1.5 Absolute Value Functions

Learning Target: Understand characteristics of absolute value functions.

Success Criteria:
- I can describe the characteristics of an absolute value function from a graph.
- I can graph an absolute value function given its characteristics.
- I can solve real-life problems involving absolute value functions.

EXPLORE IT! Describing Characteristics of Absolute Value Functions

Work with a partner.

a. Match each function with its graph. Explain your reasoning.

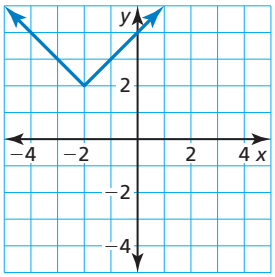
i. $f(x) = |x + 2| - 2$

ii. $g(x) = -|x - 2| + 2$

iii. $j(x) = -|x - 2| - 2$

iv. $m(x) = |x + 2| + 2$

A.



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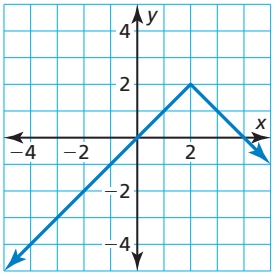

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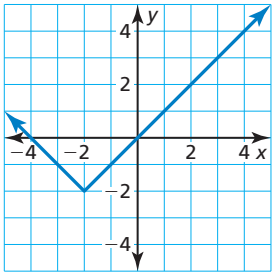

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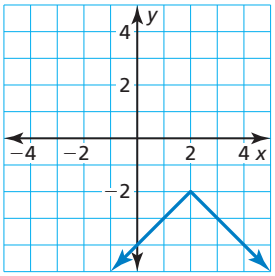

1 MTR ANALYZE A PROBLEM

You know the vertex, range, line of symmetry, and end behavior for an absolute value function p. Do you have enough information to graph p? If not, what other information do you need?

b. Complete the table below to describe characteristics of each function in part (a).

Function	Vertex	Range	Line of symmetry	End behavior
f				
g				
j				
m				

c. Explain how each of the characteristics in the table are related to the equation of an absolute value function.

Algebraic Reasoning

MA.912.AR.4.4 Solve and graph mathematical and real-world problems that are modeled with absolute value functions. Interpret key features and determine constraints in terms of the context.

Functions

MA.912.F.1.7 Compare key features of two functions each represented algebraically, graphically, in tables or written descriptions.

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Characteristics of Absolute Value Functions

Vocabulary

vertex form, *p. 38*

Recall that an absolute value function is a function that contains an absolute value expression.

KEY IDEA

Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x) = a|x - h| + k$, where $a \neq 0$, is in **vertex form**. The vertex of the graph of g is (h, k).

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

REMEMBER

The *end behavior* of a function is the behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$).

EXAMPLE 1 Describing Characteristics

Graph $f(x) = |x - 4| - 1$. Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

MAKE A PLAN

Make a plan for how to choose x-values for a table when graphing an absolute value function. Explain your reasoning.

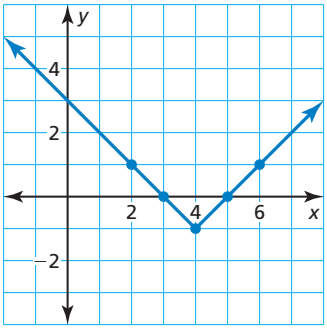
SOLUTION

Step 1 Make a table of values.

x	2	3	4	5	6
$f(x)$	1	0	−1	0	1

Step 2 Plot the ordered pairs.

Step 3 Draw the graph.



Positive and Negative: The x-intercepts are 3 and 5. The function is positive when $x < 3$, negative when $3 < x < 5$, and positive when $x > 5$.

Increasing and Decreasing: The vertex is $(4, -1)$. The function is decreasing when $x < 4$ and increasing when $x > 4$.

End Behavior: The graph shows that the function values increase as x approaches both positive and negative infinity. So, $y \to +\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph f. Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

1. $f(x) = |x + 6| + 1$

2. $f(x) = -|x - 5| + 2$

3. $f(x) = 2|x + 1| - 8$

4. STRUCTURE A function g is increasing when $x < 2$, decreasing when $x > 2$, and has a range of $(-\infty, -2)$. Use the given values to complete the function. Do not use any value more than once.

$g(x) = \square|x - \square| + \square$

−2 0 2 −1

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EXAMPLE 2 Using Characteristics to Graph an Absolute Value Function

WATCH

Graph each absolute value function f with the given characteristics.

a. f has a range of $(-\infty, 1)$, and a graph that is symmetric about the line $x = -2$, and has a y-intercept of -5.

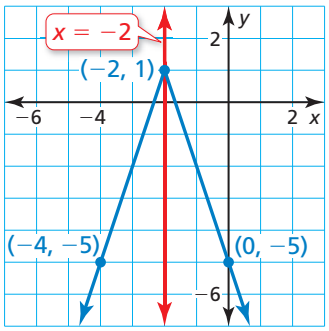
b. f is positive over the intervals $(-\infty, 0)$ and $(4, \infty)$, negative over the interval $(0, 4)$, and the minimum value is -4.

STUDY TIP

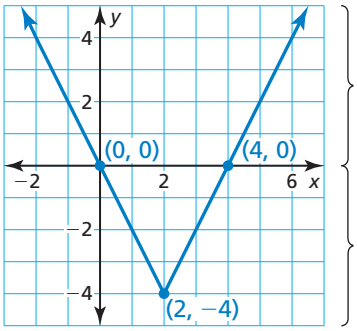
The interval in part (a) represents y-values, and the intervals in part (b) represent x-values.

SOLUTION

a. Because the graph is symmetric about $x = -2$, the x-value of the vertex is -2. Because the range is $(-\infty, 1)$, the y-value of the vertex is 1. Plot the vertex $(-2, 1)$. Because the y-intercept is -5, plot the point $(0, -5)$ and its reflection in the line of symmetry, $(-4, -5)$. Then draw the graph.



b. Because f is positive over the intervals $(-\infty, 0)$ and $(4, \infty)$, and negative over the interval $(0, 4)$, you know that the x-intercepts are 0 and 4. Because the x-intercepts are 0 and 4, the vertex has an x-coordinate of $\frac{0 + 4}{2} = 2$. So, plot the points $(0, 0)$, $(4, 0)$, and the vertex $(2, -4)$. Then draw the graph.



The graph is above the x-axis when f is positive.

The graph is below the x-axis when f is negative.

1 MTR ANALYZE A PROBLEM

In part (a), is it possible to determine the graph of f using only two of the given characteristics? Explain your reasoning.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. Graph the absolute value function f with the following characteristics.

- f is decreasing over the interval $(-\infty, -3)$ and increasing over the interval $(-3, \infty)$.
- The graph of f has a y-intercept of 1.
- The range of f is $(-2, \infty)$.

4 MTR 6. DIFFERENT WORDS, SAME QUESTION Consider the function $g(x) = |x + 5| - 3$. Which is different? Find "both" answers.

What is the vertex of the graph of g?

At what point does the graph of g change direction?

What is the minimum value of g?

What point on the graph of g lies on the line of symmetry?

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Solving Real-Life Problems

EXAMPLE 3 Modeling Real Life

WATCH

The function $m(x) = |x - 178|$ represents the absolute deviation (in centimeters) of a man's height x (in centimeters) from the average height of men in Florida. The function $w(x) = |x - 163|$ represents the absolute deviation (in centimeters) of a woman's height x (in centimeters) from the average height of women in Florida.

a. Interpret and compare the vertices of the graphs in this context.

b. Compare the absolute deviations for a man and a woman who are both 170.5 centimeters tall.

SOLUTION

a. Each function is written in vertex form. Identify h and k to find the vertex of each function.

$m(x) = |x - 178|$ $\quad h = 178, k = 0$

$w(x) = |x - 163|$ $\quad h = 163, k = 0$

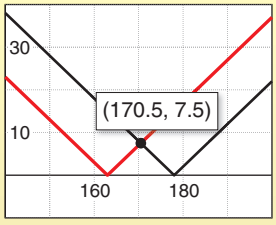
▶ The vertex of the graph of m is (178, 0). So, the average height of men in Florida is 178 centimeters. The vertex of the graph of w is (163, 0). So, the average height of women in Florida is 163 centimeters. The vertex of the graph of m is 15 units right of the vertex of the graph of w, indicating that the average height of men in Florida is 15 centimeters greater than the average height of women in Florida.

b. Evaluate each function for $x = 170.5$.

$m(x) = \|x - 178\|$	Write the function.	$w(x) = \|x - 163\|$
$= \|170.5 - 178\|$	Substitute.	$= \|170.5 - 163\|$
$= 7.5$	Simplify.	$= 7.5$

▶ The heights of the man and the woman are both 7.5 centimeters from the average heights.

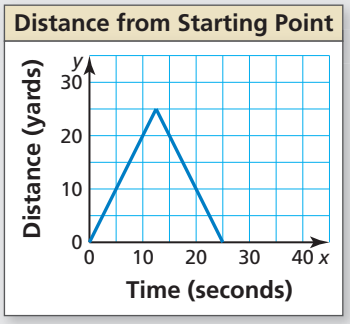
Check Graph each function to verify that the graphs intersect at (170.5, 7.5). ✓



SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

7. You and your friend race from one end of a pool to the other and then back again. The function $d(x) = -1.5\left|x - \frac{50}{3}\right| + 25$ represents your distance (in yards) from the starting point after x seconds. Your friend's distance from the starting point is shown in the graph. Who wins the race? By how many seconds does the first-place finisher win the race? Justify your answer.



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1.5 Practice WITH CalcChat® AND CalcView®

In Exercises 1–8, graph f. Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function. *(See Example 1.)*

1. $f(x) = |x + 3| + 2$

2. $f(x) = |x - 7| + 2$

3. $f(x) = -|x - 4| - 9$

4. $f(x) = |x + 8| - 1$

5. $f(x) = 3|x - 1| + 5$

6. $f(x) = -\frac{1}{4}|x + 4| + 3$

7. $f(x) = -\frac{3}{2}|x - 6| - 10$

8. $f(x) = -2|x + 2| - 8$

In Exercises 9–14, graph the absolute value function f with the given characteristics. *(See Example 2.)*

9. f has a range of $(-\infty, 7)$, and a graph that is symmetric about the line $x = 3$, and has a y-intercept of 4.

10. f has a range of $(-4, \infty)$, and a graph that is symmetric about the line $x = -1$, and has a y-intercept of -3.

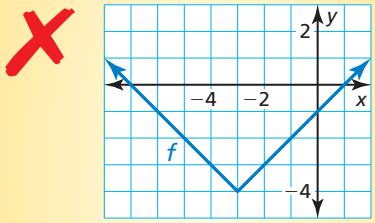
11. f is positive over the intervals $(-\infty, -1)$ and $(11, \infty)$, negative over the interval $(-1, 11)$, and the vertex of the graph of f is $(5, -3)$.

12. f is negative over the intervals $(-\infty, -7)$ and $(5, \infty)$, positive over the interval $(-7, 5)$, and the maximum value is 6.

13. f is increasing over the interval $(-\infty, 2)$ and decreasing over the interval $(2, \infty)$. The graph of f has a y-intercept of -16, and the range of f is $(-\infty, 2)$.

14. f is decreasing over the interval $(-\infty, -4)$ and increasing over the interval $(-4, \infty)$. The graph of f has a y-intercept of -8, and the range of f is $(-12, \infty)$.

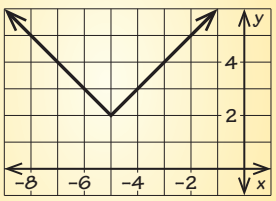
15. ERROR ANALYSIS Describe and correct the error in determining when the function is positive or negative.



The vertex is $(-3, -4)$. The function is negative when $x < -3$ and positive when $x > -3$.

16. ERROR ANALYSIS Describe and correct the error in graphing the function.

$f(x) = |x - 5| + 2$

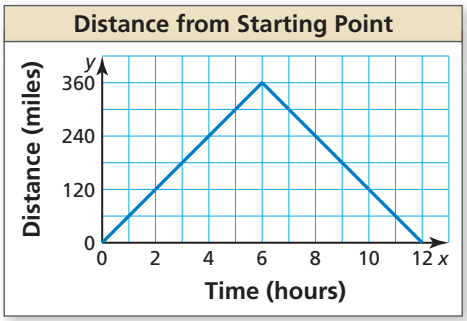


17. MODELING REAL LIFE The function $m(x) = |x - 41|$ represents the absolute deviation (in years) of a man's age x (in years) from the average age of men in Florida. The function $w(x) = |x - 44.2|$ represents the absolute deviation (in years) of a woman's age x (in years) from the average age of women in Florida. *(See Example 3.)*

a. Interpret and compare the vertices of the graphs in this context.

b. Compare the absolute deviations for a man and a woman who are both 65 years old.

18. MODELING REAL LIFE A car and a truck follow the same route on a pair of round-trip deliveries. The function $c(x) = -72|x - 5| + 360$ represents the distance (in miles) of the car from the starting point after x hours. The truck's distance from the starting point is shown in the graph.



a. How far is the delivery from the starting point?

b. Which vehicle takes longer to complete the delivery? How much longer? Justify your answers.

19. MAKING AN ARGUMENT Your friend says that if you are given the vertex of any absolute value function and the intervals where the function is positive and negative, then you are able to graph the function. Is your friend correct? Explain your reasoning.

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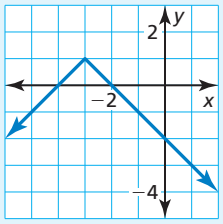
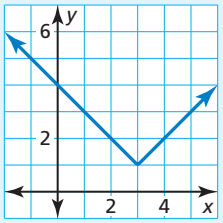
20. HOW DO YOU SEE IT?
Match each absolute value function with its graph. Explain.

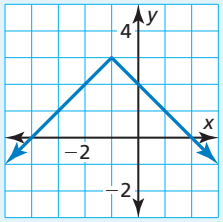
a.



b.



c.



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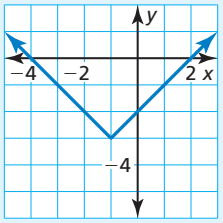


i. $f(x) = |x - 3| + 1$ **ii.** $g(x) = -|x + 1| + 3$

iii. $h(x) = -|x + 3| + 1$ **iv.** $k(x) = |x + 1| - 3$

5 MTR **STRUCTURE** **In Exercises 21 and 22, graph the absolute value function. Then find the area of the enclosed region created by the graph of the function and the x-axis.**

21. $f(x) = \frac{1}{2}|x + 2| - 6$ **22.** $f(x) = -4|x - 5| + 8$

23. REASONING An absolute value function is positive over its entire domain. How many x-intercepts does the graph of the function have?

24. THOUGHT PROVOKING
Graph an absolute value function f that represents the route a wide receiver runs in a football game. Let the x-axis represent distance (in yards) across the field horizontally. Let the y-axis represent distance (in yards) down the field. Limit the domain so the route is realistic.

REVIEW & REFRESH

WATCH

In Exercises 25–28, write a function g described by the given transformation of $f(x) = |x| - 5$.

25. translation 2 units to the left

26. reflection in the x-axis

27. translation 4 units up

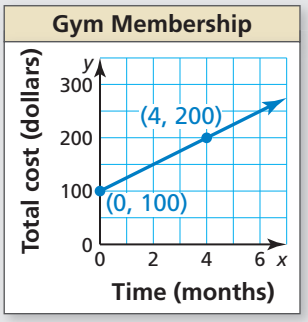
28. vertical stretch by a factor of 3

7 MTR **29. MODELING REAL LIFE** The table shows the total distance a new car travels each month after it is purchased. What type of function can you use to model the data? Estimate the mileage after 1 year.

Time (months), x	Distance (miles), y
0	0
2	2300
5	5750
6	6900
9	10,350

30. Graph $f(x) = 3|x - 4| + 2$. Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

31. Use the graph to write an equation of the line and interpret the slope.



In Exercises 32 and 33, find the product.

32. $(x - 2)^2$ **33.** $(3m + 1)^2$

In Exercises 34–39, solve the inequality. Graph the solution.

34. $|6 + w| > -15$ **35.** $5y - 2 \leq 28$

36. $6.5 \geq -\frac{n}{3}$ **37.** $2(x - 4) > 6x - 16$

38. $|2h + 3| - 3 < -1$

39. $4t + 21 < -7$ *or* $-\frac{1}{2}t \leq 2$

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