and 113, graph the system. Identify a solution, if possible.

112. $y > x - 1$
$y \leq -4$

113. $x + y \leq 3$
$y + 2 \geq -4x$