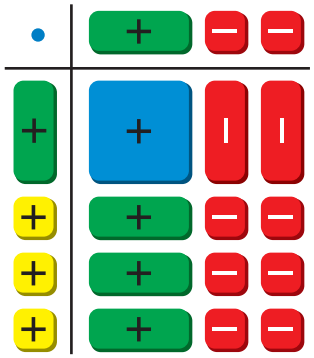
7.2 Multiplying and Dividing Polynomials

Learning Target: Multiply and divide polynomials.

Success Criteria:
- I can multiply and divide polynomials by monomials.
- I can multiply binomials using the Distributive Property.
- I can multiply binomials using the FOIL Method.
- I can multiply binomials and trinomials.

EXPLORE IT! Multiplying Polynomials Using Algebra Tiles

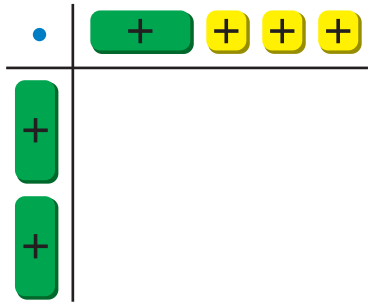
Work with a partner. You can use algebra tiles to find products of polynomials.



a. Write the equation modeled by the rectangular array of algebra tiles shown. What types of polynomials are being multiplied? What type of polynomial is the product?

b. Write the product modeled by each array of algebra tiles. Use additional algebra tiles to complete the model. Then write each product as a polynomial.

i.



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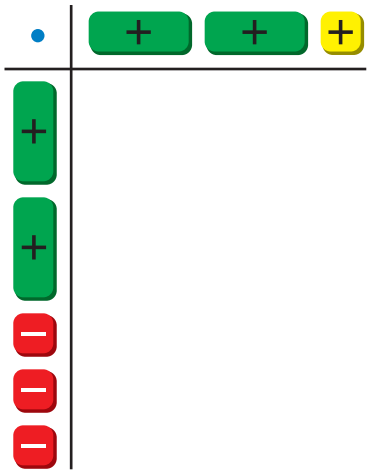

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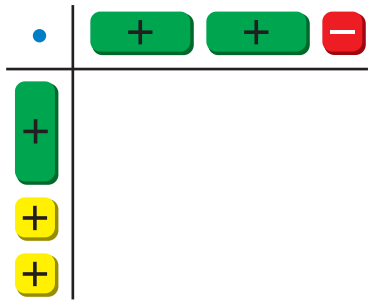

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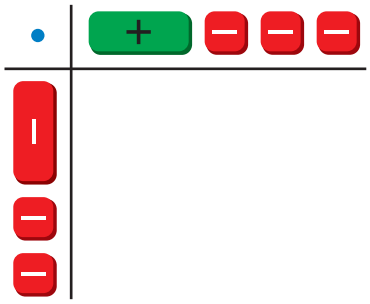

c. Explain how you can multiply two polynomials without using algebra tiles.

d. Does multiplying two polynomials result in an expression that is also a polynomial? Explain your reasoning.

5 MTR USE A SIMILAR PROBLEM

Consider some of the strategies you have previously learned to multiply numbers. Which of these might be useful when multiplying polynomials?

Algebraic Reasoning

MA.912.AR.1.3 Add, subtract and multiply polynomial expressions with rational number coefficients.

MA.912.AR.1.4 Divide a polynomial expression by a monomial expression with rational number coefficients.

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Multiplying and Dividing by Monomials and Binomials

Vocabulary AZ VOCAB

FOIL Method, *p. 344*

The product of two polynomials is always a polynomial. So, like the set of integers, the set of polynomials is *closed* under multiplication. You can use the Distributive Property to multiply polynomials.

EXAMPLE 1 Multiplying Polynomials and Monomials

WATCH

Find (a) $-2x(3x + 7)$ and (b) $3x^2\left(5x^2 - \frac{1}{3}x + \frac{2}{3}\right)$.

SOLUTION

a. $-2x(3x + 7) = -2x(3x) + (-2x)(7)$ Distribute $-2x$ to each term of $(3x + 7)$.

$= -6x^2 - 14x$ Multiply.

b. $3x^2\left(5x^2 - \frac{1}{3}x + \frac{2}{3}\right) = 3x^2(5x^2) - 3x^2\left(\frac{1}{3}x\right) + 3x^2\left(\frac{2}{3}\right)$ Distribute $3x^2$ to each term of $5x^2 - \frac{1}{3}x + \frac{2}{3}$.

$= 15x^4 - x^3 + 2x^2$ Multiply.

EXAMPLE 2 Dividing Polynomials

WATCH

Find (a) $\frac{x^4 + 4x^3 + 11x^2}{x^2}$ and (b) $\frac{6y^2 - 30y}{3y}$.

STUDY TIP

Notice in Examples 2(a) and 2(b) that the denominators cannot be equal to 0. So, $x \neq 0$ in part (a), and $y \neq 0$ in part (b).

SOLUTION

a. Divide each term in the numerator by the monomial in the denominator.

$\frac{x^4 + 4x^3 + 11x^2}{x^2} = \frac{x^4}{x^2} + \frac{4x^3}{x^2} + \frac{11x^2}{x^2}$ Divide each term in the numerator by x^2.

Use the Quotient of Powers Property. → $= x^2 + 4x + 11$ Simplify.

▶ The quotient is $x^2 + 4x + 11$.

b. Divide each term in the numerator by the monomial in the denominator.

$\frac{6y^2 - 30y}{3y} = \frac{6y^2}{3y} - \frac{30y}{3y}$ Divide each term in the numerator by $3y$.

Use the Quotient of Powers Property. → $= 2y - 10$ Simplify.

▶ The quotient is $2y - 10$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the product.

1. $(-8x^4)(11x^3)$
2. $\frac{1}{2}y^2\left(\frac{4}{5}y - 10\right)$
3. $-3b^3(6b^2 + b - 9)$

5 MTR **4. STRUCTURE** In Example 1, why is the degree of the product different from the degree of the factors?

Find the quotient.

5. $\frac{-4h^4 + 6h^3 - 2h^2}{h}$
6. $\frac{2k^5 - 2k^4}{2k^3}$
7. $\frac{4z^2 + 28z}{0.2z}$

8. **REASONING** Is the set of polynomials closed under division? Explain your reasoning.

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EXAMPLE 3 Multiplying Binomials Using the Distributive Property

WATCH

Find each product.

a. $(x + 2)(x + 5)$ **b.** $(x + 3)(x - 4)$

SOLUTION

a. Use the horizontal method.

$$(x + 2)(x + 5) = x(x + 5) + 2(x + 5)$$ Distribute $(x + 5)$ to each term of $(x + 2)$.

$$= x(x) + x(5) + 2(x) + 2(5)$$ Distributive Property

$$= x^2 + 5x + 2x + 10$$ Multiply.

$$= x^2 + 7x + 10$$ Combine like terms.

▶ The product is $x^2 + 7x + 10$.

2 MTR USE ANOTHER METHOD

Solve part (a) by distributing $(x + 2)$ to each term of $(x + 5)$.

b. Use the vertical method.

$$\begin{array}{rr} & x + 3 \\ \times & x - 4 \\ \hline & -4x - 12 \\ & x^2 + 3x \\ \hline & x^2 - x - 12 \end{array}$$

Align like terms vertically.

Multiply $-4(x + 3)$. → $-4x - 12$ Distributive Property

Multiply $x(x + 3)$. → $x^2 + 3x$ Distributive Property

Combine like terms.

▶ The product is $x^2 - x - 12$.

EXAMPLE 4 B.E.S.T. Test Prep: Multiplying Binomials Using a Table

WATCH

Which polynomial represents the product of $2x - 3$ and $x + 5$?

Ⓐ $2x^2 - 15$ Ⓒ $-x - 15$

Ⓑ $2x^2 - 3x + 5$ Ⓓ $2x^2 + 7x - 15$

SOLUTION

Write each binomial as a sum of terms.

$$(2x - 3)(x + 5) = [2x + (-3)](x + 5)$$

Then make a table of products.

	$2x$	-3
x	$2x^2$	$-3x$
5	$10x$	-15

▶ The product is $2x^2 - 3x + 10x - 15$, or $2x^2 + 7x - 15$. So, the correct answer is Ⓓ.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the Distributive Property to find the product.

9. $(y + 4)(y + 1)$ **10.** $(z - 2)(z + 6)$ **11.** $\left(4q - \frac{1}{2}\right)\left(8q - \frac{1}{4}\right)$

Use a table to find the product.

12. $(p + 3)(p - 8)$ **13.** $(r - 5)(2r - 1)$ **14.** $(1.5s - 1)(3s + 6)$

15. REASONING Explain why the set of polynomials is closed under multiplication.

16. WRITING Explain how to simplify the expression $4d(2d - 7) + (5d + 4)(4d - 1)$.

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Using the FOIL Method

The **FOIL Method**, a method of applying the Distributive Property, is a shortcut for multiplying two binomials.

KEY IDEA

FOIL Method

To multiply two binomials using the FOIL Method, find the sum of the products of the

First terms, $(x + 1)(x + 2)$ → $x(x) = x^2$

Outer terms, $(x + 1)(x + 2)$ → $x(2) = 2x$

Inner terms, and $(x + 1)(x + 2)$ → $1(x) = x$

Last terms. $(x + 1)(x + 2)$ → $1(2) = 2$

$$(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

EXAMPLE 5 Multiplying Binomials Using the FOIL Method

WATCH

Find each product.

a. $(x - 3)(x - 6)$ **b.** $(2x + 1)(3x - 5)$

SOLUTION

Use the FOIL Method.

a.

	First Outer Inner Last	
$(x - 3)(x - 6)$	$= x(x) + x(-6) + (-3)(x) + (-3)(-6)$	FOIL Method
	$= x^2 + (-6x) + (-3x) + 18$	Multiply.
	$= x^2 - 9x + 18$	Combine like terms.

▶ The product is $x^2 - 9x + 18$.

b.

	First Outer Inner Last	
$(2x + 1)(3x - 5)$	$= 2x(3x) + 2x(-5) + 1(3x) + 1(-5)$	FOIL Method
	$= 6x^2 + (-10x) + 3x + (-5)$	Multiply.
	$= 6x^2 - 7x - 5$	Combine like terms.

▶ The product is $6x^2 - 7x - 5$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the FOIL Method to find the product.

17. $(z + 5)(z + 10)$ **18.** $(m - 3)(m - 7)$ **19.** $(x - 4)(x + 2)$

20. $\left(2u + \frac{1}{2}\right)\left(u - \frac{3}{2}\right)$ **21.** $(4c - 7)(3c + 8)$ **22.** $\left(\frac{1}{2}n + 2\right)\left(n^2 + \frac{1}{3}\right)$

23. WRITING Explain why the FOIL Method is a valid method.

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Multiplying Binomials and Trinomials

EXAMPLE 6 Multiplying a Binomial and a Trinomial

WATCH

Find $(x + 5)(x^2 - 3x - 2)$.

SOLUTION

$$\begin{array}{rl} & x^2 - 3x - 2 \\ \times & \quad x + 5 \\ \hline & 5x^2 - 15x - 10 \\ & x^3 - 3x^2 - 2x \\ \hline & x^3 + 2x^2 - 17x - 10 \end{array}$$

Multiply $5(x^2 - 3x - 2)$. → Distributive Property

Multiply $x(x^2 - 3x - 2)$. → Distributive Property

Align like terms vertically.

Combine like terms.

▶ The product is $x^3 + 2x^2 - 17x - 10$.

7 MTR

EXAMPLE 7 Modeling Real Life

WATCH

In hockey, a goalie behind the goal line can play a puck only in the trapezoidal region.

a. Write a polynomial that represents the area of the trapezoidal region.

b. Find the area of the trapezoidal region when the shorter base is 22 feet.

SOLUTION

a. $\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(x - 11)[x + (x + 6)]$ Substitute.

$= \frac{1}{2}(x - 11)(2x + 6)$ Combine like terms.

$= \frac{1}{2}[2x^2 + 6x + (-22x) + (-66)]$ FOIL Method

$= \frac{1}{2}(2x^2 - 16x - 66)$ Combine like terms.

$= x^2 - 8x - 33$ Distributive Property

▶ A polynomial that represents the area of the trapezoidal region is $x^2 - 8x - 33$.

b. Find the value of $x^2 - 8x - 33$ when $x = 22$.

$x^2 - 8x - 33 = 22^2 - 8(22) - 33$ Substitute 22 for x.

$= 484 - 176 - 33$ Simplify.

$= 275$ Subtract.

▶ The area of the trapezoidal region is 275 square feet.

Florida has two NHL teams: the Tampa Bay Lightning and the Florida Panthers. The Tampa Bay Lightning won the Stanley Cup in 2004 and 2020.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the product.

24. $(x + 1)(x^2 + 5x + 8)$ **25.** $(n - 3)(n^2 - 2n + 4)$ **26.** $(2 - w)\left(\frac{1}{4}w^2 + \frac{1}{2}w - 1\right)$

27. In Example 7, the longer base is extended by 1 foot, and the shorter base remains 22 feet. Explain how the polynomial changes. Then find the percent of change in the area of the trapezoidal region.

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7.2 Practice WITH CalcChat® AND CalcView®

In Exercises 1–8, find the product. *(See Example 1.)*

1. $2c(5c^2)$
2. $6d^4(-3c^3)$
3. $-4r^2(9r + 6)$
4. $12t^3(5t^5 - 2)$
5. $7w^3(w^2 - 4w - 1)$
6. $-z^2(2z^4 + 10z^2 - 16)$
7. $(15 - 3g^2)\left(-\frac{2}{3}g^5\right)$
8. $(9h^2 - 18 + 9h^4)\left(\frac{1}{6}h^3\right)$

In Exercises 9–16, find the quotient. *(See Example 2.)*

9. $\dfrac{7b^2 + 14b}{b}$
10. $\dfrac{-9h^4 + 27h^3}{h^2}$
11. $\dfrac{2n^3 + 8n^2 - 20n}{2n}$
12. $\dfrac{-6k^4 + 15k^3 - 9k^2}{3k^2}$
13. $\dfrac{4x^5 - x^7 + 7x^4}{x^3}$
14. $\dfrac{10y^2 + 6y^4 + 8y^3}{2y^2}$
15. $\dfrac{4p^6 - 20p^4 + 16p}{12p}$
16. $\dfrac{9m^7 - 27m^3 + 81m^2}{18m^3}$

In Exercises 17–24, use the Distributive Property to find the product. *(See Example 3.)*

17. $(x + 1)(x + 3)$
18. $(y + 6)(y + 4)$
19. $(z - 5)(z + 3)$
20. $(a + 8)(a - 3)$
21. $\left(g - \frac{1}{2}\right)\left(g - \frac{3}{2}\right)$
22. $(n - 0.4)(n - 0.5)$
23. $(3m + 1)(m + 9)$
24. $(5s + 6)(s - 2)$

In Exercises 25–30, use a table to find the product. *(See Example 4.)*

25. $(x + 3)(x + 2)$
26. $(h - 8)(h - 9)$
27. $(3k - 1)(4k + 9)$
28. $(5g + 3)(g + 8)$
29. $(-3 + 0.6j)(0.4j - 7)$
30. $\left(\frac{1}{5}d - 12\right)\left(-7 + \frac{5}{4}d\right)$

In Exercises 31–40, use the FOIL Method to find the product. *(See Example 5.)*

31. $(b + 3)(b + 7)$
32. $(w + 9)(w + 6)$
33. $(k + 5)(k - 1)$
34. $(x - 4)(x + 8)$
35. $\left(q - \frac{3}{4}\right)\left(q + \frac{1}{4}\right)$
36. $\left(z - \frac{5}{3}\right)\left(z - \frac{2}{3}\right)$
37. $(9 - r)(2 - 3r)$
38. $(8 - 4x)(2x + 6)$
39. $(w + 5)(w^2 + 3w)$
40. $(v - 3)(v^2 + 8v)$

4 MTR ERROR ANALYSIS **In Exercises 41 and 42, describe and correct the error in finding the product of the binomials.**

41.

$$(t - 2)(t + 5) = t - 2(t + 5)$$
$$= t - 2t - 10$$
$$= -t - 10$$

42.

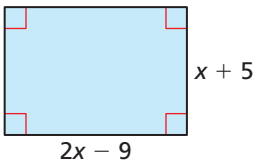
$$(x - 5)(3x + 1)$$

	$3x$	1
x	$3x^2$	x
5	$15x$	5

$$(x - 5)(3x + 1) = 3x^2 + 16x + 5$$

5 MTR CONNECTING CONCEPTS **In Exercises 43–46, write a polynomial that represents the area of the shaded region.**

43.



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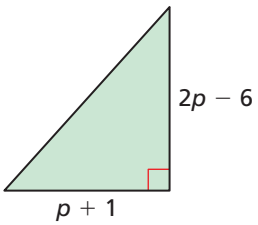

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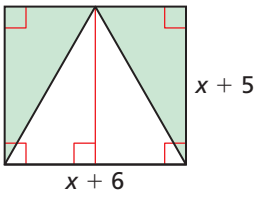

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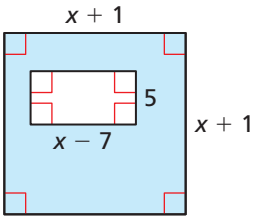

In Exercises 47–54, find the product. *(See Example 6.)*

47. $(x + 4)(x^2 + 3x + 2)$
48. $(f + 1)(f^2 + 4f + 8)$
49. $(y + 3)(y^2 + 8y - 2)$
50. $(t - 2)(t^2 - 5t + 1)$
51. $(4 - b)(5b^2 + 5b - 4)$
52. $(6 + d)(2d^2 - d + 7)$
53. $(3e^2 - 0.5e + 7)(0.8e + 1)$
54. $(0.2v^2 + 2v - 9)(4 - 0.7v)$

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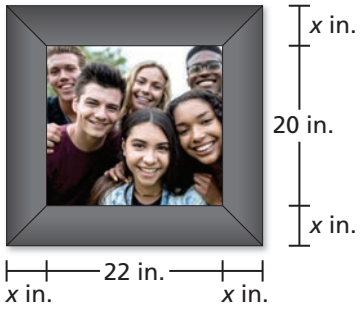
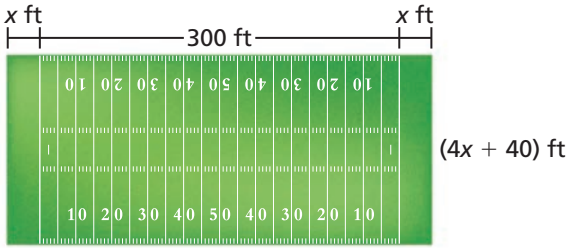
55. **MODELING REAL LIFE** You design a frame to surround a rectangular photo. The width of the frame is the same on each side, as shown. *(See Example 7.)*



a. Write a polynomial that represents the combined area of the photo and the frame.

b. Find the combined area of the photo and the frame when the width of the frame is 4 inches.

56. **MODELING REAL LIFE** The football field is rectangular.



a. Write a polynomial that represents the area of the football field.

b. Find the area of the football field when the length of the field is 360 feet.

57. **CHOOSE A METHOD** Describe two ways to find the product of two binomials. Which method do you prefer? Explain.

58. **REASONING** Can you use the FOIL Method to multiply a binomial by a trinomial? two trinomials? Explain your reasoning.

59. **MAKING AN ARGUMENT** You use the Distributive Property to multiply $(x + 3)(x - 5)$. Your friend uses the FOIL Method to multiply $(x - 5)(x + 3)$. Should your answers be equivalent? Justify your answer.

60. **STRUCTURE** Find the values of a, b, and c that make the equation true.

$$(2x - 1)(3x + 4) = ax^2 + bx + c$$

61. **WRITING** When multiplying two binomials, explain how the degree of the product is related to the degree of each binomial.

GO DIGITAL

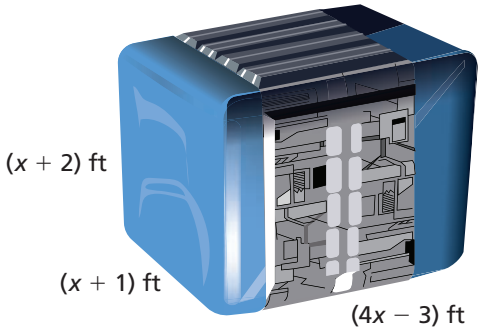
62. **HOW DO YOU SEE IT?**
The table shows one method of finding the product of two binomials.

	$-4x$	3
$-8x$	a	b
-9	c	d

a. Write the two binomials being multiplied.

b. Determine whether a, b, c, and d will be positive or negative when $x > 0$.

63. **B.E.S.T. TEST PREP** The satellite is in the shape of a rectangular prism. Which polynomial represents the volume of the satellite?



Ⓐ $4x^3 + 9x^2 - x - 6$ Ⓒ $4x^3 + 8x^2 - 3x - 6$

Ⓑ $4x^3 - 3x^2 + 12x - 9$ Ⓓ $4x^3 + 4x^2 - 6x - 6$

64. **REASONING** When dividing two monomials, is it possible for the degree of the quotient to be greater than the degree of the dividend? the divisor? Explain.

65. **MODELING REAL LIFE** The area of the tablet screen (in square centimeters) is represented by $2x^2 - 4x$.

a. Write a polynomial that represents the length of the screen.

b. Find the length of the screen when the width is 12 centimeters.

66. **DIG DEEPER** The volume of the locker (in cubic inches) is represented by $(4x^3 + 7x^2)$.

a. Write a polynomial that represents the height of the locker.

b. Find the height of the locker (in feet) when the side length of the base is 15 inches.

67. OPEN-ENDED Write two polynomials that are not monomials whose product is a trinomial of degree 3.

68. THOUGHT PROVOKING
Find the value of k that makes the equation true. Justify your answer.

$$(12x^5 + 84x^4)(k)^{-2} = 3x^3 + 21x^2$$

4 MTR 69. DISCUSS MATHEMATICAL THINKING The product of $(x + m)(x + n)$ is $x^2 + bx + c$.

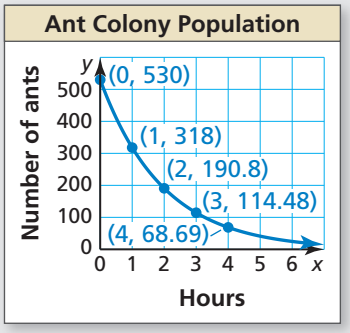
a. What do you know about m and n when $c > 0$?

b. What do you know about m and n when $c < 0$?

REVIEW & REFRESH

WATCH

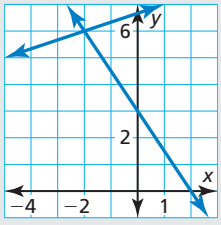
70. The graph represents the population y of an ant colony x hours after pest control sprays the colony. Find the population after 6 hours.



71. REASONING The sum of two polynomials is $3x^2 - 7x + 5$. One of the polynomials is $x - 2$. What is the product of the polynomials?

72. Find the difference of $(s^4 - 2s^2 - 4)$ and $(-9s^2 + 5s - 7)$.

73. Use the graph to solve the system $6x + 4y = 12$ and $-x + 3y = 20$. Check your solution.



In Exercises 74–77, simplify the expression. Write your answer using only positive exponents.

74. $10^2 \cdot 10^9$

75. $\dfrac{x^5 \cdot x}{x^8}$

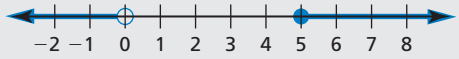
76. $(3z^6)^{-3}$

77. $\left(\dfrac{2y^4}{y^3}\right)^{-2}$

78. Determine whether the relation is a function. Explain.

Input, x	8	−2	−6	5	8
Output, y	−1	2	5	−7	2

79. Write an inequality that represents the graph.



In Exercises 80 and 81, find the sum or difference.

80. $(8y^3 - y^2 + 12) + (2y^2 + 3y - 4)$

81. $(3.8m^2 + 7.2m + 7) - (0.5m^2 - 5m - 1)$

In Exercises 82–85, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

82. $9 + z^2$

83. $3d^4 - 6d^6$

84. $-2c - 4c^3 + c^2$

85. $\frac{1}{2}w^5 + 5w^3 + 7w^8$

86. Write a function that represents a $750 laptop that decreases in value by 20% each year.

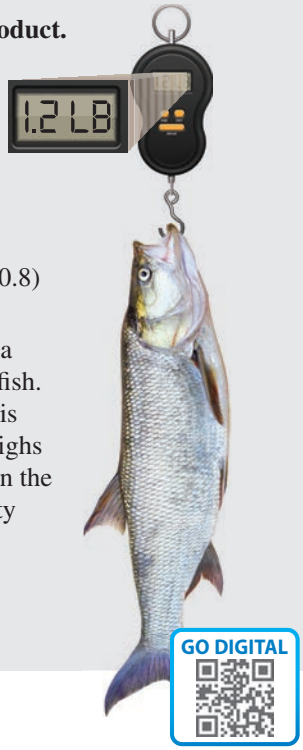
In Exercises 87–90, find the product.

87. $-2a^2(4a + 9)$

88. $(b - 3)(b - 6)$

89. $(g^2 + 8)(2g + 5)$

90. $(0.5v + 4)(-6v^2 - 6v + 10.8)$



7 MTR 91. MODELING REAL LIFE On a fishing trip, you catch two fish. The weight of the first fish is shown. The second fish weighs at least 0.5 pound more than the first fish. Write an inequality that represents the possible weights of the second fish.

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