6.5 Modeling with Exponential Functions

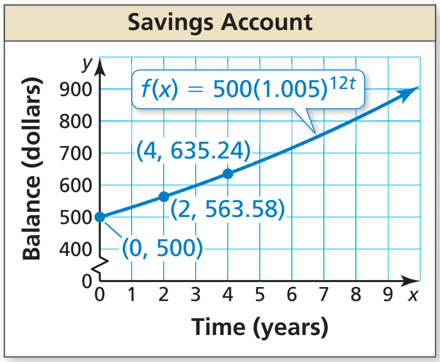
Learning Target: Model real-life problems using exponential functions.

Success Criteria:
- I can write exponential growth functions and exponential decay functions from graphs and tables.
- I can solve real-life problems using exponential growth and decay functions.
- I can solve real-life problems involving compound interest.

EXPLORE IT! Modeling with Exponential Functions

Work with a partner. Decide whether each real-life scenario represents *exponential growth*, *exponential decay*, or *neither*. Explain your reasoning.

a. The graph shows the balance of a savings account over time.



b. The table shows the battery power y (as a percent) of an electric car x hours after noon.

x	0	1	2	3	4	5	6
y	100	86	72	56	52	68	96

2 MTR MODEL A PROBLEM
How can graphing the values in the table help you answer part (b)?

c. A forensic pathologist was called to estimate the time of death of a person. At midnight, the body temperature was 80.5°F and the room temperature was a constant 60°F. One hour later, the body temperature was 78.5°F. Each hour, the difference of the body temperature and the room temperature changes by the same percent as in the first hour.

d. Your friend automatically deposits a constant amount of her paycheck into a checking account. The balance y (in dollars) of the account after x deposits can be represented by $y = 100x + 350$.

Algebraic Reasoning
MA.912.AR.5.4 Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.
Financial Literacy
MA.912.FL.3.2 Solve real-world problems involving simple, compound and continuously compounded interest.
Also MA.912.FL.3.4

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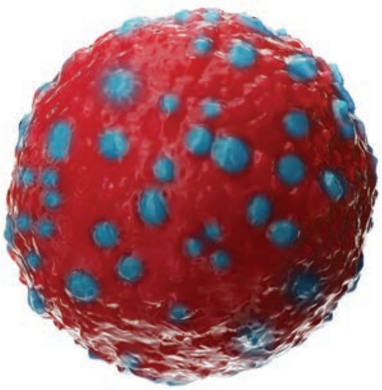
Writing Exponential Functions

For an exponential function of the form $y = ab^x$, the y-values change by a factor of b as x increases by 1. You can use this fact to write an exponential function when you know the y-intercept, a.

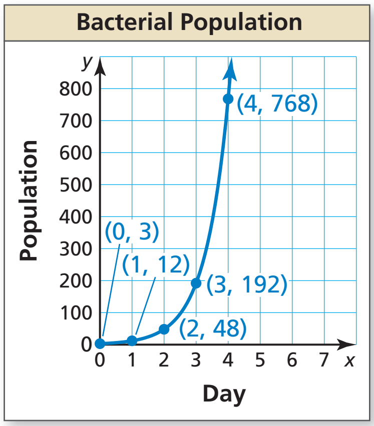
Vocabulary

compound interest, *p. 320*



EXAMPLE 1 Writing an Exponential Function

The graph represents a bacterial population y after x days. Find the population after 12 hours and after 5 days.



SOLUTION

The graph appears to show exponential growth. One way to find the populations is to write and evaluate the function represented by the graph.

Use a table of values to identify the y-intercept and growth factor.

$+1$ $+1$ $+1$ $+1$

x	0	1	2	3	4
y	3	12	48	192	768

$\times 4$ $\times 4$ $\times 4$ $\times 4$

The y-intercept is 3. As x increases by 1, y is multiplied by 4. So, the population can be modeled by $y = 3(4)^x$. Use the function to find the population after 12 hours and after 5 days.

Population after 12 hours		*Population after 5 days*
$y = 3(4)^x$	Write the function.	$y = 3(4)^x$
$= 3(4)^{1/2}$ (12 hours $= \frac{1}{2}$ day)	Substitute for x.	$= 3(4)^5$
$= 3(2)$	Evaluate the power.	$= 3(1024)$
$= 6$	Multiply.	$= 3072$

▶ There are 6 bacteria after 12 hours and 3072 bacteria after 5 days.

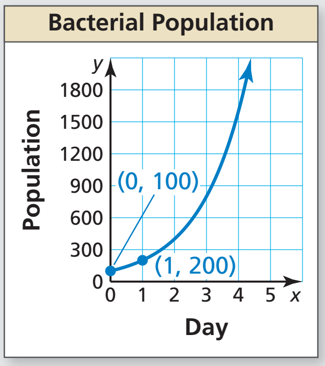
SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. A bacterial population y after x days can be represented by an exponential function whose graph is shown.

a. Find the population after 6 days.

b. Does this bacterial population grow faster than the bacterial population in Example 1? Explain your reasoning.



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EXAMPLE 2 Writing an Exponential Function

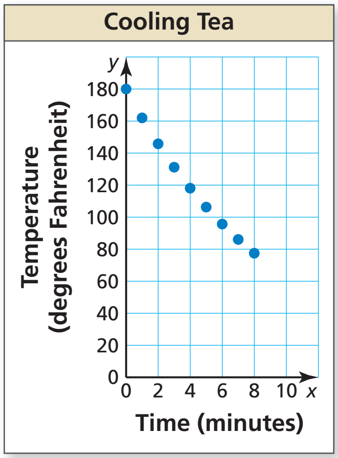
WATCH

Time, x	Temperature, y
0	180
1	162
2	145.8
3	131.2
4	118.1
5	106.3
6	95.7
7	86.1
8	77.5

The table shows the temperatures y (in degrees Fahrenheit) of tea x minutes after pouring a cup. Write a function that models the data. Predict the temperature of the tea 3.5 minutes after it is poured.

SOLUTION

Graph the data in the table. The graph appears to show exponential decay. One way to find the temperature after 3.5 minutes is to write and evaluate the function represented by the graph.



Use the table of values to identify the y-intercept and decay factor.

x	0	1	2	3	4	5	6	7	8
y	180	162	145.8	131.2	118.1	106.3	95.7	86.1	77.5

$+1 \quad +1 \quad +1 \quad +1 \quad +1 \quad +1 \quad +1 \quad +1$

$\times 0.9 \quad \times 0.9 \quad \times 0.9 \quad \times 0.9 \quad \times 0.9 \quad \times 0.9 \quad \times 0.9 \quad \times 0.9$

5 MTR RELATE CONCEPTS
How can you use ratios to find the decay factor?

The y-intercept is 180. As x increases by 1, y is multiplied by about 0.9. So, the temperature can be modeled by $y = 180(0.9)^x$. Use the function to predict the temperature after 3.5 minutes.

$y = 180(0.9)^x$ Write the function.

$= 180(0.9)^{3.5}$ Substitute for x.

≈ 124.5 Use technology.

▶ The temperature of the tea after 3.5 minutes is about 124.5°F.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

2. WHAT IF? The initial temperature of the tea is 175°F, and the percent rate of change of the temperature remains the same. Predict the temperature of the tea 3.5 minutes after it is poured.

3. The table shows the amount y (in milligrams) of caffeine in a person's bloodstream x hours after drinking coffee. Write a function that models the data. Predict the amount of caffeine in the person's bloodstream after 6 hours.

Time, x	Caffeine, y
0	125
1	100
2	80
3	64
4	51

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Using Compound Interest

Exponential growth functions are used in real-life situations involving *compound interest*. Although interest earned is expressed as an *annual* rate, the interest is usually compounded more frequently than once per year. So, the formula $y = a(1 + r)^t$ must be modified for compound interest problems.

STUDY TIP

For interest compounded yearly, you can substitute 1 for n in the formula to get $y = P(1 + r)^t$.

KEY IDEA

Compound Interest

Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P = principal (initial amount)

r = annual interest rate (in decimal form)

t = time (in years)

n = number of times interest is compounded per year

EXAMPLE 3 Using Compound Interest

WATCH

You deposit \$100 in an investment account that earns 6% annual interest compounded monthly.

a. Write a function m that represents the balance (in dollars) of the investment account after t years.

b. What is the balance of the account after 5 years?

SOLUTION

a. $m(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ — Use the compound interest formula.

$= 100\left(1 + \frac{0.06}{12}\right)^{12t}$ — Substitute 100 for P, 0.06 for r, and 12 for n.

$= 100(1.005)^{12t}$ — Simplify.

b. Find $m(t)$ when $t = 5$.

$m(5) = 100(1.005)^{12(5)}$ — Substitute 5 for t.

≈ 134.89 — Use technology.

▶ So, the balance of the account after 5 years is \$134.89.

5 MTR DECOMPOSE A PROBLEM

Notice that the function consists of the product of the principal, 100, and a factor independent of the principal, $(1.005)^{12t}$.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. WHAT IF? Repeat Example 3 when the investment account earns 3% annual interest compounded monthly.

5 MTR **5. STRUCTURE** The function $y = 800\left(1 + \frac{0.02}{12}\right)^{12t}$ represents the balance y (in dollars) of a savings account after t years. What can you determine about the account? Explain.

5 MTR **6. CONNECTING CONCEPTS** Explain the relationship between compound interest and exponential growth.

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6.5 Practice WITH CalcChat® AND CalcView®

In Exercises 1–4, write a function that models the data.

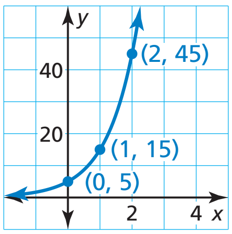
1.

x	y
0	250
1	150
2	90
3	54
4	32.4

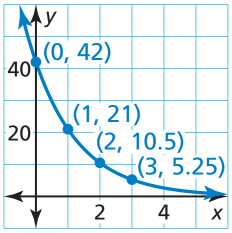
2.

j	k
0	3
5	28
10	53
15	78
20	103

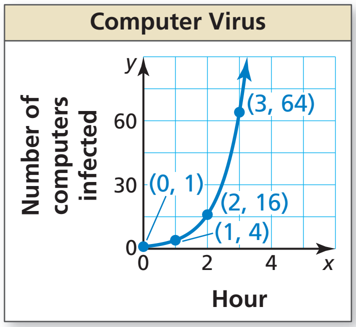
3.



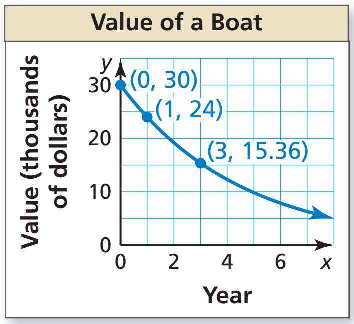
4.



5. MODELING REAL LIFE The graph represents the number y of computers infected by a virus after x hours. Find the number of computers infected after 90 minutes and after 6 hours. *(See Example 1.)*



6. MODELING REAL LIFE The graph represents the value y of a boat after x years. Find the value of the boat after 2 years and after 8 years.



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7. MODELING REAL LIFE The table shows the numbers y of views an online video receives after being online for x days. Write a function that models the data. Predict the number of views the video receives after being online for 7 days. *(See Example 2.)*

Day, x	0	1	2	3	4
Views, y	12	67	376	2107	11,801

8. MODELING REAL LIFE The table shows the coyote populations y in a national park after t decades. Write a function that models the data. Predict the coyote population after 60 years.

Decade, t	0	1	2	3	4
Population, y	16	24	36	54	81

In Exercises 9 and 10, write a function that represents the balance y (in dollars) after t years.

9. \$2000 deposit that earns 5% annual interest compounded quarterly

10. \$3500 deposit that earns 9.2% annual interest compounded annually

11. MODELING REAL LIFE You deposit \$9000 in a savings account that earns 3.6% annual interest compounded monthly. *(See Example 3.)*

a. Write a function s that represents the balance (in dollars) of your savings account after t years.

b. What is the balance of the account after 7 years?

c. How many years will it take to double the principal?

12. MODELING REAL LIFE Your checking account has a constant balance of \$500. Let the function m represent the balance of your savings account after t years. The table shows the total balance of the accounts over time.

Year, t	Total balance
0	\$2500
1	\$2540
2	\$2580.80
3	\$2622.42
4	\$2664.86
5	\$2708.16

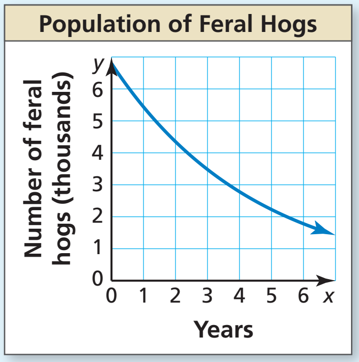
a. Write a function B that represents the total balance after t years.

b. Find and interpret $B(8)$.

c. Compare the savings account to the account in Exercise 11.

13. OPEN-ENDED Describe two account options into which you can deposit \$1000 and earn compound interest. Write a function that represents the balance of each account after t years. Which account would you rather use? Explain your reasoning.

14. HOW DO YOU SEE IT?
The graph shows the population y (in thousands) of invasive feral hogs x years after an area implements control efforts.



Would you suggest this control effort continue? Explain your reasoning.

15. DIG DEEPER Solve the compound interest formula $y = P\left(1 + \frac{r}{n}\right)^{nt}$, for P.

a. Interpret the formula.

b. An account earned 11.2% annual interest compounded monthly for 7 years and currently has a balance of \$14,840. Is there enough information to find the principal? If so, find the principal. If not, what other information do you need?

c. Repeat part (b) for an account that earned 23% annual interest compounded quarterly for 3 years and currently has a balance of \$6258. Compare this account to the account in part (b).

16. THOUGHT PROVOKING
You are deciding whether to place money into an account that accrues simple interest or one that accrues compound interest. What information do you need to decide which account to use? Explain your reasoning.

REVIEW & REFRESH

WATCH

17. Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

x	0	1	2	3
y	7	21	63	189

18. The table shows the numbers y of bacteria that grow on a sandwich x hours after it is left in the sun. Write a function that models the data. Predict the number of bacteria in the fifth hour.

Hours, *x*	1	2	3	4
Number of Bacteria, *y*	25	75	225	675

In Exercises 19 and 20, solve the inequality. Graph the solution, if possible.

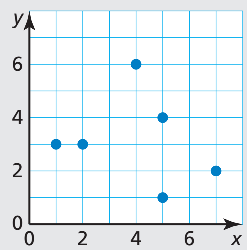
19. $3(2n - 1) < 6n - 4$ **20.** $|5t + 1| - 10 \le -6$

In Exercises 21 and 22, simplify the expression. Write your answer using only positive exponents.

21. 11^{2x-7} **22.** $\left(\frac{g^2}{2}\right)^{-3}$

23. Evaluate $y = 6(2)^x$ when $x = -3$, 0, and 2.

24. Determine whether the graph represents a function. Explain.



25. PROBLEM SOLVING You are stopped in a line of traffic that is about 1.5 miles long. How long is the line in inches?

26. Rewrite $\left(\sqrt[6]{70}\right)^5$ in rational exponent form.

In Exercises 27 and 28, write an equation in slope-intercept form of the line that passes through the given points.

27. (1, 7), (3, −3) **28.** (0, −10), (8, −4)

5 MTR **29. STRUCTURE** Complete the equation so that it has infinitely many solutions.

$$7x - 20 + 8x = -5(\square x + \square)$$

30. Solve $7 \ge |x + 3| - 10$. Graph the solution, if possible.

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