2.4 Modeling with Quadratic Functions

Learning Target: Write quadratic functions to model real-life data.

Success Criteria:
- I can write quadratic equations to model data sets.
- I can use technology to find a quadratic model for a set of data.
- I can use quadratic models to solve real-life problems.

EXPLORE IT! Modeling with Quadratic Functions

1 MTR ANALYZE A PROBLEM
What would a t-intercept of a graph indicate in this context?

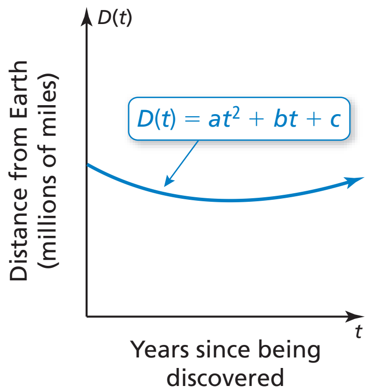
Work with a partner.

a. Explain what the graph represents.

b. What do you know about the value of a? How does the graph change if a is increased? decreased? What does this mean in this context? Explain your reasoning.

c. Write an expression that represents the year t when the comet is closest to Earth.

Comet



d. The comet is the same distance away from Earth in 2012 and 2020. Estimate the year when the comet is closest to Earth. Explain your reasoning.

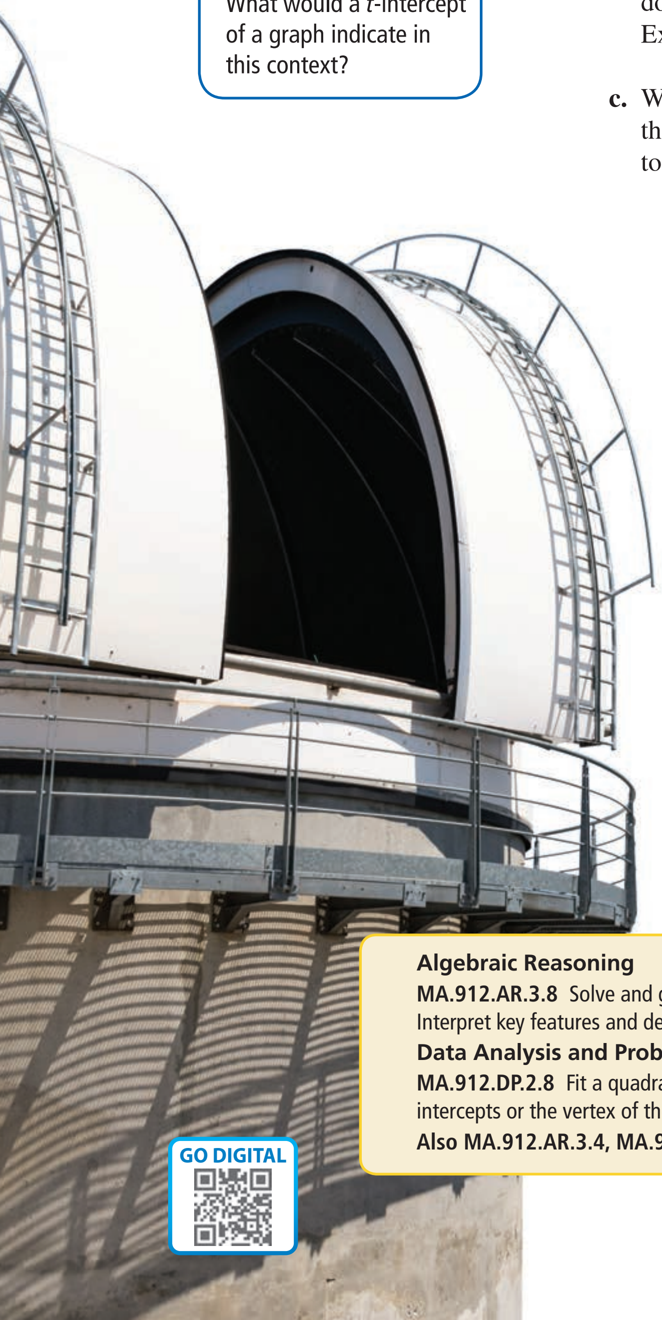
e. What does c represent in this context? How does the graph change if c is increased? decreased? Explain.

f. Assume that the model is still valid today. Is the comet's distance from Earth currently increasing, decreasing, or constant? Explain.

g. The table shows the approximate distances y (in millions of miles) from Earth for a planetary object m months after being discovered. Can you use a quadratic function to model the data? How do you know? Is this the only type of function you can use to model the data? Explain your reasoning.

Months, m	0	1	2	3	4	5	6	7	8	9
Distance (millions of miles), y	50	57	65	75	86	101	115	130	156	175

h. Explain how you can find a quadratic model for the data. How do you know your model is a good fit?



Algebraic Reasoning
MA.912.AR.3.8 Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.
Data Analysis and Probability
MA.912.DP.2.8 Fit a quadratic function to bivariate numerical data that suggests a quadratic association and interpret any intercepts or the vertex of the model. Use the model to solve real-world problems in terms of the context of the data.
Also MA.912.AR.3.4, MA.912.F.1.1

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Writing Equations to Model Data

When data have equally spaced inputs, you can analyze patterns in the differences of the outputs to determine the type of function that can be used to model the data. Linear data have constant *first differences*. Quadratic data have constant *second differences*. The first and second differences of $f(x) = x^2$ are shown below.

Equally spaced x-values

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

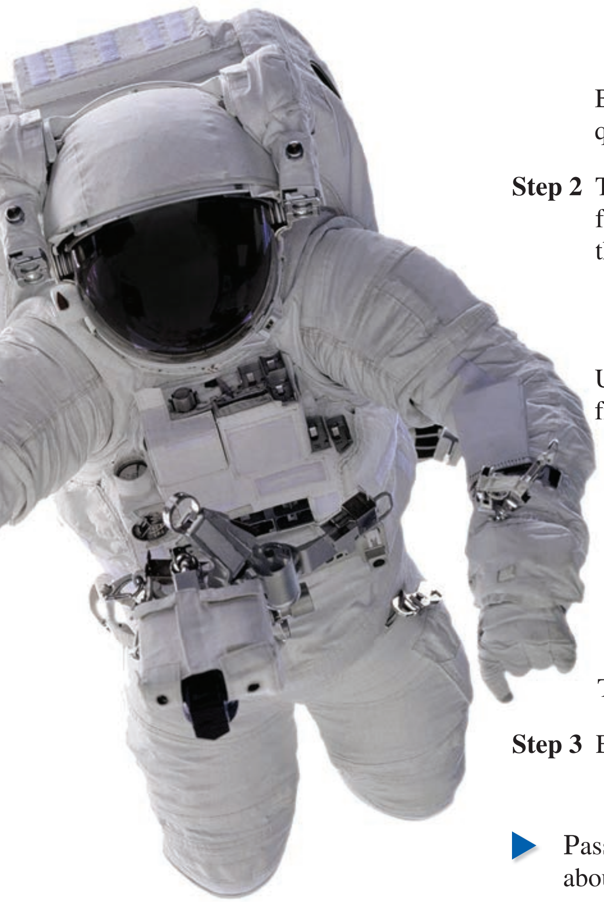
first differences: -5 $\quad -3$ $\quad -1$ $\quad 1$ $\quad 3$ $\quad 5$

second differences: 2 $\quad 2$ $\quad 2$ $\quad 2$ $\quad 2$

EXAMPLE 1 **Writing a Quadratic Function**

NASA can create a weightless environment by flying a plane in parabolic paths. The table shows the heights $h(t)$ (in feet) of a plane t seconds after starting the flight path. After about 20.8 seconds, passengers begin to experience a weightless environment. Write and evaluate a function to approximate the height at which this occurs.

Time, t	Height, $h(t)$
0	21,000
10	26,900
20	30,600
30	32,100
40	31,400
50	28,500



SOLUTION

Step 1 The input values are equally spaced. So, analyze the differences in the outputs to determine the type of function you can use to model the data.

$h(0)$	$h(10)$	$h(20)$	$h(30)$	$h(40)$	$h(50)$
21,000	26,900	30,600	32,100	31,400	28,500

first differences: 5900 $\quad$ 3700 $\quad$ 1500 $\quad$ -700 $\quad$ -2900

second differences: -2200 $\quad$ -2200 $\quad$ -2200 $\quad$ -2200

Because the second differences are constant, you can model the data with a quadratic function.

Step 2 The table shows that the y-intercept is 21,000. So, a quadratic function of the form $h(t) = at^2 + bt + 21{,}000$ models the data. Use two points $(t, h(t))$ from the table to write a system of equations.

Use (10, 26,900): $100a + 10b + 21{,}000 = 26{,}900$ Equation 1

Use (20, 30,600): $400a + 20b + 21{,}000 = 30{,}600$ Equation 2

Use the elimination method to solve the system. Begin by subtracting 21,000 from each side of each equation.

$100a + 10b = 5900$ New Equation 1

$400a + 20b = 9600$ New Equation 2

$200a = -2200$ Subtract 2 times new Equation 1 from new Equation 2.

$a = -11$ Solve for a.

$b = 700$ Substitute into new Equation 1 to find b.

The data can be modeled by the function $h(t) = -11t^2 + 700t + 21{,}000$.

Step 3 Evaluate the function when $t = 20.8$.

$$h(20.8) = -11(20.8)^2 + 700(20.8) + 21{,}000 = 30{,}800.96$$

▶ Passengers begin to experience a weightless environment at about 30,800 feet.

SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

1. The table shows the estimated profits y (in dollars) for a concert when the charge is x dollars per ticket. Write and evaluate a function to determine the maximum profit.

Ticket price, *x*	0	3	6	9	12	15
Profit, *y*	−1000	4100	7400	8900	8600	6500

Using Quadratic Regression

Real-life data that show a quadratic relationship usually do not have constant second differences, because the data are not *exactly* quadratic. Relationships that are *approximately* quadratic have second differences that are relatively "close" in value. Many technology tools have a *quadratic regression* feature that you can use to find a quadratic function that best models a set of data.

EXAMPLE 2 Using Quadratic Regression

WATCH

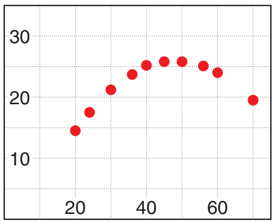
The table shows fuel efficiencies of a vehicle at different speeds. Write a function that models the data. Use the model to approximate the best gas mileage.

Miles per hour, *x*	Miles per gallon, *y*
20	14.5
24	17.5
30	21.2
36	23.7
40	25.2
45	25.8
50	25.8
56	25.1
60	24.0
70	19.5

SOLUTION

Because the x-values are not equally spaced, you cannot analyze the differences in the outputs. Use technology to find a function that models the data.

Step 1 Enter the data from the table and create a scatter plot. The data show a quadratic relationship.



Step 2 Find the quadratic equation. The values in the equation can be rounded to obtain $y = -0.014x^2 + 1.37x - 7.1$.

$y = ax^2 + bx + c$

PARAMETERS

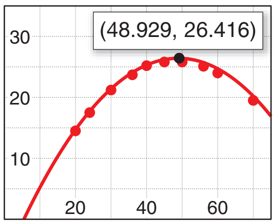
$a = -0.0141$ $b = 1.3662$

$c = -7.1441$

STATISTICS

$R^2 = 0.9992$

Step 3 Graph the regression equation with the scatter plot.



In this context, the best gas mileage is the maximum mileage per gallon. Using technology, you can see that the maximum mileage per gallon is about 26.4 miles per gallon when driving about 48.9 miles per hour.

▶ So, the best gas mileage is about 26.4 miles per gallon.

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	x	R_t
June 1	1	1.65
June 15	15	1.43
June 30	30	1.22
July 15	45	1.04
July 31	61	0.93
Aug 15	76	0.89
Aug 31	92	0.87
Sept 15	107	0.90
Sept 30	122	0.98
Oct 15	137	1.11
Oct 31	153	1.26

EXAMPLE 3 Using Quadratic Regression

WATCH

The *effective reproduction number* R_t for a virus represents the average number of people who become infected by an infectious person. The table shows the effective reproduction number for a virus x days after May 31.

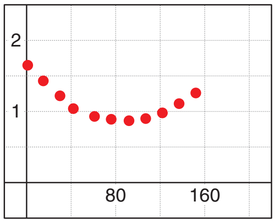
a. Write a function that models the data. When is the virus spreading slowest?

b. Approximate R_t when $x = 20$ and when $x = 170$.

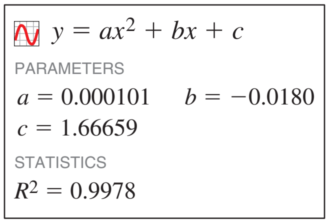
SOLUTION

a. Because the x-values are not equally spaced, you cannot analyze the differences in the outputs. Use technology to find a function that models the data.

Step 1 Enter the data from the table and create a scatter plot. The data show a quadratic relationship.

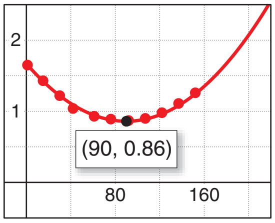


Step 2 Find the quadratic equation. The values in the equation can be rounded to obtain $R_t = 0.0001x^2 - 0.018x + 1.67$.



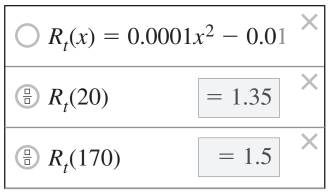
Step 3 Graph the regression equation with the scatter plot.

In this context, the virus is spreading slowest when R_t is at a minimum. Using technology, you can see that the minimum R_t-value is 0.86 when $x = 90$.



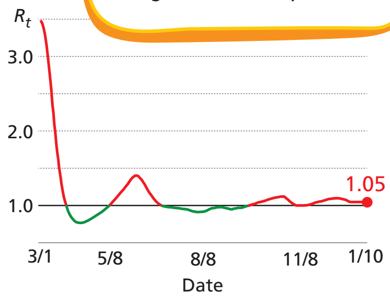
▶ So, the virus is spreading slowest 90 days after May 31, or on August 29.

b. Use technology to evaluate the function when $x = 20$ and when $x = 170$.



▶ So, the R_t-value is 1.35 on June 20. Assuming the trend continues, you can predict the R_t-value is 1.5 on November 17.

This graph represents the R_t-value over time in Florida during the Covid-19 pandemic.



SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

2. WRITING Explain when it is appropriate to use a quadratic model for a set of data.

6 MTR **3. ASSESS REASONABLENESS** Is it reasonable to use the model in Example 3 to predict the R_t-value after 2 years? Explain your reasoning.

4. The table shows the results of an experiment testing the maximum weights y (in tons) supported by ice x inches thick. Write a function that models the data. How much weight can be supported by ice that is 22 inches thick? 30 inches thick?

Ice thickness, x	12	14	15	18	20	24	27
Maximum weight, y	3.4	7.6	10.0	18.3	25.0	40.6	54.3

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2.4 Practice WITH CalcChat® AND CalcView®

1. **MODELING REAL LIFE** Every rope has a safe working load. A rope should not be used to lift a weight greater than its safe working load. The table shows the safe working loads S (in pounds) for ropes with circumferences C (in inches). Write an equation for the safe working load for a rope. Find the safe working load for a rope that has a circumference of 10 inches. *(See Example 1.)*

Circumference, C	0	1	2	3
Safe working load, S	0	180	720	1620

2. **MODELING REAL LIFE** A water balloon is thrown up in the air. The table shows the heights y (in feet) of the balloon after x seconds. Write an equation for the height of the balloon. Find the height of the water balloon after 1 second. Interpret the y-intercept.

STEM

Time, x	0	0.2	0.4	0.6
Balloon height, y	5	7.16	8.04	7.64

3. **USING TOOLS** The table shows the numbers y (in thousands) of people in a city who regularly use sharable electric scooters x weeks after the scooters are introduced. Write a function that models the data. Use the model to predict the number of users after 32 weeks. *(See Example 2.)*

Time, x	Number of users, y
1	1.5
4	2.2
6	2.4
10	3.9
12	5.5
15	6.8
20	12.3
24	16.4
25	17.6

4. **USING TOOLS** The table shows the numbers y of students absent from school x days after a flu outbreak. Write a function that models the data. Use the model to approximate the number of students absent 10 days after the outbreak.

Time (days), x	2	4	5	6	8	9	11
Number of students, y	11	17	19	19	17	14	7

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5. **REASONING** The table shows the estimated profit y (in dollars) of a lemonade stand x hours after the stand opens.

   a. Write a function that models the data. When are the profits the highest?

   b. Approximate the profits when $x = 5$ and when $x = 10.5$. *(See Example 3.)*

Time, x	Profit, y
1	10
2	30
3.5	60
4	68
6	87
7.5	90
8	88
8.5	85
9.5	75

6. **REASONING** The table shows the average wait times y (in minutes) of a new ride at Disney World x days after the ride opens.

Day, x	1	3	4	8	11	14	15	17
Wait time, y	115	135	140	155	150	130	118	93

   a. Write a function that models the data. On what day is the average wait time the longest?

   b. Interpret the y-intercept.

   c. Approximate the average wait time when $x = 7$ and when $x = 20$.

7. **MODELING REAL LIFE** The table shows the distances y a motorcyclist is from home after x hours.

Time (hours), x	0	1	2	3
Distance (miles), y	0	45	90	135

   a. Determine the type of function you can use to model the data. Explain your reasoning.

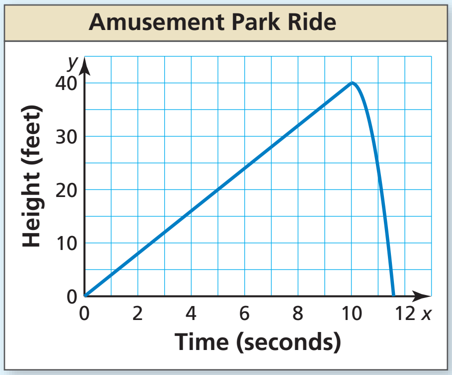
   b. Write and evaluate a function to determine the distance the motorcyclist is from home after 6 hours.

8. **PROBLEM SOLVING** The table shows the heights y of a competitive water-skier x seconds after jumping off a ramp. Write a function that models the height of the water-skier over time. Interpret the y-intercept. When is the water-skier 5 feet above the water? How long is the skier in the air?

Time (seconds), x	0	0.25	0.75	1	1.1
Height (feet), y	22	22.5	17.5	12	9.24

9. OPEN-ENDED Describe a real-life situation not mentioned in this chapter that can be modeled by a quadratic equation. Justify your answer.

10. HOW DO YOU SEE IT?
The graph models the height of a car on an amusement park ride, where x represents the time (in seconds) and y represents the height (in feet). The ride raises a car and then releases the car to fall. Approximate the height of the car after (a) 5 seconds and (b) 11 seconds. Explain your reasoning.



11. REASONING The table shows the number of tiles in each figure. Verify that the data show a quadratic relationship. Predict the number of tiles in the 12th figure.

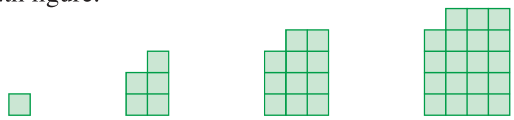


Figure 1 Figure 2 Figure 3 Figure 4

Figure	1	2	3	4
Number of tiles	1	5	11	19

12. THOUGHT PROVOKING
The table shows the temperatures y (in degrees Fahrenheit) of a cup of tea after x minutes. Write a function that models the data and can be used to predict the temperature of the tea after 20 minutes. Explain your reasoning.

Time, x	0	2	4	6	8	10
Temperature, y	190	164	146	131	120	111

REVIEW & REFRESH

WATCH

In Exercises 13–16, factor the polynomial.

13. $x^2 + 4x + 3$

14. $x^2 - 3x + 2$

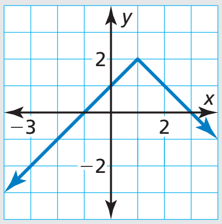
15. $3x^2 - 15x + 12$

16. $x^2 + x - 6$

17. MODELING REAL LIFE The table shows the heights y (in feet) of a firework x seconds after it is launched. The firework explodes at its highest point. Write an equation for the path of the firework. Find the height at which the firework explodes.

Time, x	0	1	2	3
Height, y	0	112	192	240

18. Determine whether the graph represents a function. Explain.



In Exercises 19 and 20, graph the function. Label the vertex and axis of symmetry.

19. $f(x) = 2(x - 1)^2 - 5$

20. $h(x) = 3x^2 + 6x - 2$

21. The table represents a quadratic function. Write an equation of the function in standard form.

x	−2	−1	0	1
$f(x)$	10	1	−2	1

22. Let the graph of g be a horizontal shrink by a factor of $\frac{1}{4}$, followed by a translation 1 unit up and 3 units right of the graph of $f(x) = (2x + 1)^2 - 11$. Write a rule for g and identify the vertex.

In Exercises 23–26, solve the inequality. Graph the solution.

23. $m + 9 \geq 13$

24. $15 - n < -6$

25. $5p > 10$

26. $-\frac{q}{4} \leq 3$

27. Determine whether the table represents a *linear* or an *exponential* function. Explain.

x	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{2}$	2	8	32

In Exercises 28 and 29, write an equation in slope-intercept form of the line that passes through the given points.

28. $(4, -1), (0, 3)$

29. $(-3, -2), (1, 4)$

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