

1.3 Modeling with Linear Functions



Learning Target: Use linear functions to model and analyze real-life situations.

Success Criteria:

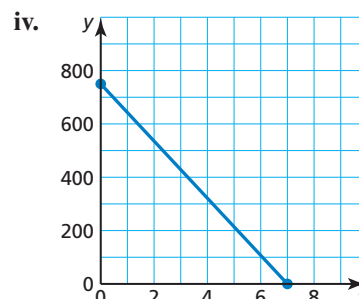
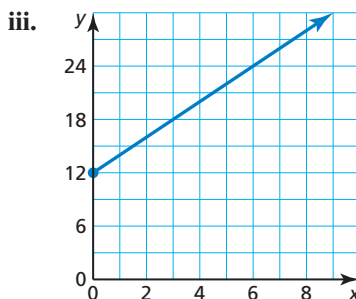
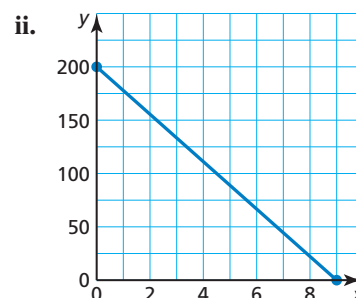
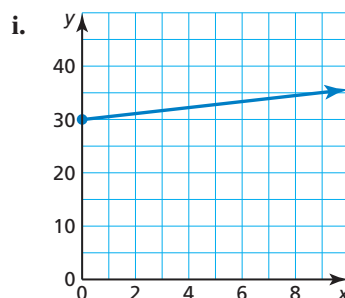
- I can write equations of linear functions.
- I can compare linear equations to solve real-life problems.
- I can determine a line of best fit.

EXPLORE IT! Modeling with Linear Functions

	A	B
1	Year, t	Value, V
2	0	\$87,000
3	1	\$79,750
4	2	\$72,500
5	3	\$65,250
6	4	\$58,000
7	5	\$50,750
8	6	\$43,500
9	7	\$36,250
10	8	\$29,000

Work with a partner. A company purchases a demolition robot for \$87,000. The spreadsheet shows how the robot depreciates over an 8-year period.

- Determine what type of function is represented by the data. Then explain how you can write a linear function to represent the value V of the robot as a function of the number t of years.
- Sketch a graph of the function. Explain why this type of depreciation is called *straight line depreciation*.
- Interpret the slope and intercepts of the graph in the context of the problem.
- Use the Internet or another resource to find a real-life example of straight line depreciation. Then write a function that models the example you found, and sketch its graph.
- Describe a real-life situation that can be modeled by each graph below. Exchange situations with your partner, and match each of your partner's situations with its corresponding graph. Explain your reasoning.



1
MTR

ANALYZE A PROBLEM

After matching a situation with its graph, label the axes to clarify their correspondence with the quantities in the situation.

- How can you use a linear function to model and analyze a real-life situation?

Functions

MA.912.F.1.1 Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

MA.912.F.1.7 Compare key features of two functions each represented algebraically, graphically, in tables or written descriptions.

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Writing Linear Equations

Vocabulary

line of fit, p. 24
line of best fit, p. 25



KEY IDEA

Writing an Equation of a Line

Given slope m and y-intercept b

Use slope-intercept form:

$$y = mx + b$$

Given slope m and a point (x_1, y_1)

Use point-slope form:

$$y - y_1 = m(x - x_1)$$

Given points (x_1, y_1) and (x_2, y_2)

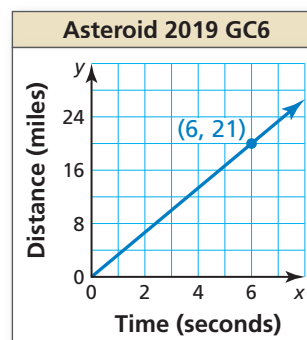
First use the slope formula to find m . Then use point-slope form with either given point.

Asteroid 2019 GC6 is about as large as a house. NASA and several international organizations are constantly scanning our skies for threats from space rocks.

EXAMPLE 1 Writing a Linear Equation from a Graph



The graph shows the distance Asteroid 2019 GC6 travels in x seconds. Write an equation of the line and interpret the slope. The asteroid came within 136,000 miles of Earth in April 2019. About how long does it take the asteroid to travel that distance?



SOLUTION

From the graph, you can see the slope is $m = \frac{21}{6} = 3.5$ and the y-intercept is $b = 0$. Use slope-intercept form to write an equation of the line.

$$y = mx + b$$

$$= 3.5x + 0$$

Slope-intercept form

Substitute 3.5 for m and 0 for b .

The equation is $y = 3.5x$. The slope indicates that the asteroid travels 3.5 miles per second. Use the equation to find how long it takes the asteroid to travel 136,000 miles.

$$136,000 = 3.5x$$

Substitute 136,000 for y .

$$38,857 \approx x$$

Divide each side by 3.5.

▶ Because there are 3600 seconds in 1 hour and

$$38,857 \text{ sec} \div \frac{3600 \text{ sec}}{1 \text{ h}} = 38,857 \text{ sec} \times \frac{1 \text{ h}}{3600 \text{ sec}} \approx 11 \text{ h},$$

it takes the asteroid about 11 hours to travel 136,000 miles.

SELF-ASSESSMENT

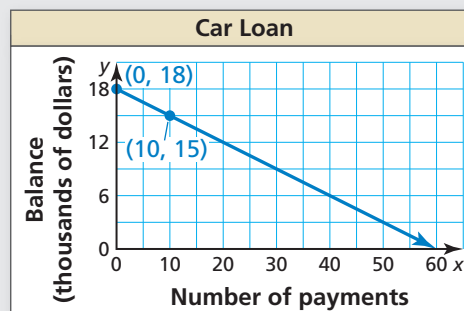
- 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.



1. **STRUCTURE** What does an equation of the form $y = mx$ indicate about x and y ?

2. The graph shows the remaining balance y on a car loan after making x monthly payments.

- Write an equation of the line, and interpret the slope and y-intercept. What is the remaining balance after 36 payments?
- Interpret the meaning of the x -intercept in this situation.
- What happens to the equation you wrote in part (a) when the y-intercept remains the same and the x -intercept is changed to 72? What changed in the problem?



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EXAMPLE 2 Comparing Linear Equations



Lakeside Inn	
Number of students, x	Total cost, y
100	\$1500
125	\$1800
150	\$2100
175	\$2400
200	\$2700

5 MTR USE A PATTERN

How can you use the table to verify that the total cost for Lakeside Inn can be represented by a linear equation?

Two prom venues charge a rental fee plus a fee per student. The table shows the total costs (in dollars) for different numbers of students at Lakeside Inn. The total cost y (in dollars) for x students at Sunview Resort is represented by the equation

$$y = 10x + 600. \quad \text{Sunview Resort}$$

Which venue charges a greater rental fee? How many students must attend for the total costs to be the same?

SOLUTION

- 1. Understand the Problem** You are given cost information for two venues. You need to compare the costs.
- 2. Make a Plan** Write an equation that represents the total cost at Lakeside Inn. Then compare the y -intercepts to determine which venue charges a greater rental fee. Equate the cost expressions and solve to determine the number of students for which the total costs are equal.
- 3. Solve and Check** The table shows a constant rate of change. Find the slope using any two points. Use $(x_1, y_1) = (100, 1500)$ and $(x_2, y_2) = (125, 1800)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 1500}{125 - 100} = \frac{300}{25} = 12$$

Write an equation that represents the total cost at Lakeside Inn using the slope of 12 and a point from the table. Use $(x_1, y_1) = (100, 1500)$.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 1500 = 12(x - 100) \quad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$y - 1500 = 12x - 1200 \quad \text{Distributive Property}$$

$$y = 12x + 300 \quad \text{Add 1500 to each side.}$$

Each equation is written in slope-intercept form. Comparing the y -intercepts, you can see that Sunview Resort charges a \$600 rental fee, which is greater than the \$300 rental fee that Lakeside Inn charges.

Equate the cost expressions and solve to determine the number of students for which the costs are equal.

$$10x + 600 = 12x + 300 \quad \text{Set cost expressions equal.}$$

$$150 = x \quad \text{Solve for } x.$$

▶ The total costs are the same when 150 students attend the prom.

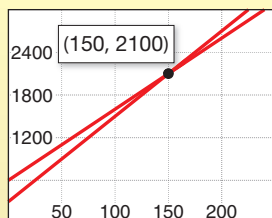
Check Notice that the table shows the total cost for 150 students at Lakeside Inn is \$2100. To check that your solution is correct, verify that the total cost at Sunview Resort is also \$2100 for 150 students.

$$y = 10(150) + 600 \quad \text{Substitute 150 for } x.$$

$$= 2100 \quad \checkmark \quad \text{Simplify.}$$

Another Way

Another way to check your solution is to graph each cost equation and find the point of intersection. The x -value of the point of intersection is 150.



SELF-ASSESSMENT

- 1 I don't understand yet.
- 2 I can do it with help.
- 3 I can do it on my own.
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- 3. WHAT IF?** Maple Ridge charges a rental fee plus a \$10 fee per student. The total cost is \$1900 for 140 students. Describe the number of students that must attend for the total cost at Maple Ridge to be less than the total costs at the other two venues. Use a graph to justify your answer.

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Finding Lines of Fit and Lines of Best Fit

Data do not always show an *exact* linear relationship. When the data in a scatter plot show an approximately linear relationship, you can model the data with a **line of fit**.



KEY IDEA

Finding a Line of Fit

- Step 1** Make a scatter plot of the data.
- Step 2** Draw the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.
- Step 3** Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.
- Step 4** Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

EXAMPLE 3 Finding a Line of Fit

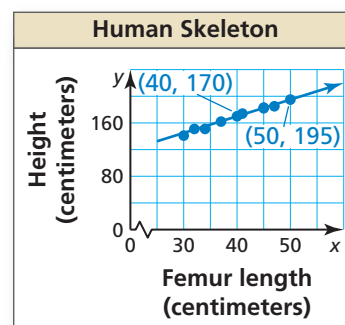


Femur length (cm), x	Height (cm), y
40	170
45	183
32	151
50	195
37	162
41	174
30	141
34	151
47	185
45	182

The table shows the femur lengths (in centimeters) and heights (in centimeters) of several people. Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a person whose femur is 35 centimeters long.

SOLUTION

- Step 1** Make a scatter plot of the data.
The data show a linear relationship.
- Step 2** Draw the line that most closely appears to fit the data. One possibility is shown.
- Step 3** Choose two points on the line.
For the line shown, you might choose (40, 170) and (50, 195).
- Step 4** Write an equation of the line.



First, find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{195 - 170}{50 - 40} = \frac{25}{10} = 2.5$$

Use point-slope form to write an equation. Use $(x_1, y_1) = (40, 170)$.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 170 = 2.5(x - 40) \quad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$y - 170 = 2.5x - 100 \quad \text{Distributive Property}$$

$$y = 2.5x + 70 \quad \text{Add 170 to each side.}$$

Use the equation to estimate the height of a person whose femur is 35 centimeters long.

$$y = 2.5(35) + 70 \quad \text{Substitute 35 for } x.$$

$$= 157.5 \quad \text{Simplify.}$$

- ▶ The approximate height of a person whose femur is 35 centimeters long is 157.5 centimeters.





1
MTR

ANALYZE A PROBLEM

How can you determine appropriate scales for your graph?

The **line of best fit** is the line that best models a set of data and lies as close as possible to all of the data points. Many technology tools have a *linear regression* feature that you can use to find the line of best fit for a set of data.

EXAMPLE 4 Finding a Line of Best Fit Using Technology



Use technology to find an equation of the line of best fit for the data in Example 3. Estimate the height of a person whose femur is 35 centimeters long. Compare this height to your estimate in Example 3.

SOLUTION

Step 1 Enter the data into two lists.

x	y
40	170
45	183
32	151
50	195
37	162
41	174
30	141

Step 2 Find an equation of the line of best fit. The values in the equation can be rounded to obtain $y = 2.6x + 65$.

$$y = mx + b$$

PARAMETERS

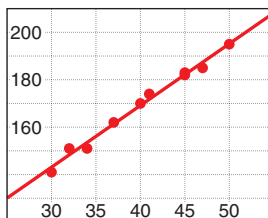
$m = 2.6036$ $b = 64.9968$

STATISTICS

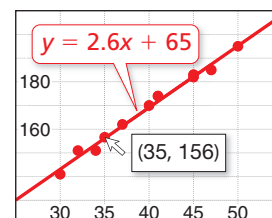
$r^2 = 0.9891$

$r = 0.9945$

Step 3 Graph the regression equation with the data.



Step 4 Find the value of y when $x = 35$.



► The height of a person with a 35-centimeter femur is about 156 centimeters. This is less than the estimate of 157.5 centimeters found in Example 3.

SELF-ASSESSMENT

- 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. The table shows the humerus lengths x (in centimeters) and heights y (in centimeters) of several females.

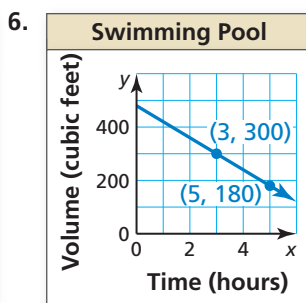
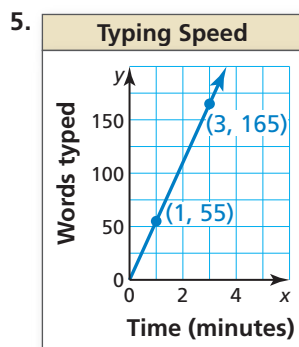
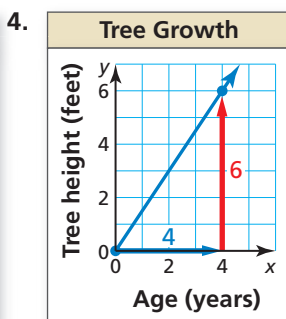
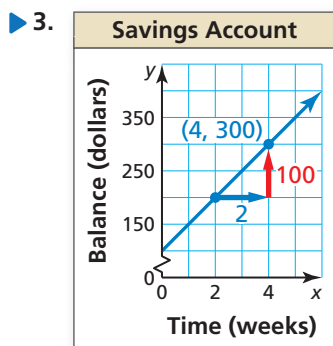
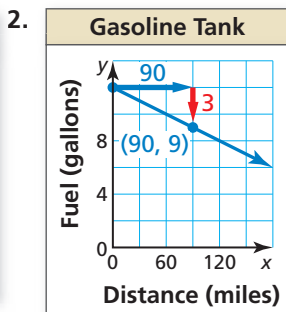
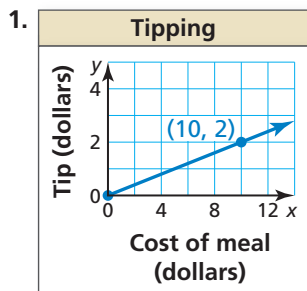
Humerus length (cm), x	33	25	22	30	28	32	26	27
Height (cm), y	166	142	130	154	152	159	141	145

- Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a female whose humerus is 40 centimeters long.
- Use technology to find an equation of the line of best fit for the data. Estimate the height of a female whose humerus is 40 centimeters long. Compare this height to your estimate in part (a).



1.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, use the graph to write an equation of the line and interpret the slope. (See Example 1.)



7. MODELING REAL LIFE

You consider buying a phone from one of two cell phone carriers. The table shows the total costs (in dollars) of the phone and service for different numbers of months at Carrier A. The total cost y (in dollars) of the phone and x months of service at Carrier B is represented by the equation $y = 55x + 300$. Which carrier has the lower initial fee? After how many months of service are the total costs the same? (See Example 2.)

Carrier A	
Months, x	Total cost, y
3	\$500
6	\$650
9	\$800
12	\$950
15	\$1100

8. MODELING REAL LIFE

You and a friend sign up for a new volunteer project to increase your service hours. The tables show your and your friend's total numbers of service hours after different numbers of weeks on the new project. Who initially has more service hours? After how many weeks do you and your friend have the same total number of service hours?

Your Service Hours	
Weeks, x	Total hours, y
4	15
6	20
8	25
10	30
12	35

Your Friend's Service Hours	
Weeks, x	Total hours, y
1	10
3	14
5	18
7	22
9	26

9. ERROR ANALYSIS

Describe and correct the error in writing an equation that models the data.

x	y
12	17
20	23
28	29
36	35
42	41

X

$$m = \frac{20 - 12}{23 - 17} = \frac{8}{6} = \frac{4}{3}$$

$$y - 17 = \frac{4}{3}(x - 12)$$

$$y - 17 = \frac{4}{3}x - 16$$

$$y = \frac{4}{3}x + 1$$

10. **PROBLEM SOLVING** You notice that temperatures in Canada are reported in degrees Celsius. You know there is a linear relationship between degrees Fahrenheit and degrees Celsius, but you have forgotten the formula. From science class, you remember that the freezing point of water is 0°C or 32°F , and its boiling point is 100°C or 212°F .
- Write a function that represents degrees Fahrenheit in terms of degrees Celsius. When is the function positive? negative?
 - A forecast for Toronto shows a high of 15°C . What is this temperature in degrees Fahrenheit?
 - Rewrite your function in part (a) to represent degrees Celsius in terms of degrees Fahrenheit. When is the function positive? negative? How does this compare to your answer in part (a)?
 - A forecast for your city shows a high of 68°F . What is this temperature in degrees Celsius?



In Exercises 11–14, determine whether the data show a linear relationship. If so, write an equation of a line of fit. Then estimate y when $x = 15$ and explain its meaning in the context of the situation. (See Example 3.)

11.

Minutes walking, x	1	6	11	13	16
Calories burned, y	6	27	50	56	70

12.

Months, x	9	13	18	22	23
Hair length (in.), y	3	5	7	10	11

13.

Hours, x	3	7	9	17	20
Battery life (%), y	86	61	50	26	0

14.

Shoe size, x	6	8	8.5	10	13
Heart rate (bpm), y	112	94	100	132	87

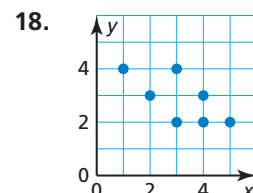
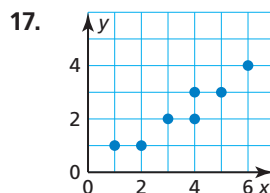
- 7 MTR** 15. **MODELING REAL LIFE** The table shows the average annual tuition and fees y (in dollars) for private nonprofit colleges and universities in the United States x years after the 2012–2013 academic year. Use technology to find an equation of the line of best fit. Interpret the slope and y -intercept in this situation. Estimate the average annual tuition in the 2023–2024 academic year. (See Example 4.)

Years after 2012–2013, x	Tuition, y
0	31,890
1	32,500
2	33,090
3	34,150
4	35,080
5	35,720
6	35,830

- 7 MTR** 16. **MODELING REAL LIFE** The table shows the population y (in millions) of Florida x years after 2013. Use technology to find an equation of the line of best fit. Interpret the slope and y -intercept in this situation. Estimate the population of Florida in 2026.

Years after 2013, x	Population, y
0	19.30
1	19.51
2	19.82
3	20.15
4	20.48
5	20.84
6	21.21

USING TOOLS In Exercises 17 and 18, use technology to find an equation of the line of best fit for the data.

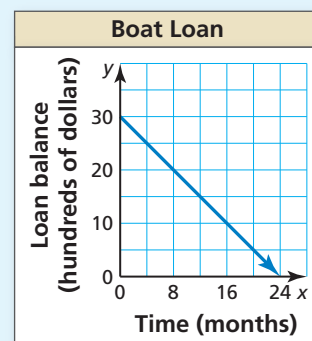


19. **REASONING** The table shows the numbers of tickets sold for a concert when different prices are charged. Write an equation of a line of fit for the data. Does it seem reasonable to use your model to predict the number of tickets sold when the ticket price is \$85? Explain.

Ticket price (dollars), x	17	20	22	26
Tickets sold, y	450	423	400	395

20. HOW DO YOU SEE IT?

You secure an interest-free loan to purchase a boat. You agree to make equal monthly payments for the next two years. The graph shows the amount of money you still owe.



- What is the slope of the line? What does the slope represent?
- What is the domain and range of the function? What does each represent?
- How much do you still owe after making payments for 12 months?

- 6 MTR** 21. **ASSESS REASONABLENESS** Two scuba divers dive in a lake that is 50 feet deep. After x minutes, Diver A's depth y (in feet) is represented by $y = -2x$ and Diver B's depth y (in feet) is represented by $y = -3.5x$. Your friend claims that the domain and range of both dives are all real numbers. Is your friend's claim reasonable? Explain.

22. **COLLEGE PREP** Which equation has a graph that is a line passing through the point $(8, -5)$ and is perpendicular to the graph of $y = -4x + 1$?

- (A) $y = \frac{1}{4}x - 5$ (C) $y = -\frac{1}{4}x - 7$
 (B) $y = -4x + 27$ (D) $y = \frac{1}{4}x - 7$

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- 4 MTR** 23. **MAKING AN ARGUMENT** The table shows the times students study x (in hours) and their grades y on a test. Your friend says the data display a linear relationship with a line of fit $y = -3x + 93$, which means studying less results in a better test grade. Is your friend's claim reasonable? Explain.

Time studying, x	1	2	4	6	8	10
Test score, y	90	55	72	86	95	63

24. THOUGHT PROVOKING

Points A and B lie on the line $y = -x + 4$. Point C is the same distance from point A as it is from point B but does not lie on the same line. Choose coordinates for points A , B , and C . Write equations for the lines connecting points A and C and points B and C .

- 7 MTR** 25. **PERFORMANCE TASK** Your family wants to purchase a new vehicle that comes in either a gasoline model or an electric model.

- Using the information shown, the approximate number of miles your family drives per year, and gas and electricity prices in your area, determine which vehicle is a better buy. Use linear equations to support your answer.
- Research other factors that affect the cost of vehicle ownership. How might these factors support or change your answer?



Gasoline
Price: \$17,950
Fuel Economy:
 26 mi/gal city
 38 mi/gal highway



Electric
Price: \$29,120
Fuel Economy:
 31 kWh/100 mi

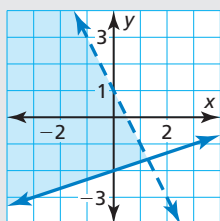
REVIEW & REFRESH

In Exercises 26–29, solve the system using any method. Explain your choice of method.

26. $3x + y = 7$ 27. $4x - 6y = 2$
 $-2x - y = 9$ $2x - 3y = 1$

28. $2x + 2y = 3$ 29. $y = x - 4$
 $x = 4y - 1$ $y = -4x + 6$

30. Write a system of inequalities represented by the graph.



31. Solve the literal equation $z = 4y + 2x + 8$ for x .
32. What percent of 25 is 14?

In Exercises 33–36, graph the function and its parent function. Then describe the transformation.

33. $f(x) = \frac{3}{2}$ 34. $f(x) = 3x$
 35. $f(x) = 2(x - 1)^2$ 36. $f(x) = -|x + 2| - 7$

In Exercises 37 and 38, find the sum or difference.

37. $(x^2 + 2x + 16) + (4x^2 - 7x - 18)$

38. $(-5n^3 + n^2 - 12n) - (6n^2 + 4n - 13)$

7 MTR 39. **MODELING REAL LIFE**

Two Internet service providers charge an installation fee plus a monthly service fee. The table shows the total costs (in dollars) for different numbers of months at Company A. The total cost y (in dollars) for

Company A	
Months, x	Total cost, y
2	\$180
3	\$220
4	\$260
5	\$300
6	\$340

x months of service at Company B is represented by $y = 45x + 50$. Which company charges less per month? After how many months of service are the total costs the same?

40. What number is 34% of 50?

In Exercises 41 and 42, write a function g whose graph represents the indicated transformation of the graph of f .

41. $f(x) = 2x + 1$; translation 3 units up
42. $f(x) = -3|x - 4|$; vertical shrink by a factor of $\frac{1}{2}$

