7.3 Special Products of Polynomials

Learning Target: Use patterns to find products of polynomials.

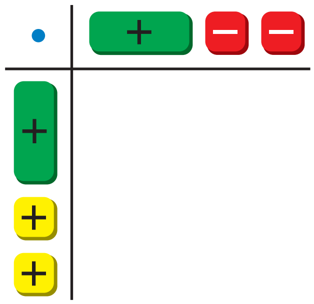
Success Criteria:
- I can use the square of a binomial pattern.
- I can multiply binomials using the sum and difference pattern.
- I can solve problems using special product patterns.

EXPLORE IT! Identifying Patterns in Products of Binomials

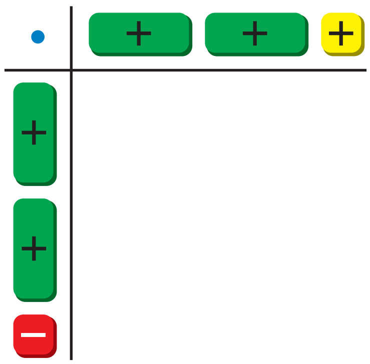
Work with a partner. You can use algebra tiles to find special products of polynomials.

a. Write the product modeled by each array of algebra tiles. Use additional algebra tiles to complete the model. Then write the product as a polynomial.

i.



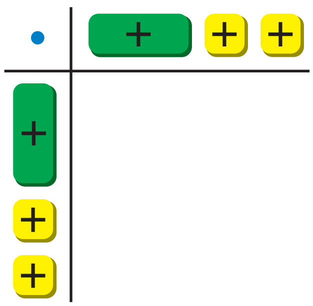
ii.



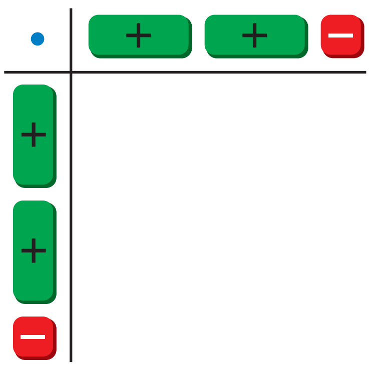
What pattern(s) do you notice? Explain your reasoning.

b. Use algebra tiles to model each product. Then write the product as a polynomial.

i. $(x + 2)^2$



ii. $(2x - 1)^2$



What pattern(s) do you notice? Explain your reasoning.

c. Use the patterns you found above to find each product. Check your answers using algebra tiles.

i. $(x + 3)(x - 3)$ **ii.** $(x - 4)(x + 4)$ **iii.** $(3x + 1)(3x - 1)$

iv. $(x + 3)^2$ **v.** $(x - 2)^2$ **vi.** $(3x + 1)^2$

5 MTR USE STRUCTURE
Why does each product in part (a) result in a binomial and not a trinomial like in part (b)?

Algebraic Reasoning
MA.912.AR.1.3 Add, subtract and multiply polynomial expressions with rational number coefficients.

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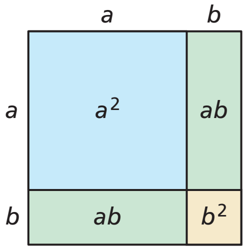
Using the Square of a Binomial Pattern

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$



This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$$ Replace b with $-b$ in the pattern above.

$$(a - b)^2 = a^2 - 2ab + b^2$$ Simplify.

STUDY TIP

The square of a binomial, a polynomial such as $x^2 + 10x + 25$ or $4x^2 - 12x + 9$, is called a *perfect square trinomial*.

KEY IDEA

Square of a Binomial Pattern

Algebra	**Example**
$(a + b)^2 = a^2 + 2ab + b^2$	$(x + 5)^2 = (x)^2 + 2(x)(5) + (5)^2$
	$= x^2 + 10x + 25$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + (3)^2$
	$= 4x^2 - 12x + 9$

EXAMPLE 1 Using the Square of a Binomial Pattern

WATCH

Find each product.

a. $(3x + 4)^2$

b. $\left(5x - \frac{1}{2}y\right)^2$

SOLUTION

STUDY TIP

In a special product pattern, a and b can be numbers, variables, or variable expressions.

a. $(3x + 4)^2 = (3x)^2 + 2(3x)(4) + 4^2$ Square of a binomial pattern

$= 9x^2 + 24x + 16$ Simplify.

▶ The product is $9x^2 + 24x + 16$.

b. $\left(5x - \frac{1}{2}y\right)^2 = (5x)^2 - 2(5x)\left(\frac{1}{2}y\right) + \left(\frac{1}{2}y\right)^2$ Square of a binomial pattern

$= 25x^2 - 5xy + \frac{1}{4}y^2$ Simplify.

▶ The product is $25x^2 - 5xy + \frac{1}{4}y^2$.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. WRITING Explain how to use the square of a binomial pattern.

Find the product.

2. $(x + 7)^2$

3. $(7x - 3)^2$

4. $(4x - y)^2$

5. $(3m + 2.7n)^2$

5 MTR **6. STRUCTURE** Explain how to find $(-3h + 6)^2$ using each pattern shown above.

7. REASONING Find the products in Exercises 2–5 without using a special product pattern. Compare the methods.

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Using the Sum and Difference Pattern

To find the product $(x + 2)(x - 2)$, you can multiply the two binomials using the FOIL Method.

$$(x + 2)(x - 2) = x^2 - 2x + 2x - 4 \qquad \text{FOIL Method}$$
$$= x^2 - 4 \qquad \text{Combine like terms.}$$

This suggests a pattern for the product of the sum and difference of two terms.

KEY IDEA

Sum and Difference Pattern

Algebra	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

EXAMPLE 2 Using the Sum and Difference Pattern

WATCH

Find (a) $(t + 5)(t - 5)$ and (b) $(3x + y)(3x - y)$.

SOLUTION

a. $(t + 5)(t - 5) = t^2 - 5^2$ Sum and difference pattern

$= t^2 - 25$ Simplify.

▶ The product is $t^2 - 25$.

b. $(3x + y)(3x - y) = (3x)^2 - y^2$ Sum and difference pattern

$= 9x^2 - y^2$ Simplify.

▶ The product is $9x^2 - y^2$.

The special product patterns can help you use mental math to find certain products of numbers.

EXAMPLE 3 Using Special Product Patterns and Mental Math

WATCH

Use special product patterns to find the product $26 \cdot 34$.

SOLUTION

Notice that 26 is 4 less than 30, and 34 is 4 greater than 30.

$26 \cdot 34 = (30 - 4)(30 + 4)$ Write as product of difference and sum.

$= 30^2 - 4^2$ Sum and difference pattern

$= 884$ Evaluate powers and simplify.

▶ The product is 884.

SELF-ASSESSMENT 1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the product.

8. $(x + 10)(x - 10)$ **9.** $(2x + 1.5)(2x - 1.5)$ **10.** $(x + 6y)(x - 6y)$

11. Describe how to use special product patterns to find $41 \cdot 39$.

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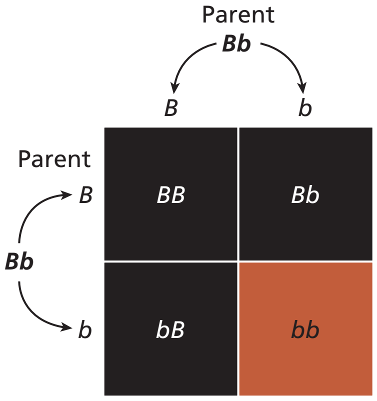
Solving Real-Life Problems

7 MTR **EXAMPLE 4** **Modeling Real Life** WATCH



A combination of two genes determines the color of the dark patches on a border collie's coat. Each offspring inherits one patch-color gene from each parent. Each parent has two patch-color genes, and the offspring has an equal chance of inheriting either one.

Each parent has the same gene combination *Bb*. The Punnett square shows the possible outcomes for the gene combinations of the offspring: black patches (*BB*), black patches (*Bb*), black patches (*bB*), and red patches (*bb*).

a. What percent of the possible gene combinations result in black patches?

b. Show how you can use a polynomial to model the possible gene combinations.

SOLUTION

a. Notice that the Punnett square shows four possible outcomes for the gene combinations of the offspring. Of these combinations, three result in black patches.

▶ So, $\frac{3}{4} = 75\%$ of the possible gene combinations result in black patches.

b. Notice that you can think of the Punnett square as a square with side length $B + b$, where B and b are the probabilities that the offspring inherits a black or a red gene from each parent. The area of each section of the Punnett square is equal to the probability of an offspring inheriting that gene combination. So, find the area of the Punnett square using the square of a binomial pattern.

$$(B + b)^2 = B^2 + 2(B)(b) + b^2$$
$$= B^2 + 2Bb + b^2$$

Because the offspring has an equal chance of inheriting either gene, $B = 0.5$ and $b = 0.5$. Evaluate each term of the polynomial.

$$B^2 + 2Bb + b^2$$

$0.5^2 = 0.25$, so 25% *BB*, black patches

$2(0.5)(0.5) = 0.5$, so 50% *Bb*, black patches

$0.5^2 = 0.25$, so 25% *bb*, red patches

▶ So, $25\% + 50\% = 75\%$ of the possible gene combinations result in black patches and 25% result in red patches.

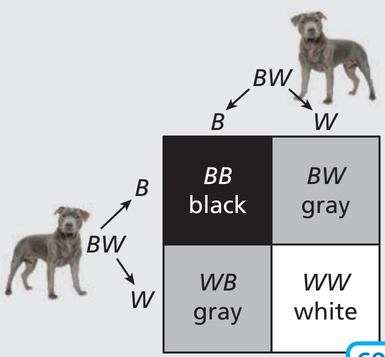
SELF-ASSESSMENT

1 I don't understand yet. **2** I can do it with help. **3** I can do it on my own. **4** I can teach someone else.

12. Each of two dogs has the same gene combination *BW*. The Punnett square shows the possible outcomes for the gene combinations of an offspring and the resulting coat colors.

a. What percent of the possible gene combinations result in a black coat?

b. Show how you can use a polynomial to model the possible gene combinations of the offspring.



5 MTR **13.** **PATTERNS** Find $(x + 1)^3$ and $(x + 2)^3$. Find a pattern in the terms, and use it to write a pattern for the cube of a binomial $(a + b)^3$.

GO DIGITAL

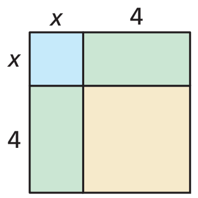
7.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–8, find the product. *(See Example 1.)*

1. $(x + 8)^2$
2. $(a - 6)^2$
3. $(2f - 1)^2$
4. $(5p + 2)^2$
5. $(-7t + 4)^2$
6. $(-12 - n)^2$
7. $\left(\frac{1}{4}a + b\right)^2$
8. $(0.5x - 0.7y)^2$

5 MTR **CONNECTING CONCEPTS** In Exercises 9–12, write a polynomial that represents the area of the square.

9.



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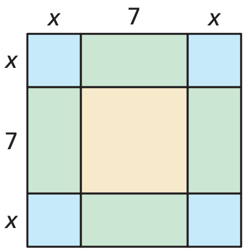

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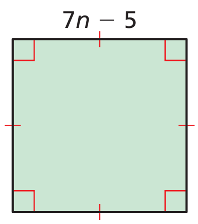

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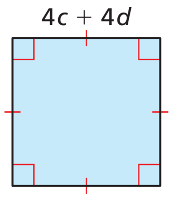


In Exercises 13–22, find the product. *(See Example 2.)*

13. $(t - 7)(t + 7)$
14. $(m + 6)(m - 6)$
15. $(4x + 1)(4x - 1)$
16. $(2k - 4)(2k + 4)$
17. $\left(\frac{1}{2} - c\right)\left(\frac{1}{2} + c\right)$
18. $(2.5 + 3a)(2.5 - 3a)$
19. $(p - 10q)(p + 10q)$
20. $(7m + 8n)(7m - 8n)$
21. $(-y + 4z)(-y - 4z)$
22. $(-5g - 2h)(-5g + 2h)$

In Exercises 23–28, use special product patterns to find the product. *(See Example 3.)*

23. $16 \cdot 24$
24. $33 \cdot 27$
25. 42^2
26. 29^2
27. 30.5^2
28. $10\frac{1}{3} \cdot 9\frac{2}{3}$

GO DIGITAL

4 MTR **ERROR ANALYSIS** In Exercises 29 and 30, describe and correct the error in finding the product.

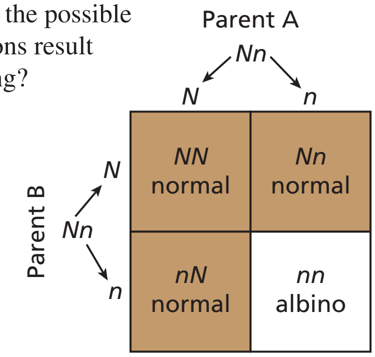
29.
$$\begin{aligned}(k + 4)^2 &= k^2 + 4^2 \\ &= k^2 + 16\end{aligned}$$

30.
$$\begin{aligned}(s + 5)(s - 5) &= s^2 + 2(s)(5) - 5^2 \\ &= s^2 + 10s - 25\end{aligned}$$

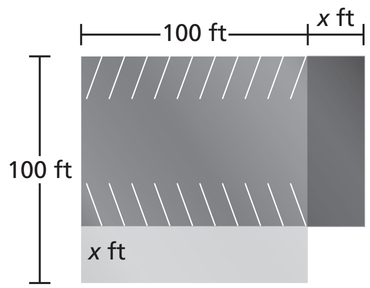
7 MTR 31. **MODELING REAL LIFE** A combination of two genes determines the coloring of a deer. Each offspring inherits one color gene from each parent. Each parent has the same gene combination Nn. The Punnett square shows the possible outcomes for the gene combinations of the offspring. *(See Example 4.)*

a. What percent of the possible gene combinations result in albino coloring?

b. Show how you can use a polynomial to model the possible gene combinations of the offspring.



7 MTR 32. **MODELING REAL LIFE** A square-shaped parking lot with 100-foot sides is reduced by x feet on one side and extended by x feet on an adjacent side.

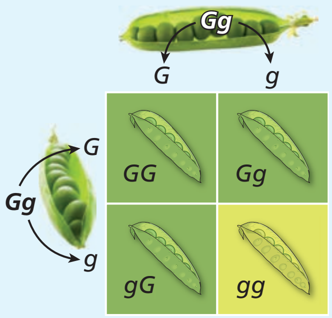


a. Write a polynomial that represents the new area of the parking lot.

b. Does the area of the parking lot increase, decrease, or stay the same? Explain.

33. **PROBLEM SOLVING** Write two binomials that have the product $x^2 - 121$. Explain.

34. HOW DO YOU SEE IT? Each offspring of two pea plants inherits one color gene from each parent. Each parent has the same gene combination Gg. The Punnett square shows the possible outcomes for the gene combinations of the offspring.



A polynomial that models the possible gene combinations of the offspring is

$$(G + g)^2 = G^2 + 2Gg + g^2.$$

Describe two ways to determine the percent of possible gene combinations that result in green pods.

In Exercises 35 and 36, find the product.

35. $(2m^2 - 5n^2)^2$

36. $(r^3 - 6t^4)(r^3 + 6t^4)$

37. REASONING Find k so that $9x^2 - 48x + k$ is the square of a binomial.

38. THOUGHT PROVOKING Modify the dimensions of the parking lot in Exercise 32 so that the area can be represented by two other types of special product patterns discussed in this section. Is there a positive x-value for which the three area expressions are equivalent? Explain.

39. DIG DEEPER In a population of 8000 people, 95% have free earlobes and 5% have attached earlobes. The possible gene combinations can be represented by $(F + a)^2$, where F and a are the probability that a randomly selected person inherited a free or attached gene from each parent. Any gene combination with F results in free earlobes. Estimate the number of people in the population who carry *both* genes. (*Hint*: Find the values of F and a. Remember that $F + a = 1$.)

REVIEW & REFRESH

WATCH

40. The points represented by the table lie on a line. Find the slope of the line.

x	-3	0	2	7
y	15	6	0	-15

41. Find the missing values in the ratio table. Then write the equivalent ratios.

Feet	18.5		92.5	
Seconds	1	2		7

In Exercises 42–44, find the sum or difference.

42. $(k + 5) - (3k - 2)$

43. $(6g^2 + 3g - 8) - (-g^2 + 12)$

44. $\left(\frac{1}{2}d - 4\right) + \left(\frac{1}{4}d^2 - 2d + 9\right)$

7 MTR **45. MODELING REAL LIFE** The value of a building is \$180,000. The value is expected to increase by 3.5% each year. Write a function that represents the value y (in dollars) of the building after x years. Then predict the value after 10 years.

In Exercises 46–53, find the product or quotient.

46. $(x - 3)(x + 5)$

47. $(y^2 + 3y - 1)(y - 6)$

48. $(x - 6)^2$

49. $(0.6n + 9y)^2$

50. $(p + 4)(p - 4)$

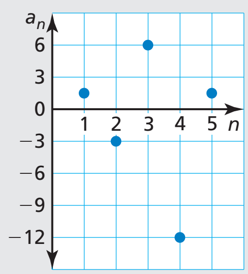
51. $(w - 5z)(w + 5z)$

52. $\dfrac{4p^5 + 5p^3 - 3p - 2}{p}$

53. $\dfrac{(24h^3 + 8h)(3h - 4)}{8h}$

7 MTR **54. MODELING REAL LIFE** A movie theater sells 12 large bags of popcorn and 25 small bags of popcorn for \$227. A large bag of popcorn costs \$3.50 more than a small bag of popcorn. How much does each size cost?

55. Determine whether the graph represents a function. Explain.



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