6.3 Exponential Functions

Learning Target: Graph and write exponential functions.

Success Criteria:

- I can identify an exponential function.
- I can evaluate and graph an exponential function.
- I can identify characteristics of exponential functions.
- I can solve real-life problems using exponential functions.

EXPLORE IT! Understanding Exponential Functions

Work with a partner. An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$.

a. Consider the exponential function $f(x) = 16(2)^x$. Complete each table. What do you notice about consecutive values of x in each table? What do you notice about consecutive values of $f(x)$?

x	$f(x) = 16(2)^x$
0	
1	
2	
3	
4	
5	

x	$f(x) = 16(2)^x$
0	
2	
4	
6	
8	
10	

b. Repeat part (a) for the exponential function $g(x) = 16\left(\frac{1}{2}\right)^x$. Do you think the statement below is true for *any* exponential function? Explain your reasoning.

"As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor."

c. Sketch the graphs of the functions given in parts (a) and (b). How are the graphs similar? How are they different?

2 MTR MAKE A CONNECTION

How do you expect the graph of $f(x) = 16(2)^x$ to compare to the graph of $g(x) = -16(2)^x$?

Algebraic Reasoning

MA.912.AR.5.6 Given a table, equation or written description of an exponential function, graph that function and determine its key features.

Functions

MA.912.F.1.1 Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.

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Identifying and Evaluating Exponential Functions

Vocabulary AZ VOCAB

exponential function, *p. 302*
asymptote, *p. 303*

An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$. As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor, which means consecutive y-values form equivalent ratios.

EXAMPLE 1 Identifying Functions WATCH

Does each table represent a *linear* or an *exponential* function? Explain.

a.

x	0	1	2	3
y	2	4	6	8

b.

x	0	1	2	3
y	4	8	16	32

SOLUTION

a.

+ 1 + 1 + 1

x	0	1	2	3
y	2	4	6	8

+ 2 + 2 + 2

▶ As x increases by 1, y increases by 2. The rate of change is constant. So, the function is linear.

b.

+ 1 + 1 + 1

x	0	1	2	3
y	4	8	16	32

× 2 × 2 × 2

▶ As x increases by 1, y is multiplied by 2. So, the function is exponential.

STUDY TIP

In Example 1(b), consecutive y-values form equivalent ratios.

$\frac{8}{4} = 2, \frac{16}{8} = 2, \frac{32}{16} = 2$

EXAMPLE 2 Evaluating Exponential Functions WATCH

Evaluate each function for the given value of x.

a. $y = -2(5)^x;\ x = 3$

b. $f(x) = 3(0.5)^x;\ x = -2$

SOLUTION

a.
$y = -2(5)^x$ Write the function.
$= -2(5)^3$ Substitute for x.
$= -2(125)$ Evaluate the power.
$= -250$ Multiply.

b.
$f(x) = 3(0.5)^x$
$f(-2) = 3(0.5)^{-2}$
$= 3(4)$
$= 12$

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Does the table represent a *linear* or an *exponential* function? Explain.

1.

x	0	1	2	3
y	8	4	2	1

2.

x	-4	0	4	8
y	1	0	-1	-2

Evaluate the function when $x = -2$, 0, and $\frac{1}{2}$.

3. $y = 2(9)^x$

4. $f(x) = 1.5(2)^x$

5. $g(x) = -3\left(\frac{1}{4}\right)^x$

6. REASONING For each function in Example 2, what happens to the y-values as $x \to +\infty$? as $x \to -\infty$? Explain.

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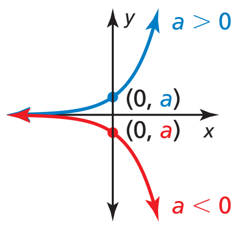
Graphing Exponential Functions

The graph of a function $y = ab^x$ is a vertical stretch or shrink by a factor of $|a|$ of the graph of the parent function $y = b^x$. When $a < 0$, the graph is also reflected in the x-axis. The y-intercept of the graph of $y = ab^x$ is a.

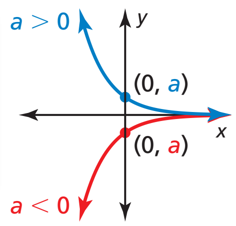
KEY IDEA

$y = ab^x$, where $b > 1$



An **asymptote** is a line that a graph approaches more and more closely. The x-axis is an asymptote of the graph of $y = ab^x$.

$y = ab^x$, where $0 < b < 1$



EXAMPLE 3 Graphing $y = ab^x$, where $b > 1$

WATCH

Graph $f(x) = 4(2)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Find the domain and range of f.

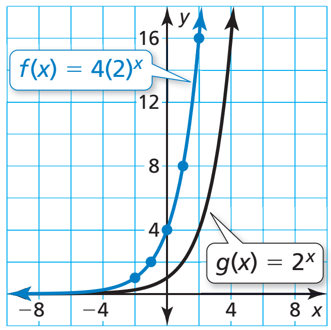
SOLUTION

Step 1 Make a table of values.

x	-2	-1	0	1	2
$f(x)$	1	2	4	8	16

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



▶ The parent function is $g(x) = 2^x$. The graph of f is a vertical stretch by a factor of 4 of the graph of g. The y-intercept of the graph of f, 4, is greater than the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g. From the graph of f, you can see that the domain is all real numbers and the range is $y > 0$.

1 MTR HELP A CLASSMATE
Explain to a classmate how to determine the end behavior of f.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the function. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Find the domain and range of f.

7. $f(x) = -3^x$

8. $f(x) = -2(4)^x$

9. $f(x) = \frac{1}{2}(2)^x$

10. OPEN-ENDED Sketch an increasing exponential function whose graph has a y-intercept of 2.

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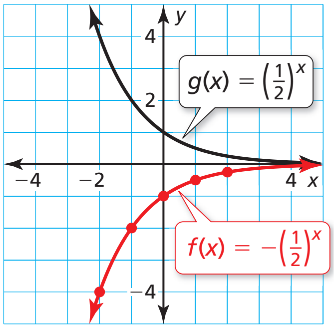
EXAMPLE 4 Graphing $y = ab^x$, when $0 < b < 1$

WATCH

Graph $f(x) = -\left(\frac{1}{2}\right)^x$. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Find the domain and range of f.

SOLUTION



Step 1 Make a table of values.

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

x	-2	-1	0	1	2
$f(x)$	-4	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$

▶ The parent function is $g(x) = \left(\frac{1}{2}\right)^x$. The graph of f is a reflection in the x-axis of the graph of g. The y-intercept of the graph of f, -1, is less than the y-intercept of the graph of g, 1. The x-axis is an asymptote of both the graphs of f and g. From the graph of f, you can see that the domain is all real numbers and the range is $y < 0$.

EXAMPLE 5 B.E.S.T. Test Prep: Identifying Characteristics

WATCH

Which statement about the functions in Example 4 is false?

(A) f is increasing and negative over its entire domain.

(B) g is decreasing and positive over its entire domain.

(C) $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

(D) $g(x) \to +\infty$ as $x \to -\infty$ and $g(x) \to 0$ as $x \to +\infty$

(E) The line $y = 0$ is an asymptote of the graphs of f and g.

SOLUTION

The functions are of the form $y = ab^x$, so the asymptote of each graph is the x-axis, or $y = 0$. The graphs show that

- f is increasing and negative over its entire domain.
- g is decreasing and positive over its entire domain.
- $f(x)$ decreases as x approaches negative infinity, and $f(x)$ approaches 0 as x approaches positive infinity. So, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to +\infty$.
- $g(x)$ increases as x approaches negative infinity, and $g(x)$ approaches 0 as x approaches positive infinity. So, $g(x) \to +\infty$ as $x \to -\infty$ and $g(x) \to 0$ as $x \to +\infty$.

▶ So, (C) is false.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Graph the function. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Find the domain and range of f.

11. $f(x) = \left(\frac{1}{3}\right)^x$

12. $f(x) = 2\left(\frac{1}{4}\right)^x$

13. $f(x) = -10\left(\frac{1}{2}\right)^x$

14. REASONING Explain why the graph of an exponential function is not a line.

15. Determine when $f(x) = 0.5(4)^x$ is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

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Solving Real-Life Problems

7 MTR

EXAMPLE 6 Modeling Real Life

WATCH

5 MTR DECOMPOSE A PROBLEM
Why does it make sense that the base of the power in the function is 0.9?

A real-estate researcher estimates that a coastal property in Florida will lose about 10% of its value each decade from 2020 to 2050. The function $f(x) = 250(0.9)^x$ represents the estimated value (in thousands of dollars) of the property x decades after 2020.

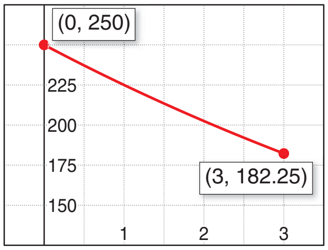
a. Graph f. Find the domain and range.

b. What is the estimated decrease in property value from 2020 to 2050?

Climate concerns, such as increasing sea level, are expected to decrease coastal property values in Florida and other states over time.

SOLUTION

a. The function is defined for the years 2020 to 2050, which represent 3 decades. So, use technology to graph the function for $0 \le x \le 3$.



The greatest y-value is $f(0) = 250$, and the least y-value is $f(3) = 182.25$.

▶ So, the domain is $\{x \mid 0 \le x \le 3\}$ and the range is $\{f(x) \mid 182.25 \le f(x) \le 250\}$.

b. The estimated value of the property in 2020 is \$250,000. The estimated value of the property in 2050 is \$182,250.

▶ So, the estimated decrease in property value from 2020 to 2050 is \$250,000 − \$182,250 = \$67,750.

SELF-ASSESSMENT

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

16. An entomologist expects an insect population to increase by about 20% each month from May 1 to September 1. The function $f(x) = 100(1.2)^x$ represents the estimated population (in thousands) x months after May 1.

a. Graph f. Find the domain and range.

b. What is the estimated increase in population from May 1 to September 1?

17. WHAT IF? In Example 6, the researcher uses new information to update the function to $g(x) = 250(0.88)^x$. Does the property lose more or less value by 2050? How much more or less?

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6.3 Practice WITH CalcChat® AND CalcView®

In Exercises 1–6, determine whether the equation represents an exponential function. Explain.

1. $y = (0.5)^{x+8}$
2. $y = x + 6$
3. $y = (x - 7)^3$
4. $y = -2^x$
5. $y = (0.5)^x - 4$
6. $y = \left(\frac{1}{2}\right)^x$

In Exercises 7–10, determine whether the table represents a *linear* or an *exponential* function. Explain. *(See Example 1.)*

7.

x	y
1	−2
2	0
3	2
4	4

8.

x	y
1	6
2	12
3	24
4	48

▶ 9.

x	−1	0	1	2	3
y	0.25	1	4	16	64

10.

x	−3	0	3	6	9
y	10	1	−8	−17	−26

In Exercises 11–18, evaluate the function for the given value of x. *(See Example 2.)*

11. $y = 3^x;\ x = 2$
12. $f(x) = 3(2)^x;\ x = -1$
13. $y = -4(5)^x;\ x = 2$
14. $f(x) = 0.5^x;\ x = -3$
15. ▶ $f(x) = \frac{1}{3}(6)^x;\ x = 3$
16. $y = \frac{1}{4}(4)^x;\ x = \frac{3}{2}$
17. $f(x) = -18(9)^x;\ x = -\frac{1}{2}$
18. $f(x) = 8(8)^x;\ x = \frac{1}{3}$

In Exercises 19 and 20, the table represents an exponential function. Graph the function.

19.

x	y
−2	1
−1	2
0	4
1	8

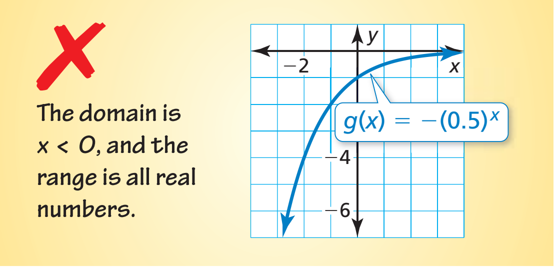
20.

x	y
1	$-\frac{1}{3}$
3	−3
5	−27
7	−243

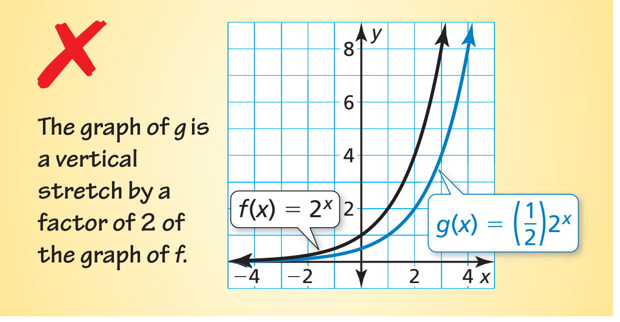
In Exercises 21–30, graph the function. Compare the graph to the graph of the parent function. Identify the y-intercepts and asymptotes of the graphs. Find the domain and range of f. *(See Examples 3 and 4.)*

21. $f(x) = 8^x$
22. $f(x) = 3(6)^x$
23. $f(x) = -4^x$
24. $f(x) = -2(7)^x$
25. ▶ $f(x) = 3(0.5)^x$
26. $f(x) = 6\left(\frac{1}{3}\right)^x$
27. $f(x) = \frac{1}{2}(8)^x$
28. $f(x) = \frac{3}{2}(0.25)^x$
29. $f(x) = 2\left(\frac{3}{4}\right)^x$
30. $f(x) = 4(0.8)^x$

4 MTR **31. ERROR ANALYSIS** Describe and correct the error in finding the domain and range of g.



4 MTR **32. ERROR ANALYSIS** Describe and correct the error in comparing the graph of g to the graph of the parent function f.



In Exercises 33–38, determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function. *(See Example 5.)*

33. $y = -9^x$
34. $f(x) = 7(5)^x$
35. $g(x) = \frac{2}{3}(0.2)^x$
36. $y = -4\left(\frac{1}{6}\right)^x$
37. $y = -3\left(\frac{2}{7}\right)^x$
38. $h(x) = 3(1.5)^x$

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In Exercises 39–44, describe values of a and b for which the function $f(x) = ab^x$ meets the given requirement(s).

39. f is positive for its entire domain

40. f is negative for its entire domain

41. f is increasing for its entire domain

42. f is decreasing for its entire domain

43. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to +\infty$

44. $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

45. **MODELING REAL LIFE** Organizers of the Gasparilla Pirate Festival in Tampa estimate that the parade attracts about 6% more people each year. The function $f(x) = 300(1.06)^x$ represents the estimated number of people (in thousands) who attend the parade x years after 2020. *(See Example 6.)*

a. Graph f. Find the domain and range.

b. What is the estimated number of people who attend the parade in 2024?

46. **MODELING REAL LIFE** A cell phone company estimates that a cell phone loses about 15% of its value each year after it is purchased. The function $f(x) = 900(0.85)^x$ represents the estimated value (in dollars) of a cell phone x years after its purchase.

a. Graph f. Find the domain and range.

b. What is the estimated decrease in value 5 years after a cell phone is purchased?

47. **MODELING REAL LIFE** You are deciding between two jobs. Job A offers \$35,000 with a 5% raise each year. The function $f(x) = 35{,}000(1.05)^x$ represents the salary of Job A after x years. Job B offers \$45,000 with a \$1500 raise each year. The function $g(x) = 45{,}000 + 1500x$ represents the salary of Job B after x years. Which job would you choose if you plan to keep it for no more than 10 years? Explain.

48. **REASONING** Explain why a is the y-intercept of the graph of $y = ab^x$.

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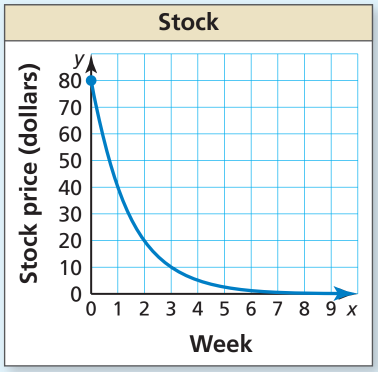
49. **STRUCTURE** Does the table represent a *linear function*, an *exponential function*, or *neither*? Explain.

x	0	1	3	6
y	2	10	50	250

50. **HOW DO YOU SEE IT?**
The exponential function $y = V(x)$ represents the projected value of a stock x weeks after a corporation loses an important legal battle. The graph of the function is shown.



a. After how many weeks will the stock be worth \$20?

b. Describe the change in the stock price from Week 1 to Week 3.

51. **REASONING** Determine whether each situation can be represented by a *linear* or an *exponential* function.

a. A checking account receives a \$500 deposit each month.

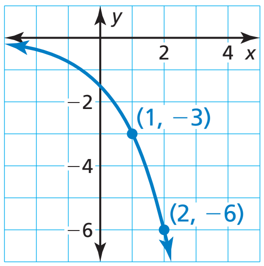
b. The number of people infected by a virus triples each week.

c. An unpaid credit card balance accrues 1.75% interest each month.

d. A radio station chooses two new game contestants each day.

52. **REASONING** Let $f(x) = 3(2)^x$ and $g(x) = 3x + 3$. Describe when (a) $f(x) < g(x)$, (b) $f(x) > g(x)$, and (c) $f(x) = g(x)$. Justify your answer.

53. **STRUCTURE** The graph represents the exponential function f. Find $f(3)$.



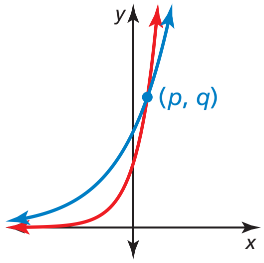
54. **MAKING AN ARGUMENT** Your friend says that for the function $f(x) = -4\left(\frac{1}{2}\right)^x$, $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to +\infty$. Is your friend correct? Explain.

55. **REASONING** Is an exponential function always increasing or always decreasing over its entire domain? Explain.

56. **THOUGHT PROVOKING**
Write an exponential function f so that the slope of a line from the point $(0, f(0))$ to the point $(2, f(2))$ is equal to 12.

57. **PROBLEM SOLVING** A function g models a relationship in which the dependent variable is multiplied by 4 for every 2 units the independent variable increases. By what number is the dependent variable multiplied by when the independent variable increases by 1?

58. **DIG DEEPER** The graphs of the functions $f(x) = n(2)^x$ and $g(x) = m(5)^x$, where $n > 0$ and $m > 0$, are shown. They intersect at the point (p, q).



a. Complete the inequality n ☐ m. Explain your reasoning.

b. Determine the value of the ratio of $f(p + 2)$ to $g(p + 2)$. Justify your answer.

REVIEW & REFRESH

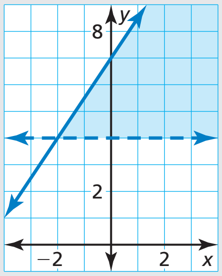
WATCH

In Exercises 59 and 60, write the percent as a decimal.

59. 4%

60. 128%

61. Write a system of linear inequalities represented by the graph.



In Exercises 62 and 63, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

62. $f(x) = -x + 1$; $g(x) = f(x) - 4$

63. $f(x) = \frac{1}{2}x - 3$; $g(x) = f(x + 2)$

64. Write an equation in slope-intercept form of the line that passes through $(-4, 9)$ and $(16, -6)$.

In Exercises 65 and 66, evaluate the expression.

65. $\left(\frac{1}{32}\right)^{1/5}$

66. $(-27)^{5/3}$

67. Write an inequality that represents the graph.

In Exercises 68 and 69, graph the function. Compare the graph to the graph of the parent function. Identify the *y*-intercepts and asymptotes of the graphs. Find the domain and range of *f*.

68. $f(x) = -5^x$

69. $f(x) = -\frac{1}{2}(4)^x$

70. Tell whether $(8, -1)$ is a solution of $-\frac{1}{2}x + 3y > -1$.

71. **MODELING REAL LIFE** There are 430 people in a wave pool. Write an inequality that represents how many more people can enter the pool.

72. Solve the system.

$$y = 4x - 5$$
$$3x + 2y = 12$$

73. **WRITING** Describe the effect of a on the graph of $y = a \cdot 2^x$ when a is positive and when a is negative.

In Exercises 74 and 75, simplify the expression. Write your answer using only positive exponents.

74. $\left(\frac{-5d^4}{9d^0}\right)^3$

75. $\left(\frac{4x^{-1}y^3}{-8x^2y^2}\right)^4$

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