1.1 Parent Functions and Transformations

Learning Target: Graph and describe transformations of functions.

Success Criteria:
- I can identify the function family to which a function belongs.
- I can graph transformations of functions.
- I can explain how translations, reflections, stretches, and shrinks affect graphs of functions.

EXPLORE IT! Identifying Basic Parent Functions

2 MTR MAKE A CONNECTION
Can considering the domain and range help you identify the graphs of any of the functions?

Work with a partner.

a. Graphs of six basic parent functions are shown below. Classify each function as *constant*, *linear*, *absolute value*, *quadratic*, *square root*, or *exponential*. Justify your reasoning.

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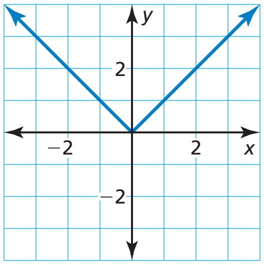

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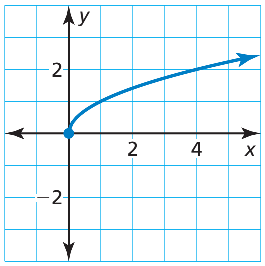

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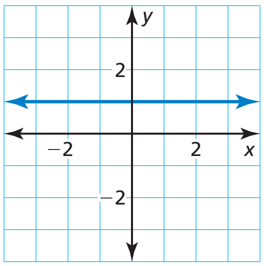

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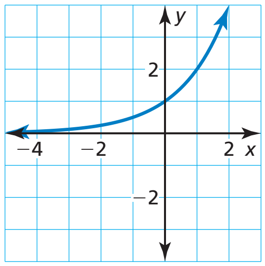

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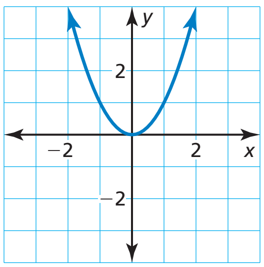

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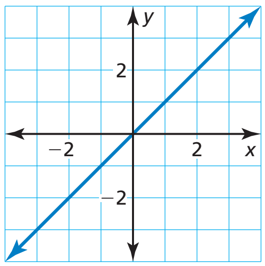

b. Sort the parent functions in part (a) into groups. Explain how you grouped the functions.

c. What are the characteristics of the graphs of some of the basic parent functions?

Functions

MA.912.F.1.1 Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

MA.912.F.2.2 Identify the effect on the graph of a given function of two or more transformations defined by adding a real number to the x- or y-values or multiplying the x- or y-values by a real number.

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Identifying Function Families

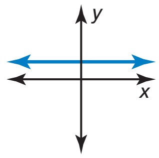
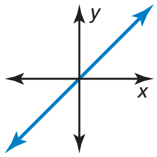
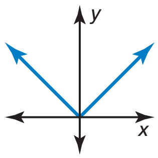
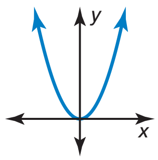
Functions that belong to the same *family* share key characteristics. The **parent function** is the most basic function in a family. Functions in the same family are *transformations* of their parent function.

Vocabulary (AZ VOCAB)

parent function, *p. 4*
transformation, *p. 5*
translation, *p. 5*
reflection, *p. 5*
vertical stretch, *p. 6*
vertical shrink, *p. 6*

KEY IDEA

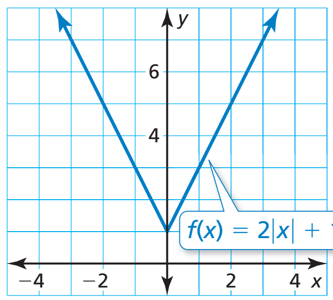
Parent Functions

Family	Constant	Linear	Absolute Value	Quadratic
Rule	$f(x) = 1$	$f(x) = x$	$f(x) = \lvert x \rvert$	$f(x) = x^2$
Graph				
Domain	All real numbers	All real numbers	All real numbers	All real numbers
Range	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

EXAMPLE 1 Identifying a Function Family

WATCH

Identify the function family to which f belongs. Compare the graph of f to the graph of its parent function.



SOLUTION

The graph of f is V-shaped, so f is an absolute value function.

The graph is shifted up and is narrower than the graph of the parent absolute value function. The domain of each function is all real numbers, but the range of f is $y \geq 1$ and the range of the parent absolute value function is $y \geq 0$.

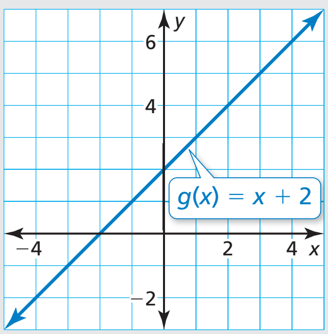
5 MTR USE STRUCTURE
How can you use a function rule to identify the function family?

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Identify the function family to which g belongs. Compare the graph of g to the graph of its parent function.

1.



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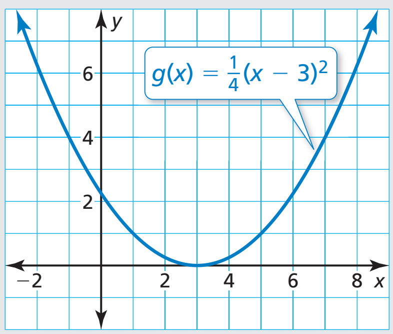

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Describing Transformations

A **transformation** changes the size, shape, position, or orientation of a graph. A **translation** is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.

EXAMPLE 2 Graphing and Describing Translations

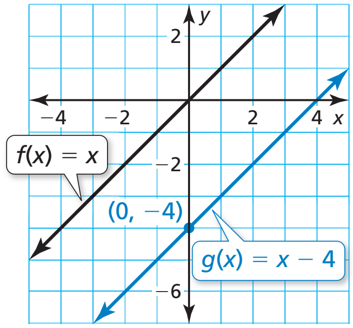
WATCH

Graph $g(x) = x - 4$ and its parent function. Then describe the transformation.

SOLUTION

> **REMEMBER**
> The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.

The function g is a linear function with a slope of 1 and a y-intercept of -4. So, draw a line through the point $(0, -4)$ with a slope of 1.



The graph of g is 4 units below the graph of the parent linear function f.

▶ So, the graph of $g(x) = x - 4$ is a vertical translation 4 units down of the graph of the parent linear function.

A **reflection** is a transformation that flips a graph over a line called the *line of reflection*. A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

EXAMPLE 3 Graphing and Describing Reflections

WATCH

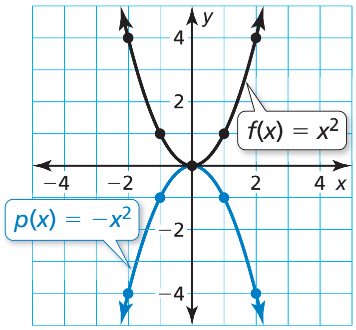
Graph $p(x) = -x^2$ and its parent function. Then describe the transformation.

SOLUTION

The function p is a quadratic function. Use a table of values to graph each function.

> **REMEMBER**
> The function $p(x) = -x^2$ is written in *function notation*, where $p(x)$ is another name for y.

x	$y = x^2$	$y = -x^2$
-2	4	-4
-1	1	-1
0	0	0
1	1	-1
2	4	-4



The graph of p is the graph of the parent function flipped over the x-axis.

▶ So, the graph of $p(x) = -x^2$ is a reflection in the x-axis of the graph of the parent quadratic function.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the function and its parent function. Then describe the transformation.

3. $g(x) = x + 3$ **4.** $h(x) = (x - 2)^2$ **5.** $n(x) = -|x|$

6. REASONING Can you describe the transformation in Example 2 in a different way? Explain your reasoning.

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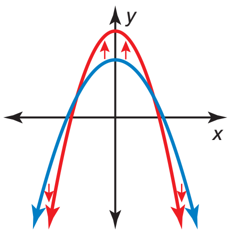
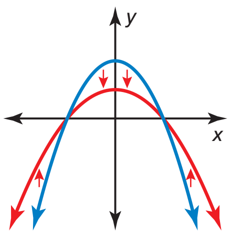
Another way to transform the graph of a function is to multiply all of the y-coordinates by the same positive factor. When the factor is greater than 1, the transformation is a **vertical stretch**. When the factor is greater than 0 and less than 1, it is a **vertical shrink**.

STUDY TIP

To visualize a vertical stretch, imagine *pulling* the points away from the x-axis.



To visualize a vertical shrink, imagine *pushing* the points toward the x-axis.



EXAMPLE 4 Graphing and Describing Stretches and Shrinks

Graph each function and its parent function. Then describe the transformation.

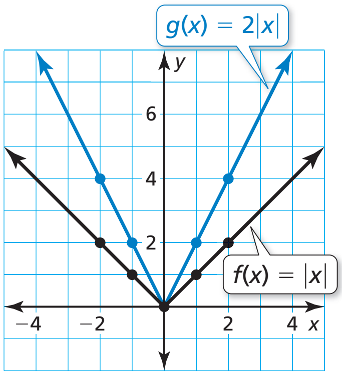
WATCH

a. $g(x) = 2|x|$ **b.** $h(x) = \frac{1}{2}x^2$

SOLUTION

a. The function g is an absolute value function. Use a table of values to graph the functions.

x	$y = \lvert x \rvert$	$y = 2\lvert x \rvert$
-2	2	4
-1	1	2
0	0	0
1	1	2
2	2	4

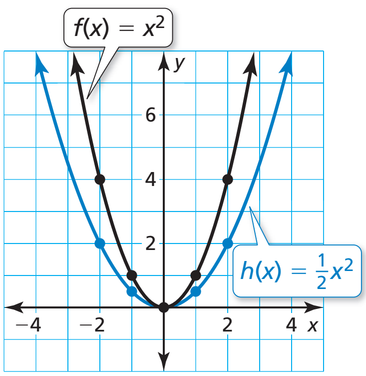


The y-coordinate of each point on g is two times the y-coordinate of the corresponding point on the parent function.

▶ So, the graph of $g(x) = 2|x|$ is a vertical stretch of the graph of the parent absolute value function by a factor of 2.

b. The function h is a quadratic function. Use a table of values to graph the functions.

x	$y = x^2$	$y = \frac{1}{2}x^2$
-2	4	2
-1	1	$\frac{1}{2}$
0	0	0
1	1	$\frac{1}{2}$
2	4	2



The y-coordinate of each point on h is one-half of the y-coordinate of the corresponding point on the parent function.

▶ So, the graph of $h(x) = \frac{1}{2}x^2$ is a vertical shrink of the graph of the parent quadratic function by a factor of $\frac{1}{2}$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the function and its parent function. Then describe the transformation.

7. $g(x) = 3x$ **8.** $h(x) = \frac{3}{2}x^2$ **9.** $c(x) = 0.2|x|$

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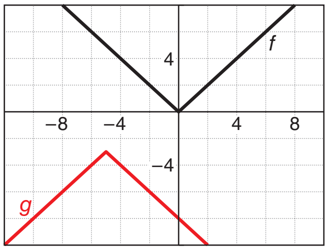
Combinations of Transformations

You can use more than one transformation to change the graph of a function.

EXAMPLE 5 Describing Combinations of Transformations

WATCH

Use technology to graph $g(x) = -|x + 5| - 3$ and its parent function. Then describe the transformations.



SOLUTION

The function g is an absolute value function.

▶ The graph of $g(x) = -|x + 5| - 3$ is a reflection in the x-axis followed by a translation 5 units left and 3 units down of the graph of the parent absolute value function.

EXAMPLE 6 Modeling Real Life

WATCH STEM 7 MTR

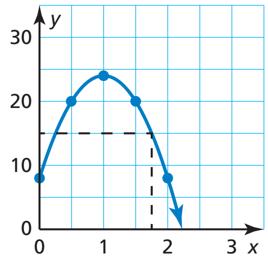
Time (seconds), x	Height (feet), y
0	8
0.5	20
1	24
1.5	20
2	8

The table shows the height y of a dirt bike x seconds after jumping off a ramp. What type of function can you use to model the data? Estimate the height after 1.75 seconds.

SOLUTION

1. **Understand the Problem** You are asked to identify the type of function that can model the table of values and then to find the height at a specific time.
2. **Make a Plan** Create a scatter plot of the data. Then use the relationship shown in the scatter plot to estimate the height after 1.75 seconds.
3. **Solve and Check** Create a scatter plot.

   The data appear to lie on a curve that resembles a quadratic function. Sketch the curve.



▶ So, you can model the data with a quadratic function. The graph shows that the height is about 15 feet after 1.75 seconds.

Check Reasonableness To check that your solution is reasonable, analyze the values in the table. Because 1.75 is between 1.5 and 2, the height is between 20 feet and 8 feet, and 8 ft < 15 ft < 20 ft. ✓

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use technology to graph the function and its parent function. Then describe the transformations.

10. $h(x) = -\frac{1}{4}x + 5$

11. $d(x) = 3(x - 5)^2 - 1$

12. $g(x) = |2x| - 3$

13. The table shows the amount of fuel in a chain saw after x minutes. What type of function can you use to model the data? When will the tank be empty?

Time (minutes), x	0	10	20	30	40
Fuel remaining (fluid ounces), y	15	12	9	6	3

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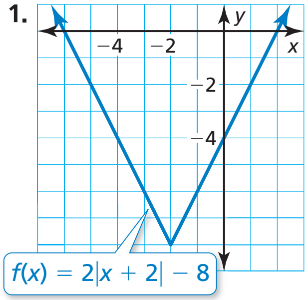
1.1 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, identify the function family to which f belongs. Compare the graph of f to the graph of its parent function. *(See Example 1.)*

1.



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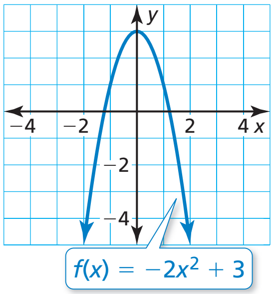

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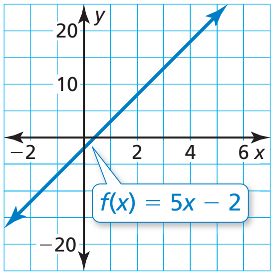

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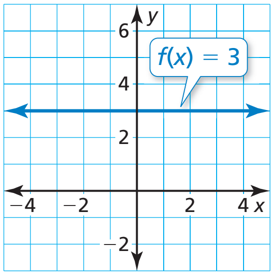


5. MODELING REAL LIFE The function $f(t) = -16t^2 + 22t + 4$ represents the height (in feet) of a bean bag t seconds after it is tossed up into the air. Identify the function family to which f belongs.

6. MODELING REAL LIFE You purchase a car from a dealership for \$10,000. The trade-in value of the car each year after the purchase is given by the function $f(x) = 10{,}000 - 1250x$, where x is the number of years after the purchase. Identify the function family to which f belongs.

In Exercises 7–16, graph the function and its parent function. Then describe the transformation. *(See Examples 2 and 3.)*

7. $g(x) = x + 4$

8. $f(x) = x - 6$

9. $f(x) = x^2 - 1$

10. $h(x) = (x + 4)^2$

11. $g(x) = |x - 5|$

12. $f(x) = 4 + |x|$

13. $g(x) = -x$

14. $h(x) = (-x)^2$

15. $f(x) = 3$

16. $f(x) = -2$

In Exercises 17–24, graph the function and its parent function. Then describe the transformation. *(See Example 4.)*

17. $f(x) = \frac{1}{3}x$

18. $g(x) = 4x$

19. $f(x) = 2x^2$

20. $h(x) = \frac{1}{3}x^2$

21. $h(x) = \frac{3}{4}x$

22. $g(x) = \frac{4}{3}x$

23. $h(x) = 3|x|$

24. $f(x) = \frac{1}{2}|x|$

In Exercises 25–32, use technology to graph the function and its parent function. Then describe the transformations. *(See Example 5.)*

25. $f(x) = 3x + 2$

26. $h(x) = -x + 5$

27. $h(x) = -3|x| - 1$

28. $f(x) = \frac{3}{4}|x| + 1$

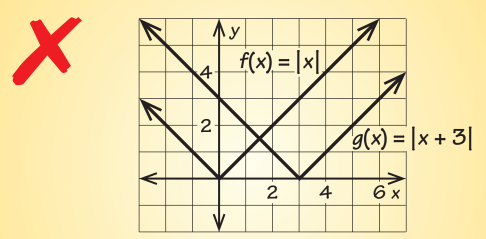
29. $g(x) = \frac{1}{2}x^2 - 6$

30. $f(x) = 4x^2 - 3$

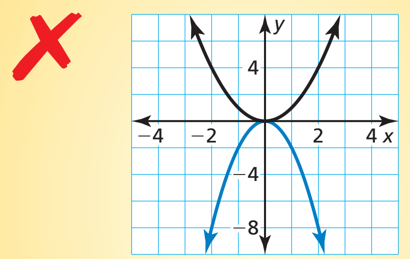
31. $f(x) = -(x + 3)^2 + \frac{1}{4}$

32. $g(x) = -|x - 1| - \frac{1}{2}$

33. ERROR ANALYSIS Describe and correct the error in graphing $g(x) = |x + 3|$ and its parent function.



34. ERROR ANALYSIS Identify and correct the error in describing the transformation of the parent function.



The graph is a reflection in the x-axis and a vertical shrink of the graph of the parent quadratic function.

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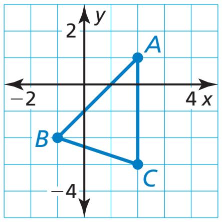
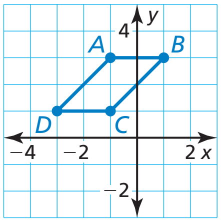
5 MTR **CONNECTING CONCEPTS** **In Exercises 35 and 36, find the coordinates of the figure after the transformation.**

35. Translate 2 units down.



36. Reflect in the x-axis.



USING TOOLS **In Exercises 37–42, identify the function family to which the function belongs. Then find the domain and range. Use technology to verify your answer.**

37. $g(x) = |x + 2| - 1$

38. $h(x) = |x - 3| + 2$

39. $g(x) = 3x + 4$

40. $f(x) = -4x + 11$

41. $f(x) = 5x^2 - 2$

42. $f(x) = -2x^2 + 6$

7 MTR **43.** **MODELING REAL LIFE** The table shows the speeds of a car as it travels through an intersection with a stop sign. What type of function can you use to model the data? Estimate the speed of the car when it is 20 yards past the intersection. *(See Example 6.)*

Displacement from sign (yards), x	Speed (miles per hour), y
−100	40
−50	20
−10	4
0	0
10	4
50	20
100	40

7 MTR **44.** **MODELING REAL LIFE** The table shows the battery life of a robotic vacuum over time. What type of function can you use to model the data? Interpret the meaning of the x-intercept in this situation.

Time (minutes), x	Battery life remaining, y
30	75%
90	25%
120	0%
180	50%
240	100%

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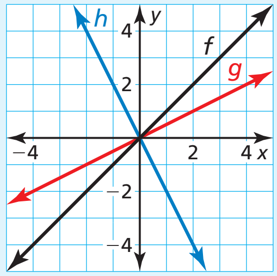
5 MTR **45.** **STRUCTURE** Are the graphs of the functions $f(x) = |x - 4|$ and $g(x) = |x| - 4$ the same? Explain.

46. **HOW DO YOU SEE IT?**
Consider the graphs of f, g, and h.

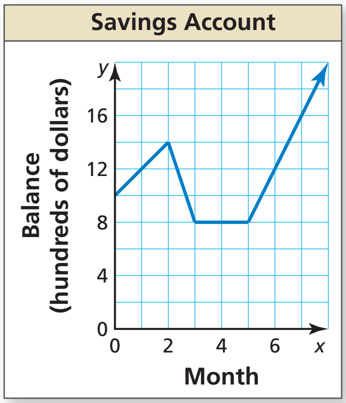


a. Does the graph of g represent a vertical stretch or a vertical shrink of the graph of f? Explain your reasoning.

b. Describe how to transform the graph of f to obtain the graph of h.

4 MTR **47.** **DISCUSS MATHEMATICAL THINKING** A person swims at a constant speed of 1 meter per second. What type of function can be used to model the distance the swimmer travels? If the person has a 10-meter head start, what type of transformation does this represent? Explain.

48. **REASONING** The graph shows the balance of a savings account over time.



a. Write a function that represents the account balance for the domain shown. Identify the function type.

b. Can you use your function to predict the account balance after 1 year? Explain.

c. What is the initial balance? How would the graph change if the account had an initial balance of $2000?

49. **PROBLEM SOLVING** You are playing bossaball, a volleyball-like game that originated in Spain and is played on an inflatable court with a trampoline section. The height (in feet) of the ball above the court t seconds after you hit the ball is modeled by the function $f(t) = -16t^2 + 26t + 6.5$.

a. Without graphing, identify the type of function that models the height of the bossaball.

b. What is the value of t when you hit the ball? Explain your reasoning.

c. How many feet above the ground is the ball when you hit the bossaball? Explain.

50. THOUGHT PROVOKING The graph of $f(x) = mx + b$ is transformed to obtain the graph of $g(x) = mx + c$, where m is a rational number and b and c are integers. Describe the transformation from the graph of f to the graph of g in terms of b and c.

51. REASONING Compare each function with its parent function. State whether each represents a *horizontal translation*, *vertical translation*, *both*, or *neither*. Explain your reasoning.

a. $f(x) = 2|x| - 3$ **b.** $f(x) = (x - 8)^2$

c. $f(x) = |x + 2| + 4$ **d.** $f(x) = 4x^2$

52. COLLEGE PREP Which of the following function types *cannot* have a range of all real numbers? Select all that apply. Explain your reasoning.

Ⓐ constant Ⓒ absolute value

Ⓑ linear Ⓓ quadratic

53. NUMBER SENSE Use the values −1, 0, 1, and 2 to complete each function so their graphs intersect the x-axis. Explain your reasoning.

a. $f(x) = 3x$ ▢ $+ 1$ **b.** $f(x) = |2x - 6| -$ ▢

c. $f(x) =$ ▢ $x^2 + 1$ **d.** $f(x) =$ ▢

REVIEW & REFRESH

WATCH

54. Tell whether (5, 2) is a solution of $y \leq x - 3$.

In Exercises 55 and 56, find the x- and y-intercepts of the graph of the equation.

55. $y = x + 2$ **56.** $x - 2y = 8$

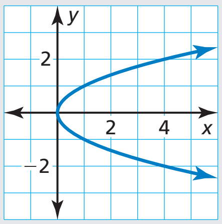
57. NUMBER SENSE The sum of three-halves a number and eight is seventeen. What is the number?

In Exercises 58–61, graph the function and its parent function. Then describe the transformation.

58. $f(x) = x^2 + 5$ **59.** $g(x) = |x - 2|$

60. $h(x) = \frac{3}{2}x$ **61.** $f(x) = 3x^2$

62. Determine whether the graph represents a function. Explain.



In Exercises 63–66, solve the equation.

63. $x + 11 = 28$ **64.** $2|x - 8| = 18$

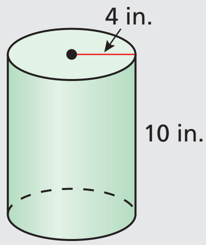
65. $x^2 - 64 = 0$ **66.** $2x^2 - 5x = 9$

67. Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function.

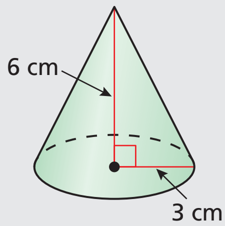
x	−1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$

In Exercises 68 and 69, find the volume of the solid.

68.



69.



70. Determine which of the lines, if any, are parallel or perpendicular.

Line a: $2y + x = 12$
Line b: $y = 2x - 3$
Line c: $y + 2x = 1$

7 MTR **71. MODELING REAL LIFE** The growth rate of a bacterial culture is 125% each hour. Initially, there are 5 bacteria. Find the number of bacteria in the culture after 6 hours.

72. The two-way table shows the results of a survey. Make a two-way table that shows the joint and marginal relative frequencies.

		Use Social Media	
		Yes	No
Own a Cell Phone	Yes	132	59
	No	87	46

In Exercises 73–76, factor the polynomial completely.

73. $x^2 - x - 30$ **74.** $3x^2 + 15x + 12$

75. $x^2 - 18x + 81$ **76.** $2x^3 - 2x$

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